# Optimizing Density of Development With Respect to Transportation Cost 

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-THE Tri-State Transportation Commission has a broad responsibility encompassing land-use planning as well as transportation facility planning. The possibilities of guiding land-use development towards some more desirable pattern are being exploredboth because of the effect on transportation and as an end in itself. Tri-State's planning staff is thus faced with the question: What would be the best possible pattern of future land use? The city planning profession has not yet agreed on the answer to this question, and it is still largely a matter for speculation. The planner lacks the tools to make an objective evaluation of a land-use plan; he must rely on judgment, intuition, and some venerable precepts which have increasingly come under attack.

It is hoped to develop some more scientific procedures-preferably some measurable criteria-to use in trying to formulate the best possible land-use and transportation plan. As one approach to this problem, I have abstracted a few of the essential parameters of a city and have shown how they might be optimized. Although this was a theoretical analysis performed on an idealized city, the results hopefully can provide some guidance to planners charged with drawing specific plans for real cities.

Other areas of human endeavor do not suffer the same lack as city planning; they have developed well-accepted criteria for determining what is good. The businessman has one clear, overall objective: to maximize his profits. In engineering and welfare economics, benefit-cost criteria have been widely utilized. Benefit-cost analysis was applied to metropolitan transportation planning by the Chicago Area Transportation Study (CATS). Of particular note was the technique developed for calculating the optimal spacing of highways (1). This kind of "ideal city" analysis found cogent practical applications in developing a highway plan for the Chicago region.

The so-called "benefit-cost analysis, " as used in transportation planning, does not actually distinguish between benefits and costs. Benefits are merely savings in costs, and thus the objective is really to minimize costs. The costs which are affected by the land-use pattern might be divided between transportation costs and other costs. Little progress has been made in identifying and measuring the non-transportation costs. Presumably, such things as utility costs, construction costs and land costs are affected by the land-use pattern, but there are also myriad indirect and elusive social and economic costs. Considerable work has been done in measuring transportation costs, so this seemed the best place to begin. Since the optimal spacing work dealt only with highways, the initial study has been limited to private vehicle transportation.

There are many aspects to the land-use pattern, and again it was necessary to select one parameter as a starting point. The most logical and convenient one was the density of development. Density can be measured in various ways, but since the study deals with the costs of motor-vehicle transportation, I measured the density of vehicle trip ends. This has the advantage that it represents both residential and nonresidential activities; population density tells only part of the story.

To summarize, the influence of the density of vehicle trip ends on highway transportation costs was investigated. The specific objective of the study was to minimize the total transportation cost per vehicle trip.

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## DEVELOPMENT OF THE METHOD

Total transportation costs for a motor-vehicle system are divided into investment costs and travel costs. Investment costs cover all expenditures involved in providing fixed physical facilities (i.e., highways), right-of-way acquisition, clearance, utility relocations, construction, etc. Travel costs include those costs borne by users of the facilities in operating their vehicles on them. These are subdivided into operating costs, accident costs, and time costs. This presumes that there is a monetary value to users' time; this value has been estimated by observing how much money people pay to save time. Expressing all of these costs in dollars does not reflect a materialistic bias, but rather the necessity of having a single common denominator.

In general, an increase in investment will produce a better highway network and result in lower travel costs. Investment cost is a one-time capital expenditure, while travel cost forms a daily recurring stream extending over an indefinite time period. Any savings in travel cost can be considered to be a return on invested capital. This puts the problem into a suitable format for benefit-cost analysis (2, 3 ).

To proceed, it is necessary to establish the relationship between the density of trip ends and the several costs. For a real city this would be a forbiddingly complex task, but it can be done for a theoretical, idealized city. Fortunately, such a city has been founded by Morton Schneider, and he has described it in a paper which forms another key block in the foundation of this work (4, 5). Schneider had a different object in mind when he set up his city-namely, to estimate traffic-but it is readily amenable to the problem here, and the ability to estimate traffic volumes is essential to this analysis. The stipulations surrounding the idealized city are fully described in Schneider's paper. To understand the current argument, one must know the following assumptions:

1. The city is absolutely regular and homogeneous, extends infinitely in all directions, and has a uniform density of trip ends throughout.
2. The city has three street systems of distinctly different quality. These can be regarded as expressways, arterials, and local streets.
3. Each street system forms a perfect gridiron with uniform spacing everywhere. The spacing of the three different systems need not be the same.
The major task is to develop equations expressing the several elements of transportation cost. This is largely a matter of synthesizing previous work done by Schneider, George Haikalis, and others at CATS.

## Investment Cost

In the hypothetical city, each square mile is exactly like every other square mile. If a gridiron street network has a spacing of $z$ miles, then in a square mile there will be $2 / \mathrm{z}$ miles of that type of street. The total mileage of streets in a square mile of the city will be

$$
\begin{equation*}
2\left(\frac{1}{\mathrm{z}_{1}}+\frac{1}{\mathrm{z}_{2}}+\frac{1}{\mathrm{z}_{s}}\right) \tag{1}
\end{equation*}
$$

where $z_{1}$ is the spacing of expressways, $z_{2}$ the spacing of arterials, and $z_{3}$ the spacing of local streets.

If $\mathrm{C}_{1}$ is the per mile investment cost of constructing expressways, with $\mathrm{C}_{2}$ and $\mathrm{C}_{3}$ representing the same for arterials and local streets, then the total investment cost for a square mile will be

$$
\begin{equation*}
2\left(\frac{\mathrm{C}_{1}}{\mathrm{Z}_{1}}+\frac{\mathrm{C}_{2}}{\mathrm{z}_{2}}+\frac{\mathrm{C}_{3}}{\mathrm{z}_{3}}\right) \tag{2}
\end{equation*}
$$

We are interested in the cost per trip, so we must divide this total by $\rho$, which is the density of vehicle trip destinations per square mile per day. One other factor, K, must be added. This is merely a conversion factor which transforms the one-time
investment cost into an equivalent daily cost, assuming some interest rate and facility life span. Now, the investment cost per trip is

$$
\begin{equation*}
\frac{2}{\mathrm{~K} \rho}\left(\frac{\mathrm{C}_{1}}{\mathrm{Z}_{1}}+\frac{\mathrm{C}_{2}}{\mathrm{Z}_{2}}+\frac{\mathrm{C}_{3}}{\mathrm{Z}_{3}}\right) \tag{3}
\end{equation*}
$$

It is possible to assume that the C's are constant. This probably does little violence to the truth for local and arterial streets; their share of the total cost is small, anyway. But it clearly is not true for expressways-their construction and, particularly, right-of-way costs are very dependent on the kind of area through which they pass. I have hypothesized that expressway investment cost follows the following formulation:

$$
\begin{equation*}
\mathrm{C}_{1}=\alpha+\beta \rho \tag{4}
\end{equation*}
$$

in which $\alpha$ and $\beta$ are coefficients whose values must be determined empirically. There is a certain minimum cost which exists even in rural areas of zero density, and cost increases as density increases. Inserting Eq. 4 for $\mathrm{C}_{1}$ in Eq. 3 results in the final expression for investment cost per trip:

$$
\begin{equation*}
\frac{2}{\bar{K}}\left(\frac{\alpha}{Z_{1} \rho}+\frac{\beta}{Z_{1}}+\frac{C_{2}}{\mathrm{Z}_{2} \rho}+\frac{\mathrm{C}_{3}}{\mathrm{Z}_{3} \rho}\right) \tag{5}
\end{equation*}
$$

## Estimating Traffic Volumes

A prerequisite to determining travel cost is a method for estimating traffic volumes on each of the three street networks (under the assumptions made, the volume on each street type is the same everywhere). As the volume on any street increases, the travel cost increases because congestion slows the traffic. Furthermore, our problem stipulated three markedly different street types, and the difference would be reflected in different travel costs, thus it is necessary to know the distribution of traffic among the street types.

Schneider addressed himself to this problem of estimating traffic in his paper on direct assignment (4). Later he made a minor revision in his technique which eliminated certain bugs but did not greatly alter the estimates yielded (6). The revision did produce equations which differ from those in the original paper, and I have used these revised equations for the traffic volumes:

$$
\begin{gather*}
V_{1}=\frac{\rho \overline{\mathbf{r}}^{3} z_{1}}{2\left(\bar{r}+z_{1}\right)\left(\bar{r}+z_{2}\right)}  \tag{6}\\
V_{2}=\frac{z_{2}}{\bar{r}} V_{1}  \tag{7}\\
V_{3}=\frac{z_{3}\left(\bar{r}+z_{1}\right)}{\bar{r} z_{1}} V_{2} \tag{8}
\end{gather*}
$$

In these equations, $V_{1}$ is the average daily volume on expressways, and $V_{2}$ and $V_{3}$ represent the volumes on arterials and locals. The symbol $\overline{\mathrm{r}}$ is the average trip length in miles. The other symbols should be familiar. Notice that the traffic volumes are dependent on the density of trip destinations, the average trip length, and the spacing of the street networks.

## Travel Cost

The average travel cost per trip is the sum of the costs on each of the three street systems. It can be represented thus:

$$
\begin{equation*}
P_{1} T_{1}+P_{2} T_{2}+P_{3} T_{3} \tag{9}
\end{equation*}
$$

$P_{1}$ is the average distance traveled by a trip on the expressway system, and $T_{1}$ is the travel cost per mile on expressways. Their product represents the average cost incurred by a motorist on the expressway system. The second and third terms of the expression represent the same thing for arterial and local streets. It may be helpful to point out that

$$
\begin{equation*}
\mathrm{P}_{1}+\mathrm{P}_{2}+\mathrm{P}_{3}=\overline{\mathrm{r}} \tag{10}
\end{equation*}
$$

Now, $P_{1}$ can be readily calculated from $V_{1}$, the relationship being

$$
\begin{equation*}
\mathrm{P}_{1}=\frac{2 \mathrm{~V}_{1}}{\mathrm{Z}_{1} \rho} \tag{11}
\end{equation*}
$$

The vehicle-miles on expressways per square mile is the product of the average volume $\left(V_{1}\right)$ times the miles of expressways per square mile $\left(2 / z_{1}\right)$. Dividing this by the number of trip destinations per square mile ( $\rho$ ) gives the average per trip. Similar equations hold for arterials and locals, namely:

$$
\begin{align*}
& \mathrm{P}_{2}=\frac{2 \mathrm{~V}_{2}}{\mathrm{Z}_{2} \rho}  \tag{12}\\
& \mathrm{P}_{3}=\frac{2 \mathrm{~V}_{3}}{\mathrm{Z}_{3} \rho} \tag{13}
\end{align*}
$$

There remains the problem of determining the T's. In view of uncertainty about the relationship of operating and accident costs to average speed, it seemed wisest to assume that the sum of operating and accident costs per mile is constant. Let A represent this constant.

The only variable, then, is time cost. This increases as speed falls, which happens as volume rises. Time cost is the product of the value of time and the amount of time. I assume the value of time is constant; let it be represented by B.

Another convention is to break time into two parts: the amount of time required to travel at free speed (i. e., if there were no interference from other traffic) and the time delays resulting from congestion. If $S_{1}$ is the free speed on expressways and $D_{1}$ is the delay per mile on expressways, then the total travel cost per mile on expressways is

$$
\begin{equation*}
\mathrm{T}_{1}=\mathrm{A}+\mathrm{B}\left(\frac{1}{\mathrm{~S}_{1}}+\mathrm{D}_{1}\right) \tag{14}
\end{equation*}
$$

and the travel cost per mile on arterials is

$$
\begin{equation*}
\mathrm{T}_{2}=\mathrm{A}+\mathrm{B}\left(\frac{1}{\mathrm{~S}_{2}}+\mathrm{D}_{2}\right) \tag{15}
\end{equation*}
$$

This is as far as the abstract reasoning can be carried. In the illustration of the method that follows, I assume free speeds to be constant and formulate specific expressions for the delays, based on empirical findings. Observe that free speeds are different on different street types, and by definition they are independent of traffic volumes. Delays should vary in response to the capacity of a street and to the volume it carries. As volume increases, delay rises very gradually at first, and then more and more sharply as capacity is approached and exceeded. Capacity is not taken as an absolute maximum, but rather a kind of standard or milepost which generally indicates the ability of a certain physical highway facility to pass vehicles.

Local streets, $\mathrm{T}_{3}$, have been omitted from the discussion. As a rule, traffic volumes on local streets are so low that congestion rarely results. There would appear to be little reward from making a sophisticated analysis of $\mathrm{T}_{3}$, so it was assumed to be constant.

## Finding the Minimum Cost

An equation was developed which represents the total transportation cost per trip in the hypothetical city. The equation is not recapped here because it is rather
cumbersome. When plotted against density, it yields a U-shaped curve. As density increases, the investment cost per trip declines, but the travel cost per trip rises. The problem is to find that density at which the curve has its minimum; this would be the optimal density. The problem is readily soluble by differential calculus. By holding all other factors constant and differentiating with respect to $\rho$, one can secure an equation which locates the optimal value of $\rho$. Since $\rho$ occurs at many places in the original equation, the work is rather involved, and I shall not bore the reader with the mathematics entailed.

With this tool, it is possible to determine the optimal density for any given expressway spacing. However, this is not totally sufficient; the planner would naturally want to manipulate both density and expressway spacing and to find the best combination. Consequently, the two variables (density and expressway spacing) were optimized simultaneously and the minimum cost per trip with respect to both was determined. This problem is also amenable to calculus by taking two partial derivatives, setting both equal to zero, and solving them simultaneously. The problem can be visualizedinthree dimensions, in which density and expressway spacing are the orthogonal horizontal axes and cost per trip is the vertical axis. The equation for total cost forms a surface shaped something like a pit, and by calculus one can locate the minimum point on that surface.

A logical extension is to consider arterial spacing, and to optimize three variables simultaneously. However, each partial derivative added causes a considerable increase in the mathematical work required. While an analytical solution for the triple optimum may be possible, I have been content to approximate it by selecting several different values for arterial spacing, finding the double optimum for each, and comparing the resulting costs per trip.

## ILLUSTRATION OF THE METHOD

It is difficult for the reader to grasp the technique fully without a concrete example utilizing numbers instead of symbols. It is also of interest to the investigator to examine how the results react as various factors take different values. Consequently, an illustration using numbers and producing concrete results was developed. It would be ideal to utilize values and relationships taken from the real world, and in particular from the Tri-State region. However, Tri-State's data analysis had not reached the stage where such information was available, therefore, hypothetical values and relationships were used. In some cases, these were based on data from CATS, and in some cases things were fabricated which seemed intuitively reasonable. Therefore, this must be regarded as purely an academic exercise, and the specific results should not be accepted at face value. However, in general the substitution of empirical findings for hypothetical data would only change the particular values of the results; the method itself would remain valid.

## Values Assumed

In the example, assume that the average trip length $(\overline{\mathrm{r}})$ is 6 miles, a value determined by CATS, which appears to be approximately the same in all major metropolitan areas. For simplicity, local spacing $\left(z_{3}\right)$ is held constant at $1 / 10$ mile. The particular value assumed for K is 3081.6 ; this is based on a 10 percent interest rate, 25 -year facility life, and 339.5 equivalent weekdays to a year. In accordance with custom, this analysis deals with trips for an average weekday. Since on the average, weekend and holiday traffic is less, the number of equivalent weekdays in a year comes out to less than 365.

For expressway investment cost, an equation developed at CATS was used:

$$
\begin{equation*}
C_{1}=1,120,000+520 \rho \tag{16}
\end{equation*}
$$

I assumed that arterial investment cost $\left(C_{2}\right)$ was $\$ 500,000$ per mile, and that local street investment cost ( $\mathrm{C}_{3}$ ) was zero. This is to argue that since the function of local streets is to provide access to land, their cost might properly be assigned to land development rather than the transportation system.

TABLE 1
TOTAL COST PER TRIP IN CENTS (When Arterial Spacing $=0.5$ Miles)

| Densitya | Expressway Spacing In Miles |  |  |  |  |
| ---: | :---: | :---: | :---: | :---: | :---: |
|  | 2 |  | 4 | 6 | 8 |

$a_{\text {In vehicle trip destinations per square mile. }}$

The sum of operating and accident costs (A) was 3.5 cents per mile and the value of time (B), \$1. 50 per hour. For $\mathrm{T}_{3}$, which was assumed to be constant, a value of 14 cents per mile was selected. All these values were based on CATS findings. Free speed on expressways ( $\mathbf{S}_{1}$ ) was assumed to be 50 miles per hour, and free speed on arterials ( $\mathbf{S}_{2}$ ) 30 miles per hour.

The investigation of Haikalis was consulted to secure expressions for delay, although all his equations were not adopted verbatim because of their complexity (2). Equations which approximated the curves he presented were formulated. The following equation was used for expressway delay per mile (in hours):

$$
\begin{equation*}
\mathrm{D}_{1}=0.001+0.00122 \mathrm{R}_{1}{ }^{3} \tag{17}
\end{equation*}
$$

in which $R_{1}$ is the volume-to-capacity ratio on expressways. Capacity of expressways was assumed to be 127,000 vehicles per day.

The delay on arterial streets occurs principally at intersections with other arterial streets (which are normally signalized). Therefore, it is logical to determine the average delay at an intersection and multiply it by the number of intersections per mile (which is the inverse of the spacing). The result of this was the following equation:

$$
\begin{equation*}
\mathrm{D}_{2}=\frac{0.0032+0.003 \mathrm{R}_{2}{ }^{3}}{\mathrm{Z}_{2}} \tag{18}
\end{equation*}
$$

in which $R_{2}$ is the volume-to-capacity ratio on arterials. The capacity was assumed to be 20,000 .

These are all the values needed to carry out the calculations and determine the optimum conditions. It may be of interest to show the resulting equation for the optimal density:

$$
\begin{equation*}
\rho=\frac{978.282\left(6+\mathrm{Z}_{1}\right)\left(6+\mathrm{Z}_{2}\right)}{\mathrm{Z}_{1}} \sqrt[4]{\frac{2.24+\frac{\mathrm{Z}_{1}}{\mathrm{Z}_{2}}}{2.058+\mathrm{Z}_{1} \mathrm{Z}_{2}{ }^{2}}} \tag{19}
\end{equation*}
$$

This equation is very easy to use. Unfortunately, the companion equation for the optimal expressway spacing is not so simple. It is not amenable to a direct algebraic solution and must be solved by trial-and-error or graphical means.

## Results

Optimal combinations of density and expressway spacing for a number of different arterial spacings were calculated. Before examining these optima, it will be instructive to see how cost per trip is affected by variations in density and expressway spacing. Table 1 gives the situation when arterial spacing is held constant at one-half mile. Reading down a column, one can locate the optimal density for any given expressway spacing. Thus, for a spacing of 2 miles, it is 30,000. Reading across a row, one can locate the optimal spacing for any given

TABLE 2
INVESTMENT COST PER TRIP IN CENTS
(When Arterial Spacing $=0.5$ Miles)

| Density $^{2}$ | Expressway Spacing In Miles |  |  |  |  |
| ---: | :---: | ---: | ---: | ---: | ---: |
|  | 2 |  | 4 | 6 | 8 |
| 5,000 | 37.12 | 25.05 | 21.03 | 19.02 | 17.81 |
| 10,000 | 27.00 | 16.74 | 13.33 | 11.62 | 10.59 |
| 15,000 | 23.62 | 13.98 | 10.76 | 9.15 | 8.19 |
| 20,000 | 21.94 | 12.59 | 9.48 | 7.92 | 6.98 |
| 25,000 | 20.92 | 11.76 | 8.71 | 7.18 | 6.26 |
| 30,000 | 20.25 | 11.21 | 8.19 | 6.69 | 5.78 |
| 35,000 | 19.77 | 10.81 | 7.83 | 6.33 | 5.44 |
| 40,000 | 19.41 | 10.51 | 7.55 | 6.07 | 5.18 |
| 45,000 | 19.12 | 10.28 | 7.34 | 5.86 | 4.98 |
| 50,000 | 18.90 | 10.10 | 7.17 | 5.70 | 4.82 |

${ }^{a}$ In vehicle trip destinations per square mile.

TABLE 3
TRAVEL COST PER TRIP IN CENTS (When Arterial Spacing $=0.5$ Miles)

| Density $^{\mathrm{a}}$ | Expressway Spacing In Miles |  |  |  |  |
| ---: | :---: | :---: | :---: | :---: | ---: |
|  | 2 | 4 | 6 | 8 | 10 |
| 5,000 | 47.19 | 49.53 | 51.10 | 52.23 | 53.08 |
| 10,000 | 47.22 | 49.65 | 51.32 | 52.54 | 53.48 |
| 15,000 | 47.29 | 49.95 | 51.90 | 53.39 | 54.57 |
| 20,000 | 47.45 | 50.55 | 53.04 | 55.05 | 56.70 |
| 25,000 | 47.70 | 51.54 | 54.91 | 57.78 | 60.21 |
| 30,000 | 48.08 | 53.01 | 57.69 | 61.85 | 65.44 |
| 35,000 | 48.60 | 55.07 | 61.59 | 67.53 | 72.75 |
| 40,000 | 49.30 | 57.81 | 66.77 | 75.09 | 82.47 |
| 45,000 | 50.20 | 61.32 | 73.42 | 84.79 | 94.95 |
| 50,000 | 51.32 | 65.70 | 81.73 | 96.92 | 110.54 |

${ }^{a}$ In vehicle trip destinations per square mile.
have minima, except at extreme values of zero and infinity. It is only when the two are superimposed that a minimum occurs at some meaningful point.

To compute all the values shown in these tables by hand would be quite laborious. Therefore, a FORTRAN program was written and the values were calculated by a 1401 computer. Tables 1 through 3 are actually excerpts from much larger tables which the computer produced.

It is possible to find the optimal combination of density and expressway spacing for any given arterial spacing by hand calculations in a reasonable length of time. This was done for a number of arterial spacings ranging from one-quarter mile to 2 miles. The results are shown in Table 4, with the cost per trip occurring at each optimum. The lowest cost per trip in this table is associated with arterial spacing of threequarters of a mile, expressway spacing of 7.7 miles, and density of 13,900 . The triple optimum is apparently in this vicinity.

This analytical method yields may interesting by-products. Various other parameters of the hypothetical city are calculated along the way, or can easily be derived. Table 5 gives some of the more significant characteristics associated with the optimal solutions given in Table 4. The speeds are average speeds including the delays due to congestion. It is also possible to calculate the distribution of vehicle-miles among the three street networks, volumes for certain turning movements, the average time for a trip, and the portion of that time caused by congestion.

## ANALYSIS AND INTERPRETATION

Some background information may help the reader to put the results of the illustration in scale. The CATS surveys showed that, outside of the CBD, densities of vehicle trip destinations in the central city mostly fell between 15,000 and 35,000 . In the close-in suburbs, densities of 10,000 to 20,000 were typical, while figures from 5,000 to 10,000 were common in suburban communities further out. An exclusively residential area with one-acre lots would probably have a density between 1,500 and 3,000 .

The optimal spacing work at CATS resulted in recommended expressway spacings of 3 miles in Chicago and 6 miles in the suburbs. Arterial spacing of one-half mile already prevailed in Chicago and 1 -mile spacing was recommended for the suburbs.

Inspection of Table 1 shows that the minimum cost is only slightly below many neighboring values (the differences would certainly be within the margin of error due to the grossness of estimated inputs). Thus, there is a rather large "'region of indifference" embracing widely varying conditions. Costs below 70 cents are obtained for conditions ranging from $\rho=45,000$ and $z_{1}=2$ to $\rho=10,000$ and $z_{1}=10$. This type of result is

[^0]common in optimization problems, and the curves have a $U$ shape rather than a V shape. I feel that this is an advantage rather than a disadvantage. It provides considerable leeway within which other factors (perhaps social, political and aesthetic) may be allowed to influence any concrete decision. What the optimization study really shows are what extremes to avoid, because when you go beyond the region of indifference, costs do rise steeply.

Table 4 indicates that there is also a considerable region of indifference with regard to arterial spacing. Quite different combinations of arterial and expressway spacing and density produce very similar. costs per trip. Again, this gives the planner considerable leeway for choice. It is important that he select a good combination of density and spacing, but there are many combinations of approximately equal merit (from the standpoint of transportation cost).

Looking at Table 5, the reader may wonder why arterial speed goes up at the same time as arterial volume. The reason is that arterial spacing is also increasing at the same time, so that while there are more vehicles on the highway, there are fewer stops for intersections.

One of the important findings of this exercise is that an optimum does exist, where cost is minimized, at conditions which are meaningful and reasonable. An optimization study must remain suspect until it is shown that it produces results which bear some relation to the real world. The reason why this optimum exists is because there are several opposing forces at work with certain trade-offs among them. It may be helpful to recapitulate how these forces operate.

As the density of trip ends increases: (a) investment cost is distributed over more trips, so the cost per trip declines; (b) there is no effect on the distribution of traffic among the three street systems (this is totally dependent on spacing); and (c) the average volumes on the streets rise, causing more delay, and so the time cost rises.

As the expressway spacing increases (becomes wider): (a) expressway investment cost goes down; (b) some traffic is shifted from expressways to arterials, which have a lower free speed, so time cost rises; (c) the average expressway volume rises, causing greater delay and increasing time cost further; and (d) the average arterial volume also rises, again causing greater delay and still further increasing time cost.

As the arterial spacing increases: (a) arterial investment cost goes down; (b) some traffic is shifted from expressways and arterials to local streets, with a consequent increase in time cost; (c) the average expressway volume declines, which raises average expressway speed and lowers time cost; (d) the average arterial volume increases, causing an increase in time cost; and (e) the frequency of delay points on arterials drops, which lowers time cost.

TABLE 5
SOME CHARACTERISTICS OF OPTIMAL SOLUTIONS
(As Given in Table 4)

| Arterial <br> Spacing <br> (miles) | Expressway <br> Volume | Arterial <br> Volumea | Local <br> Volumea | Expressway <br> Speed <br> $(\mathrm{mph})$ | Arterial <br> Speed <br> $(\mathrm{mph})$ |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 0.25 | 181,200 | 7,550 | 294 | 40.7 | 21.4 |
| 0.50 | 150,690 | 12,560 | 403 | 43.5 | 24.3 |
| 0.75 | 124,970 | 15,620 | 463 | 45.1 | 25.3 |
| 1.00 | 106,100 | 17,680 | 496 | 46.1 | 25.9 |
| 1.25 | 92,210 | 19,210 | 516 | 46.6 | 26.3 |
| 1.50 | 81,680 | 20,420 | 526 | 46.9 | 26.6 |
| 1.75 | 73,400 | 21,410 | 529 | 47.1 | 26.8 |
| 2.00 | 66,760 | 22,250 | 531 | 47.2 | 27.0 |

[^1]
## IMPROVEMENTS AND EXTENSIONS

While the limitations of the various assumptions on the applicability of the results were realized from the start, a number of additional weaknesses came to light as the exercise proceeded and the sample calculations were made.

In some cases, volumes and travel costs reached unrealistically high figures. Sometimes arterial speed exceeded expressway speed and local speed exceeded arterial speed. These difficulties showed up only under rather extreme conditions, and never in the vicinity of the optima. It appears that they always led to an underestimate of cost, and never an overestimate. It would be desirable to have some kind of capacity restraint feature in the traffic estimation procedure. As traffic on one street type exceeds capacity, there should be a way of redistributing some of it to the other street types which still have spare capacity. The local street system almost always has spare capacity.

I had some concern over the correct nature of the relationship between expressway investment cost and density. I assumed that the cost rises slower than density, causing a lower cost per trip as density increases. This is certainly true at low densities, but at high densities near the city center, it may well be that cost increases faster than density. A curvilinear equation depicting such a relationship could be formulated. It would produce a minimum in each column of Table 2. Undoubtedly, an overall minimum of total transportation cost would still exist.

Because of the planning context of the study, there is some question as to what density should be included in the equation for expressway investment cost. This cost is probably influenced more by existing density than by ultimate density. Yet it is ultimate density that is considered in this exercise.

In general, the utility of a theoretical solution is inversely proportional to the number and importance of assumptions it is necessary to make. A natural course for improving the method, therefore, is to attempt to remove some of the assumptions and to deal with a more realistic case. Obviously, it would be desirable to be able to handle a real city in which density does vary, highway networks are not regular, and there is no artificial distinction among street types. These improvements have apparently been accomplished by Schneider for the problem of estimating traffic (7). As yet this new methodology has not been applied to optimization problems, but it may be suitable.

One of the questionable assumptions is that of a constant average trip length. Density may have some effect on average trip length, but the precise nature of the relationship remains mysterious. Another candidate for elimination is the assumption that free speeds are constant. It seems reasonable to argue that free speeds vary as a function of the density of trip ends. The higher the density of surrounding development, the lower posted speed limits are likely to be. For arterials, higher densities are apt to mean more side frictions from driveways, parking, and pedestrians. For that matter, probably capacities should also be varied as a function of density. Capacity and free speed are both really aspects of the same thing: the ability of a highway to pass traffic.

A major extension of this study would be to consider the transit mode. This would require breaking some new ground in optimization of transit systems, which have not received as much attention as highway systems. Perhaps the first cut would be to consider an alternative hypothetical universe in which transit is the only mode. Later the two worlds could be merged, requiring some treatment of model split-a subject which has generated heat but little light.

For the land-use planner, it would be interesting to go to some measure of density which is more familiar to him, such as population, employment, or floor area. This brings in the whole problem of trip generation, which as yet is only dimly understood. Certainly this transition is necessary at some point.

There is the matter of non-transportation costs, which would be important to any comprehensive evaluation of land-use costs. This will be a tough nut to crack, and perhaps it will never be possible to make more than a partial accounting of these costs.

A final point to consider is whether minimization of costs is the proper criterion for selection of the best plan. Is there some way to measure benefits and compare them to costs? Is there some alternative to this approach? It is obvious from the behavior of
people, at least in our affluent society, that they do not necessarily attempt to minimize costs any more than they attempt to minimize travel.

## CONCLUSIONS

The emphasis in early transportation studies was on finding the optimal transportation network for an assumed land-use pattern. Now the goal at Tri-State and many other studies is to find the optimal combination of land-use pattern and transportation system. Both land-use and transportation facilities are considered to be planning variables which are subject to control. It is necessary for the planner to get some idea of the interaction of these two realms-not just to see what happens when one is held constant and the other changes, but to see what happens when both change simultaneously. This study has attempted to establish a beachhead on this uncharted and perhaps unfriendly continent. A land-use variable (density) and transportation variables (highway spacing) are considered to be joint determinants of an optimal solution. The method provides a bridge between land-use planning and transportation planning and indicates the kind of theoretical analysis by which it may be possible to narrow in on the best land-use and transportation plan.

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[^0]:    ${ }^{1}$ 'For densities of 5,000 and 10,000, the values shown in the table do not turn up. Extension of the calculations indicates that for 5,000 the minimum cost is 70.69 cents at a spacing of 15 miles. For 10,000 the minimum turns out to occur at the 10 -mile spacing.

[^1]:    a In vehicles per 24 hours.

