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Number 207

Urban Land Use:
Concepts
and
Models

6 Reports

Subject Area

83 Urban Land Use

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Foreword

The six papers presented in this RECORD represent some of the ideas and concepts that are being investigated by various researchers concerning urban area growth and structure. Concepts and aspects of models are presented that should be of particular interest to those concerned with varying model techniques.

Swerdloff reports on an investigation of residential density structure of smaller sized urban areas and postulates that residential density structure of small urban areas exhibits a rather uniform exponential decline with distance from the activity center, and that simplified analysis of the existing structure can yield meaningful forecasts of the future.

Black's paper investigates the influence of density (measured in vehicle trip ends) on highway transportation cost. Working with an idealized city with uniform density and a uniform gridiron highway network, an equation is developed expressing the relationship between density and total transportation cost, consisting of investment, operating, accident, and travel-time costs. Optimal density is calculated at which total cost per trip is minimized. The author also shows how to optimize density and highway spacing simultaneously by assuming particular values for the variables and calculating optima. The results indicate a fairly large region of indifference around the optimum—many combinations of density and spacing yield approximately the same cost. Black concludes that the region provides leeway for the planner to consider other factors (perhaps social, political, or aesthetic); as one goes beyond the region, however, cost rises steeply.

Hemmens presents a progress report on a model for examining the impact of changes in components of urban form on urban spatial structure. The purpose of the effort is to test the utility of using a simple linear programming function as an allocation rule for evaluating urban form alternatives by two criteria: (a) the efficiency of the alternatives in terms of minimal travel requirements, and (b) the equity of the alternatives in terms of locational advantage of residence locations. The criteria are evaluated by the primal and dual problem of a "transportation problem" in linear programming.

Ellis describes a two-phase model to stimulate the behavior of households in choosing their homes. During the first phase, the housing preferences of the locating families are determined. In the second phase, the search process by which the household picks a location possessing these characteristics is simulated. A number of multivariate statistical techniques are employed in the partial calibration of the model.

The Brand, Barber and Jacob paper describes the EMPIRIC land-use forecasting model developed and calibrated for two metropolitan regions in Massachusetts, in one instance using two different data sets. The authors state that on the basis of the results of three calibrations, a generalized set of equations has been abstracted which attempts to explain causal relationships between activities and activity growths occurring under conditions exemplified by the calibration areas. The model as described is designed not only as a device for forecasting, but also as a design tool. That is, since the model is sensitive to transportation and other public works policies, the planner or

designer is presented with the opportunity to forecast the consequences of alternative sets of selected actions.

Fidler describes a model for predicting future location of commercial activity in an urban area. The model uses a gravity formula to allocate commercial trips to commercial land in traffic analysis zones. The model may be used to predict future locations of commercial land growth and the trip-drawing potential of these sites. It also may be used to determine the feasibility of planned commercial sites. The model has been applied to the Niagara Frontier area in New York.

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Residential Density Structure: An Analysis and Forecast With Evaluation

CARL N. SWERDLOFF, Highway Research Engineer, U. S. Bureau of Public Roads

This report is on a practical investigation of the residential density structure of a typical smaller sized urban area, Greensboro, N. C. Utilizing a rather extensive supply of land-use and transportation data for 1948 and 1960, the analysis followed two major lines of pursuit: (a) analysis of the existing 1948 residential density structure with particular emphasis on investigation of classical mathematical expressions of a distance-gradient nature, and (b) comparative analysis of the outcomes of several simplified attempts at forecasting the 1960 density structure.

•THE study of urban population density has absorbed the energies of professionals in a number of related disciplines for a great number of years. Ever since Clark (3) published his now universally familiar exponential decay formulation of the spatial arrangement of urban population densities, economists, ecologists, geographers, city planners, and others have been intrigued as to the actual universality of the hypothesis. Remaining unanswered are questions concerning the factors contributing to interregional variability in the expression's parameters and the temporal stability, or lack thereof, of the relationship for any particular urban region.

The onrush of urban transportation planning studies in the late 1950's brought with it the requirement to estimate the future activity distribution pattern in urban areas as a necessary prelude to estimating future transportation demands. This impetus of attention to the analysis and forecasting of urban activity structure logically carried along with it an increased concern and interest in the population density question.

Recent concern with population or residential density has primarily stemmed from an accounting requirement of the land-use or activity distribution process of the traditional transportation planning program. Having made the distribution of the resident population to individual geographic analysis zones, utilizing some sort of mathematical model or distributive device, the analyst usually relies on estimates of population density (a) to ascertain whether the distribution results imply unrealistic zonal residential densities, or (b) to compute the consumption of previously vacant land by the increment of residential growth in each zone. The latter process is of particular importance if the urban simulation process is performed in a finite number of incremental time periods requiring an updating of the relevant data files at the conclusion of each simulation period in preparation for the sequential increment. The density configuration has been integrated as an active and interrelated element of the distributive mechanism. In particular, reference is made (9) to the inclusion of individual density submodels as part of the total activity forecasting model system developed at the Delaware Valley Regional Planning Commission (DVRPC). Under this simulation system, the distribution pattern of activities is responsive to the existing density patterns in the region, just as the forecast density pattern is sensitive to the existing activity location pattern. There are numerous examples of activity distribution relationships which contain as

explanatory variables measures of activity density; however, unlike the DVRPC procedure, the values of such variables are normally exogenously determined rather than being forecast using separate relationships.

Activity density is useful in the trip generation analysis of a great many transportation studies. Trip generation, or the estimating of the total numbers of trips originating or terminating in each analysis zone, is frequently accomplished using a multiple-regression relationship equating trip production to a number of measurable characteristics of the analysis zone, one of them often being a measure of existing density. It can be inferred from trip generation analysis of this kind that the manner in which activities, for example households, arrange themselves will in and of itself influence the total volume of daily trip-making. The external economies associated with more dense activity arrangements undoubtedly are of some influence.

Transportation planners are currently interested in the question of how the community distributes its daily travel demands between available private and public transportation facilities, better known as the question of mode split. The activity density pattern has been observed to be of importance to this whole area of analysis. One could also refer to a substantial amount of professional speculation in the literature as to the future role, function, form, and viability of our great urban regions. These speculations, almost without exception, are heavily contingent upon the individual author's assessment of the levels of activity densities that the future populace is both willing and desirous of sustaining.

The question naturally arises as to current knowledge and technical competence in this area insofar as the existence of operational procedures or normative guides which can be utilized by those confronted with the overall task of estimating the future urban region is concerned. In fact the urban analyst and transportation planner will find little assistance with regard to the whole question of the future distribution of urban densities other than a number of less than completely satisfactory statements prognosticating the continued growth of the suburbs. This condition has prompted the present study, which attempts to investigate the household or residential density pattern existing in a small-sized urban region for two time periods, 12 years apart. The author has attempted to establish the conformity of the observed density structure to the previously mentioned universal formulation of urban population density, an exercise that may be of questionable payoff value but which arouses one's curiosity and is difficult to avoid. A considerable amount of multiple-regression analysis has also been undertaken. There should be some interest in the observed shifts in the parameters of these derived relationships over the 12-year interval. Finally, an attempt has been made to examine the relative order of accuracy associated with each of several forecasts of the residential density structure. The analysis and forecasting procedures examined have been purposely kept simple in an effort to maintain a balance with the kind of effort which a transportation planning study could realistically undertake in such a moderate-sized urban area.

RESIDENTIAL DENSITY: A DEFINITION OF TERMS

Residential density might at first appear to be a fairly clear and unambiguous quantity which should not require any extensive definition. However, a perusal of the literature reveals an extraordinary amount of confusion resulting from the avoidance in many cases of a rigorous definition of terms. Although this confusion cannot be settled here, it is necessary that the terms and concepts referred to in the remainder of this paper be defined at this point.

The notion of residential density refers to the ratio of some measure of the volume of residential activity per unit of land or space. From this very basic concept there arises a host of possibilities stemming primarily from the manner in which these two quantities are defined and measured (6). The numerator of the ratio presents far less difficulty than the denominator. The volume of residential activity can refer either to numbers of persons, households, or dwelling units. While there is room for ambiguity here, these quantities are familiar and have fairly strict definitions. For the purposes of this study, residential activity is always expressed in total dwelling unit terms. The

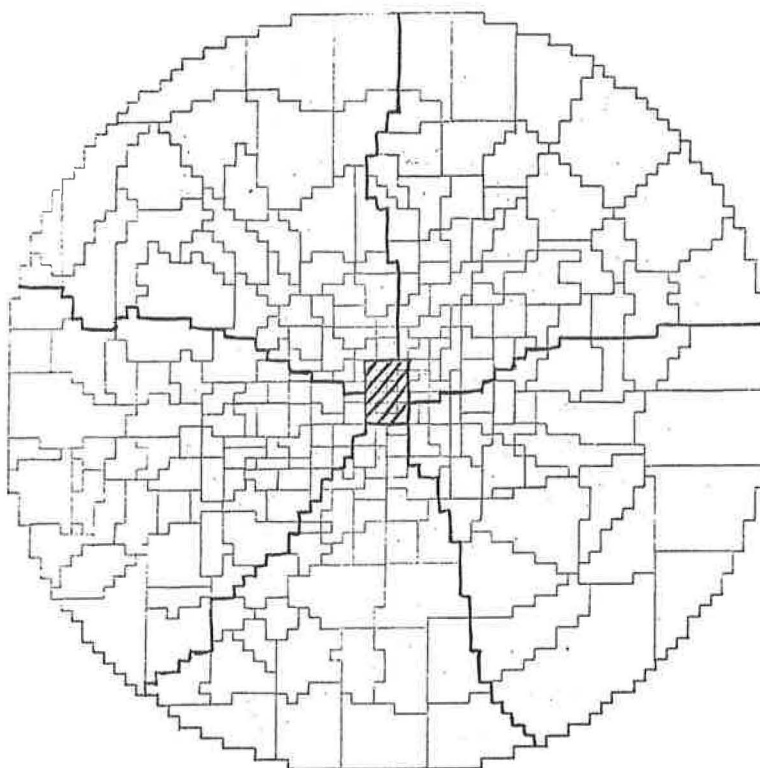


Figure 1. Study area showing zone and sector boundaries, Greensboro, North Carolina.

denominator of the ratio, on the other hand, has been the major source of definitional inconsistency. If we eliminate from consideration all third-dimensional possibilities, such as cubic feet of living space, we are still left with considerable room for variation using second dimension terms. A major breakdown here distinguishes between areal measures on the land itself, such as square feet, square miles, or acres, as opposed to areal quantities measured in man-made structures utilized by the resident population—for example, square feet of housing floor area. Discounting quantities of this latter nature and considering only areal units referenced to the earth's surface, one is confronted by a final major division which segregates residential density measures into what are commonly referred to as net and gross quantities. There is a fuzzy and shifting line which separates the two; however, the utility of each differs significantly. This point will be discussed later. In general, and for the purposes of this paper, the denominator of the gross residential ratio refers to the total area of the analysis unit, i. e., the area arrived at by planimetering its boundaries. Gross census tract dwelling unit density would then be calculated by dividing the total number of dwelling units in a census tract by the total area contained within the boundaries of the tract. Net residential density is a sharper measure than is gross density, and differs from the latter primarily as a result of a classification or stratification of the land uses contained within the boundaries of the analysis unit. For example, all land constituting a given census tract may be classified as either being used or vacant, and a net census tract dwelling unit density per square foot of used land may be computed. Used land may be further divided into residential and nonresidential usage terms and a dwelling unit density per square foot of residentially used land computed. In this paper, net residential density is defined as total number of dwelling units per unit of residentially used land, including street area.

TEST CONDITIONS AND METHODOLOGY

The Study Region

The Greensboro, North Carolina, metropolitan region served as the locale for this study. A rather complete and detailed data supply was available at a detailed geographic level for the years 1948 and 1960 (10, pp. 5-7; 2). The primary data source contained measures of total dwelling units, land area measures by use and unusable land, assessed land value, proximity to a variety of urban activities, and to the central business district (CBD) all coded to 3,980 thousand-foot-square grid cells which covered the circular study area (approximately 8 miles in radius) centering about Greensboro's CBD. Additional data consisting of total employment and an index of accessibility to total employment were also available on a travel analysis zone level. Figure 1 shows the entire study area structured into 249 analysis zones. Average family income was not present in these original data sources, but was available for both 1950 and 1960 from the U. S. Bureau of the Census (11, 12). Each zone was assumed to exhibit an average family income equal to the mean for the census tract into which it fell.

The study area was aggregated further into 5 sectors radiating out from the center of Greensboro and into circular rings each one mile wide. The sector boundaries of the study area (which were forced to analysis zone boundaries) are shown superimposed in Figure 1. The first ring (shaded area in Fig. 1), which was one-half mile in radius, circumscribed the central core of Greensboro City. The primary areal analysis unit utilized in this study was the district defined as that area contained within the inter-section of successive sector and ring boundaries. The study area could have been structured into "driving time to the CBD" time increment rings as opposed to distance increments. However, earlier work (10) with this same data indicated little advantage to either. Therefore, distance units were selected primarily for correspondence with the bulk of earlier structural density analysis reported on in the literature.

Excluding the central core ring, the remaining 8-mile rings and 5 sectors totaled 40 districts. A primary reason for selecting the district as the basic analysis unit is that it most nearly approximated, in average resident population terms (the average district in Greensboro had 660 dwelling units in 1948), the typical traffic analysis zone used in traffic simulation analysis. The analysis zone had previously been shown to be too fine in tests of residential location models (10) and was therefore judged to be inappropriate for net density analysis.

In order that some justification for this rather coarse aggregation level could be provided, a one-way analysis of variance on the 1948 net residential density was performed. Table 1 gives the results of the analysis and the finding of statistically significant between-column variance, indicating that the district aggregation of zones did not mask out the prevailing zonal net density variability and was therefore not an inappropriate analysis unit. The reader may have noticed that the central core district and its composite analysis zones were not included in the analysis of variance. In fact, the central core district was removed from all analysis in this study. The density quantities computed for the central core district consistently deviated substantially and quite illogically from what would be expected from the findings for the remainder of the study area. The central core area is traditionally quite distinct in residential terms and for that reason is often treated as such in transportation simulation analysis. Beyond this intrinsic difference, the quality of the residential land-use and dwelling unit data is often less reliable than that for the remainder of the urban region and could account for further difficulty.

The sectors and rings have an historical analytic attraction and were investigated as additional levels of geographic analysis of residential density structure. A two-way analysis of variance was also performed on the same 1948 zonal net density values stratified by ring and sector. Table 2 gives the results of this analysis, revealing that significant between-ring variability existed but not significant inter-sector variability. Apparently distance out from the region's center is a more appropriate indicator of prevailing residential density than is the angular direction with respect to some reference axis. Based on these results, the sector was abandoned as a potentially fruitful analysis unit.

TABLE 1
ANALYSES OF VARIANCE RESULTS FOR
INTERDISTRICT NET RESIDENTIAL DENSITY

Source of Variation	Sum of Squares	Degrees of Freedom	Estimated Variance
Between districts	6,860.1	39	175.9
Within districts	11,494.4	191	60.2

$$F = \frac{175.9}{60.2} = 2.92 \text{ (significant at 0.001 level)}$$

TABLE 2
ANALYSES OF VARIANCE RESULTS FOR
INTER-SECTOR AND INTER-RING NET DENSITY

Source of Variation	Sum of Squares	Degrees of Freedom	Estimated Variance
Between rings	703.1	7	100.4
Between sectors	14.8	4	3.7
Residual	98.8	29	3.4

$$F \text{ for column means} = \frac{100.4}{3.4} = 29.5 \text{ (significant at 0.001 level)}$$

$$F \text{ for row means} = \frac{3.7}{3.4} = 1.1 \text{ (not significant)}$$

In summary, all reported analysis of the distribution of gross and net residential density is for the district and ring units with the central Greensboro core area having been removed from consideration.

The Analysis Methodology

The strategy followed in this study centered about an investigation of the residential density structure (and the observed change in this structure over a 12-year period) of what was considered a fairly typical small-sized urban region. The study area sustained a 52 percent increase in numbers of dwelling units over the 12-year period, a rate of growth which is well above the average for the nation as a whole. Density-distance gradients were developed for both the 1948 and 1960 regions using the least squares criterion. Multiple-regression relationships for net density were calibrated for both 1948 and 1960. The analysis concluded with investigations of expected error in forecasting density. The following summarizes the primary objective of the study: (a) to investigate the appropriateness of several simple techniques which could be undertaken by a small planning study and staff with a minimal data supply; (b) to provide comparative quantitative measures of forecasting accuracy for each procedure or method investigated; and (c) to present some indication of any apparent advantage or disadvantage in selecting between gross or net residential density as the unit of measurement. The absence of any substantial amount of material in the literature on forecasting trends of residential density patterns coupled with the availability of data for only two time periods severely limited the selection of even simple forecasting techniques.

Because of personal bias, the bulk of the analysis concentrated on net residential density. Two district multiple-regression relationships were developed for the 1948 condition and were tested as valid forecasting devices. The initial regression formulation was modeled after the general form of the SPACEC I submodel of the previously mentioned Delaware Valley Regional Planning Commission's Activities Allocation Model system. The second regression relationship examined was of the usual multiple linear form. All calibration and forecasting errors are reported in coefficient of determination (R^2) terms¹ and therefore maintain comparability for cross-comparisons.

Gross Residential Density Analysis

Gross residential density has been defined as total numbers of dwelling units per unit of total land and thus avoids any consideration of the actual usage of the total land stock. This probably accounts for the historical orientation of previous density analysis to gross density measures. However, this simplicity is not achieved without a price, namely, a rather superficial measure of the individual household's consumption of land. Gross residential density (D_G) is defined as

¹ $R^2 = \frac{\text{original variance} - \text{explained variance}}{\text{original variance}}$; where estimates by particular techniques are transformations of density (e.g., logarithmic), they have been converted to density prior to the computation of residual error.

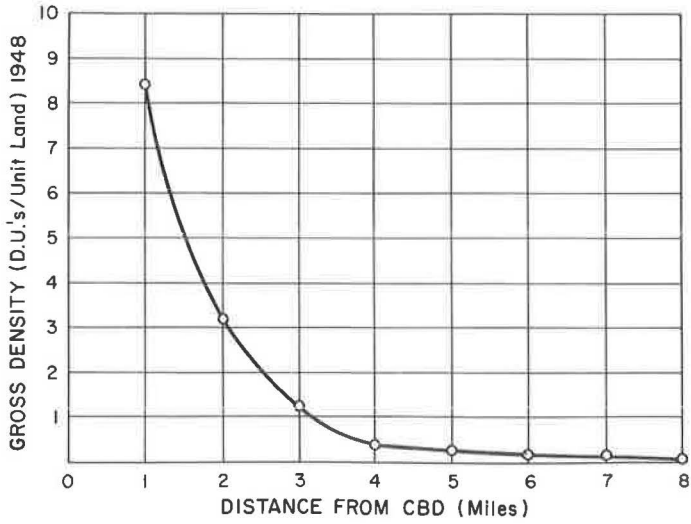


Figure 2. Gross density gradient (1948).

$$D_G = \frac{D. U. 's}{A} \tag{1}$$

where

D. U. 's = total number of dwelling units in the analysis unit, and
 A = total land area of the analysis unit.

That is, a general expression of gross density for any geographic unit i may be expressed as follows:

$$D_{G_{it}} = (c) D. U. 's_{it} \tag{2}$$

where

$D_{G_{it}}$ = gross density in analysis unit i at time t,

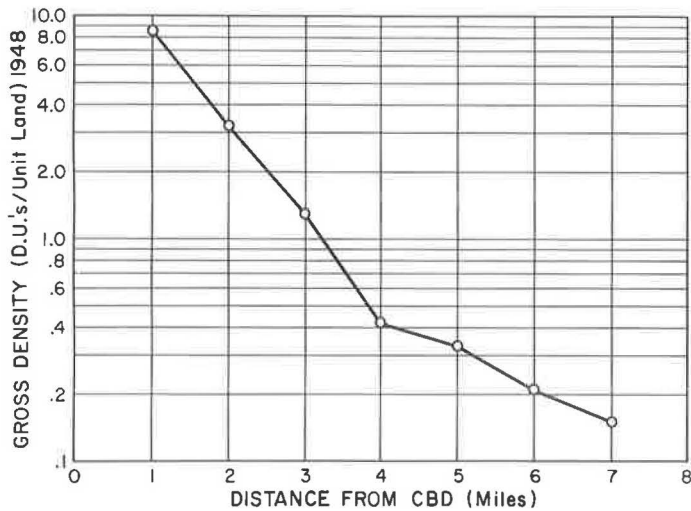


Figure 3. Semilogarithmic gross density plot (1948).

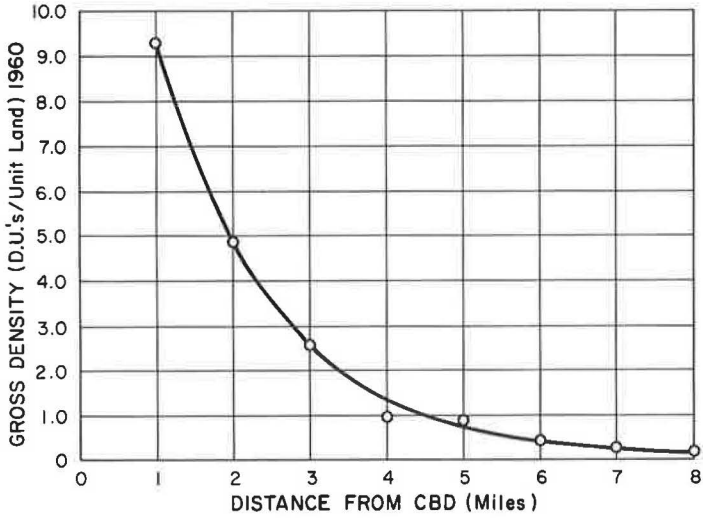


Figure 4. Gross density gradient (1960).

D. U. 's_{it} = number of dwelling units in unit *i* at time *t*, and

c = constant equal to $\frac{1}{A}$.

Gross density is then proportional to the dwelling unit stock in the analysis unit, and as such provides little gain in a time series analysis over a simple accounting of the fluctuations in the dwelling unit stock. It provides little information concerning the actual living compactness of the population.

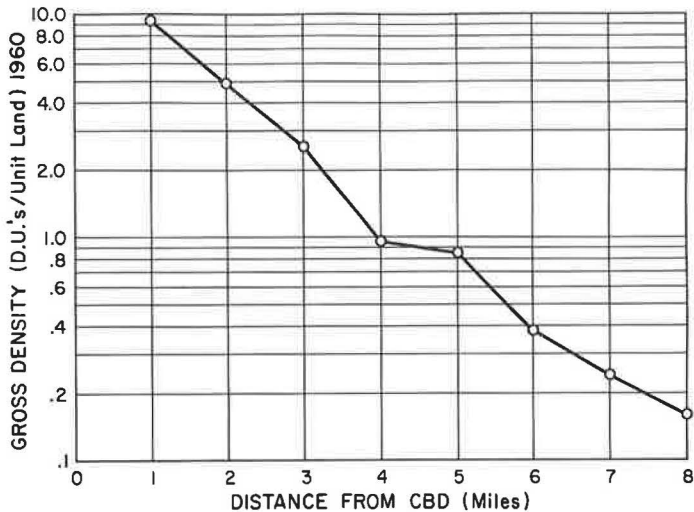


Figure 5. Semi-logarithmic gross density plot (1960).

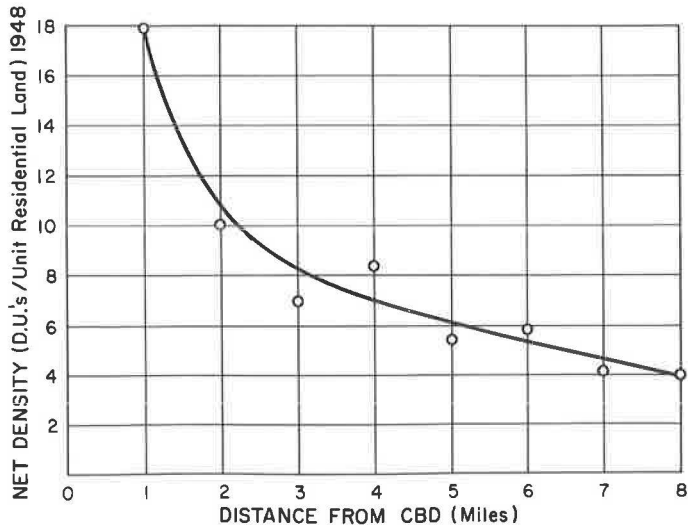


Figure 6. Net density gradient (1948).

The gross residential density of each of the 8-mile wide rings was computed for the study area for both 1948 and 1960.² The 1948 results were then plotted on regular graph paper as a function of the distance of the ring from the CBD of Greensboro in miles (Fig. 2). Clearly, a nonlinear relationship is in evidence. Figure 3 shows a replotting of the same data on semilogarithmic paper. A straight line fit in Figure 3 would give evidence of a negative exponential relationship. A definite straight line tendency is observed. A simple linear regression line fit to the points in Figure 3 resulted in the following:

$$\ln D_{G, 48} = 2.43 - 0.648X \quad (3)$$

where

X = miles from the CBD, and
 $D_{G, 48}$ = gross residential density (1948).

Transforming this regression equation to its antilog form yields:

$$D_{G, 48} = 11.36 e^{-0.648X} \quad (4)$$

which is in the general negative exponential form. Note that the least squares fit obtained for the dependent variable in logarithmic form will not necessarily yield the best equation in terms of minimum residual variance when the relationship is solved for the dependent variable in antilogarithmic terms.

The R^2 for Eq. 4 was computed as 0.886. This same relationship, calibrated on the ring gross density values, was then examined as a fit of the gross density values at the district level. Solutions to Eq. 4 for the districts yielded an R^2 of 0.834. A reduced

²The coding of the land-use data from the 1,000-ft-sq grid file was in units of ninth's of development of the total area of the grid for the particular use category. For this reason all of the density analysis of this paper is in dwelling units per ninth of 1,000-ft-sq grid. This rather awkward dimension does not, of course, affect any of the structural analysis or measures of calibration and forecasting accuracy. Any of the density values reported in this paper can be converted to D.U.'s per acre by multiplying by the constant 0.392, or to D.U.'s per sq mi by multiplying by the constant 250.9.

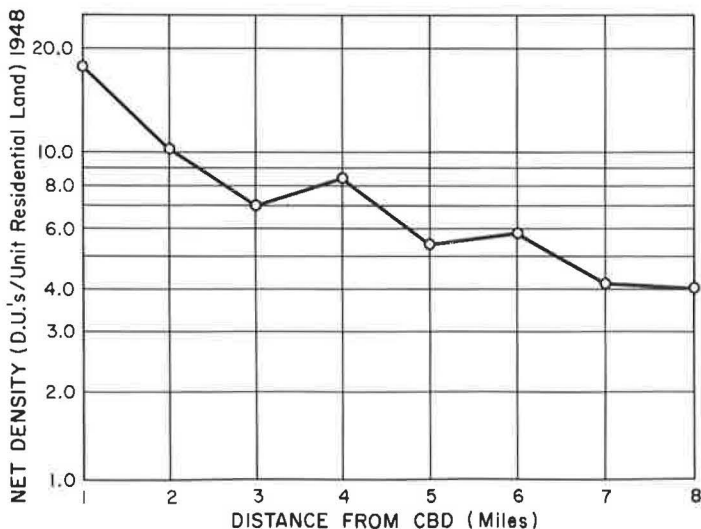


Figure 7. Semilogarithmic plot of 1948 net densities.

R^2 is to be expected if only because of the disaggregation and the resultant introduction of greater variability.

The ring gross densities for 1960 were then plotted on regular graph paper (Fig. 4). The nonlinear relationship suggested by the 1948 plot is again present in Figure 4. Replotting the 1960 points on semilogarithmic paper (Fig. 5) established the following least squares relationship:

$$\ln D_{G, 60} = 2.75 - 0.585X \quad (5)$$

which transforms to

$$D_{G, 60} = 15.58 e^{-0.585X} \quad (6)$$

R^2 's for Eq. 6 for both 1960 ring and district analysis levels are 0.989 and 0.923 respectively.

The marginal shifting of parameters observed between the 1948 and 1960 gradients suggested a test of the utility of the 1948 relationship as a predictor of 1960 densities. Solutions of the 1948 equation were then used as estimates for 1960 again at both a ring and district level. The resultant R^2 's were then computed as 0.784 and 0.743 respectively.

Negative exponential relationships of the general form

$$D_G = a (X)^b$$

were also investigated as potentially useful gross density gradients. While the data did plot in a linear fashion on log-log paper, the calibration and forecast R^2 's associated with these relationships were consistently below those previously reported.

Net Residential Density Analysis

Those most concerned with the residential density structure of urban areas are fundamentally pursuing indications or measures of the living compactness of households. A substantial amount of discussion exists in the literature, of fairly recent origin, which is directed toward the theoretical workings of household space consumption and residential location processes. Residential land consumption is treated as a resolution of an economic equilibrium between demand and supply. Viewed as one of many economic transactions engaged in by the urban household, the selection of a residential site is

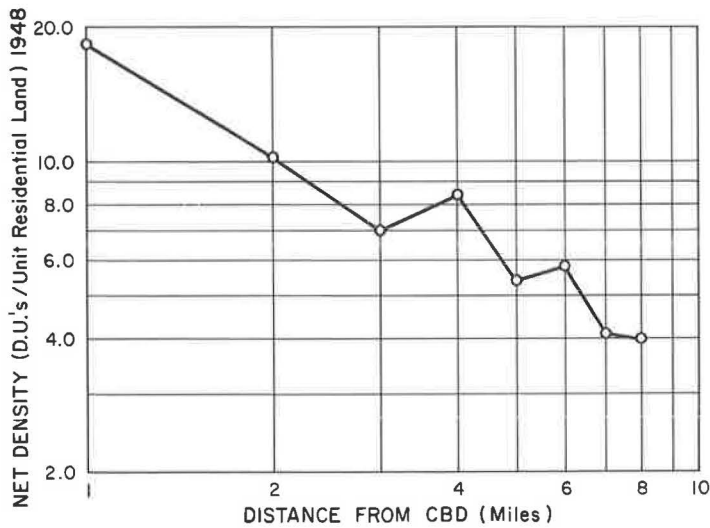


Figure 8. Logarithmic plot of 1948 net density.

determined by the economic condition of the household, its preference pattern in terms of trade-offs with other commodities, the state of the housing market and its relationship to the transportation system (7, 13, and 14). The household is provided with an income which it must allocate in the purchase of goods and services in such a way as to achieve as much total satisfaction as possible. For simplicity, let us assume that all household expenditures fall into three general classifications: transportation, housing and other. If we assume further that the "other" purchases absorb a fixed proportion of total income, the urban household faces the problem of purchasing housing and transportation such that composite satisfaction is maximized and total purchases do not exceed a fixed amount. As a first solution the household head might elect to buy housing where land costs are cheapest, thereby getting the most space for his money; however, it is likely that this location is remote from the remaining activities of the urban area with which he must interact, thereby leaving him with an extravagant transportation bill. On the other hand, he might elect to locate where transportation service is best but where housing cost is so high that to stay within his fixed expenditure allowance he is constrained to the purchase of an undesirably small housing package. Contained within this total theoretical framework is a causal relationship between the land value distribution in the region and transportation service. Areas which are highly accessible are most desirable and can therefore command a higher price. It is primarily this final consideration which directly links the urban transportation planner's decisions with the course of urban development (15, pp. 256-257).

This somewhat tangential discussion has been made to show the appropriateness of net as opposed to gross density data and analysis. Net density analysis can contribute to as well as draw upon this theoretical framework. Gross density techniques, with their vague tie to land consumption, cannot so contribute. Conceivably, analytic tools will be forthcoming, incorporating these theoretical relationships, which will provide the transportation planner with direct assignments of the form and composition of marginal development to the areas of influence about proposed transportation routes or improvements thereto.

Net residential density was computed for each district and ring in the study region by totaling the dwelling units and dividing by the total area of land existing in residential use. A much more desirable procedure for computing average net density would have been to average the density of each individual dwelling; however, this requires consumed land on an individual dwelling basis. The computed average net density must be

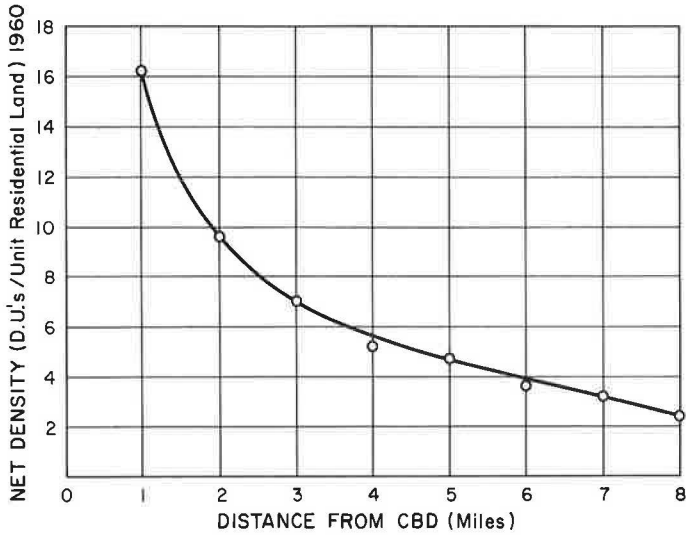


Figure 9. Net density gradient (1960).

treated as representative of the average condition in the analysis unit. Its representativeness is dependent on the variability of the individual dwelling densities within the unit. It should also be noted that unlike the gross density measure, net density is not monotonically related to the total dwelling stock; it can rise or fall both with increases or decreases in the contained dwelling unit total.

Figure 6 shows a plot of the computed ring net densities for 1948 as a function of distance to the CBD. The general conformity in shape with the equivalent gross density plot is evident. Figure 7 reveals the general linear relationship obtained by a replotting on semilogarithmic paper. However, as suggested partially by the evidence of non-linearity in the plot in Figure 7 and from Kramer's work (4), the net residential density

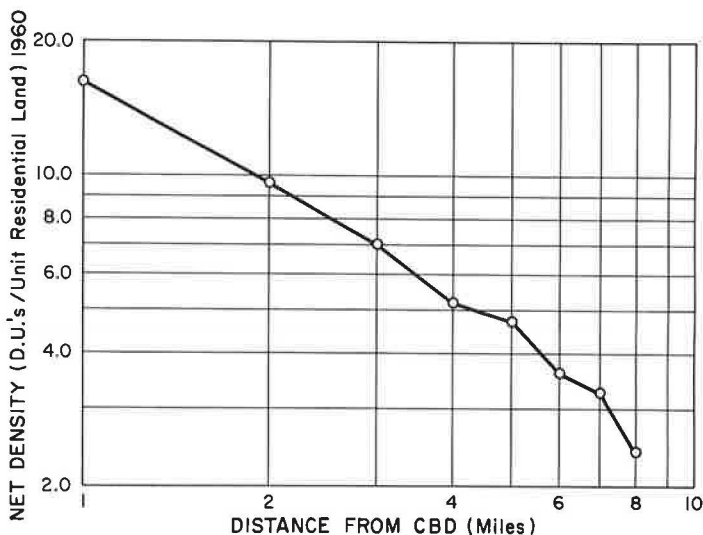


Figure 10. Logarithmic plot of 1960 net density.

data were plotted on log-log paper (Fig. 8). Least squares fits were computed for both scatter diagrams, Figures 7 and 8, and R^2 values computed. The doubly logarithmic relationship proved to be a better linear fit.

The linear equation fit to the 1948 net ring densities was

$$\ln D_{N. 48} = 2.850 - 0.688 \ln X \quad (7)$$

which in nonlogarithmic form is

$$D_{N. 48} = 17.29 (X)^{-0.688} \quad (8)$$

The R^2 associated with Eq. 8 was 0.957. Eq. 8 was then examined as an estimator of the 1948 district net densities and yielded an R^2 of 0.835.

Under the assumption of stability in the net density structure of the test region over the 12-year period, Eq. 8 was tested as a valid predictor of the 1960 net densities at both the ring and district level. The resulting R^2 's were respectively computed to be 0.927 and 0.844.

To complete this particular line of investigation, the 1960 net ring densities were plotted first on regular graph paper and then on log-log paper (Figs. 9 and 10). The least squares regression fit to Figure 10 resulted in the following relationship:

$$\ln D_{N. 60} = 2.855 - 0.876X \quad (9)$$

or

$$D_{N. 60} = 17.4 (X)^{-0.876} \quad (10)$$

with R^2 's of 0.986 and 0.934 at the ring and district aggregation levels.

The relative success of these investigations suggested the testing of the following less involved procedure; ring densities in 1960 were estimated to remain exactly as they were computed to be in 1948. This simplifying assumption implies that the added dwelling units over the test period consumed, on the average, the same amount of land as the average dwelling unit existing in the ring in 1948. The computed R^2 for the 1960 net ring densities was 0.849. Carrying this procedure down to the districts, incremental dwelling growth in each district was assumed to locate at the same average 1948 net density as for the particular ring to which it fell. Implicit in this trial is that the intra-ring net density variability is diminishing over time with each district's net density approaching its ring average. The percent of 1960 net district variance explained utilizing this technique was 0.533.

Finally, each district was assumed to maintain constant average net density from 1948 to 1960, the 1948 values then serving as 1960 estimates. An R^2 of 0.640 was computed for this case.

The concluding analysis of the net residential density pattern involved the development of multiple-regression equations utilizing as independent explanatory variables selected data items from the rather extensive list available. However, because the majority of these data were already available at the analysis zone level, the decision was made to calibrate the net density multiple regressions at this level, and to utilize them as estimators for both districts and rings. While this procedure violates strict regression procedure, the errors introduced were thought not to be severe, partially relying on the results of the interdistrict analysis of variance reported on earlier which revealed that the intradistrict variability was minor in relation to the interdistrict variance. Additionally, this approximating procedure required that the dependent variables for each of the regressions be an intensive quantity, and thereby independent of the size of the analysis observation unit.

The functional form of the DVRPC's density submodel, SPACEC I, was investigated as representative of the study region's density pattern. In a much simplified form the relationship is

$$D_N = a e^{\sum b_i X_i} \quad (11)$$

where

X_i = independent variable i ,
 b_i = the coefficient of variable i ,
 a = constant, and
 D_N = net residential density.

This relationship transforms by logarithmic conversion to a standard linear multiple-regression relationship:

$$\ln D_N = \ln a + \sum b_i X_i \quad (12)$$

Using a stepwise regression program, least squares relationships of the form of Eq. 12 were developed for both the 1948 and 1960 net densities. A considerable number of trials were attempted before two final relations were accepted which were logically sound and which contained only statistically significant explanatory variables. The 1948 equation computed was

$$\ln D_{N.48} = 1.534 + 0.005X_1 + 0.017X_2 + 0.109X_3 \quad (13)$$

(1.92) (2.12) (6.62)

or

$$D_{N.48} = (4.64) e^{(0.005X_1 + 0.017X_2 + 0.109X_3)}$$

where

X_1 = land value 1948 (\$/sq ft),
 X_2 = percent developed land in industrial use (1948), and
 X_3 = gross residential density (1948) = $D_{G.48}$.

The numbers in parenthesis below each coefficient are the regression "t" values. Eq. 13 was then used to estimate the 1948 ring and district net densities by a simple substitution of the appropriate values for the independent variables. The R^2 's computed for the rings and districts were 0.802 and 0.714.

The stability of the relationship developed for 1948 was investigated by using Eq. 13 as a predictor for 1960, substituting 1960 values for the explanatory variables. Values of variable X_1 , land value, did not change inasmuch as these data were only available for 1948. The R^2 's resultant from this predictive effort were 0.902 and 0.802, respectively, for the rings and districts. Solutions to Eq. 13 were transformed to non-logarithmic form prior to the calculation of residual errors.

A least squares regression of the general form of Eq. 12 was then made for the 1960 net ring density distribution. The measures of accuracy for this relationship could then be used to evaluate how well the 1948 relationship held up. In addition, the changes in the variable makeup of this new relationship might provide some interesting comparisons with the 1948 equation. The 1960 least squares relationship was

$$\ln D_{N.60} = 0.086 + 0.123X_1 + 0.542X_2 \quad (14)$$

(6.83) (8.21)

or

$$D_{N.60} = (1.9) e^{0.123X_1 + 0.542X_2}$$

where

$D_{N.60}$ = net residential density (1960),
 X_1 = gross residential density (1960), and

X_2 = logarithm of net residential density (1948) = $\ln D_{N. 48}$.

The ring and district R^2 's were 0. 805 and 0. 832.

Multiple-regression estimates were then developed for a nontransformed dependent variable. Again a stepwise procedure was used in testing a number of independent variable combinations before two relationships, for 1948 and 1960, were accepted. The two equations were

$$D_{N. 48} = 6. 25 + 0. 085X_1 + 0. 206X_2 + 0. 922X_3 \quad (15)$$

(4. 04) (2. 96) (6. 49)

where

X_1 = land value - (\$/sq ft) 1948,

X_2 = percent of developed land in industrial use (1948), and

X_3 = gross residential density (1948) = $D_{G. 48}$;

and

$$D_{N. 60} = 1. 960 + 0. 012X_1 + 0. 053X_2 + 1. 082X_3 + 0. 119X_4 \quad (16)$$

(2. 00) (3. 12) (28. 47) (7. 00)

where

X_1 = land value (\$/sq ft) 1948,

X_2 = percent developed land in industrial use (1960),

X_3 = gross residential density (1960) = $D_{G. 60}$, and

X_4 = net residential density (1948) = $D_{N. 48}$.

The R^2 values for Eq. 15 were 0. 755 and 0. 561 at the ring and district. Eq. 15 was then tested as a predictor of 1960 ring district and densities. R^2 values of 0. 264 and 0. 151 were determined for solutions of Eq. 15, substituting where possible 1960 values for the independent variables. Eq. 16 was then solved and ring and district R^2 values of 0. 963 and 0. 938 computed.

The independent variables and the signs of the coefficient in Eq. 15 appear logical and causatively related to the quantity being estimated. The positive coefficient of assessed land cost reflects the economic supply and demand process at work. The higher the cost of land, the less the individual family can afford to consume, and the resultant increase in net density (7). The positive coefficient of percent industrial land is probably a reflection of the tendency for low income families to settle in the marginal residential areas which are often characterized by a heavy mixture of industrial development. This result is somewhat at odds with that found by Muth (7) who states that his finding of a negative relationship between net population density and proximity to local manufacturing centers is probably due to a net decline in housing price resulting from a generally undesirable neighborhood effect overcoming a coincident positive pricing effect associated with the increased accessibility of such areas.³ Muth's findings suggest at least two alternative explanations of the positive relationship in Eqs. 15 and 16:

1. The transportation network and manufacturing sites are so located in the study area as to afford those areas of substantial manufacturing activity a decided accessibility advantage. This advantage would then be reflected in inflated housing pricing, enough to overcome any deteriorating effect of an unfavorable environment; and
2. The areas of manufacturing concentration have substantially remained in the older sections of the city where surrounding housing is traditionally of a higher density than in newer developing residential areas.

³Preliminary calibration results of the SPACEC I model by DVRPC have shown a similar positive relation between residential density and industrial activity. See internal staff memorandum of February 2, 1966, titled SPACEC I Parameters.

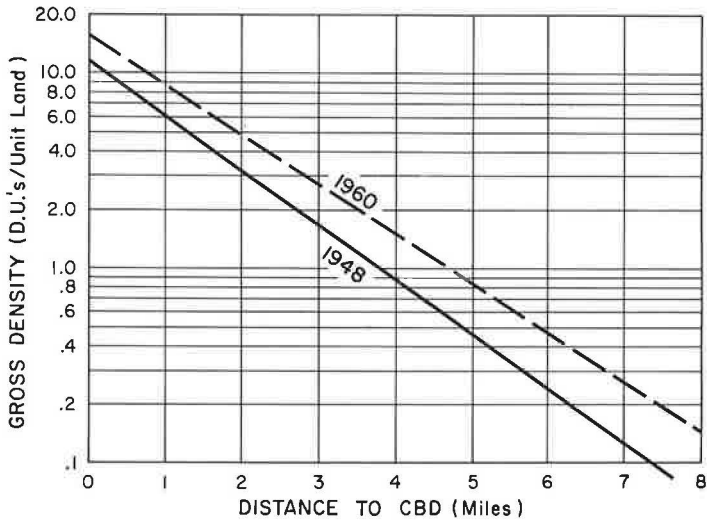


Figure 11. Calibrated gross density gradients (1948 and 1960).

These propositions are partially supported by the entries in the analysis zone simple correlation matrix. The "accessibility to employment" and "percent of developed land in industrial use" variables exhibit a moderate positive correlation of 0.37, substantiating to some degree the first possible explanation. The employment accessibility index is a regional measure such that the built-in collinearity in the two variables is probably not overly biasing. However, recent findings (5) tend to suggest that the urban worker is giving only secondary consideration to access to the workplace in selecting his place of residence. As the mobility of labor increases, along with the eventual shortening of the work week, it is unlikely that this trend will reverse. It is therefore doubtful that the pricing effect of superior accessibility would be so substantial as to overcome the general nuisance effect of proximate industrial activity. This

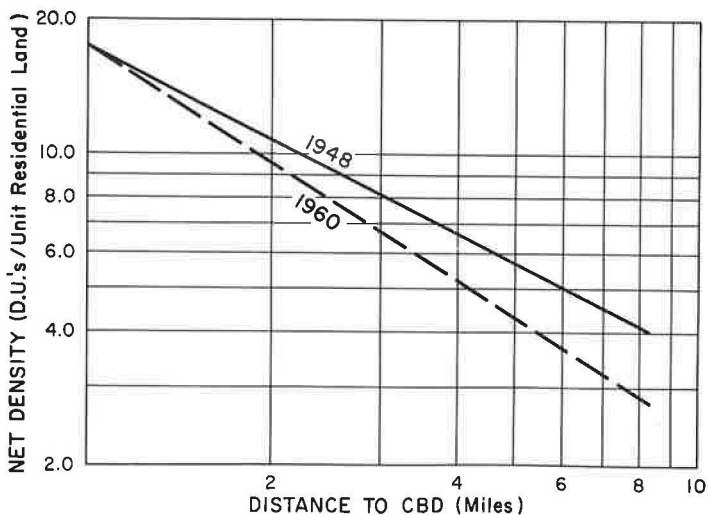


Figure 12. Calibrated net density gradients (1948 and 1960).

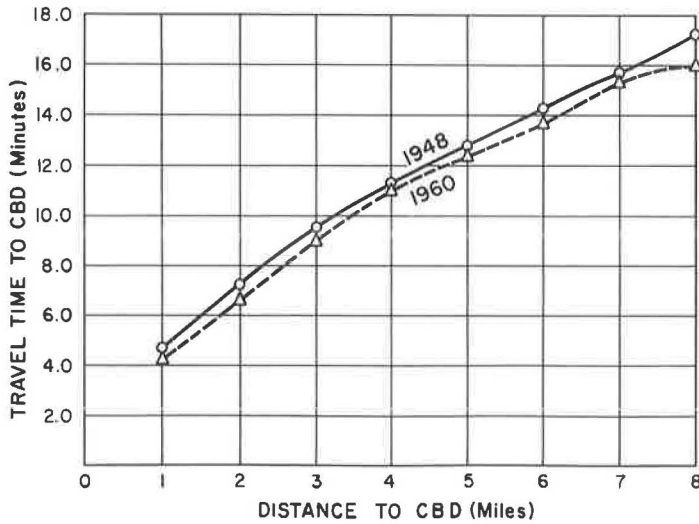


Figure 13. Travel times to the CBD (1948 and 1960).

is especially true for a smaller sized urban region where the influence of travel impedance as a locational factor has been observed to be below that found in larger urban areas.

There was no simple data item available which specifically measured the age of development in an area; however, the variable "percent of total land area not in use" was thought to be highly correlated with such an index. A negative correlation of 0.38 was found for this variable and "percent of developed land in industrial use." This relationship tends to support the second extended explanation for the positive correlations found between net density and degree of industrialization. However, these arguments must remain inconclusive, requiring a much more detailed analysis with additional data.

The strong positive correlation between gross and net density is revealed by the coefficient of the last independent variable in Eq. 15. As greater numbers of families locate in a given area, it is expected that increased demand for the relatively fixed stock of available land will result in increased land costs and increased densities.

The independent variable set in Eq. 16 is subject to the exact same interpretation as Eq. 15 with the addition of a fourth variable, net 1948 residential density. A substantial proportion of the dwelling stock in 1960 existed in 1948 in spite of the significant growth over this period. A strong serial correlation in the two net density variables is clearly evident, the correlation being 0.53. However, the strong explanatory relation between the two measures may also reflect a general inertia characteristic of new residential development. That is, the existing density pattern in an area influences the density characteristics of incremental dwelling development so that there is a tendency to avoid any great contrasts over relatively small geographic areas. This correlation, if it in fact exists, would probably be dependent on the degree of development existing in the area and the rate of growth. Further investigation of this point would require density data for the dwelling units locating over the study period, data which were not available for this study.

Observations on the Change in Density Structure

It may be both interesting and informative to briefly examine the actual shifting in the residential density structure of the study area over the 12-year period of analysis.

Figure 11 contains the two density distance gradients (Eqs. 3 and 5) fit to the ring gross density observations for both 1948 and 1960. Two obvious changes have occurred in the gross density configuration manifest in the deviations of the two linear gradients.

Gross density has consistently increased in each ring. This is to be expected in light of the definition of gross density and the occurrence of a 52 percent growth in total dwellings in the study region over the test period. A decline in gross density could only have occurred under the condition of an absolute loss in total dwellings, a remote possibility in a region experiencing such a rapid expansion. Also the gradient has flattened out slightly, indicative of a less compact population distribution in 1960 than in 1948, and characteristic of a suburbanizing region. Berry et al (1) have found this phenomenon to be generally true for western cities and observed that density gradients tend to decline over time for a given city and tend to be flatter for larger sized cities.

The net residential gradients for both 1948 and 1960 are shown in Figure 12. In contrast to the gross density gradients, note the consistent decline in net density from 1948 to 1960 regardless of distance from the CBD. This observation is not simply explained in terms of absolute population growth and requires an extensive economic analysis of existing market conditions and consumer preferences. Clearly though, a major factor contributing to this overall density decline could be the substantial improvement in the transportation service in the region. Figure 13 shows average over-the-road travel time to the CBD for each ring in both 1948 and 1960. Highway service to the CBD has apparently shown consistent improvement over the 12-year span. The reduced travel costs associated with such improvements can provoke profound shifts in the locational equilibrium position of households. Reduced transportation costs can provide for decreasing land rent and also produce income effects which probably will increase the household's housing expenditure. Combining these two effects likely results in the consumption of more living space per household, perhaps explaining in part the results observed in Figure 12. However, this trend is not necessarily irreversible in spite of continued transportation improvements. As net densities continue to decline, the marginal worth of increased space necessarily falls with the distinct possibility of it reaching a point where it no longer is to the household's benefit to consume more (7, pp. 28-29). Housing space may in fact become an inferior good at some point (different for each household or household group). An interesting recent finding may provide some empirical justification for this prognostication. Lansing (5) has found that a majority of households unsatisfied with their present lot sizes prefer larger lots up to $\frac{3}{10}$ of an acre. On the other hand, the majority of households living on lots larger in size than one-half in acre, and expressing dissatisfaction, would prefer smaller sized lots.

DISCUSSION OF RESULTS

Test Findings

Table 3 contains the results of all the analysis of gross residential density. The R^2 entries in the table reveal that the negative exponential formulation, equating gross density with distance outward from the center of the urban area, provides an effective description of the existing pattern. As expected, the results at the higher level of geographic aggregation show less residual error; however, considering that the reported district errors result from the application of the relationship calibrated to the ring values, it appears that the intra-ring variance is relatively minor in comparison with the inter-ring gross density variability. Using the exponential relationship fitted to the 1948 distribution as a forecasting device for 1960 proved to be moderately successful. The drop in explained variance from 1948 to 1960 was on the order of 11 percent at both the ring and district level. As was pointed out previously, the distance gradient fit to the 1960 gross density distribution was flatter than that for 1948 and, as shown by the R^2 values, accounted for approximately 11 percent more variance at both the ring and district aggregation levels. It should be noted that the 1948 gross density distance exponent of 0.648 is probably low for the size of the study area in comparison with the findings of Muth (7, p. 221).

The comparative results for the net density analysis are contained in Table 4. The procedures utilized in the net densities analysis fall under four general headings, corresponding to the four major sections in Table 4: distance gradients, multiple regressions (with both transformed and untransformed dependent variables), and assumed stability of 1948 values to 1960.

TABLE 3
CALIBRATION AND FORECAST R²'S FOR
GROSS DENSITY DISTANCE GRADIENTS

Equation	1948		1960	
	Ring	District	Ring	District
$D_G = 11.36 e^{-0.648X}$	0.886	0.834	0.784*	0.743*
$D_G = 15.58 e^{-0.585X}$	—	—	0.989	0.923

*Forecast results.

There was only a minor falling off in explained ring net density variance in 1960 and surprisingly a 10 percent increase in explained interdistrict variance. The latter result points to some of the peculiarities associated with the use of linear estimating procedures in fitting essentially nonlinear relationships by the expedient of logarithmic transformation. The R²'s calculated for the 1960 net density gradient are quite high and match almost exactly the equivalent gross density results. Once again the increase in the slope of the net density distance gradient from 1948 to 1960 is noted. Overall, the density-distance gradient formulations provided comparable accuracy at both aggregation levels for both net and gross values, with the single outstanding result being the superiority of the 1948 net gradient as a predictor of 1960 conditions.

The multiple-regression R²'s developed for the net density distributions are also presented in Table 4. For nonlinear formulation, the results indicate moderate explanatory success in calibration for 1948. The residual variance is greater than for the distance gradient trial at both the ring and district levels, there being a considerable decrease in explained variance for the ring analysis. Quite unexpectedly the equation produced higher R²'s in a projection role than it did in calibration. In fact, solutions to the equation with 1960 values of the independent variables (with the exception of land value) resulted in less residual error after transformation to nonlogarithmic form than the equation calibrated to the 1960 data.

TABLE 4
CALIBRATION AND FORECAST R²'S FOR NET DENSITY ANALYSIS

Analysis Procedure	Model Form	1948		1960	
		Ring	District	Ring	District
Distance gradient	$D_n = (17.3) X^{-0.688}$	0.957	0.761	0.927*	0.844*
	$D_n = (17.4) X^{-0.876}$	—	—	0.986	0.934
Log linear multiple regression	$\ln D_{n,48} = 1.534 + 0.005 \text{ land value} + 0.017 \% \text{ industrial land} + 0.109 \text{ gross density}_{48}$	0.802	0.714	0.902*	0.820*
	$\ln D_{n,60} = 0.086 + 0.123 \text{ gross density}_{48} + 0.542 \ln D_{n,48}$	—	—	0.805	0.832
Linear multiple regression	$D_{n,48} = 6.257 + 0.085 \text{ land value} + 0.206 \% \text{ industrial land} + 0.922 \text{ gross density}_{48}$	0.755	0.561	0.264*	0.151*
	$D_{n,60} = 1.960 + 0.012 \text{ land value} + 0.053 \% \text{ industrial land} + 1.082 \text{ gross density}_{60} + 0.119 D_{n,48}$	—	—	0.963	0.938
Assumed no change in net densities	Assume 1960 ring densities same as 1948 ring densities	—	—	0.849*	0.533*
	Assume 1960 district densities same as 1948 district densities	—	—	—	0.640*

*Forecast results.

The distance gradients fit to the 1948 net and gross density distributions show significant R²'s at both the ring and district level. However, the reduction in accuracy at the district level is substantially greater in the net density case than was found in the gross density analysis. On the other hand, the stability of the 1948 net density gradient is considerably above that found for gross density as indicated by the comparative R² values found in utilizing the 1948 gradient as a projection tool to 1960.

The results in Table 4 for the linear regression estimating equation without exception show that the accuracy of the 1948 calibrated relationship is inferior, at both aggregation levels, to those found for either the nonlinear regression or the distance gradient formulations. This is true both with respect to calibration and projection. These results are particularly interesting considering that the exact same explanatory variables compose both the linear and nonlinear 1948 regressions. However, the calibration R^2 's for the 1960 linear regressions are quite high, comparable to those obtained for the distance gradient and considerably better than those resulting from the nonlinear regression relationship. The strong serial correlation between 1948 and 1960 net densities most probably accounts for the sharp increase in explanatory accuracy of the 1960 equation as compared to that for 1948, inasmuch as the two equations have precisely the same independent variable composition with the exception of the inclusion of 1948 net density in the 1960 equation.

The final two entries in Table 4 testify somewhat to the point of temporal stability in aggregate net density patterns. These last two sets of R^2 's coincide with a forecast of 1960 ring and district densities under the assumption of no change in average ring densities over the 12-year period. It is apparent that considerable net density variability can be explained as a carry-over from the base time period. However, the accuracy of this forecasting procedure falls off considerably in going to lower levels of geographic aggregation. Even when account is taken of the intra-ring variability and 1960 district densities are assumed to remain as they were in 1948, only 64 percent of the variance is accounted for. While the simple forecasting technique of projecting no change in the density distribution is effective, it does not do as well as the assumption of stability in a density gradient relationship. However, by introducing the possibility of simulating the effects of temporal changes in the character or nature of the urban region, as is the case in the development of properly structured regression equations, considerable improvement can be expected insofar as accounting for density variability. This appears to be even more true as the level of aggregation falls. It is well to reiterate at this point that the regression relationships developed in this study were calibrated at a lower level of aggregation than at which the indices of accuracy were calculated and reported on in Table 4. It is quite likely that had the regressions been developed on district data, higher calibration and projection R^2 's would have been obtained.

The comparative results shown in Tables 3 and 4 indicate that the analysis and projection of net residential patterns can be made at least as accurately as for gross density, although personal bias of the author resulted in only limited investigation of gross density. The results contained in Table 4 substantiate that considerable success can be expected in the projection of urban net density configurations through the development of simple distance gradients. Conclusions concerning the development of multiple-regression relationships are difficult to construct from the somewhat inconsistent results obtained. While the nonlinear regression formulation proved quite superior to the linear equation in calibration to 1948 conditions and in projection to 1960, the reverse was found to be true with respect to the 1960 calibrated relationships. In any case it is apparent the significant regression relationships can be developed which contain explanatory variables with rational causative justification. It is unfortunate that the family income measure did not enter as a significant variable in any of the regression relationships in light of the apparent theoretic importance of this factor in the explanation of urban settlement. Unfortunately, the income data available for this analysis were census tract medians, too aggregated for the analysis zone level at which the remaining data were available and at which the actual regression calibrations were conducted. Perhaps zonal household income would have contributed to the explanatory relationships.

SUMMARY

There are some major points to be made concerning the analysis of small-sized urban residential structure resultant from this one limited test. To the analyst or planner concerned with developing a single best estimate of the future population density distribution, it is apparent that it is worth the limited amount of added effort to develop a

best-fitting distance gradient as opposed to simply projecting base year conditions blindly into the future on a small-area basis. The utility of distance gradients as effective representations of the density surface quite likely diminishes as the geographic analysis unit becomes finer. At gross levels of analysis, residential density patterns are apparently well correlated with distance outward from the region's center; however, there exists an underlying pattern of small-area heterogeneity superimposed upon this gross pattern of exponential decay.⁴ Accurate simulation of this lower level variability will likely depend on the development of causal relationships incorporating many of the notions currently contained in location theory. Additionally, distance gradients are quite useless in reproducing the likely fluctuations in residential development compactness resulting from alternations in one or a number of key policies or planning standards or from shifts in the socioeconomic character of the population. Only through the development of sound and logical models which simulate these interrelationships can such planning flexibility be established.

Finally, through the exchange medium of land value the urban transportation planner is able to contribute actively in the total effort aimed at bringing order and efficiency to the urban space. By providing and depriving transportation access spatially, heterogeneity is induced in the land value surface. The transportation planner thus participates directly in the alteration of the residential density configuration, a significant parameter of the urban mechanism.

CONCLUSIONS

It should be apparent that the results in this paper cannot be viewed as being conclusive coming from a single analysis of a particular urban area over a single time period. However, these results will hopefully contribute to and advance existing analytic and forecasting facility with respect to the residential density structure of small- and medium-sized urban areas. The following are then a brief listing of several of the most important conclusions to be drawn from this analysis:

1. Future analytic work in the general area of urban residential structure should concentrate on net as opposed to gross density measures. Net density is a much richer and more exact unit directly compatible with a substantial body of existing theory and apparently is as conducive, if not more so, to meaningful analysis and projection as gross density.
2. The analysis district, the basic unit of analysis for this study, having an average internal population of 2,000 persons and thereby being comparable to the familiar urban transportation travel analysis zone, is a useful level of aggregation for studying urban residential density structure and does not subsume the most significant variability in net density within the urban region.
3. Density-distance gradients are useful tools in analyzing the density structure of the urban area and can also serve as appropriate projection devices. However, it is clear that density gradients are not static, suggesting that additional research be devoted towards developing rational explanations of, and procedures for estimating, these parametric shifts. Such knowledge would greatly improve the forecasting potential of distance-gradient relationships.
4. The development of accurate models of net residential density, which are logically structured in terms of existing theories of economic equilibrium and activity location, should be actively pursued. Results obtained in the present study are encouraging in this regard in spite of obvious informational deficiencies.
5. It is probable that the central core areas should be treated separately from the remainder of the urban region in the development of simple models of residential density. Also, considerable distortion can be introduced by the inclusion of substantially

⁴Witness the recent development of high-density high-rise apartment developments in what have traditionally been areas solely developed to typical suburban single-family dwelling densities.

rural areas, which are not expected to sustain significant urbanization, in the development of residential density relationships.

6. Future residential density configurations should not serve merely as exogeneously determined constraints to simulation models of residential location. If density patterns can be functionally related to socioeconomic characteristics, then it would appear that the future density structure should be responsive to, as well as influence, forecasts of the location of the urban area's activities.

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Optimizing Density of Development With Respect to Transportation Cost

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•THE Tri-State Transportation Commission has a broad responsibility encompassing land-use planning as well as transportation facility planning. The possibilities of guiding land-use development towards some more desirable pattern are being explored—both because of the effect on transportation and as an end in itself. Tri-State's planning staff is thus faced with the question: What would be the best possible pattern of future land use? The city planning profession has not yet agreed on the answer to this question, and it is still largely a matter for speculation. The planner lacks the tools to make an objective evaluation of a land-use plan; he must rely on judgment, intuition, and some venerable precepts which have increasingly come under attack.

It is hoped to develop some more scientific procedures—preferably some measurable criteria—to use in trying to formulate the best possible land-use and transportation plan. As one approach to this problem, I have abstracted a few of the essential parameters of a city and have shown how they might be optimized. Although this was a theoretical analysis performed on an idealized city, the results hopefully can provide some guidance to planners charged with drawing specific plans for real cities.

Other areas of human endeavor do not suffer the same lack as city planning; they have developed well-accepted criteria for determining what is good. The businessman has one clear, overall objective: to maximize his profits. In engineering and welfare economics, benefit-cost criteria have been widely utilized. Benefit-cost analysis was applied to metropolitan transportation planning by the Chicago Area Transportation Study (CATS). Of particular note was the technique developed for calculating the optimal spacing of highways (1). This kind of "ideal city" analysis found cogent practical applications in developing a highway plan for the Chicago region.

The so-called "benefit-cost analysis," as used in transportation planning, does not actually distinguish between benefits and costs. Benefits are merely savings in costs, and thus the objective is really to minimize costs. The costs which are affected by the land-use pattern might be divided between transportation costs and other costs. Little progress has been made in identifying and measuring the non-transportation costs. Presumably, such things as utility costs, construction costs and land costs are affected by the land-use pattern, but there are also myriad indirect and elusive social and economic costs. Considerable work has been done in measuring transportation costs, so this seemed the best place to begin. Since the optimal spacing work dealt only with highways, the initial study has been limited to private vehicle transportation.

There are many aspects to the land-use pattern, and again it was necessary to select one parameter as a starting point. The most logical and convenient one was the density of development. Density can be measured in various ways, but since the study deals with the costs of motor-vehicle transportation, I measured the density of vehicle trip ends. This has the advantage that it represents both residential and nonresidential activities; population density tells only part of the story.

To summarize, the influence of the density of vehicle trip ends on highway transportation costs was investigated. The specific objective of the study was to minimize the total transportation cost per vehicle trip.

DEVELOPMENT OF THE METHOD

Total transportation costs for a motor-vehicle system are divided into investment costs and travel costs. Investment costs cover all expenditures involved in providing fixed physical facilities (i. e., highways), right-of-way acquisition, clearance, utility relocations, construction, etc. Travel costs include those costs borne by users of the facilities in operating their vehicles on them. These are subdivided into operating costs, accident costs, and time costs. This presumes that there is a monetary value to users' time; this value has been estimated by observing how much money people pay to save time. Expressing all of these costs in dollars does not reflect a materialistic bias, but rather the necessity of having a single common denominator.

In general, an increase in investment will produce a better highway network and result in lower travel costs. Investment cost is a one-time capital expenditure, while travel cost forms a daily recurring stream extending over an indefinite time period. Any savings in travel cost can be considered to be a return on invested capital. This puts the problem into a suitable format for benefit-cost analysis (2, 3).

To proceed, it is necessary to establish the relationship between the density of trip ends and the several costs. For a real city this would be a forbiddingly complex task, but it can be done for a theoretical, idealized city. Fortunately, such a city has been founded by Morton Schneider, and he has described it in a paper which forms another key block in the foundation of this work (4, 5). Schneider had a different object in mind when he set up his city—namely, to estimate traffic—but it is readily amenable to the problem here, and the ability to estimate traffic volumes is essential to this analysis. The stipulations surrounding the idealized city are fully described in Schneider's paper. To understand the current argument, one must know the following assumptions:

1. The city is absolutely regular and homogeneous, extends infinitely in all directions, and has a uniform density of trip ends throughout.
2. The city has three street systems of distinctly different quality. These can be regarded as expressways, arterials, and local streets.
3. Each street system forms a perfect gridiron with uniform spacing everywhere. The spacing of the three different systems need not be the same.

The major task is to develop equations expressing the several elements of transportation cost. This is largely a matter of synthesizing previous work done by Schneider, George Haikalis, and others at CATS.

Investment Cost

In the hypothetical city, each square mile is exactly like every other square mile. If a gridiron street network has a spacing of z miles, then in a square mile there will be $2/z$ miles of that type of street. The total mileage of streets in a square mile of the city will be

$$2 \left(\frac{1}{z_1} + \frac{1}{z_2} + \frac{1}{z_3} \right) \quad (1)$$

where z_1 is the spacing of expressways, z_2 the spacing of arterials, and z_3 the spacing of local streets.

If C_1 is the per mile investment cost of constructing expressways, with C_2 and C_3 representing the same for arterials and local streets, then the total investment cost for a square mile will be

$$2 \left(\frac{C_1}{z_1} + \frac{C_2}{z_2} + \frac{C_3}{z_3} \right) \quad (2)$$

We are interested in the cost per trip, so we must divide this total by ρ , which is the density of vehicle trip destinations per square mile per day. One other factor, K , must be added. This is merely a conversion factor which transforms the one-time

investment cost into an equivalent daily cost, assuming some interest rate and facility life span. Now, the investment cost per trip is

$$\frac{2}{K\rho} \left(\frac{C_1}{z_1} + \frac{C_2}{z_2} + \frac{C_3}{z_3} \right) \quad (3)$$

It is possible to assume that the C's are constant. This probably does little violence to the truth for local and arterial streets; their share of the total cost is small, anyway. But it clearly is not true for expressways—their construction and, particularly, right-of-way costs are very dependent on the kind of area through which they pass. I have hypothesized that expressway investment cost follows the following formulation:

$$C_1 = \alpha + \beta\rho \quad (4)$$

in which α and β are coefficients whose values must be determined empirically. There is a certain minimum cost which exists even in rural areas of zero density, and cost increases as density increases. Inserting Eq. 4 for C_1 in Eq. 3 results in the final expression for investment cost per trip:

$$\frac{2}{K} \left(\frac{\alpha}{z_1\rho} + \frac{\beta}{z_1} + \frac{C_2}{z_2\rho} + \frac{C_3}{z_3\rho} \right) \quad (5)$$

Estimating Traffic Volumes

A prerequisite to determining travel cost is a method for estimating traffic volumes on each of the three street networks (under the assumptions made, the volume on each street type is the same everywhere). As the volume on any street increases, the travel cost increases because congestion slows the traffic. Furthermore, our problem stipulated three markedly different street types, and the difference would be reflected in different travel costs, thus it is necessary to know the distribution of traffic among the street types.

Schneider addressed himself to this problem of estimating traffic in his paper on direct assignment (4). Later he made a minor revision in his technique which eliminated certain bugs but did not greatly alter the estimates yielded (6). The revision did produce equations which differ from those in the original paper, and I have used these revised equations for the traffic volumes:

$$V_1 = \frac{\rho \bar{r}^3 z_1}{2(\bar{r} + z_1)(\bar{r} + z_2)} \quad (6)$$

$$V_2 = \frac{z_2}{\bar{r}} V_1 \quad (7)$$

$$V_3 = \frac{z_3(\bar{r} + z_1)}{\bar{r} z_1} V_2 \quad (8)$$

In these equations, V_1 is the average daily volume on expressways, and V_2 and V_3 represent the volumes on arterials and locals. The symbol \bar{r} is the average trip length in miles. The other symbols should be familiar. Notice that the traffic volumes are dependent on the density of trip destinations, the average trip length, and the spacing of the street networks.

Travel Cost

The average travel cost per trip is the sum of the costs on each of the three street systems. It can be represented thus:

$$P_1T_1 + P_2T_2 + P_3T_3 \quad (9)$$

P_1 is the average distance traveled by a trip on the expressway system, and T_1 is the travel cost per mile on expressways. Their product represents the average cost incurred by a motorist on the expressway system. The second and third terms of the expression represent the same thing for arterial and local streets. It may be helpful to point out that

$$P_1 + P_2 + P_3 = \bar{r} \quad (10)$$

Now, P_1 can be readily calculated from V_1 , the relationship being

$$P_1 = \frac{2V_1}{z_1\rho} \quad (11)$$

The vehicle-miles on expressways per square mile is the product of the average volume (V_1) times the miles of expressways per square mile ($2/z_1$). Dividing this by the number of trip destinations per square mile (ρ) gives the average per trip. Similar equations hold for arterials and locals, namely:

$$P_2 = \frac{2V_2}{z_2\rho} \quad (12)$$

$$P_3 = \frac{2V_3}{z_3\rho} \quad (13)$$

There remains the problem of determining the T 's. In view of uncertainty about the relationship of operating and accident costs to average speed, it seemed wisest to assume that the sum of operating and accident costs per mile is constant. Let A represent this constant.

The only variable, then, is time cost. This increases as speed falls, which happens as volume rises. Time cost is the product of the value of time and the amount of time. I assume the value of time is constant; let it be represented by B .

Another convention is to break time into two parts: the amount of time required to travel at free speed (i. e., if there were no interference from other traffic) and the time delays resulting from congestion. If S_1 is the free speed on expressways and D_1 is the delay per mile on expressways, then the total travel cost per mile on expressways is

$$T_1 = A + B \left(\frac{1}{S_1} + D_1 \right) \quad (14)$$

and the travel cost per mile on arterials is

$$T_2 = A + B \left(\frac{1}{S_2} + D_2 \right) \quad (15)$$

This is as far as the abstract reasoning can be carried. In the illustration of the method that follows, I assume free speeds to be constant and formulate specific expressions for the delays, based on empirical findings. Observe that free speeds are different on different street types, and by definition they are independent of traffic volumes. Delays should vary in response to the capacity of a street and to the volume it carries. As volume increases, delay rises very gradually at first, and then more and more sharply as capacity is approached and exceeded. Capacity is not taken as an absolute maximum, but rather a kind of standard or milepost which generally indicates the ability of a certain physical highway facility to pass vehicles.

Local streets, T_3 , have been omitted from the discussion. As a rule, traffic volumes on local streets are so low that congestion rarely results. There would appear to be little reward from making a sophisticated analysis of T_3 , so it was assumed to be constant.

Finding the Minimum Cost

An equation was developed which represents the total transportation cost per trip in the hypothetical city. The equation is not recapped here because it is rather

cumbersome. When plotted against density, it yields a U-shaped curve. As density increases, the investment cost per trip declines, but the travel cost per trip rises. The problem is to find that density at which the curve has its minimum; this would be the optimal density. The problem is readily soluble by differential calculus. By holding all other factors constant and differentiating with respect to ρ , one can secure an equation which locates the optimal value of ρ . Since ρ occurs at many places in the original equation, the work is rather involved, and I shall not bore the reader with the mathematics entailed.

With this tool, it is possible to determine the optimal density for any given expressway spacing. However, this is not totally sufficient; the planner would naturally want to manipulate both density and expressway spacing and to find the best combination. Consequently, the two variables (density and expressway spacing) were optimized simultaneously and the minimum cost per trip with respect to both was determined. This problem is also amenable to calculus by taking two partial derivatives, setting both equal to zero, and solving them simultaneously. The problem can be visualized in three dimensions, in which density and expressway spacing are the orthogonal horizontal axes and cost per trip is the vertical axis. The equation for total cost forms a surface shaped something like a pit, and by calculus one can locate the minimum point on that surface.

A logical extension is to consider arterial spacing, and to optimize three variables simultaneously. However, each partial derivative added causes a considerable increase in the mathematical work required. While an analytical solution for the triple optimum may be possible, I have been content to approximate it by selecting several different values for arterial spacing, finding the double optimum for each, and comparing the resulting costs per trip.

ILLUSTRATION OF THE METHOD

It is difficult for the reader to grasp the technique fully without a concrete example utilizing numbers instead of symbols. It is also of interest to the investigator to examine how the results react as various factors take different values. Consequently, an illustration using numbers and producing concrete results was developed. It would be ideal to utilize values and relationships taken from the real world, and in particular from the Tri-State region. However, Tri-State's data analysis had not reached the stage where such information was available, therefore, hypothetical values and relationships were used. In some cases, these were based on data from CATS, and in some cases things were fabricated which seemed intuitively reasonable. Therefore, this must be regarded as purely an academic exercise, and the specific results should not be accepted at face value. However, in general the substitution of empirical findings for hypothetical data would only change the particular values of the results; the method itself would remain valid.

Values Assumed

In the example, assume that the average trip length (\bar{r}) is 6 miles, a value determined by CATS, which appears to be approximately the same in all major metropolitan areas. For simplicity, local spacing (z_3) is held constant at $\frac{1}{10}$ mile. The particular value assumed for K is 3081.6; this is based on a 10 percent interest rate, 25-year facility life, and 339.5 equivalent weekdays to a year. In accordance with custom, this analysis deals with trips for an average weekday. Since on the average, weekend and holiday traffic is less, the number of equivalent weekdays in a year comes out to less than 365.

For expressway investment cost, an equation developed at CATS was used:

$$C_1 = 1,120,000 + 520\rho \quad (16)$$

I assumed that arterial investment cost (C_2) was \$500,000 per mile, and that local street investment cost (C_3) was zero. This is to argue that since the function of local streets is to provide access to land, their cost might properly be assigned to land development rather than the transportation system.

TABLE 1
TOTAL COST PER TRIP IN CENTS
(When Arterial Spacing = 0.5 Miles)

Density ^a	Expressway Spacing In Miles				
	2	4	6	8	10
5,000	84.31	74.59	72.13	71.25	70.89
10,000	74.22	66.39	64.65	64.16	64.07
15,000	70.92	63.93	62.66	62.54	62.76
20,000	69.38	63.14	62.51	62.97	63.68
25,000	68.62	63.30	63.61	64.96	66.47
30,000	68.33	64.22	65.89	68.53	71.22
35,000	68.37	65.88	69.41	73.86	78.19
40,000	68.70	68.32	74.32	81.16	87.65
45,000	69.32	71.60	80.76	90.66	99.93
50,000	70.22	75.80	88.89	102.61	115.36

^aIn vehicle trip destinations per square mile.

The sum of operating and accident costs (A) was 3.5 cents per mile and the value of time (B), \$1.50 per hour. For T₃, which was assumed to be constant, a value of 14 cents per mile was selected. All these values were based on CATS findings. Free speed on expressways (S₁) was assumed to be 50 miles per hour, and free speed on arterials (S₂) 30 miles per hour.

The investigation of Haikalis was consulted to secure expressions for delay, although all his equations were not adopted verbatim because of their complexity (2). Equations which approximated the curves he presented were formulated. The following equation was used for expressway delay per mile (in hours):

$$D_1 = 0.001 + 0.00122 R_1^3 \quad (17)$$

in which R₁ is the volume-to-capacity ratio on expressways. Capacity of expressways was assumed to be 127,000 vehicles per day.

The delay on arterial streets occurs principally at intersections with other arterial streets (which are normally signalized). Therefore, it is logical to determine the average delay at an intersection and multiply it by the number of intersections per mile (which is the inverse of the spacing). The result of this was the following equation:

$$D_2 = \frac{0.0032 + 0.003 R_2^3}{Z_2} \quad (18)$$

in which R₂ is the volume-to-capacity ratio on arterials. The capacity was assumed to be 20,000.

These are all the values needed to carry out the calculations and determine the optimum conditions. It may be of interest to show the resulting equation for the optimal density:

$$\rho = \frac{978.282 (6 + z_1) (6 + z_2)}{z_1} \sqrt[4]{\frac{2.24 + \frac{z_1}{z_2}}{2.058 + z_1 z_2^2}} \quad (19)$$

This equation is very easy to use. Unfortunately, the companion equation for the optimal expressway spacing is not so simple. It is not amenable to a direct algebraic solution and must be solved by trial-and-error or graphical means.

Results

Optimal combinations of density and expressway spacing for a number of different arterial spacings were calculated. Before examining these optima, it will be instructive to see how cost per trip is affected by variations in density and expressway spacing. Table 1 gives the situation when arterial spacing is held constant at one-half mile. Reading down a column, one can locate the optimal density for any given expressway spacing. Thus, for a spacing of 2 miles, it is 30,000. Reading across a row, one can locate the optimal spacing for any given

TABLE 2
INVESTMENT COST PER TRIP IN CENTS
(When Arterial Spacing = 0.5 Miles)

Density ^a	Expressway Spacing In Miles				
	2	4	6	8	10
5,000	37.12	25.05	21.03	19.02	17.81
10,000	27.00	16.74	13.33	11.62	10.59
15,000	23.62	13.98	10.76	9.15	8.19
20,000	21.94	12.59	9.48	7.92	6.98
25,000	20.92	11.76	8.71	7.18	6.26
30,000	20.25	11.21	8.19	6.69	5.78
35,000	19.77	10.81	7.83	6.33	5.44
40,000	19.41	10.51	7.55	6.07	5.18
45,000	19.12	10.28	7.34	5.86	4.98
50,000	18.90	10.10	7.17	5.70	4.82

^aIn vehicle trip destinations per square mile.

TABLE 3
TRAVEL COST PER TRIP IN CENTS
(When Arterial Spacing = 0.5 Miles)

Density ^a	Expressway Spacing In Miles				
	2	4	6	8	10
5,000	47.19	49.53	51.10	52.23	53.08
10,000	47.22	49.65	51.32	52.54	53.48
15,000	47.29	49.95	51.90	53.39	54.57
20,000	47.45	50.55	53.04	55.05	56.70
25,000	47.70	51.54	54.91	57.78	60.21
30,000	48.08	53.01	57.69	61.85	65.44
35,000	48.60	55.07	61.59	67.53	72.75
40,000	49.30	57.81	66.77	75.09	82.47
45,000	50.20	61.32	73.42	84.79	94.95
50,000	51.32	65.70	81.73	96.92	110.54

^aIn vehicle trip destinations per square mile.

have minima, except at extreme values of zero and infinity. It is only when the two are superimposed that a minimum occurs at some meaningful point.

To compute all the values shown in these tables by hand would be quite laborious. Therefore, a FORTRAN program was written and the values were calculated by a 1401 computer. Tables 1 through 3 are actually excerpts from much larger tables which the computer produced.

It is possible to find the optimal combination of density and expressway spacing for any given arterial spacing by hand calculations in a reasonable length of time. This was done for a number of arterial spacings ranging from one-quarter mile to 2 miles. The results are shown in Table 4, with the cost per trip occurring at each optimum. The lowest cost per trip in this table is associated with arterial spacing of three-quarters of a mile, expressway spacing of 7.7 miles, and density of 13,900. The triple optimum is apparently in this vicinity.

This analytical method yields many interesting by-products. Various other parameters of the hypothetical city are calculated along the way, or can easily be derived. Table 5 gives some of the more significant characteristics associated with the optimal solutions given in Table 4. The speeds are average speeds including the delays due to congestion. It is also possible to calculate the distribution of vehicle-miles among the three street networks, volumes for certain turning movements, the average time for a trip, and the portion of that time caused by congestion.

ANALYSIS AND INTERPRETATION

Some background information may help the reader to put the results of the illustration in scale. The CATS surveys showed that, outside of the CBD, densities of vehicle trip destinations in the central city mostly fell between 15,000 and 35,000. In the close-in suburbs, densities of 10,000 to 20,000 were typical, while figures from 5,000 to 10,000 were common in suburban communities further out. An exclusively residential area with one-acre lots would probably have a density between 1,500 and 3,000.

The optimal spacing work at CATS resulted in recommended expressway spacings of 3 miles in Chicago and 6 miles in the suburbs. Arterial spacing of one-half mile already prevailed in Chicago and 1-mile spacing was recommended for the suburbs.

Inspection of Table 1 shows that the minimum cost is only slightly below many neighboring values (the differences would certainly be within the margin of error due to the grossness of estimated inputs). Thus, there is a rather large "region of indifference" embracing widely varying conditions. Costs below 70 cents are obtained for conditions ranging from $\rho = 45,000$ and $z_1 = 2$ to $\rho = 10,000$ and $z_1 = 10$. This type of result is

density. Thus, for a density of 25,000, it is 4 miles. Scanning the entire table, the lowest value found is 62.51 cents at a density of 20,000 and spacing of 6 miles. This is the optimum combination of density and expressway spacing.¹ Of course, this is a coarse-grained table; a more precise calculation of the optimum finds it to occur at $\rho = 17,440$ and $z_1 = 6.5$ miles, with a cost per trip of 62.37 cents.

Tables 2 and 3, in the same format but showing investment cost and travel cost separately, should make it easier to understand what is happening. The values in Table 1 are the sums of values in Tables 2 and 3. Note that the last two tables do not

¹For densities of 5,000 and 10,000, the values shown in the table do not turn up. Extension of the calculations indicates that for 5,000 the minimum cost is 70.69 cents at a spacing of 15 miles. For 10,000 the minimum turns out to occur at the 10-mile spacing.

common in optimization problems, and the curves have a U shape rather than a V shape. I feel that this is an advantage rather than a disadvantage. It provides considerable leeway within which other factors (perhaps social, political and aesthetic) may be allowed to influence any concrete decision. What the optimization study really shows are what extremes to avoid, because when you go beyond the region of indifference, costs do rise steeply.

Table 4 indicates that there is also a considerable region of indifference with regard to arterial spacing. Quite different combinations of arterial and expressway spacing and density produce very similar costs per trip. Again, this gives the planner considerable leeway for choice. It is important that he select a good combination of density and spacing, but there are many combinations of approximately equal merit (from the standpoint of transportation cost).

Looking at Table 5, the reader may wonder why arterial speed goes up at the same time as arterial volume. The reason is that arterial spacing is also increasing at the same time, so that while there are more vehicles on the highway, there are fewer stops for intersections.

One of the important findings of this exercise is that an optimum does exist, where cost is minimized, at conditions which are meaningful and reasonable. An optimization study must remain suspect until it is shown that it produces results which bear some relation to the real world. The reason why this optimum exists is because there are several opposing forces at work with certain trade-offs among them. It may be helpful to recapitulate how these forces operate.

As the density of trip ends increases: (a) investment cost is distributed over more trips, so the cost per trip declines; (b) there is no effect on the distribution of traffic among the three street systems (this is totally dependent on spacing); and (c) the average volumes on the streets rise, causing more delay, and so the time cost rises.

As the expressway spacing increases (becomes wider): (a) expressway investment cost goes down; (b) some traffic is shifted from expressways to arterials, which have a lower free speed, so time cost rises; (c) the average expressway volume rises, causing greater delay and increasing time cost further; and (d) the average arterial volume also rises, again causing greater delay and still further increasing time cost.

As the arterial spacing increases: (a) arterial investment cost goes down; (b) some traffic is shifted from expressways and arterials to local streets, with a consequent increase in time cost; (c) the average expressway volume declines, which raises average expressway speed and lowers time cost; (d) the average arterial volume increases, causing an increase in time cost; and (e) the frequency of delay points on arterials drops, which lowers time cost.

TABLE 4
OPTIMAL SOLUTIONS FOR
VARIOUS ARTERIAL SPACINGS

Arterial Spacing (miles)	Optimal Density ^a	Optimal Expressway Spacing (miles)	Cost per Trip (cents)
0.25	24,470	4.5	66.42
0.50	17,440	6.5	62.37
0.75	13,900	7.7	61.70
1.00	11,560	8.8	61.85
1.25	9,980	9.8	62.29
1.50	8,770	11.0	62.82
1.75	7,820	12.4	63.39
2.00	7,080	13.9	63.96

^aIn vehicle trip destinations per square mile.

TABLE 5
SOME CHARACTERISTICS OF OPTIMAL SOLUTIONS
(As Given in Table 4)

Arterial Spacing (miles)	Expressway Volume ^a	Arterial Volume ^a	Local Volume ^a	Expressway Speed (mph)	Arterial Speed (mph)
0.25	181,200	7,550	294	40.7	21.4
0.50	150,690	12,560	403	43.5	24.3
0.75	124,970	15,620	463	45.1	25.3
1.00	106,100	17,680	496	46.1	25.9
1.25	92,210	19,210	516	46.6	26.3
1.50	81,680	20,420	526	46.9	26.6
1.75	73,400	21,410	529	47.1	26.8
2.00	66,760	22,250	531	47.2	27.0

^aIn vehicles per 24 hours.

IMPROVEMENTS AND EXTENSIONS

While the limitations of the various assumptions on the applicability of the results were realized from the start, a number of additional weaknesses came to light as the exercise proceeded and the sample calculations were made.

In some cases, volumes and travel costs reached unrealistically high figures. Sometimes arterial speed exceeded expressway speed and local speed exceeded arterial speed. These difficulties showed up only under rather extreme conditions, and never in the vicinity of the optima. It appears that they always led to an underestimate of cost, and never an overestimate. It would be desirable to have some kind of capacity restraint feature in the traffic estimation procedure. As traffic on one street type exceeds capacity, there should be a way of redistributing some of it to the other street types which still have spare capacity. The local street system almost always has spare capacity.

I had some concern over the correct nature of the relationship between expressway investment cost and density. I assumed that the cost rises slower than density, causing a lower cost per trip as density increases. This is certainly true at low densities, but at high densities near the city center, it may well be that cost increases faster than density. A curvilinear equation depicting such a relationship could be formulated. It would produce a minimum in each column of Table 2. Undoubtedly, an overall minimum of total transportation cost would still exist.

Because of the planning context of the study, there is some question as to what density should be included in the equation for expressway investment cost. This cost is probably influenced more by existing density than by ultimate density. Yet it is ultimate density that is considered in this exercise.

In general, the utility of a theoretical solution is inversely proportional to the number and importance of assumptions it is necessary to make. A natural course for improving the method, therefore, is to attempt to remove some of the assumptions and to deal with a more realistic case. Obviously, it would be desirable to be able to handle a real city in which density does vary, highway networks are not regular, and there is no artificial distinction among street types. These improvements have apparently been accomplished by Schneider for the problem of estimating traffic (7). As yet this new methodology has not been applied to optimization problems, but it may be suitable.

One of the questionable assumptions is that of a constant average trip length. Density may have some effect on average trip length, but the precise nature of the relationship remains mysterious. Another candidate for elimination is the assumption that free speeds are constant. It seems reasonable to argue that free speeds vary as a function of the density of trip ends. The higher the density of surrounding development, the lower posted speed limits are likely to be. For arterials, higher densities are apt to mean more side frictions from driveways, parking, and pedestrians. For that matter, probably capacities should also be varied as a function of density. Capacity and free speed are both really aspects of the same thing: the ability of a highway to pass traffic.

A major extension of this study would be to consider the transit mode. This would require breaking some new ground in optimization of transit systems, which have not received as much attention as highway systems. Perhaps the first cut would be to consider an alternative hypothetical universe in which transit is the only mode. Later the two worlds could be merged, requiring some treatment of model split—a subject which has generated heat but little light.

For the land-use planner, it would be interesting to go to some measure of density which is more familiar to him, such as population, employment, or floor area. This brings in the whole problem of trip generation, which as yet is only dimly understood. Certainly this transition is necessary at some point.

There is the matter of non-transportation costs, which would be important to any comprehensive evaluation of land-use costs. This will be a tough nut to crack, and perhaps it will never be possible to make more than a partial accounting of these costs.

A final point to consider is whether minimization of costs is the proper criterion for selection of the best plan. Is there some way to measure benefits and compare them to costs? Is there some alternative to this approach? It is obvious from the behavior of

people, at least in our affluent society, that they do not necessarily attempt to minimize costs any more than they attempt to minimize travel.

CONCLUSIONS

The emphasis in early transportation studies was on finding the optimal transportation network for an assumed land-use pattern. Now the goal at Tri-State and many other studies is to find the optimal combination of land-use pattern and transportation system. Both land-use and transportation facilities are considered to be planning variables which are subject to control. It is necessary for the planner to get some idea of the interaction of these two realms—not just to see what happens when one is held constant and the other changes, but to see what happens when both change simultaneously. This study has attempted to establish a beachhead on this uncharted and perhaps unfriendly continent. A land-use variable (density) and transportation variables (highway spacing) are considered to be joint determinants of an optimal solution. The method provides a bridge between land-use planning and transportation planning and indicates the kind of theoretical analysis by which it may be possible to narrow in on the best land-use and transportation plan.

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Experiments in Urban Form and Structure

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●EVALUATION of alternate land development patterns is an important, unsolved task in urban planning. There are many reasons for the rather slow progress in developing methods for evaluating alternate development plans. Perhaps chief among them is disagreement on what are the proper criteria for evaluation. However, part of the difficulty lies in the limited understanding of the relationships among the components of urban form. For example, what difference does it make in the operation of an urban area if workplaces are concentrated downtown, or concentrated in a few suburban locations, or scattered throughout the urban area? What difference would it make if residential density were highest in the suburbs and lowest near the downtown area instead of the reverse pattern which exists today?

We have difficulty in answering these questions because we are not able to examine the various alternatives in nature, nor do we have the freedom to reconstitute cities according to our designs in order to observe the effects of variations in urban form. The solution to a part of this problem will be found, I think, in the development of fairly simple experimental models of an urban community which are designed specifically for the exploration of the relationships among elements of urban form, and which can be easily manipulated and readily understood by urban planners.

This paper is a report on a simple model for examining the impact of changes in components of urban form on urban spatial structure. The distinction made here between urban form and urban structure is quite simple. Urban form is the physical arrangement of residences, work places, etc. Urban structure is the pattern formed by the connection of these elements in the daily activities of the area's residents. Urban structure implies an allocation rule. Given a physical pattern of places, the connections between them—from home to work, from home to shopping center—must be established. Another way of making the distinction is to say that urban form describes the static, physical setting itself and that urban structure describes the dynamics of a particular physical setting. The nomenclature is arbitrary, but the distinction is necessary.

The approach developed and examined here is only one of many possible approaches to the problem. The purpose of this effort has been to test the utility of using a simple linear programming formulation as an allocation rule for evaluating urban form alternatives by two criteria: the efficiency of the alternatives in terms of minimal travel requirements, and the equity of the alternatives in terms of locational advantage of residence locations. These criteria are evaluated by the primal and the dual problem of a "transportation problem" in linear programming.

The problem with which we deal is this. Given alternate distributions among sub-areas of an urban area of each of the urban form elements of workplaces, shopping places, and residences; alternate systems of transportation service; and an allocation rule which specifies the way residences will be linked with workplaces and shopping places—what is the impact of changes in the components of urban form on urban spatial structure? The basic question might better be put as a series of questions. What effect do changes in the components of urban form have on travel requirements, given a particular allocation rule? What is the relative impact of individual elements of urban form on urban spatial structure? Do changes in the residential pattern have more or less

impact than changes in transportation service? Is there a best combination of elements of urban form in the sense that this particular combination requires less travel than any other combination of elements? The list of questions could be continued almost indefinitely. They all add up to the same concern: Can we demonstrate the effect of changes in urban form on urban spatial structure?

THE ALLOCATION RULE

The allocation rule is a linear programming allocation to minimize total travel required for establishing a linkage between each residence and a workplace and a shopping place. The LP allocation is used as a diagnostic of urban form in this application and is not intended to simulate in realistic detail the behavior of persons in urban areas. As a diagnostic, the LP allocation provides an evaluation of the potential efficiency of alternate urban forms under conditions of aggregate optimizing behavior. It is true that a person does not always go to the nearest shopping center on each shopping trip, nor does every family choose to live in the house meeting its requirements which is closest to the head of the household's place of work. Furthermore, the LP allocation produces a community or system minimization of travel requirements rather than an individual minimization. However, it has been shown that the majority of daily work and shopping trips in a large urban area conform closely to the time requirements of an LP minimizing allocation (1). The output of the allocation model provides three kinds of information about the activity structure for a particular urban form—the travel required by the minimum solution; the linkage pattern selected; and from the dual of the minimizing problem, the comparative locational advantage of residential zones.

The formal statement of the problem is:

find the X_{ij} such that

$$\sum \sum C_{ij} X_{ij} \text{ is a minimum} \quad (1)$$

subject to

$$\sum_{j=1}^n X_{ij} = O_i \quad i = 1 \dots m \quad (2)$$

$$\sum_{i=1}^m X_{ij} = D_j \quad j = 1 \dots n \quad (3)$$

$$X_{ij} \geq 0, C_{ij} \geq 0 \quad (4)$$

and

$$\sum_{i=1}^m O_i = \sum_{j=1}^n D_j$$

where

- C_{ij} = travel time from zone i to zone j ,
- X_{ij} = trips from zone i to zone j ,
- O_i = trip origins in zone i , and
- D_j = trip destinations in zone j .

The dual problem is

$$\sum_{j=1}^n r_j v_j - \sum_{i=1}^m s_i u_i = \text{maximum} \quad (5)$$

where the constraints are

$$v_j - u_i \leq C_{ij}$$

and

$$u_i, v_j \geq 0$$

and where

s_i = trips sent from zone i , and
 r_j = trips received at zone j .

The value of u_i is the rental value of location in zone i as an origin point for trips to a particular activity. We interpret v_j as the value of the trip maker of the activity in zone j (2). The values are measured in travel-time units, since these are the cost data of the original problem. The rental value of a site is a measure of its attractiveness as a location point. A high rental value means that the zone has a relatively advantageous location. Since the values assigned to the dual variables are based on minimization of total travel time in the system, the values assigned to residential origins measure the comparative locational advantage of residential locations under conditions of efficient travel.

EXPERIMENTAL DESIGN

The components of the urban area model are a set of zones comprising the urban area, a set of alternate residential patterns, a set of patterns of work places, a set of patterns of shopping places, and alternate systems of transportation service. The number of residences equals the capacity of the work places and the capacity of the shopping places. In other words, one trip is to be made from each residence to a work place and to a shopping place.

The hypothetical urban area is shown in Figure 1. There are 37 zones of equal size. Thirty-two of these zones may contain residences. No residences are permitted in zones containing work centers. There are seven commercial centers. One is in the center of the urban area and the other six are distributed regularly around the center. There are five work centers. Again, one is in the center of the urban area and the others are regularly spaced around the center. Three zones contain both work centers and commercial centers.

This is obviously a highly simplified representation of an urban area. However, it does resemble the general pattern of many large urban areas. The central zone can be interpreted as the central business district. The outlying commercial centers

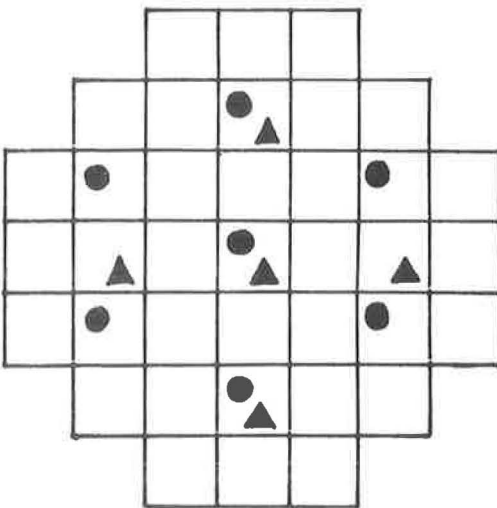


Figure 1. Experimental urban form: ● = commercial centers; ▲ = work centers.

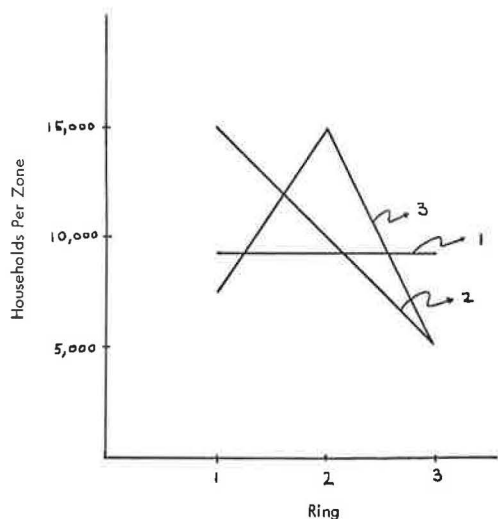


Figure 2. Alternate residential density patterns.

R2, high central density declining regularly with distance from the center; and R3, crested density rising from a low value in the center to a high point and then declining (Fig. 2). There are 300,000 residences. This places the population of the urban area at about one million persons. There are two alternative patterns of work center and commercial center capacity, and they are similar. In the first (W1 and C1), 70 percent of the jobs and 70 percent of the shopping opportunities are in the (geographic) center zone. The remaining 30 percent of the jobs are equally divided among the four outlying work centers, and the remaining 30 percent of the shopping opportunities are equally divided among the six outlying commercial centers. The second alternative (W2 and C2) is the reverse of the first. Thirty percent of the work and shopping opportunities are in the central zone and the remaining 70 percent are divided among the outlying centers. These alternatives have obvious interpretations. In the first case,

there is a traditional strong metropolitan core complemented by relatively weak suburban centers. The second case depicts a sharp decline in the relative importance of the core and a corresponding increase in the importance of suburban centers. However, even in the latter case the core capacity is greater than the capacity of an individual suburban center.

There are three alternate systems of transportation service. The only routes permitted are in north-south and east-west directions from the center of a zone to an adjacent zone. So a diagonal path through the area is composed of zigzag right-angle links. The travel time or cost of travel from one zone to another is defined in terms of level of service provided rather than in terms of the design capacity and speed of physical facilities. Since the allocation model will impose different loads on different links, the network of physical transportation facilities must be differentiated. For convenience, assume

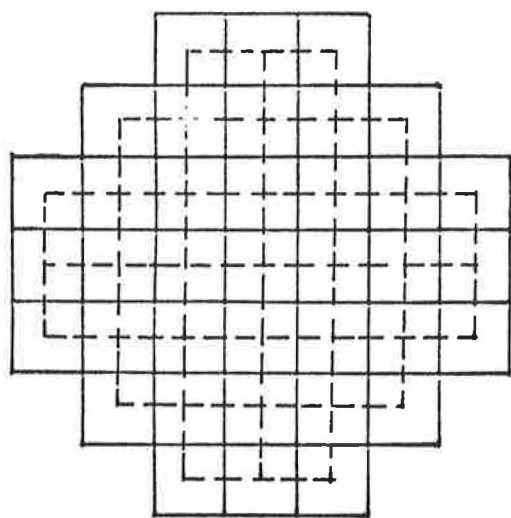


Figure 3. Transportation alternative 1: travel time on each link = 2.

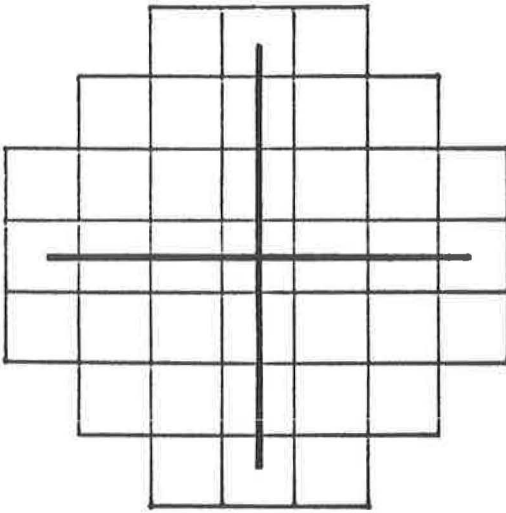


Figure 4. Transportation alternative 2: travel time on major links = 1; all others = 2.

facilities which have been developed in many metropolitan areas.

The three transportation alternatives, three residential alternatives, two commercial center alternatives, and two work center alternatives can be combined into 36 different urban forms. To clarify the alternative urban forms possible, each is given a description. Basically, all combinations with the first residential alternative are variants of a spread city. With the second residential alternative, all combinations are variants on a cone-shaped form which is called a centric city. Combinations with the third residential alternative are called variations of a ring city. The alternative forms are

- R1, C1, W1: Spread city with strong core,
- R1, C1, W2: Spread city with spread employment, but strong commercial core,
- R1, C2, W1: Spread city with spread commercial, but strong employment core,
- R1, C2, W2: Spread city,
- R2, C1, W1: Centric city,
- R2, C1, W2: Centric city with dispersed employment,
- R2, C2, W1: Centric city with dispersed commercial,
- R2, C2, W2: Centric city with dispersed commercial and employment,
- R3, C1, W1: Ring city with strong commercial and employment core,
- R3, C1, W2: Ring city with commercial core,
- R3, C2, W1: Ring city with employment core, and
- R3, C2, W2: Ring city with weak core.

The alternative transportation systems can be intuitively related to the alternate development patterns. The first system, providing uniform transportation service is essentially neutral. It is indifferent to urban form. We would expect the second system, featuring high-level radial access to the center of the urban area, to be well matched with the centric city. The third system provides a high level of service through the outer ring and might be expected to best match the dispersed forms of both the spread and ring city.

IMPACT OF ALTERNATE URBAN FORMS ON MINIMUM TRAVEL REQUIREMENTS

First we will look at the minimum travel requirements of alternate urban forms when the transportation system is constant. Figure 6 shows the travel requirements

that the roads are the only elements of the system and all travel is by individuals in private vehicles.

The first transportation system consists of uniform transportation service (Fig. 3). The travel cost of all zone-to-zone links is given the same arbitrary value of 2 time units. It is assumed that sufficient capacity to maintain this level of service will be provided. The second and third transportation systems superimpose higher service level facilities over this basic transportation surface. In the second system, north-south and east-west links through the central zone from the periphery are established at a travel cost of 1. This creates four high service level radial routes (Fig. 4). The third transportation system adds to the first and second a ring of high service level links (Fig. 5). Taken as a sequence over time these transportation service systems resemble the radial-circumferential networks of transportation

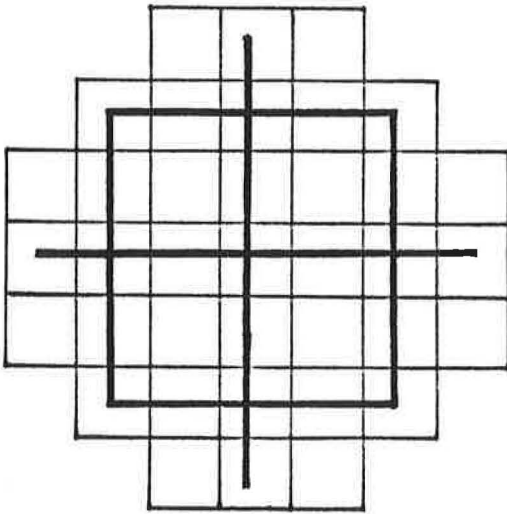


Figure 5. Transportation alternative 3: travel time on major links = 1; all others = 2.

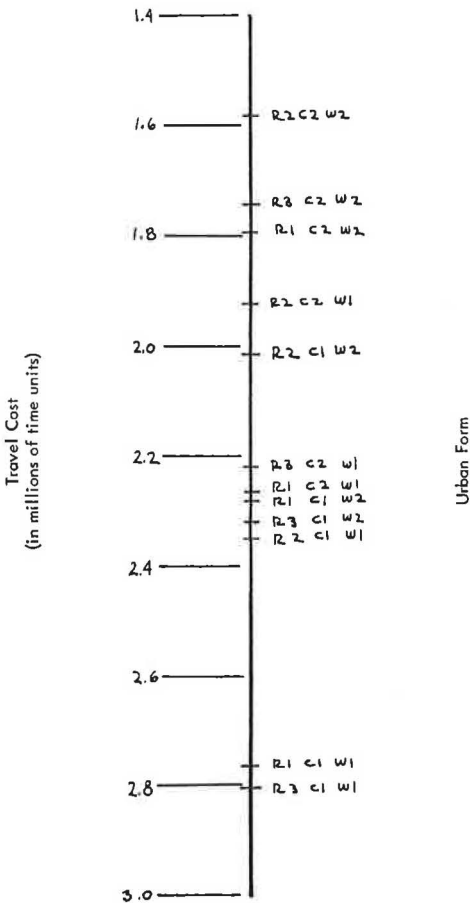


Figure 6. Minimum travel requirements of alternate urban forms.

of all 12 possible urban forms with the system of uniform transportation service.¹ The least cost solution is the centric city with dispersed commercial and employment opportunities. The most costly form is the ring city with a strong core, and it is closely followed by the spread city with a strong core. In general, the urban forms with a weak commercial and employment core have the lowest travel requirements, and those with a strong core have the greatest travel requirements.

In the individual elements, a change in the commercial pattern when the residence and workplace pattern are the same has the greatest impact on travel requirements. Next in significance is a change in workplaces. Changes in the residential pattern have the least effect on travel requirements. In the travel requirements of alternatives of each element, a weak commercial and employment core always requires less travel than a strong core under the travel minimizing allocation. Any combination of the commercial and employment opportunities requires less travel with the centric residential pattern than does the same employment and commercial pattern with either the ring or spread residential pattern.

These results suggest that, given uniform transportation service, the most efficient urban form couples dispersed employment and commercial opportunities with residential density that is high in the center and declines with distance from the center. The results also suggest that major variations in the residential pattern do not have a very significant influence on travel requirements.

It is difficult to evaluate these results because the differences in the alternatives of the several elements are not necessarily of the same magnitude. For example, the difference between uniform residential density and a regular density gradient does not necessarily involve the same proportional change as the difference between a spread

¹Each experiment contains two allocations—trips to a given distribution of work places and trips to a given distribution of commercial centers—from a common residential distribution. The travel times for work and shopping trips are summed to give the total travel time for the specified urban form.

TABLE 1
TIME UNITS REQUIRED FOR MINIMAL LINKAGES IN
URBAN FORM EXPERIMENTS

Experiment	Commercial	Work	Total	Rank
T1 R1 C2 W2	827, 500	960, 000	1, 787, 500	3
T1 R1 C2 W1	827, 500	1, 440, 000	2, 267, 500	7
T1 R1 C1 W2	1, 320, 000	960, 000	2, 280, 000	8
T1 R1 C1 W1	1, 320, 000	1, 440, 000	2, 760, 000	11
T1 R2 C2 W2	680, 000	900, 000	1, 580, 000	1
T1 R2 C2 W1	680, 000	1, 240, 000	1, 920, 000	4
T1 R2 C1 W2	1, 112, 000	900, 000	2, 012, 000	5
T1 R2 C1 W1	1, 112, 000	1, 240, 000	2, 352, 000	10
T1 R3 C2 W2	760, 000	980, 000	1, 740, 000	2
T1 R3 C2 W1	760, 000	1, 460, 000	2, 220, 000	6
T1 R3 C1 W2	1, 340, 000	980, 000	2, 320, 000	9
T1 R3 C1 W1	1, 340, 000	1, 460, 000	2, 800, 000	12
T2 R1 C2 W2	629, 375	742, 500	1, 371, 875	2
T2 R1 C2 W1	629, 375	982, 500	1, 611, 875	5
T2 R1 C1 W2	894, 375	742, 500	1, 636, 875	6
T2 R1 C1 W1	894, 375	982, 500	1, 876, 875	9
T2 R2 C2 W2	580, 000	700, 000	1, 280, 000	1
T2 R2 C2 W1	580, 000	880, 000	1, 460, 000	3
T2 R2 C1 W2	815, 000	700, 000	1, 515, 000	4
T2 R2 C1 W1	815, 000	880, 000	1, 695, 000	8
T2 R3 C2 W1	612, 000	1, 040, 000	1, 652, 000	7
T3 R1 C2 W2	545, 000	592, 500	1, 137, 500	2
T3 R1 C2 W1	545, 000	832, 500	1, 377, 500	7
T3 R1 C1 W2	772, 500	592, 500	1, 365, 000	6
T3 R1 C1 W1	772, 500	832, 500	1, 605, 000	9
T3 R2 C2 W2	460, 000	540, 000	1, 000, 000	1
T3 R2 C2 W1	460, 000	720, 000	1, 180, 000	3
T3 R2 C1 W2	690, 000	540, 000	1, 230, 000	4
T3 R2 C1 W1	690, 000	720, 000	1, 410, 000	8
T3 R3 C2 W1	495, 000	800, 000	1, 295, 000	5

commercial pattern and a concentrated pattern of shopping opportunities. So we must qualify the statement that changes in the commercial pattern have a greater influence on minimum travel time than changes in the residential pattern by saying that this has been shown to be so if the changes are comparable.

Table 1 gives the minimal travel requirements for all the experiments conducted. In addition to the full 12 form combinations with the uniform transportation service, experiments have been conducted with 9 form combinations with each of the other transportation alternatives. The most important finding is that the general ranking of urban forms by travel requirements found with uniform transportation service holds for all transportation alternatives. This means that at least for the particular alternatives we have examined, the system of transportation service has little influence on the relative efficiency of alternate urban forms. If this is generally true, i. e., if it holds for other transportation systems and other residential, commercial, and employment patterns that we have examined, it is a significant finding.

The obvious implication for urban planning is that the spatial pattern of land use and the pattern of transportation service can be planned somewhat more independently than is commonly thought. Independence is implied in a peculiar sense. The results do not imply that the land-use pattern and the transportation system are not interrelated. They imply that evaluation of alternative land-use patterns may be considered without reference to particular transportation systems. The reverse situation is clearly not implied. If this implication is correct, then the proper order of attack on the problem of selecting an efficient urban form is to examine alternative land-use patterns and then to examine alternate transportation systems to serve the selected land-use pattern.

While alternate transportation systems do not significantly affect the relative efficiency of alternate land-use patterns, they do affect the absolute efficiency of these patterns. Figure 7 shows the range of minimum travel requirements for all the experiments with the three transportation systems. For any urban form the minimum travel requirements are reduced as the quality of transportation service is improved. This is not surprising. Any other result would make us suspect that the model was totally irrelevant to the conditions it is being used to examine. Two results are

worthy of note however. First, improvement of the quality of transportation service results in a reduction of the absolute difference in travel requirements between alternate land-use forms. The total range of travel requirements is reduced. This also is to be expected. But it is interesting to note that after the first improvement, the addition of higher level radial service, the range of travel time required is not further reduced by the addition of more high-level service in the third alternative.

Second, the results of the experiments begin to suggest ways in which changes in the land-use pattern can be traded off against changes in the transportation system to achieve the same level of improvement in minimum travel requirements. For example, if we start with the centric city with a strong core, approximately the same improvement in minimum travel requirements can be achieved by improving the quality of radial transportation service to the core as by dispersing commercial and employment opportunities to the outer zones. The potential for this type of trade-off is shown by the areas of overlap in Figure 7.

These conclusions may seem somewhat at odds with the earlier observation of independence of the transportation system and the land-use pattern, but there is no conflict. Our earlier observation was that changes in the transportation system do not appear to affect the relative efficiency of alternate land-use patterns. These second observations simply show that a superior transportation system can make an inferior land-use pattern as efficient as a superior land-use pattern. The implication for planning is equally clear. If, for example, a level of minimum travel requirements is specified as an objective, alternate means of achieving it can be demonstrated, and a clear policy choice between investment in transportation service and control and direction of land development can be formulated.

Locational Advantage as a Measure of Urban Form

Thus far our experiments have shown that alternate residential patterns have relatively little effect on minimum travel requirements of the experimental urban forms. However, alternate residential patterns may nevertheless represent significantly different locational qualities for residents of individual zones. To examine this question, we turn to an aggregate statistic—the range of locational advantage.

The range of values of locational advantage is simply the difference between the highest zonal value and the lowest zonal value defined in a particular experiment. The significance of the choice of a residential zone increases with increases in the range of values of locational advantage. If the range were zero, i. e., if all zones had an equal value of locational advantage, there would be no reason to select one zone over another as a residential location. If the range of values were very large, the choice of a residential zone would be more significant, since it would involve the potential for travel savings.

The range of values of comparative locational advantage defined by all 30 experiments conducted is given in Table 2. As expected, the range of locational advantage decreases with improvements in the quality of transportation service. This is simply a result of decreases in the average travel expenditure. Alternate urban forms with any one transportation system show considerable stability in range of locational advantage. This stability is due in part to the grossness of the experiments. The small number of zones and the small range of possible travel requirements limit the variations in locational advantage. The centric city with dispersed work and commercial opportunities has the smallest range of values.

One further outcome should be noted. The dual problem, as we have said,

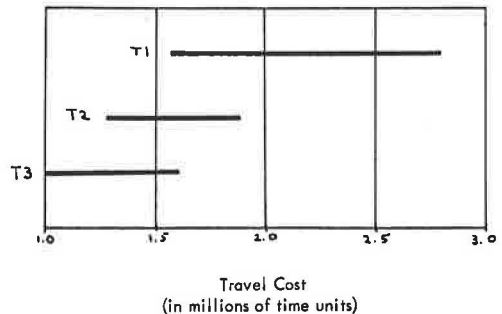


Figure 7. Range of travel requirements with alternate transportation systems.

TABLE 2
RANGE OF VALUES OF COMPARATIVE LOCATIONAL
ADVANTAGE OF ALTERNATE URBAN FORMS

Experiment	Commercial	Employment	C + W/2
T1 R1 C2 W2	6	6	6
T1 R1 C2 W1	6	6	6
T1 R1 C1 W2	6	6	6
T1 R1 C1 W1	6	6	6
T1 R2 C2 W2	6	6	6
T1 R2 C2 W1	6	6	6
T1 R2 C1 W2	6	6	6
T1 R2 C2 W2	6	6	6
T1 R3 C2 W2	6	6	6
T1 R3 C2 W1	6	6	6
T1 R3 C1 W2	6	6	6
T1 R3 C1 W1	6	6	6
T2 R1 C2 W2	4	5	4.5
T2 R1 C2 W1	4	5	4.5
T2 R1 C1 W2	4	5	4.5
T2 R1 C1 W1	4	5	4.5
T2 R2 C2 W2	4	3	3.5
T2 R2 C2 W1	4	5	4.5
T2 R2 C1 W2	4	3	3.5
T2 R2 C1 W1	4	5	4.5
T2 R3 C2 W1	4	5	4.5
T3 R1 C2 W2	4	4	4
T3 R1 C2 W1	4	4	4
T3 R1 C1 W2	4	4	4
T3 R1 C1 W1	4	4	4
T3 R2 C2 W2	2	2	2
T3 R2 C2 W1	2	4	3
T3 R2 C1 W2	4	2	3
T3 R2 C1 W1	4	4	4
T3 R3 C2 W1	4	4	4

calculates value or prices at both origin and destination. The price at the destination is traditionally interpreted as the delivered price of the item being shipped. In our experiments, the shipped item is persons transporting themselves to work. So the price at the destination may be interpreted as the input cost of labor to the several employment centers. It can be interpreted as the average price in travel time which must be "paid" by each employment center to attract its work force, given the distribution of employment opportunities, the residential pattern, and the transportation system. Examination of these prices for the dual problem of all experiments conducted shows that for the centric city with dispersed employment and only for that urban form the prices are equal. In other words each work place "pays" the same price for its labor input. We can interpret this to mean that the locations of the employment centers are equally efficient.

CONCLUSIONS

This report has discussed some beginning efforts at one approach to examining the relationships among elements of urban form as a first step toward developing more satisfactory analytic methods of evaluating alternatives of the form and performance of cities. The allocation used provides a means for examining the effects of changes in urban form under conditions of travel minimizing behavior. Two criteria were used in the analysis: the potential efficiency of alternate urban forms, measured by the total travel required in the system; and the equity with which this efficiency is distributed, measured by the comparative locational advantage of residential locations. The allocation rule is used as a diagnostic of urban form and not as a simulation of behavior.

The results of the experiments performed show that, under the conditions established, the system of transportation service and alternate residential patterns have little influence on the relative efficiency of alternate urban forms. In the very simple experiments performed, the same urban form was selected as most satisfactory by the two criteria used. The potential for trade-offs between changes in the system of transportation service and the arrangement of land-based activities to achieve a given level of efficiency was identified in the experiments.

The results of these experiments should not be taken as conclusive. They are only intended to be suggestive of the approach to urban analysis, which I believe is necessary for improving the quality of public investment decisions. We need to supplement our often hortatory urban development plans with measured alternatives which spell out the usually general objectives of such plans in programmatic terms, and assess the cost and effectiveness of public actions proposed to achieve the objectives. But before we can do this we need a much better understanding of how cities function and how people use the physical city in the conduct of their daily activities. Because we cannot reconstitute cities or change the behavior of city dwellers in order to evaluate unexplored alternatives, and because past behavior of city dwellers may not be a reliable guide to their behavior in quite different environments, we can more profitably approach this problem through a form of laboratory experiments rather than observation or trend estimates alone.

One of the first tasks in developing a more satisfactory experimental method is the investigation of a variety of allocation rules. The one used here is somewhat unrealistic. Its virtues are simplicity, ease of use, and fidelity to a straightforward behavioral hypothesis. In the "as if" world of this diagnostic, experiments are easily performed and results are easy to interpret. On the other hand, most of the experience with mathematical models in urban analysis has been with statistical models or gravity and opportunity models which are carefully fitted to observed behavior. Transfer of these "fitted" models to new urban alternatives is conceptually difficult. Experiments should be made with different allocation rules to determine their relative merits. There is reason to suspect that the kind of allocation rule most useful for simulating urban behavior in order, for example, to validate a transportation scheme may not be the most useful allocation rule for examining the more abstract problem of urban form alternatives.

The results of these experiments do suggest that it may be worthwhile to reexamine some current emphases in urban analysis. Most attempts at mathematical models of urban development have concentrated on simulation of the residential pattern. This is in part due to a traditional preoccupation with residential settlement, but it is perhaps also partly because residential patterns are more amenable to aggregate statistical analysis than industrial and commercial location decisions. If the results of these experiments are indicative of the relative importance of alternate residential patterns on the functional structure of urban communities then, perhaps, the emphasis is misplaced. Similarly, there has been a great deal of emphasis recently in transportation analysis on the potential influence of the system of transportation facilities on the spatial structure of urban communities. The experiments suggest that this influence may be smaller than is often argued, and perhaps some further analysis of this hypothesis is in order.

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Modeling of Household Location: A Statistical Approach

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The purpose of this study is to simulate the behavior of households in choosing their homes. A two-phase model is proposed. During the first phase, the housing preferences of the locating family are determined. In the second phase, the search process by which the household picks a location possessing these characteristics is simulated. A number of multivariate statistical techniques are employed in the partial calibration of the model.

•RECENT progress in land-use modeling has included the implementation of macro-level procedures for estimating future spatial distributions of urban activity. Although these techniques have fulfilled the initial requirement for land-use projections as inputs to the transportation planning process, they do not meet the general planning requirement for a policy oriented land-use planning methodology. Such a methodology should furnish planners with the information necessary to evaluate alternative policy sets. Further, it should be adaptable to changes in behavior, technology, resource availability, and policy.

At present, progress towards this goal can best be achieved by research leading to the development of a more micro-level model of residential location behavior. Advances in sociology and economics have resulted in a substantial body of theory upon which such work can be based. These developments, in conjunction with the increasing quality and quantity of survey findings, provide a firm foundation for modeling efforts. In addition to leading toward a desired goal, a concentration of effort on modeling residential location has intrinsic merit. The largest single use of land in any metropolitan area is for residential purposes. Since the total amount of land is fixed, this allocation has important implications for the structure of land prices. Further, the individual is vitally affected by the residential land-use consequences of alternative policies, and any difference in the welfare level attained is therefore an important criterion for evaluating plans.

This study was designed as an exploratory investigation into the location behavior of households. The development is based on the premise that the family's choice of a home is an essentially rational decision which is reached through a consideration of preferences, financial resources, and the market. A prototypical micro-level model is developed and partially calibrated using data from Tucson, Arizona. Ultimately, of course, such an analysis must be integrated with an overall simulation, although not necessarily a Monte Carlo simulation, of the urban development process.

FORMULATION OF MODEL

The proposed model of residential location behavior operates in two steps. First, a description of the environment desired by a given household is determined through consideration of its socioeconomic characteristics. In effect, the quality and quantity

of the housing which the household will select is fixed in an environmental space. Numerous sites meeting the environmental requirements specified by the household normally exist throughout a metropolitan area. Second, a search process is conducted to select one of these sites as the new location for the family.

The choice of a house usually involves a simultaneous decision about the future trip set of the family and the style of life which the family will follow. Since each location is characterized by a set of accessibilities to all trip destinations, the household, in choosing a site, is also making a decision about desired levels of these accessibilities. By choosing the job site as the origin of the search process, the oft-cited relationship between the location of the work site and the residential location choice is introduced into the model. Alternatively, if the work trip does not appear to strongly influence the housing choice, the influence of information about opportunities can be entered into the model by choosing the existing homesite as the origin for the search. In practice, either of these origins may be chosen depending on the circumstances of the searching family.

Mathematically, the premise that the housing environment which a locator selects is a function of his socioeconomic characteristics may be expressed as

$$E^i = f \left(x_1^i, x_2^i, \dots, x_j^i, \dots, x_n^i \right) \quad (1)$$

where E^i is a scalar representing the environment selected by the i th household, and x_j^i is the j th socioeconomic characteristic influencing the location behavior of the i th household.

For purposes of clarity, the environment was characterized in Eq. 1 by a scalar value. In reality, the environment must be characterized by a vector whose elements completely describe the quality of the housing and the area in which the housing is located. For example, the environmental vector might include measures of housing cost, number of persons per room, and type of house to characterize the housing, and measures of social status, racial composition, and quality of educational and recreational facilities to characterize the area in which the housing is located. Eq. 1 may now be written as

$$\begin{aligned} y_1^i &= f \left(x_1^i, \dots, x_j^i, \dots, x_n^i \right) \\ &\vdots \\ y_k^i &= f \left(x_1^i, \dots, x_j^i, \dots, x_n^i \right) \\ &\vdots \\ y_m^i &= f \left(x_1^i, \dots, x_j^i, \dots, x_n^i \right) \end{aligned} \quad (2)$$

where y_k^i is the k th element of the environment which is considered by household i .

Any given household characteristic j does not necessarily influence the level of the measure of the environment, mathematically:

$$\frac{\partial y_k^i}{\partial x_j^i} > 0 \quad (3)$$

Eq. 2 may be written in matrix form as

$$[Y] = f[X] \quad (4)$$

where Y is an $m \times 1$ matrix characterizing the environment, and X is an $n \times 1$ matrix characterizing the household.

The functional relationship may take any of a number of forms. The form adopted for this analysis involves a linear transformation of elements. Rewriting Eq. 2:

$$\begin{aligned} y_1^i &= a_{11} x_1^i + \dots + a_{1j} x_j^i + \dots + a_{1n} x_n^i \\ &\vdots \\ y_k^i &= a_{k1} x_1^i + \dots + a_{kj} x_j^i + \dots + a_{kn} x_n^i \\ &\vdots \\ y_m^i &= a_{m1} x_1^i + \dots + a_{mj} x_j^i + \dots + a_{mn} x_n^i \end{aligned} \quad (5)$$

This formulation may be represented in matrix form as

$$[Y] = [A] [X] \quad (6)$$

where A is an $m \times n$ matrix of coefficients.

Eq. 3 states that an element a_{kj} of matrix A can assume any real value. To summarize, the model may be characterized as follows:

$$\begin{bmatrix} y_1 \\ \vdots \\ y_m \end{bmatrix} = \begin{bmatrix} a_{11} & \dots & a_{1n} \\ \vdots & & \vdots \\ a_{m1} & \dots & a_{mn} \end{bmatrix} \begin{bmatrix} x_1 \\ \vdots \\ x_n \end{bmatrix} \quad (7)$$

environmental
desire
household
vector
coefficients
vector

matrix

Estimates of the parameters of the coefficients matrix are obtained by an analysis of the current location structure. The model, therefore, does not predict what a household desires, but rather what the household must settle for under current conditions. These conditions may change, and it is implicitly assumed in this development that the values in the coefficients matrix are not fixed. The values of the matrix may be altered to take account of changing group norms, as revealed by studies of historical trends, consumer studies of the housing desires of various groups, and other types of analyses. For example, the group characterized by the vector (blue-collar, white, income less than \$5000, stage three in the family life cycle, six members in the family) might be placing an increasingly important emphasis on recreation relative to the general population. This change in group behavior could be taken into account through the alteration of the appropriate coefficients.

An environmental bundle which will be demanded by each group of families has been developed, but the households have not been located in physical space nor has the decision component of work site accessibility been considered. At each time stage, the number of opportunities of each environment in each subarea can be estimated. In effect, an opportunity surface for each environment is constructed for the metropolitan area. It is assumed that a finite number of employment centers have been located within the metropolitan region in the industrial allocation phase of the overall land-use simulation model. The characteristics of families whose principal wage earner works at each

work site could then be determined, and the residential demand relative to each employment center estimated using the demand equations.

The process by which each family uses its employment center as an origin to search the environmental opportunity surface for a homesite could then be simulated through models analogous to those used in the distribution phase of a synthetic traffic study. Trip distribution models use one or more of the following factors to estimate the probability of an interaction between zone k and zone n :

1. The intensity of activity at zone n , being in the residential location model the number of opportunities for family i at zone n ;
2. The number of opportunities between zone k and zone n which the family must pass up to locate in zone n ; and
3. The cost of interaction between zone k and zone n , in the residential location model this being the time distance from the homesite to the work site.

In the absence of further information relating to the relative importance of any one of these factors, all three will be included in the allocation model. The probability that a given zone will be accepted is, therefore, a function of both the cost of interaction with the zone and the opportunity surface for the given housing environment. Mathematically, this may be characterized as:

$$P_{ikn} = K \left[\frac{O_{mn}}{\sum_{j=1}^n O_{mj}} \right] t_{kn}^b \quad (8)$$

where

P_{ikn} is the probability that members of socioeconomic group i working in zone k will locate in subdivision n ,

O_{mn} is the number of opportunities of environmental set m in subdivision n ,

t_{kn}^b is the time distance from employment zone k to subdivision n ,

b is a parameter which must be calibrated, and

K is a normalizing constant.

After the coefficients matrix has been estimated, the parameter b is approximated by an analysis of the current pattern of residential location. Note that the model implicitly assumes that everyone desiring a certain type of environment can find a homesite with this environment within the metropolitan area and within a commuting range which the head of the household will accept. There is no feedback, in this sense, from the accessibility of the desired environments to the actual desire for the environments. The feedback is considered by recalibrating the coefficients matrix for each metropolitan area and calibrating b for each zone. The effect of accessibility on the choice of environment is therefore implicitly but crudely considered in the desire coefficients.

The model can be operated in either an iterative or a single-pass mode, depending on the degree of accuracy desired. If only a reasonable degree of accuracy is desired, the employment centers from which locators are distributed would be randomly selected, the locators assigned, and the opportunity surface appropriately adjusted. The alternative procedure would involve distributing the locators from all employment zones, determining those zones in which the number of assigned locators exceeds the available supply, appropriately reducing the available supply at the zones in which the supply was exceeded so as to reduce their attractiveness, and iterating until a stable situation develops. It would appear that the first procedure would be adequate in most situations.

To recapitulate, the location model operates in two stages (Fig. 1). In the first stage, the household vector is manipulated to yield the housing environments which the family will desire. Prototypical vectors for representing the environment and the household are developed in the next section. In the second stage, households are distributed to available housing sites using the work site of the principal worker as the

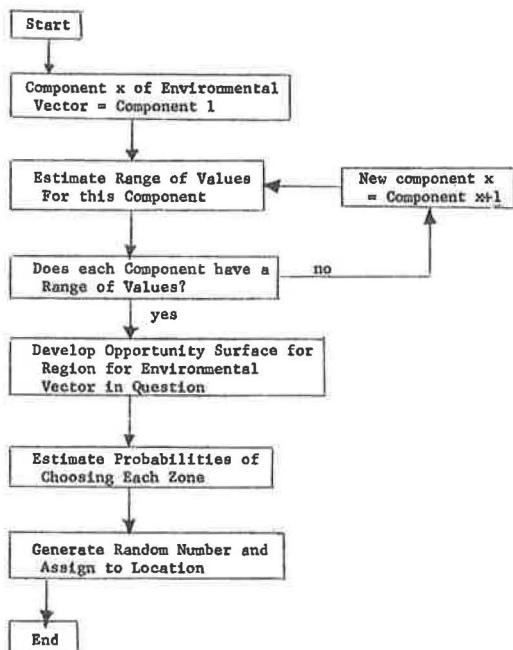


Figure 1. Flow chart for location process for an individual household.

origin and a distribution procedure which considers both the opportunity surface and the actual distance from work site to homesite.

DEVELOPMENT OF ENVIRONMENTAL AND HOUSEHOLD VECTORS

Prior to the calibration of the desire matrix, it is necessary to define the environmental and household vectors. Fulfillment of the following criteria is suggested as necessary for the development of an effective environmental vector:

1. The components of the model must be definable at the level at which the model is to be applied, the level of the travel analysis zone;
2. It is necessary that the variables form a relatively small group, since they will be frequently manipulated and interpreted;
3. As a set, the variables must exhaust the possible variation in environment among alternate sites;
4. The variables used at the census tract or travel analysis zone level must be redefinable at the level of the individual household; and
5. The variables used at the travel analysis zone level of aggregation must be related to all and exhaust at least the major factors actually considered by a given household in choosing a site.

This study will focus at the level of aggregation—the travel analysis zone—and determine if a defining environmental vector for each area can be developed. The analysis is concerned with developing measures which fulfill the first three criteria. The limited resources available for the study precluded demonstrating that the suggested measures of physical differentiation do indeed correspond to the individual's perception of site amenities, or that the proposed measures "exhaust" the individual's perception of that environment. The first goal, definition at the census tract-analysis zone level, is achieved by defining the variables at that level. The other two criteria will be achieved through operations on the proposed indices.

Measures of the environment are proposed below which are defined at the level of the census tract-travel analysis zone. Difficulties in obtaining the necessary data resulted in the exclusion of measures of shopping facilities. The following indices of environment are suggested:

1. The population density of the tract (persons per acre);
2. The proportion of land in parks;
3. The proportion of land in open space;
4. The proportion of the units built before 1939;
5. The proportion of the units built after 1950;
6. The proportion of the units in sound condition and containing all of the standard plumbing facilities;
7. The median value of the homes;
8. The median gross rent;
9. The proportion of single family dwelling units;
10. The median rooms per dwelling unit;
11. The proportion of units with 0.50 persons per room or less;

TABLE 1
LISTING OF ORIGINAL ENVIRONMENTAL INDICES

Variable Number	Mnemonic	Definition
1	DENSIT	Density of census tract in persons per acre
2	PCPARK	Proportion of area of tract which is park
3	PCOPSP	Proportion of area of tract which is open space
4	PCOLDH	Proportion of homes in tract built before 1939
5	PCNEWH	Proportion of homes in tract built after 1950
6	PCGOOD	Proportion of homes in good condition with all plumbing
7	MEDVAL	Median value of homes in tract in dollars
8	MEDRNT	Median gross rent in dollars
9	PCSFUD	Proportion of units in tract which are single-family dwelling units
10	MEDRMS	Median number of rooms per house
11	LOWDEN	Proportion of units in tract with less than 0.50 persons per room
12	HGHDEN	Proportion of units in tract with more than 1.01 persons per room
13	PCWHIT	Proportion of the tract's population which is white
14	SBSOCR	Shevky and Bell social rank index
15	SBOCCI	Shevky and Bell occupation index
16	SBEDUI	Shevky and Bell education index
17	MEDYRE	Median years of education of people in tract
18	MEDINC	Median income of inhabitants of the tract

12. The proportion of units with 1.01 persons per room or more;
13. The proportion of the population which is white;
14. The social rank of the area as defined by Shevky and Bell (1);
15. The Shevky and Bell occupation ratio for the area;
16. The Shevky and Bell education ratio;
17. The median years of education of the residents; and
18. The median family income.

Table 1 contains a listing of the assigned variable number, a mnemonic and a description of the variable.

Tucson, Arizona, was chosen as the study city. Those census tracts outside the legal limits of the city contain areas which are functionally unrelated to the city. Two

TABLE 2
ENVIRONMENTAL VECTOR: NORMAL VARIMAX ROTATED FACTOR MATRIX

Variables	Loadings on Factor ^a							Communality
	1	2	3	4	5	6	7	
1 DENSIT	—	—	—	—	-0.938	—	—	0.959
2 PCPARK	—	—	0.967	—	—	—	—	0.973
3 PCOPSP	—	-0.631	—	—	—	—	0.522	0.941
4 PCOLDH	—	0.811	—	—	—	—	—	0.938
5 PCNEWH	—	-0.808	—	—	—	—	—	0.941
6 PCGOOD	0.777	—	—	—	—	0.491	—	0.952
7 MEDVAL	0.852	—	—	—	—	—	—	0.963
8 MEDRNT	0.781	—	—	—	—	—	—	0.922
9 PCSFUD	—	-0.894	—	—	—	—	—	0.849
10 MEDRMS	0.750	—	—	—	—	—	—	0.881
11 LOWDEN	0.811	—	—	—	—	—	—	0.933
12 HGHDEN	-0.957	—	—	—	—	—	—	0.962
13 PCWHIT	—	—	—	-0.935	—	—	—	0.996
14 SBSOCR	0.978	—	—	—	—	—	—	0.980
15 SBOCCI	0.970	—	—	—	—	—	—	0.966
16 SBEDUI	0.916	—	—	—	—	—	—	0.941
17 MEDYRE	0.885	—	—	—	—	—	—	0.927
18 MEDINC	0.621	—	—	—	—	—	—	0.946
Eigenvalue	8.342	3.565	1.443	1.255	1.316	0.502	0.546	
Cumulative percent common variance	49.2	70.2	78.7	86.1	93.8	96.8	100.0	
Interpretation of factor	Socio-economic	Single-family dwelling units	Percent parks	Racial composition	Density	Percent good	Percent open space	

^aFactor loadings between +0.49 and -0.49 have been omitted to ease reading.

TABLE 3
LISTING OF INDEPENDENT VARIABLES USED IN CALIBRATION
OF COEFFICIENTS MATRIX

Variable Number	Mnemonic	Description	Dummy (D) or Continuous (C)
1	ZERCAR	Household owns no cars	D
2	ONECAR	Household owns one car	D
3	TWOCAR	Household owns two cars	D
4	ONEPER	One person in household	D
5	TWOPER	Two persons in household	D
6	THFRPR	Three or four persons in household	D
7	FVSXPR	Five or six persons in household	D
8	NOPEMP	No persons employed	D
9	ONEEMP	One person employed	D
10	TWOEMP	Two persons employed	D
11	RACEHH	Race of head of household	D
12	OCCPHH	Occupation of head of household	D
13	LENRES	Length of residence at this site	C
14	PCEMPD	Proportion of household employed	C

of the five tracts, for example, consist of Indian reservations. The observations were therefore limited to those census tracts within the city limits. For each of these tracts, the previously described set of observations was obtained from the 1960 Census report for Tucson and from Volume One of the Final Study Report of the Tucson Area Transportation Study.

Following preliminary analysis, a seven-component environmental vector was hypothesized which contained the following factors:

1. A socioeconomic status factor;
2. A factor pertaining to proportion of single family dwelling units;
3. A recreational facilities factor;
4. A racial composition factor;
5. A density (of population) factor;
6. An age of housing factor; and
7. A proportion of open space factor.

A factor analysis was carried out to test the power of this model. The rotated factor matrix is given in Table 2. Since the results generally confirmed the hypothesized model, a prototypical environmental vector containing seven components was adopted for use in the analysis.

The requirement for a vector whose mutually independent components precisely define the household is implicit in the presentation of the model. A common-sense approach suggests that the following factors which influence the location behavior of the household should be considered for inclusion in the vector:

1. Income;
2. Number of years of education of the head of the household;
3. Proportion of the household which is employed;
4. Size of the household;
5. Number of children in the household;
6. Stage in the family life cycle;
7. Race of the head of the household;
8. Occupation of the head of the household (blue-collar or white-collar); and
9. Sex of the head of the household.

Unfortunately, data limitations precluded the development and analysis of these indices. Based on available data, and following an analysis similar to that performed for the environmental vector, a seven-component household vector was developed. The following indices were used to measure socioeconomic differentiation among households:

1. The number of cars owned by the family, a surrogate for income;
2. The total number of persons in the family;
3. The number of persons employed;

4. The proportion of the household employed;
5. The length of residence in the area, introduced because of its potential relevance to the ex post facto analysis of the household's location behavior;
6. The race of the head of the household; and
7. The occupation of the head of the household.

CALIBRATION OF PREFERENCE MODEL

The calibration of the desire coefficients matrix could have been performed at either the household level, which is essentially non-aggregated data, or at the level of the census tract. Operating at the census tract level implies the concept of a single composite family representing all the families in the tract by the average value within the tract for each parameter. The principal argument against the use of such data involves the conceptual difficulty which emerges in defining an average family for a census tract. A tract family does not exist and it is difficult to make statements of a behavioral nature about a nonexistent entity. Further, the desire coefficients become a function of the artificial set of boundaries which are used to define a census tract.

The alternate and chosen approach focuses at an appropriate level of behavior, the household. Problems of a different sort emerge in operating at this level. The race and the occupation of the head of the household are measured on ordinal scales, but the technique of linear regression which will be used to calibrate the coefficients matrix considers the relationship between interval scales. Utilizing the technique of dummy variables (2), race and occupation can be included in a regression equation. Further difficulties result from the nonlinear effect of other variables, particularly number of cars owned, number of people in the family, and number of people employed. These nonlinearities may be taken into account by coding these variables as dummy variables.

Converting the noncontinuous and nonlinear variables identified previously into dummy variables results in 14 independent variables, which are given in Table 3. The criterion for choosing among the infinite number of possible combinations of the elements a_{kj} is the maximization of the explained variance of the dependent variable y_k . Since the equation will not be forced through the origin, an $m \times 1$ vector of coefficients must be added to take account of the y intercept. Rewriting Eq. 7:

$$\begin{bmatrix} y_1 \\ \cdot \\ \cdot \\ \cdot \\ y_7 \end{bmatrix} = \begin{bmatrix} a_{1,1} \cdots a_{1,14} \\ \cdot \\ \cdot \\ \cdot \\ a_{7,1} \cdots a_{7,14} \end{bmatrix} \begin{bmatrix} x_1 \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ x_{14} \end{bmatrix} + \begin{bmatrix} b_1 \\ \cdot \\ \cdot \\ \cdot \\ b_7 \end{bmatrix} \quad (9)$$

TABLE 4
LISTING OF DEPENDENT VARIABLES

Variable Number	Name	Description
1	Social rank	Shevly and Bell social rank index
2	Proportion of single-family dwelling units	Proportion of dwelling units in tract which house only one household
3	Recreational facilities	Proportion of land area of tract taken up by parks
4	Racial composition	Proportion of population of tract which is white
5	Density	Density of tract in person per acre
6	Age	Proportion of dwelling units in tract which were built between 1950 and 1960
7	Open space	Proportion of land area of tract taken up by open space

Variable	Constant	ZERCAR	ONECAR	TWOCAR	ONEPER	TWOPER	THFRPR	FVSKI
Social rank	47.2	-15.4 (4.0)	-4.8 (2.4)	—	—	—	—	—
Proportion of single-family homes	86.2	-8.6 (1.4)	—	—	—	—	—	3.1 (1.0)
Recreational facilities	0.2	—	—	0.3 (0.1)	—	—	—	—
Racial composition	72.3	-3.0 (1.3)	—	—	—	—	—	—
Density	6.7	—	—	—	1.7 (0.5)	0.8 (0.4)	—	—
Age of housing	51.3	-20.8 (5.1)	—	—	—	-6.6 (3.3)	—	—
Open space	39.1	-8.5 (3.0)	—	—	—	-6.2 (2.0)	—	—

All coefficients are significant at the 0.05 level.

The calibration of the coefficients matrix is equivalent to a series of seven multiple regressions, one regression for each of the seven components of the vector Y. Since there is no reason to assume that every variable enters into every equation, a stepwise form of multiple regression was employed with the conditions that a variable must attain a level of significance of 0.05 or greater to be included in the equation, and that the variable be removed from the equation if the addition of subsequent variables causes it to fall below a 0.05 level of significance. Table 4 lists the chosen dependent variables.

Table 5 summarizes the equations which were developed. The standard error for each regression coefficient is shown in parentheses beneath the coefficient. The correlation coefficient (r), the standard error of estimate, and the level of significance of each equation are shown to the right. It is observed that the correlation coefficients are low; this implies that the independent variables used in the calibration phase of the study do not give sufficient insight into the housing choices made by the household. The availability and utilization of other measures of household characteristics, such as income, stage in the family life cycle, education, and number of children, might have improved the results obtained from the calibration of the model. Nonetheless, the results are encouraging, particularly in view of the disaggregate nature of the observations.

Several generalizations can be based on the results. Each of the independent variables entered into at least one of the equations. In this sense, the hypothesis that these factors do relate to the household's location behavior is supported. Only four independent variables enter into three or more equations. Since these four factors could be interpreted as influences which cause location desires to deviate from the norm, each will be discussed separately.

The dummy variable "household owns no cars" entered into five of the seven equations. It was previously suggested that car ownership is a surrogate for income. The analysis supports this contention by showing that households not owning a car tend to live in older, nonwhite areas containing fewer single-family homes and having less open space and a lower social rank. An ex post facto hypothesis is advanced that the dummy variable "two persons in household" represents the influence of retired couples. Having recently moved to Tucson and not owning a car, these people would tend to live in older areas with a higher density and less open space.

The dummy variable "race of the head of the household" entered strongly into four of the seven equations. The dummy variable was coded one if the head of the household was white and zero otherwise. Those coded one tended to live in newer, less dense, and white areas which had a considerably higher social rank. The length of residence variable entered into four equations. Not surprisingly, those who have lived at a site for a longer period of time live in older areas and have less open space.

In order to implement the model developed here, the future attributes of each site in the metropolitan area must be known. It is observed that all of the environmental variables except social rank are either predictable, e. g., age and racial composition, or

IONS DEVELOPED

NOPEMP	ONEEMP	TWOEMP	RACEHH	OCCPHH	LENRES	PCEMPD	r	Standard Error	Significant at Level
—	—	-6.1 (2.4)	27.7 (2.6)	—	—	—	0.57	18.4	0.0005
15.2 (7.0)	—	—	—	4.5 (2.0)	-0.3 (0.1)	—	0.46	6.8	0.0005
—	—	-0.2 (0.1)	—	—	—	—	0.21	0.65	0.001
—	—	—	6.5 (1.0)	21.0 (2.0)	-3.0 (1.3)	—	0.69	6.5	0.0005
—	—	—	-1.0 (0.4)	—	—	—	0.25	2.9	0.0005
57.4 (25.6)	—	—	23.6 (3.6)	—	-1.1 (0.3)	-22.9 (5.2)	0.54	24.8	0.0005
—	-5.0 (2.0)	—	—	—	-0.5 (0.2)	-11.5 (3.3)	0.40	15.2	0.0005

can be planned for, e. g., density level, proportion of single-family homes, recreational facilities, and open space. Although an effort could be made to predict social rank, the alternative strategy of substituting a plannable factor, median value of housing, appears to be more feasible, particularly since these measures are highly correlated. A multiple regression was performed using median value of housing as the dependent variable, and approximately the same variables and levels of quality were obtained.

CONCLUSIONS AND RESEARCH IMPLICATIONS

Two tentative conclusions can be drawn from this analysis. Three factors, income, stage in the family life cycle, and race, appear to be correlated with variations in the environments selected by different households. These factors, and additional factors identified in future work, should be considered in future efforts to predict the environmental preferences of households.

The second conclusion is based on the observed relationship between length of residence and characteristics of the environment. As households change through time, their housing preferences also vary. This relationship suggests that, in lieu of constantly moving to obtain environments which satisfy their existing preferences, households accept certain gaps between their preferred and their existing environment.

It has been the purpose of this paper to explore selected issues relating to the micro-level simulation of residential location behavior. The most important research implication of this work is the need for more sophisticated data on both consumer behavior and consumer preferences in the housing market. For example, there is presently little data available concerning the effect of variations in the level of information available to the consumer on the consumer's behavior in the housing market. The amount of information on which households actually base their location decisions is unknown. It is therefore impossible to build these considerations into new location models. Behavioral data on the levels of information achieved by different locators could be obtained through in-depth surveys of households which have recently selected a new location.

Surveys of consumer preferences based on data about actual behavior may handle the preference-reality gap emphasized previously in either of two ways. They may sample only those households which have recently moved or they may attempt to measure dissatisfaction with the existing environment. Neither of these approaches, however, explicitly probes the vital question of the play-offs among preferred attributes which households make in selecting an environment and a site. Such information can best be achieved through the development of games in which households would be asked to choose among locations with varying attributes and to explain their choices. While the implementation of such games is undoubtedly difficult, the additional empirical insight thereby gained should be considerable.

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Technique for Relating Transportation Improvements and Urban Development Patterns

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•THIS paper describes a land-use forecasting model which embodies desirable features not heretofore available for use in planning transportation facilities. In the process of developing and calibrating the model, it was assumed that there are limited controls available at the regional scale for guiding the development of an urban area. One of these controls is the transportation system. It is hypothesized that there exists a partial ability to influence the development of a region by means of the transportation system. This is an ability which the planner should utilize, both for the promotion of a more desirable region in which to live, and for planning the transportation system in a more complete and efficient manner.

In the past, the required plan or forecast of the future pattern of land uses has normally been prepared somewhat independently of the planned transportation facilities. An important missing link in the overall urban plan-making process has been a systematic measurement of the effect that future transportation facilities themselves have in shaping the land-use pattern. This is an effect which generally leads to higher usage of transportation facilities than would otherwise be expected, since transportation facilities often attract land uses which require such facilities. It is, therefore, imperative that the planner and engineer plan transportation facilities to accommodate not only those land-use activities already in place and those expected owing to urban expansion, but also those activities which will be induced by the proposed facilities to redistribute themselves.

In this paper, attention is focused primarily on the information which the calibration of the EMPIRIC model reveals on the relative and absolute effect of transportation and community facility improvements on land development patterns. Secondary attention is focused on some recent results of production forecasts with the model. The (production) EMPIRIC model, to date, has been structured and the equations estimated, for three data sets involving two different urban regions. Production forecasts have been carried out for the two different urban regions for which the model was calibrated.

The remainder of this paper describes (a) the formulation of the EMPIRIC model, (b) the estimation of coefficients for the equations comprising the model, (c) generalized equations reflecting knowledge gained to date with the model on the forces underlying urban development patterns, and (d) some results of forecasting with the EMPIRIC model.

FORMULATION OF THE MODEL

The EMPIRIC land-use forecasting model is a technique, programmed for the computer, which was designed for use in the planning process. It does not apply optimization techniques nor does it restrict freedom of choice; rather, it attempts to make planning a more meaningful procedure by forecasting one important consequence of a set of alternative policies and plans: namely, the future distribution of population, employment and other socioeconomic activities in the region.

The model was formulated such that it would satisfy several criteria, some of which were felt to be important theoretical constraints, and others of which were the operational realities of applying the model in the Eastern Massachusetts region. These criteria,¹ which are largely applicable to any North American metropolitan area, were the abilities:

1. To recognize the simultaneous and interacting nature of metropolitan development;
2. To take as direct input, planned changes in the transportation system (both highway and transit);
3. To output important categories of population, employment, and automobile ownership (i. e., the model must provide data for forecasting trip origins, destinations, and modal splits);
4. To provide forecasts for areas sufficiently small to allow meaningful forecasting of trip origins, destinations, and modal splits; and
5. To be applied recursively (in steps) over relatively short time intervals to allow inputting new values of staged construction of facilities (i. e., the model should produce information directly useful for public works programming).

Criteria of a second order were:

1. The model should accept other important non-transportation policy decisions as inputs. In effect, its output should be a systematic estimate of how a region would develop under the influence of regional growth rates and planning policies relative, not only to transportation, but also to utilities, zoning, open space, etc.
2. The model should allow for reasonable budget limits on operating costs of the model.
3. Input and output to the model should be compatible with other needs; e. g., input transportation networks should be the same as those needed for traffic work.

The framework decided on for the EMPIRIC model consists of a set of simultaneous linear regression equations. That is, more than one output variable is contained in a single equation, and the relationships embodied in the model between the input and output variables are linear and additive. The simultaneous nature of the model (the coefficients of the equations are estimated using simultaneous regression techniques) is a major innovation, getting around the problem of having to decide which activities to locate or forecast first.

All variables in the equations are expressed as shares of regional totals, and the model forecasts changes in shares of activities, between base year and forecast year, in each of the zones or subregions into which the region is divided. Mathematically, a change in subregional share may be expressed as

$$\frac{R_{ih}(t)}{\sum_{h=1}^H R_{ih}(t)} - \frac{R_{ih}(t-1)}{\sum_{h=1}^H R_{ih}(t-1)}$$

where R_{ih} is the level of activity i in zone h , H is the total number of zones in the region, (t) indicates the forecast year, and $(t-1)$ indicates the base year.

Data from two points in time are used to calibrate the model. The formulation of the variables enables both growths and declines of activity levels to be easily handled. Having forecasted changes in shares, the model adds these changes to the shares at the beginning of the forecast interval to obtain the new zonal shares, and then multiplies the new shares by regional totals at the end of the forecast interval to obtain the actual activity levels in each traffic zone. The regional totals are forecast independently of

¹This list is similar in many respects to the list of criteria presented by Lathrop and Hamburg (10).

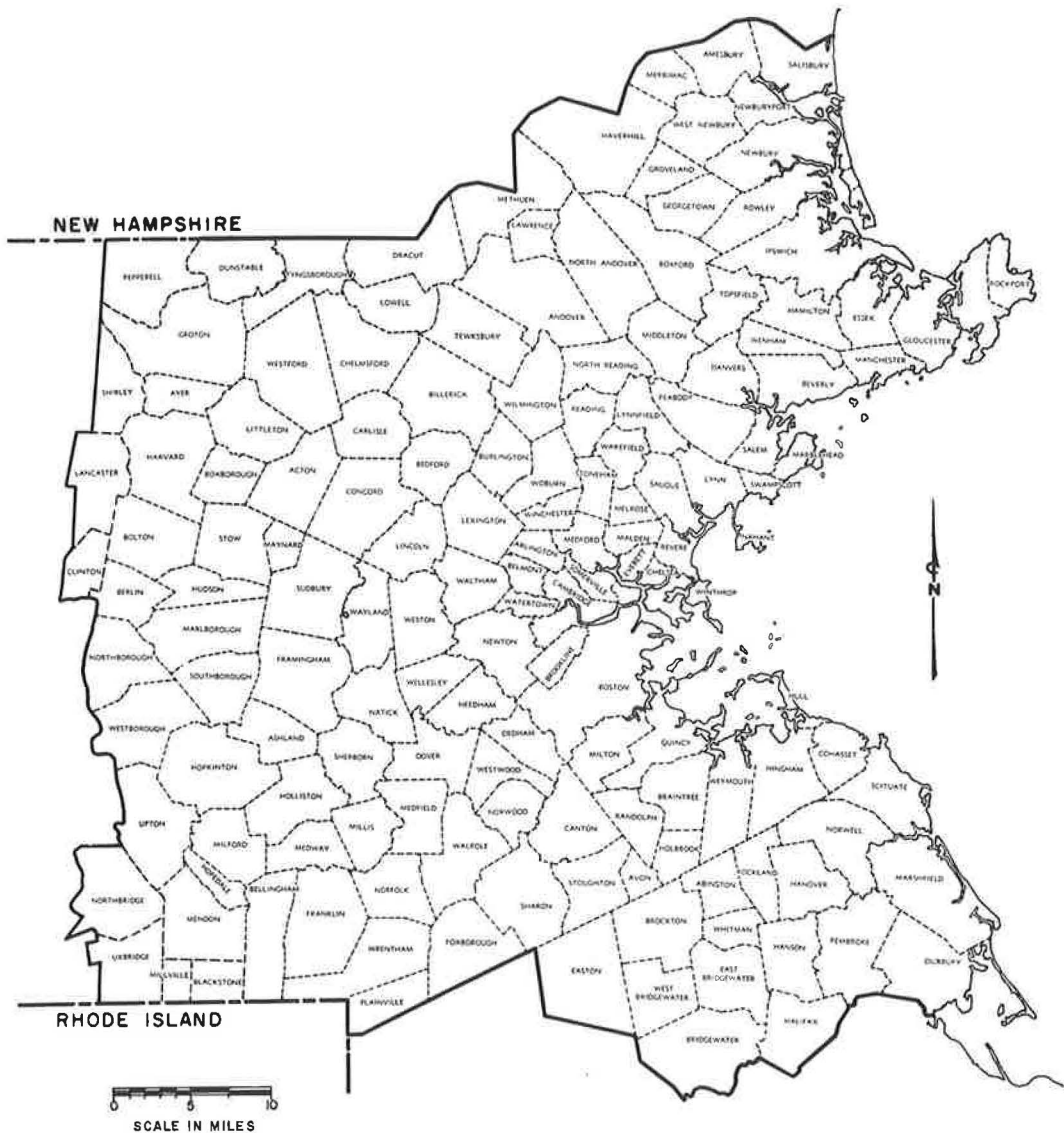


Figure 1. The Eastern Massachusetts region.

the model so that, with this formulation of output variables, the model is strictly a distributional model.

There are two classes of input variables used in the **EMPIRIC** model: policy variables, and non-policy variables. Policy variables may be manipulated or preset by planners, and therefore they enter the model as terminal or forecast year data. Examples are the transportation system (in the form of accessibilities) and sewage disposal and water supply service levels. Non-policy variables are base-year data, such as families-by-income categories and employment-by-industry categories. Also defined as non-policy variables are various measures of the capacity of a zone to house development of the various types of activities. These, however, could be used as policy variables by reserving land in zones in accordance with recreation and/or open space policies.

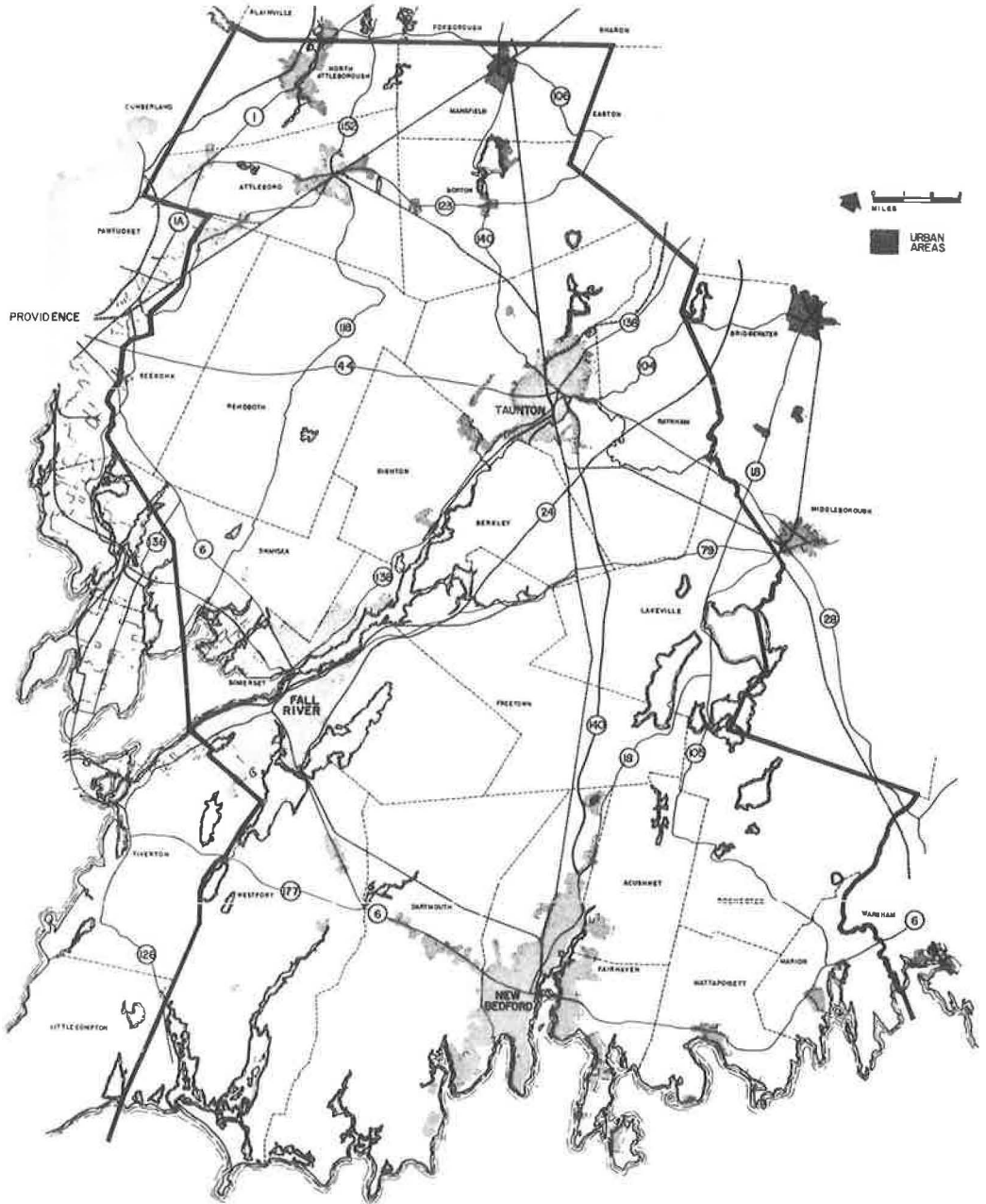


Figure 2. Southeastern Massachusetts planning region.

ESTIMATION OF EMPIRIC MODEL EQUATIONS

To date, the EMPIRIC model has been calibrated for three data sets. Two of these data sets were for the 3.4 million population (in 1960) Eastern Massachusetts region (Fig. 1). The first involved the region disaggregated into 626 traffic zones (i. e., observations for each variable), whereas the second divided the region into 97 subregions. The third data set divided the 400,000 population (in 1964) Southeastern Massachusetts Regional Planning District (Fig. 2) into 71 land-use forecasting districts.

The regression analyses used to estimate the coefficients of the models were preceded by intensive theoretical studies as to the proper structure of the model. These studies, based on a priori reasoning as well as on knowledge gained from prototype EMPIRIC model development work (2) and the literature, resulted in initial or preliminary structuring of each model.

The theoretical studies were augmented by data-analysis techniques programmed as part of the EMPIRIC model, notably factor analysis, which provides insight into the proper grouping of data categories to form model variables that are as independent of one another as possible, and bivariate correlation analysis, which provides insights into the nature and strengths of the correlations or relationships between pairs of model variables. Using these analyses and the theoretical studies, coefficients for several models for each area (data set) were estimated, each successive model showing improvement over the preceding one. The improvements exhibited were not so much in the "goodness of fit" of the data, but in the stability, the conformance with theory, and the improved significance levels of the variables in the model. These factors are important criteria which must be met if the model is to be a valid and reliable forecasting tool.

The estimated coefficients for the most disaggregated version of the model will be described in detail here. This version of the model comprises a set of nine simultaneous equations, and forecasts four categories of population and five categories of employment for a set of 626 traffic zones. The categories are:

1. Families with less than \$5,000 annual income (1959 dollars);
2. Families with between \$5,000 and \$9,999 annual income;
3. Families with between \$10,000 and \$14,999 annual income;
4. Families with greater than \$14,999 annual income;
5. Manufacturing and construction employment (Standard Industrial Classification codes 15-39);
6. Wholesale, transportation, communication, utilities, government, and other employment (SIC codes 01-14, 40-50, 91-99);
7. Retail employment (SIC codes 52-59);
8. Service employment (SIC codes 70-89); and
9. Finance, insurance, and real estate employment (SIC codes 60-67).

The estimated equations in this calibrated model are described in detail in the Appendix.

Data for two points in time (1950 and 1963) were used to calibrate this model. Due to insufficient data for the earlier year, the model was calibrated using data from 453 of the 626 traffic zones (representing about 80 percent of the 1960 regional population of about 3.4 million persons). Forecasting, however, is being done for all 626 zones.

The statistical significance of each of the variables in the equations of this model is measured with the t-test, which provides an index of the degree to which the effect of a variable upon an output variable is either random or systematic. The t values were computed, for all input and output variables, by the same programs which applied the regression techniques for the estimation of the coefficients in the equations. For 453 sampling points (i. e., traffic zones), a t value of 1.96 or greater is indicative of a variable which is significant to the 95 percent confidence level—a level which is felt to be a very stringent test of significance. Fifty-one of the 63 input and output variables in the nine equations of the model met this high standard. Of the other 12, ten are significant to a level of confidence of 75 percent or greater (the two exceptions being significant at the 56 percent and 58 percent levels).

The few variables which were significant to less than the 95 percent confidence level were still felt, therefore, to be statistically acceptable, and were retained in the final model structure because they, along with the other variables in the model, had regression coefficients whose signs and relative magnitudes satisfactorily expressed the hypothesized relationships between the variables.

An additional test of the model was its "goodness of fit" over the calibration period. That is, an indication of the model's reliability as a forecasting tool was obtained by using the calibrated model and the calibration base year (1950) data, and "forecasting" to the terminal year of the calibration period (1963) to see how well the model reproduced

TABLE 1
STATISTICAL SUMMARIES OF OBSERVED VS CALCULATED POPULATION AND EMPLOYMENT LEVELS

Category	453 Zones			104 Districts		
	RMS Error	RMS Error Ratio	R ²	RMS Error	RMS Error Ratio	R ²
Families, < \$5,000	108	0.249	0.951	232	0.123	0.990
Families, \$5,000-\$9,999	209	0.269	0.906	685	0.203	0.950
Families, \$10,000-\$14,999	82	0.380	0.793	233	0.250	0.915
Families, ≥ \$15,000	61	0.578	0.826	150	0.328	0.946
Mfg and construction employment	1,031	1.23	0.549	2,301	0.630	0.862
Wholesale, TCU ^a Govt., and other employment	412	0.782	0.876	969	0.422	0.982
Retail employment	310	0.781	0.860	846	0.490	0.949
Service employment	677	1.43	0.500	1,958	0.949	0.880
FIR ^b employment	224	1.33	0.953	260	0.352	0.997

^aTransportation, communication and utilities.

^bFinance, insurance, and real estate.

the activity growths occurring during the calibration interval. Statistical summaries were then prepared comparing observed and calculated (forecast) 1963 zonal values of the output variables.

These summaries include the root-mean-square (RMS) error, the RMS error ratio, and the coefficient of determination (R²). The RMS error is computed in the following manner:

$$\text{RMS error} = \sqrt{\frac{\sum_{h=1}^H (O_{ih} - C_{ih})^2}{H}}$$

where O_{ih} is the observed value of variable i in zone h , C_{ih} is the calculated value of variable i in zone h , and H is the total number of zones in the region. Assuming normality, the observed value does not differ from the calculated value for about 67 percent of the zones by more than plus or minus the RMS error. The RMS error ratio is the ratio of the RMS error to the mean or arithmetic average of the observed output variables (\bar{O}_i).

The coefficient of determination (R²) is computed as follows:

$$R^2 = \frac{\sum_{h=1}^H (O_{ih} - \bar{O}_i)^2 - \sum_{h=1}^H (O_{ih} - C_{ih})^2}{\sum_{h=1}^H (O_{ih} - \bar{O}_i)^2}$$

As R² approaches unity, the reliability of the model is regarded to be quite high, and conversely, as R² approaches zero, the reliability is said to be quite low. These summaries, for the 453 traffic zones in the calibration area, are given in Table 1 for the nine equations in the final calibrated model. In addition, the corresponding statistics have been recomputed following the aggregation of the 453 traffic zones into 104 calibration analysis districts. This procedure was designed to provide some indication of the sensitivity of these reliability statistics to zonal aggregation.

It can be seen that the model fits the population data better than the employment data. This is to be expected, since a statistical model fits large numbers of small locating units (e. g., households) better than the "lumpier" activities which typify the employment locating units. The fit to the geographically small 453 zones appears highly

satisfactory, and compares favorably with similar error measures calculated for home interview survey origin-destination data, and for various types of traffic models, e. g., gravity models (11). In addition, the model in the Appendix appears quite sound from the standpoints of statistical significance (high t values), and logic (conformance with hypothesized relationships).

GENERALIZED LAND-USE FORECASTING EQUATIONS

The true measure of the EMPIRIC model's worth as a forecasting and plan-making tool is in the empirical and logical reliability of the regression coefficients. Because the variables are formulated as zonal shares or changes in zonal shares, these coefficients may be interpreted as indicators of the relative effects of the variables in influencing relative growths or declines of an output variable at the zonal level. The sign of the coefficient (positive or negative) indicates whether the variable induces or hinders the growth in zonal share of the output variable, while the magnitude of the coefficient indicates the importance of this influence relative to the influence of the other variables in the equation on the growths of the output variables. Coefficient stability, therefore, becomes an important indicator of the success achieved in producing true relationships in the model; relationships from which one may learn about influencing the shape of metropolitan development, and, consequently, the usage of transportation facilities.

Two types of coefficient stability may be described. The first is coefficient stability as successive model structures are estimated in the model calibration process using a single data set. The coefficients in the model described in the Appendix behaved extremely well in this regard over the successive equation estimations (8). In the few instances when coefficients in the final model exhibited appreciable changes from the corresponding coefficients in earlier models, it was almost always attributable to a problem of collinearity between independent variables in the earlier models. The situation was remedied by the deletion in the final model of all but one of the related independent variables, or by the substitution of a single variable for the complete set of collinear independent variables.

The second type of coefficient stability pertains to the similarity of the relationships expressed by the coefficients, as different data sets for the same region or for different regions are used to estimate the same structural equations. The three calibrations of the EMPIRIC model just described did not use the same structural equations because of the purposes for which the models were developed, and because of the differences in the data available for calibration. It is hoped that future work will allow the estimation of the same EMPIRIC model structural equations for different data sets.

Nevertheless, the three models all distributed classes of population and employment to relatively large numbers of small areas. And the types of independent variables used in each model were similar. The results indicate that there is enough coefficient similarity between corresponding input and output variables for the differing data sets and areas to warrant an attempt to generalize the results of the three models. In recording these results, it is recognized that there should indeed be different relationships between variables with differing zone sizes. Also, different urban areas have different regional growth rates and different compositions of activities comprising the urban development pattern. In fact, if the coefficient set were completely stable it would not be necessary to recalibrate the model for different zone systems and areas.

The generalized equations are written out completely below. The following notation is employed:

(Δ) = change in subregional share over the time interval

(t) = subregional share at the end of the time interval

($t - 1$) = subregional share at the beginning of the time interval

(All variables are formulated as shares or as changes in shares.)

POPL, POPM, and POPU = lower-, middle-, and upper-income population

MFG, RTL, SVC, and OTH = manufacturing, retail, service, and other employment

UTIL = measure of utilities service

CAPP, CAPM, and CAPR = measures of the capacity and propensity of a zone to house new population (i. e., residential), manufacturing, and retail development (the measures are defined in the Appendix)

VACC and QACC = measures of vehicle (automobile) and transit accessibility (accessibility is defined in the Appendix)

The magnitudes of the coefficients are indicated as s, m, or b—small, medium, and big (<0.1, 0.1 to 0.4, and >0.4).

The equations follow:

$$(\Delta) \text{POPL} = b (\Delta) \text{POP}M - m (\Delta) \text{POP}U + s (\Delta) \text{SVC} + m (t - 1) \text{POPL} - s (t - 1) \text{POP}U + s (t) \text{UTIL} - m (\Delta) \text{VACC}$$

$$(\Delta) \text{POP}M = -s (\Delta) \text{POPL} + m (\Delta) \text{POP}U + s (\Delta) \text{RTL} + s (\Delta) \text{SVC} - m (t - 1) \text{POP}M + m (t) \text{UTIL} + m (\Delta) \text{VACC} + s (\Delta) \text{QACC}$$

$$(\Delta) \text{POP}U = -m (\Delta) \text{POPL} + m (\Delta) \text{POP}M - m (t - 1) \text{POP}U + m (t) \text{UTIL} + s (t - 1) \text{CAPP} - m (\Delta) \text{VACC} + s (\Delta) \text{QACC}$$

$$(\Delta) \text{MFG} = -m (\Delta) \text{POP}M - b (\Delta) \text{POP}U + m (\Delta) \text{OTH} - b (t - 1) \text{MFG} + m (\Delta) \text{CAP}M + m (t) \text{VACC} + m (\Delta) \text{QACC}$$

$$(\Delta) \text{RTL} = m (\Delta) \text{OTH} - s (t - 1) \text{POP}U - m (t - 1) \text{RTL} + m (t - 1) \text{CAP}R + m (\Delta) \text{VACC}$$

$$(\Delta) \text{SVC} = -s (\Delta) \text{OTH} - m (t - 1) \text{SVC} + m (\Delta) \text{UTIL} + m (t) \text{VACC} + m (\Delta) \text{QACC}$$

$$(\Delta) \text{OTH} = m (\Delta) \text{MFG} + s (\Delta) \text{RTL} - m (t - 1) \text{OTH} + s (\Delta) \text{QACC}$$

It must be reemphasized that these equations are for discussion and theory building purposes only, and are abstracted from only three models calibrated for two areas: the relatively slow-growing Eastern and Southeastern Massachusetts regions.

The equations generalize the interrelationships among activities in this type of area for this scale of zonal disaggregation (i. e., for an average zonal population of from about 5,000 to about 35,000), and for this type of model (linear and share). Each of the equations describes hypothesized relationships designed to explain the growth of a particular output activity. For example, the first equation states that growth of lower-income population in a zone is induced by a simultaneous growth of middle-income population but hindered by the growth of and presence of (at the beginning of the time interval) upper-income population. It is also induced by the simultaneous growth of service employment, by the presence at the beginning of the time interval of low-income population (the ghetto effect), and by the presence at the end of the time interval of utilities services. It is hindered by the (relative) growth of vehicle accessibility of the zone (since they compete for more accessible land with higher-income groups, as explained later).

An examination of the equations indicates that the accessibility variables are the most important of the policy variables for forecasting the location of population and employment. However, the non-policy variables, over which the planner has no direct control, are generally stronger determinants of locational patterns than are the policy variables. In particular, growths in the various population-by-income groupings are strongly related to growths in the adjacent population-by-income groupings. It is also observed that in the employment equations among the strongest variables are one or more of the other output variables. These observations provide evidence of the realism of this type of simultaneous model.

In all equations, one of the more important determinants of growth is the "lagged" variable, i. e., the value of the output variable at the beginning of the forecast interval. In every instance but one, the lagged variable carries a medium or large negative sign. The single exception is important in that it is in the (first) equation for the low-income population. In only that instance does the presence of the (same) activity at the beginning of the time interval induce increased growth in the zone in the regional share of the activity. This is striking statistical evidence of the increasing ghettoism of the low-income family, about which there is much discussion today.

Many of the coefficients capture other relationships worthy of examination. In the low-income population equation again, the medium-sized negative coefficient modifying growth in vehicle accessibility indicates that these low-income families do not have the resources to take their full share of the advantages of improvements in the regional highway system. However, it may also be noted that the highest income group (in the third equation) exhibits the same medium-sized negative sign for this variable. This appears to indicate that they would rather pay increased transportation costs to enjoy the other residential amenities which they desire. The very large middle-income group, on the other hand (in the second equation), exhibits the concern for improved highways with which we are familiar.

It is also of interest to note that the middle- and high-income groups take advantage in a small but noticeable way of transit improvements, which in this case were rapid transit and commuter railroad service changes.

The position of the accessibility variables as the most influential of the policy variables is especially significant because there seems to be considerably greater control at the regional level over the transportation system than over any of the other policy variables relating to the development and physical arrangement of land patterns. This is partly because most land development policies are determined at the local level by the citizens of the localities affected. Transportation policies, on the other hand, cannot be so isolated at the local level. The function of transportation is to connect places (which may have differing transportation desires), and major transportation policies must be decided on a broader (e. g., regional) level. At best, planners can plan and promote transportation improvements which reinforce development decisions made at the local level.

FORECASTING WITH THE EMPIRIC MODEL

The capabilities of the EMPIRIC land-use forecasting model to manipulate data, to reproduce significant parts of the environment, and to quickly simulate complex relationships between the forces which shape the environment, provide the model with the ability to predict the future distribution of land-use activities with varying sets of input public works policies. This ability is essential for providing information for judging alternative plans, i. e., for determining (a) how well each plan functions, (b) how well each plan achieves its desired set of values, and (c) whether a particular programming (scheduling) strategy has been successful. A means of using the model in conjunction with travel forecasting techniques for evaluating alternative transportation policies and programs is outlined as follows:

1. The model is calibrated (i. e., the equations structured and the coefficients estimated) using data from two historical time points; say, time t and time $t + x$, where time t is x years earlier than time $t + x$. (The x -year forecasting interval would normally be about 5 or 10 years.)

2. Estimates of regional growth for an x -year period would be made for each activity to be predicted, and regional forecasts of these activities would be made for time $t + 2x$.

3. The land-use model would be applied for an x -year forecast from time $t + x$ to time $t + 2x$. The input data required for forecasting would include base year (time $t + x$) values of activity levels, and base year and forecast year (time $t + 2x$) travel times (the latter times being based on the anticipated or proposed completion of new transportation facilities and the closure of old facilities). Also input would be base year and forecast year values of other policy variables, such as utilities service.

4. The traffic model would be applied to forecast for time $t + 2x$, traffic flows, times and costs, based on the predicted land-use pattern and the travel facilities scheduled for completion at time $t + 2x$.

5. The procedure outlined in steps 3 and 4 would be repeated if the travel times and costs found in 4 differed substantially from final year values used in 3.

6. The procedures outlined in steps 2 through 5 would be repeated for successive x -year periods, using activity levels estimated by the land-use model at the end of each

TABLE 2
FUTURE REGIONAL CONTROL TOTALS (IN THOUSANDS)

Year	Population	Mfg Employment	Non-Mfg Employment
1963	3,540.5	426.8	870.0
1975	3,924.0	433.5	1,073.2
1990	4,733.0	478.7	1,322.4

period as starting levels for forecasting the next period. This step would be continued until the final target year had been reached.

This process thus provides a systematic representation of the anticipated sequential stages of development of a region under the influence of a set of public policies relating to the transportation system, utilities

service, etc. Repeating the process for different sets of policies will produce different anticipated development patterns. The planners and decision-makers can study these various patterns, analyze their relative merits and costs, and can more knowledgeably make decisions as to which sets of policies will be most effective in furthering the social and economic goals of the region. Especially valuable would be the exploration of alternative public works programming strategies. This process allows the program to be developed as an integral part of, and at the same time as, the overall plan.

At this writing, the EMPIRIC model has been used to make four sets of production forecasts at the 97 subregion level for the Eastern Massachusetts region and one set of forecasts at the 71 district level for the Southeastern Massachusetts region.

SUBREGION FORECAST RESULTS

For the purpose of exploring patterns of urban growth which are considered feasible for the future development of the Eastern Massachusetts region, an application of the 97 subregion EMPIRIC model was made. Forecasts were prepared for four regional growth alternatives. Each alternative pursued different basic physical objectives for structuring future urban growth. The alternatives are called (a) the composite plan, (b) the radial corridor plan, (c) the spread city plan, and (d) the nucleated plan. In this application the EMPIRIC model is viewed as a design tool; i. e., the designer is able to determine the consequences of selected programs. This in turn enables him to choose which program (set and schedule of actions) to propose for implementation or to subject to more detailed analysis.

Values for each of the policy variables were altered in this model application. For each of the four plans, appropriate "test" future highway, transit, water and sewer networks were designed. There were differences between the test networks only for the period 1975 to 1990 owing to the region's strong commitment to the 1975 programs for highway and transit networks. Identical regional "control" totals of population and employment were used for each plan. These are listed in Table 2.

Forecast results for 1990 showed an average difference between the highest and lowest subregional values among the four plans of 9 percent for population, 42 percent for manufacturing employment and 13 percent for non-manufacturing employment. The range of differences between the high and low 1990 forecasts was 1 percent to 46 percent for population, with 14 of the 97 subregions having differences over 15 percent. The corresponding figures for manufacturing employment were from 2 percent to 500 percent with 11 subregions over 50 percent, and for non-manufacturing employment, from 1 percent to 89 percent with 7 subregions over 30 percent. However, certain patterns are common to each of the four forecasts. First, the regional core area continues to decline, although at a slower rate than during the model calibration period 1950-1963. Second, each geographic sector retains an almost constant share of regional population and employment. Third, change in share by ring is greater than change in share by sector as would be expected (growth is moving outward from the regional center or core).

By identifying and comparing subregions in which only the highway or transit network input data have been changed, it is possible to measure the impact of transportation facilities. It appears in some cases that good highway connections will result in 10 to 15 percent more population than poorer highway connections. Similar observations are possible with respect to employment. Many such observations would have to be made and investigated before any verified generalizations could be made. Sufficient differences

existed between plans to warrant exploration of alternatives at a more detailed level (i. e., with the 626 traffic zone EMPIRIC model).

That sufficient differences occurred was not surprising. The hypothesis that the design of the transportation system plays a large and important role in the shaping of metropolitan development was borne out by a test carried out with a prototype version of the EMPIRIC model (11). This model was used to simulate the effect on the locational pattern of population and employment in the Eastern Massachusetts region of two different design policies of transportation facilities over the 1950-1960 decade. The first design policy simulated was exactly that which took place in the region between 1950 and 1960 insofar as highway and mass transportation improvements or closures were concerned. The second simulated design policy was that no changes were made in the highway and mass transportation systems between 1950 and 1960.

The major transportation improvements consisted of radial expressway sections plus Route 128, a major circumferential expressway which passes through a tier of suburban communities. The simulated policy of transportation improvements resulted in expected increases in population and employment in the third and fourth tiers or rings of subregions, relative to results with the simulated policy of no transportation improvements. However, it is interesting to note that relative increases in population and employment were also obtained in the older core cities of Boston, Cambridge, and Somerville, due to the new radial expressways and the extension of the rapid transit system to the periphery of Newton (i. e., to Route 128).

CONCLUSIONS

From the results obtained thus far with the three versions of the EMPIRIC land-use forecasting model, several observations may be drawn. The model has in each instance been satisfactorily calibrated in terms of logical relationships expressed by the variables and their coefficients (i. e., conformance with hypotheses), high statistical significance (as measured with t values), good fit with the data, and stability of the coefficients within each model (as observed by tracing variables through the successively estimated models).

The model thus far has been successfully used for forecasting to relatively large numbers of zones in two instances: (a) with the 97 subregion version calibrated for the (Boston) Metropolitan Area Planning Council, and (b) with the 71 district version calibrated for the Southeastern Massachusetts Regional Planning District. It is expected that the model will be able to successfully forecast to very large numbers of zones, as will be soon tested when forecasts are made using the 626 zone version of the model calibrated for the Eastern Massachusetts Regional Planning Project.

In addition, it appears that the model is properly sensitive to varying public policy inputs. The four sets of forecasts produced with the 97 subregion version of the model were intended to reflect widely ranging transportation policies, and the results displayed substantial and logical differences in the forecast values of population and employment. It is felt that this is in large part due to the fact that the model deals primarily with growths of activities rather than with absolute levels of activities at one point in time.

While these substantial findings have been made from the research and development work completed to date, further research into and with the EMPIRIC model would be useful. Moreover, future calibrations and applications of the model are warranted. Such calibrations and applications, with data from other metropolitan areas, would contribute to a better understanding of land-use development patterns in urban areas.

There are at least five major areas of research. First, the questions of coefficient stability could be investigated. Second, possibilities for designing optimal sets of inputs (e. g., accessibility variables) to produce desired plans could be undertaken through mathematical reformulation of the model. Third, the potential for developing programs for public investment using the staging capabilities of the model could be investigated. Fourth, further application of the model as a design tool is worth exploring. Fifth, the possibility of joining the EMPIRIC computer programming system to a plan evaluation system should be investigated. Such a joint or tandem system would be extremely

desirable since it would increase our capacity for exploration of alternative policies and programs.

Finally, a more intensive analysis of the forecast results produced by the (Boston) Metropolitan Area Planning Council may yield more support for generalizations of the type attempted in this paper.

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Appendix

THE 626 ZONE EASTERN MASSACHUSETTS EMPIRIC MODEL

The following variables are used in the model:

Population variables (All income figures are given in terms of 1959 dollars.)

- F_{<5k} = Number of families with an annual income less than \$5,000.
- F_{5-10k} = Number of families with an annual income between \$5,000 and \$9,999.
- F_{10-15k} = Number of families with an annual income between \$10,000 and \$14,999.
- F_{≥15k} = Number of families with an annual income equal to, or greater than, \$15,000.

Employment variables (All employment variables are measured at the zone of employment.)

- M & C = Manufacturing and construction employment (SIC codes 15-39).
 Other = Wholesale, transportation, communication, utilities, government and other employment (SIC codes 1-14, 40-50, 91-99).
 Ret = Retail employment (SIC codes 52-59).
 Svc = Service employment (SIC codes 70-89).
 FIR = Finance, insurance, and real estate employment (SIC codes 60-67).

Land developability variables (The nomenclature used to define these variables includes: NAP = net residential area; NAM = net manufacturing area; NAR = net retail area; UA = total used area of a zone = NAP + NAM + NAR + other developed area; and GA = gross area = UA + developable area.)

- CI Pop = Capacity or land developability index for population = (NAP/GA) (GA-UA).
 CI Mfg = Capacity or land developability index for manufacturing = (NAM/GA) (GA-UA).
 CI Ret = Capacity or land developability index for retail = (NAR/GA) (GA-UA).

Utilities service variables

- Water = Index, from 1 through 7, of water supply service, multiplied by UA.
 Sewer = Index, from 1 through 5, of sewage disposal service, multiplied by UA.

Accessibility variables (The accessibility of zone g to activity i is equal to

$$\sum_{h=1}^H R_{ih} e^{-\beta t_{gh}} \text{ where } R_{ih} \text{ is the quantity of activity } i \text{ in zone } h, H \text{ is the total}$$

number of zones, e is the base of natural logarithms, t_{gh} is the travel time between zones g and h, and β (the beta factor) is an empirically derived factor. All accessibilities were then multiplied by UA for use in the model.)

- VaccTF = Vehicle accessibility of a zone to total families.
 QaccTF = Transit accessibility of a zone to total families.
 VaccF_{≥10} = Vehicle accessibility of a zone to total families with an annual income equal to, or greater than, \$10,000 (1959 dollars).
 QaccF_{<10} = Transit accessibility of a zone to families with an annual income less than \$10,000 (1959 dollars).
 VaccTE = Vehicle accessibility of a zone to total employment.
 QaccTE = Transit accessibility of a zone to total employment.
 VaccM & C = Vehicle accessibility of a zone to manufacturing and construction employment.
 VaccR & S = Vehicle accessibility of a zone to retail and service employment.

Variables measured at the forecast year are preceded by (t). Variables measured at the base year are preceded by (t - 1). Variables representing changes between the base year and forecast year are preceded by Δ . All (t) and (t - 1) variables are formulated as subregional shares. The " Δ " variables are formulated as changes in subregional shares. The number in parentheses following the accessibility variables indicates the value of the beta factor used for the calculation of that accessibility. The model, then, is comprised of the following equations:

Equation 1: $\Delta F_{<5k} = 0.637 \Delta F_{5-10k} - 0.295 \Delta F_{10-15k} + 0.018 \Delta Svc$
 $+ 0.133 (t - 1) F_{<5k} - 0.109 (t - 1) F_{10-15k} + 0.044 (t - 1) Water - 0.298$
 $\Delta VaccTE (0.05) - 0.068 (t - 1) VaccTE (0.15)$

$$\begin{aligned} \text{Equation 2: } \Delta F_{5-10k} &= 0.530 \Delta F_{<5k} + 0.337 \Delta F_{10-15k} + 0.022 \\ &\Delta \text{Ret} + 0.060 \Delta \text{Svc} - 0.101 (t-1) F_{5-10k} + 0.036 (t-1) \text{Svc} + \\ &0.044 (t) \text{Sewer} + 0.025 (t-1) \text{CI Pop} + 0.302 \Delta \text{VaccTE} (0.05) + 0.114 \\ &\Delta \text{QaccTE} (0.005) \end{aligned}$$

$$\begin{aligned} \text{Equation 3: } \Delta F_{10-15k} &= -0.125 \Delta F_{<5k} + 0.637 \Delta F_{5-10k} + 0.294 \\ &\Delta F_{>15k} - 0.224 (t-1) F_{10-15k} + 0.196 (t-1) \text{Sewer} + 0.145 \Delta \text{Sewer} \end{aligned}$$

$$\begin{aligned} \text{Equation 4: } \Delta F_{>15k} &= -0.282 \Delta F_{5-10k} + 0.603 \Delta F_{10-15k} - 0.278 \\ &(t-1) F_{>15k} + 0.145 (t-1) \text{Water} + 0.118 (t-1) \text{Sewer} + 0.046 (t-1) \\ &\text{CI Pop} - 0.384 \Delta \text{Vacc} F_{>10} (0.15) + 0.093 \Delta \text{QaccTE} (0.15) \end{aligned}$$

$$\begin{aligned} \text{Equation 5: } \Delta M \&C = 0.220 \Delta \text{Other} - 0.302 (t-1) M \&C - 0.015 (t-1) \\ &\text{FIR} + 0.138 (t-1) \text{CI Mfg} + 0.278 \Delta \text{Qacc} F_{<10} (0.05) + 0.121 (t-1) \\ &\text{VaccTF} (0.05) \end{aligned}$$

$$\begin{aligned} \text{Equation 6: } \Delta \text{Other} &= 0.456 \Delta M \&C + 0.081 \Delta \text{Ret} - 0.132 \Delta \text{FIR} \\ &+ 0.106 (t-1) M \&C - 0.194 (t-1) \text{Other} - 0.414 \Delta \text{VaccTE} (0.15) + 0.095 \\ &(t-1) \text{QaccTF} (0.05) \end{aligned}$$

$$\begin{aligned} \text{Equation 7: } \Delta \text{Ret} &= 0.440 \Delta \text{Other} - 0.117 (t-1) F_{>15k} + 0.126 (t-1) \\ &\text{Other} - 0.363 (t-1) \text{Ret} + 0.165 (t-1) \text{CI Ret} + 0.213 \Delta \text{VaccTF} (0.15) \\ &- 0.064 (t-1) \text{QaccTF} (0.05) \end{aligned}$$

$$\begin{aligned} \text{Equation 8: } \Delta \text{Svc} &= -0.252 \Delta \text{Other} - 0.510 (t-1) \text{Svc} + 0.022 (t-1) \\ &\text{FIR} + 0.620 \Delta \text{Water} + 0.240 \Delta \text{Sewer} + 0.564 \Delta \text{QaccTF} (0.05) + \\ &0.390 (t-1) \text{VaccTF} (0.05) \end{aligned}$$

$$\begin{aligned} \text{Equation 9: } \Delta \text{FIR} &= 0.614 \Delta \text{Other} + 0.020 (t-1) \text{Svc} - 0.159 \\ &(t-1) \text{FIR} + 0.110 (t-1) \text{QaccTF} (0.05) \end{aligned}$$

THE 626 ZONE EASTERN MASSACHUSETTS EMPIRIC SUB-MODEL

In addition to the nine output variables contained in the model, there were four additional variables for which forecasts were desired: total population (Pop); automobile ownership (Autos); school enrollment in grades K-8 (School, K-8); and school enrollment in grades 9-12 (School, 9-12). These variables were not included in the main model owing either to their being highly correlated with other output variables, or to suitable data being available for only one of the calibration time points.

These variables, consequently, were incorporated into a sub-model which was calibrated using data from only one point in time (1963). The equations comprising the sub-model are written out below. The notation is the same as that used earlier for describing the main model structure, with the additional variables TF (total number of families) and Med FI (median family income in terms of 1959 dollars multiplied by TF).

$$\begin{aligned} \text{Equation 1: } (t) \text{Pop} &= 0.944 (t) \text{TF} + 0.016 (t) \text{Water} + 0.034 (t) \\ &\text{QaccTE} (0.15) \end{aligned}$$

Equation 2: (t) Autos = 0.871 (t) Med FI + 0.164 (t) Water -
0.042 (t) QaccTF (0.15)

Equation 3: (t) School, K-8 = 0.918 (t) TF + 0.154 (t) Water -
0.065 (t) QaccTF (0.15)

Equation 4: (t) School, 9-12 = 0.874 (t) TF + 0.095 (t) Sewer +
0.037 (t) QaccTF (0.15)

The sub-model is forecast following forecasts with the main model. These latter forecasts are used to derive (t) TF and (t) Med FI for use in the sub-model. The other input variables required for sub-model forecasting (utility service and accessibilities) represent policy variables.

The reliability check performed on the sub-model (i. e., the comparison of observed with "forecast" 1963 values) yielded the following results:

Category	453 Zones			104 Districts		
	RMS Error	RMS Error Ratio	R ²	RMS Error	RMS Error Ratio	R ²
Total population	643	0.104	0.984	2,477	0.092	0.991
Automobile ownership	410	0.229	0.915	1,179	0.151	0.963
School enrollment, K-8	211	0.220	0.929	632	0.151	0.969
School enrollment, 9-12	71	0.211	0.939	239	0.164	0.966

Commercial Activity Location Model

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•ONE important component of the transportation planning process is the forecasting of future travel demand. Usually, this is closely tied to several land-use forecasts; one of these is a forecast of commercial land. This paper describes a model to predict the most likely locations of future commercial activity in an urban area.

People are consumers. They satisfy their demand for commercial goods and services chiefly by traveling to commercial land. An increasing population will cause a corresponding increase in the demand for commercial goods. It is the magnitude and location of this demand increase which will determine the size and location of future commercial centers.

This model defines a measure of consumer demand for commercial goods. It simulates the movement of people traveling to satisfy this demand. If the locations of future demand are known (forecast independently), the places where this future demand is satisfied can then be found by simulation. The model determines locations where the expected growth in satisfied demand is high; these are potential sites for future commercial development.

DEFINITIONS AND THEORY

The first task is to define an accurate measure of the demand for commercial goods. Because we are dealing with the movement of people to land, one obvious measure of commercial demand is person trips to commercial land; that is, person trips which are measured at traffic origin zones and that are known to have a commercial land use at zone of destination. (For convenience, the traffic analysis zone was chosen as the geographical unit of measurement.) These trips should also be constrained by trip purpose; for example, work trips to commercial land cannot reasonably be included in a measure of consumer demand. Furthermore, these person trips might be weighted by household or family income to add a "spending power" dimension to the measure of demand. Thus, we shall define the demand for commercial goods as a special class of person-trip origins weighted (optionally) by income; the particular trip purposes and land-use types used in applications of the model are described later.

Commercial establishments compete for consumer demand. A measure of the competition that the commercial establishments in one zone exert on the person trips in another zone is defined; the sizes of the establishments are measured by land area. Suppose that D_j units of commercial land exist in zone j . Suppose also that t_{ij} is the trip-driving time between zone i and zone j . Then the competition on the trips in zone i due to the commercial land in zone j is defined by the expression D_j/t_{ij}^x ; x is an exponent which measures the relative importance of driving time and will be examined in detail in the next section. Because all zones which have commercial land compete for the trips in zone i , the equation

$$C_i = \sum_{\text{all } j} \left(\frac{D_j}{t_{ij}^x} \right)$$

defines an expression for the total competition on the trips in zone i , C_i , from all study area zones. The zone table of C_i values is referred to as the competition surface.

The model allocates the trips which originate in a given zone to destination zones according to the proportion of total competition on the origin zone which is due to commercial land in a destination zone. For example, if zone i contains 200 trip origins, if C_i equals 100 units of competition on zone i , and if 25 of these 100 units are due to commercial land in zone j , then $(25/100)$ or one-fourth of the 200 trips will be allocated from zone i to zone j .

In symbols, suppose that T_i trips originate in zone i and that C_i is the total competition on these trips (if the option to weight trips by family income is used, T_i would represent "trip-dollars"). We know that $(D_j/t_{ij}^x)/C_i$ is the fraction of total competition on zone i which is due to commercial land in zone j . Thus, the number of trips allocated to zone j from zone i is the foregoing fraction multiplied by T_i . The total trips allocated to zone j from all study area zones can be calculated in this way. The equation

$$AT_j = \sum_{\text{all } i} \left\{ \left[\left(\frac{D_j}{t_{ij}^x} \right) C_i \right] T_i \right\}$$

is the symbolic representation of the total trips allocated to zone j . This method of trip distribution is the familiar "gravity" formula. The zone table of AT_j values is referred to as the trip surface.

The preceding material describes how the model simulates the movement of people to commercial establishments—people traveling to satisfy their demand for commercial goods and services. This simulation technique is employed in three distinct model phases which are described in the following sections of this paper.

It should be noted that it is not strictly necessary for the demand and competition variables to be defined as they have been. For example, one might wish to use population instead of trip origins as the measure of demand, or airline distance instead of travel time as the measure of spatial separation. In fact, the use of employment instead of land as a measure of commercial establishment size (competition variable) has been examined; the results of this research are encouraging and are given in the next section. The calibration phase of the model is a handy tool for testing the accuracy of particular variables.

THE CALIBRATION PHASE

Description

The purpose of this phase is to check the accuracy of the model using present data. A selected class of zone trip origins is obtained (from survey), and these trips are allocated by the model to zones of destination. This is done according to the simulation technique previously described.

Allocated trips are then compared to the actual trip destinations (from survey). The comparison is made on a district basis (aggregates of zones), and several measures of estimating accuracy are obtained. In this way, the variables which give the best estimates can be selected. For example, one important function of this phase is to determine the "best" value of the travel-time exponent—assuming that the trip class, land-use type, and interzonal travel times have been previously specified and are held constant. This phase proved valuable in examining relationships among variables and in evaluating the usefulness and accuracy of the model.

Application

The first tests of this model were made using 1962 data from the files of the Niagara Frontier Transportation Study. The following base-period inputs are required for the calibration phase:

1. Person trips by zone of origin by land use at zone of destination.
2. Person trips by zone of destination by land use at zone of destination.

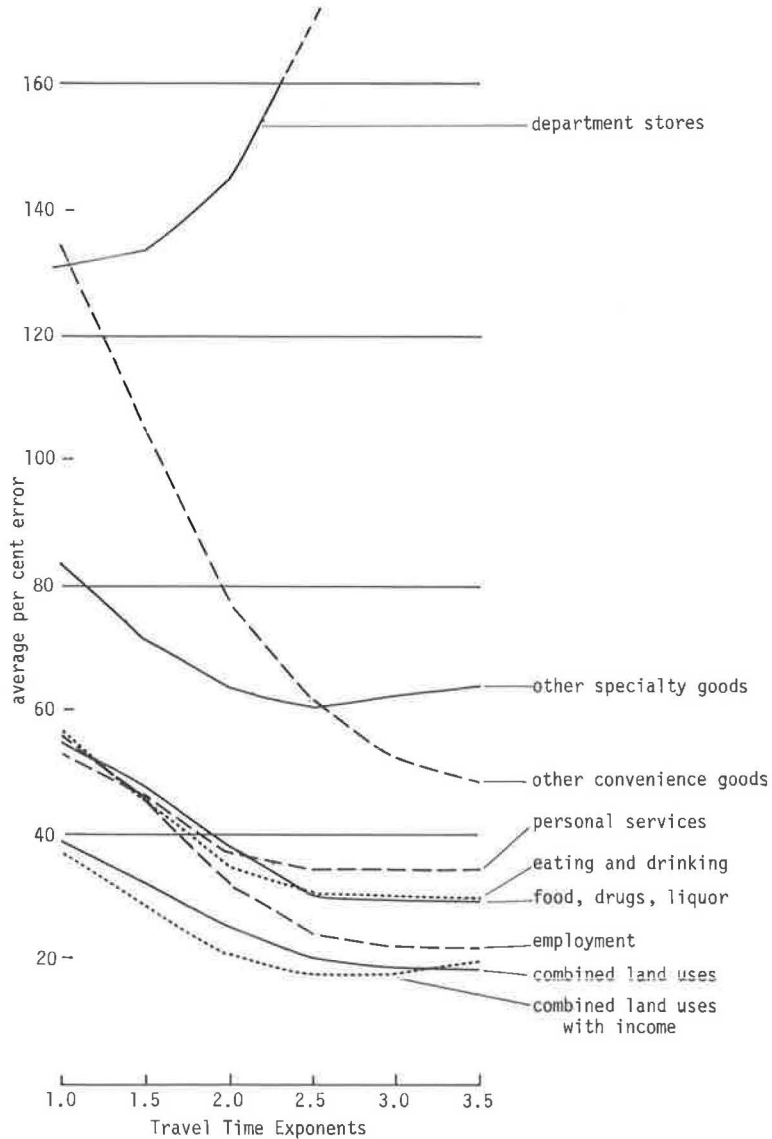


Figure 1. Simulation of nonwork trips to commercial land, using 1962 travel times.

3. Commercial land use by zone.
4. Interzonal travel times for all possible pairs of traffic zones.
5. Income factors (optional) by zone.

Land-use data were available by detailed categories, the following of which were selected for extensive testing: (a) food, drug, and liquor stores; (b) eating and drinking places; (c) department stores; (d) other specialty goods (shopping goods) stores; (e) other convenience goods stores; and (f) stores providing personal services. These data did not include large parking areas.

Person-trip information was obtained by origin and destination zone and was separated into categories according to the land uses just mentioned (at zone of destination). Moreover, all work-purpose trips (purpose at destination) were excluded from these

tests; the trip purposes remaining included the following: (a) shopping, (b) social-recreation, (c) eat meal, (d) personal business, (e) serve passenger, and (f) ride as a passenger. For the selected land-use categories, most of the trips were shopping trips.

Inter-zonal travel-time data for the 1962 highway network were obtained from the Schneider traffic assignment program, using the capacity restraint feature. This input was constant for all tests of the calibration phase. For the zone income factors, U. S. Bureau of the Census data for 1960 were used. The median incomes of families and unrelated individuals were obtained by census tract and were converted into zone factors.

With travel times invariate, tests were made to determine the responsiveness of the model to land-use type, travel-time exponent, and income. In general, all of these factors were significant. As was mentioned previously, the calibration phase compares model-allocated trip destinations with actual trip destinations on a district basis; the average absolute percentage difference of actual trips vs estimated trips is one criterion used to measure accuracy.

Figure 1 shows some of the results of these tests. Model accuracy is plotted against the travel-time exponent for different land-use types. As an example, consider the curve for the food, drug, and liquor stores category. The points for this curve were obtained by running the calibration phase for this land-use type for the six travel-time exponents. In particular, person trips at zone of origin (known to have a food, drug, or liquor store at destination) were allocated by the model to destination zones, using food, drug, and liquor store land as the competition variable. There were 195,584 nonwork trips in this category.

As Figure 1 shows, the accuracy of simulation improves as the travel-time exponent increases—average percent difference declines from 56 percent for exponent 1.0 to 30 percent for exponent 3.5. The fact that simulation accuracy improves as the exponent increases is a meaningful result. Higher exponents increase the importance of driving time; therefore, model accuracy should improve for higher exponents if the trip type under consideration was one for which driving times are relatively significant. Previous research (1) has indicated that driving times are more significant for shopping trips to convenience goods land than for shopping trips to specialty goods land (convenience goods are those purchased frequently and are usually low-cost items).

Because the food, drug, and liquor store category is one of the convenience goods land uses, the model responds in a reasonable way for this category. Indeed, the same statement can be made for other categories. All of the so-called convenience goods land uses have simulation accuracies which improve as the travel-time exponent increases. These include: (a) eating and drinking places (88,246 trips); (b) person services land (23,818 trips); and (c) other convenience goods stores (9,954 trips). Specialty goods land uses exhibit an opposite effect. The simulation of trip movements to department stores (99,043 trips) decreases in accuracy as the travel-time exponent increases; this is to be expected, since driving times are less significant for these trips. The "other specialty goods" category (44,466 trips) appears to be an intermediate category with respect to the importance of driving time.

One disturbing element of these results is the uniformly low accuracy in simulating department store trips. Two possible explanations can be put forward: (a) the model is inappropriate for these trips, or (b) survey sampling may have been ineffective for these trips, since Saturday was not included as a sampling day in this study area. The importance of Saturday as a shopping day is well-known. If most department store trips are made on this day, commercial trips may be underrepresented in this category. Since the model did respond in a reasonable way to the travel-time exponent for these trips, there is some justification for claiming the second explanation instead of the first.

The high inaccuracy in the other convenience goods category has little significance, for the trips in this group constitute only 3 percent of all convenience goods trips.

Considerable improvements in the accuracy of simulation occur when the model is run for combinations of land-use categories. One example is presented here: the demand variable used is person trips to convenience goods land uses (food, drugs, and liquor; eating and drinking; person services; and other convenience goods) while the competition variable is total commercial land (convenience goods land plus specialty

goods land). There was a total of 317,602 nonwork trips in the combined group. This particular combination implies that specialty goods land has a role in attracting convenience goods trips—an assumption which is not unreasonable for large clusters of commercial activity. By making this choice of categories, we are able to make forecasts of commercial activity which include specialty goods land; moreover, this eliminates the large amount of error due to department store trips. Of course, the trips to specialty goods land uses will then have to be predicted by some other method.

Figure 1 shows that average percentage error of simulation is reduced to 19 percent for this combination of categories (lowest unbroken curve). The combining of land-use categories in this way appears to have a strong effect on simulation accuracy; this is probably due to smaller errors of sampling variability.

The effect of weighting trip origins by income was also tested, using the above combination of categories. The result was a small reduction in percentage error for most travel-time exponents. Most of this improvement in accuracy occurred in central business district zones, where the model was overestimating trips significantly; one general effect of income weighting in these tests was to remove trips from the downtown area and allocate them elsewhere.

On the basis of the calibration tests, it was decided to produce a forecast of commercial activity using the preceding combination of categories. Trip origins were weighted by incomes and a travel-time exponent of 3.0 was chosen. Figure 2 shows a comparison of model-estimated trips with actual (survey) trips for this choice of parameters. This simulation has a 17.5 average percent error.

Employment as Competition Variable

Some preliminary tests of the model were made using employment as the competition variable. Employment was measured by the number of work trips having particular commercial land uses at zone of destination. These trips replace commercial land as the "attractor" of nonwork trips in the model.

A dashed curve in Figure 1 shows the results of these tests. In this case, the work trips used were those having any convenience goods or specialty goods land use at destination. The demand variable (trip origins) was nonwork trips having convenience goods land at destination; these trips were also weighted by income. Again, the accuracy of simulation improves as the travel-time exponent increases: 57 percent error for exponent 1.0 to 23 percent error for exponent 3.0.

We conclude from this that employment appears to be an accurate alternative to land as the competition variable. Of course, if it is desired to use the model for land-use forecasting, an additional step would be required to make the conversion from employees to land. Otherwise, commercial activity could be forecast in units of employment instead of in land units. However, it will be assumed in the remainder of this paper that land is the competition variable.

THE INITIALIZATION PHASE

Description

In this phase of the model, the region is examined under "present" conditions to determine whether additional commercial land can be supported. Every zone is considered as a possible location for new land. An initial allocation of trip origins is made to zones using the present land-use pattern and traffic network; this determines the

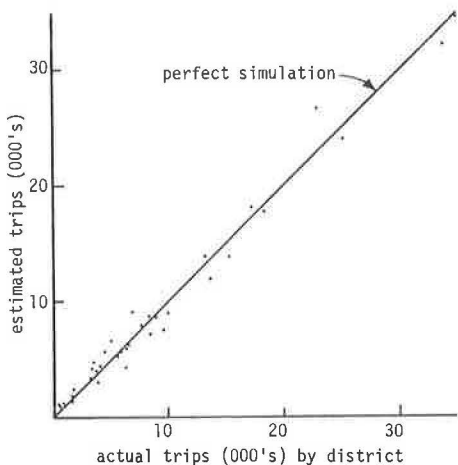


Figure 2. Comparison of model-allocated trips with actual trips for combined land-use categories.

base or initial trip surface. Next, an independently defined commercial activity center size is selected; this commercial center, or "commercial unit," may be defined in terms of floor area or site area. This amount of commercial land is then temporarily added to the existing land in a particular zone i , and a second allocation of trips to destination zones is performed. If the trips attracted to zone i in the second allocation are compared to those attracted in the initial allocation, the number of trips which have been drawn to the zone because of the new commercial center can be measured. We shall refer to the difference between the two allocation values as the trip potential of this size commercial center in zone i .¹ The trip potential is then determined independently for each zone.

The foregoing procedure yields a set of numerical values representing a trip-potential surface. Zones having high trip-potential values represent possible sites for the commercial center in question. From this trip potential surface, the "best" zone in which to locate the new center can be determined. In this connection, the zone trip-potential values are first aggregated into districts, and an average trip-potential value is calculated for each district; the best zone is then chosen to be the zone with the highest trip potential within the district having the highest average potential. This technique was used to overcome possible inaccuracies due to sampling variability.

This selection process guarantees that a best zone is determined. However, the best may not be good enough. Some criterion is needed to determine whether the new trips attracted to the zone justify the new center. Accordingly, a minimum trip-generation rate for the specified size of commercial center is required as an independent input. The minimum rate represents the smallest number of new trips required to travel to a zone on an average travel day in order for the new center to be established; it is specified in units of trips per 1000 sq ft of land and is referred to in this paper as the trip sufficiency rate. If the trip potential in the selected best zone equals or exceeds this minimum rate, the center may be located there. If not, another zone will be tried.

Other criteria may be used to determine the feasibility of locating a new commercial center in the selected zone. The zone may be required to contain a sufficient amount of vacant usable land. Also, zoning laws or land cost may be such as to prohibit commercial development in certain locations. These criteria may be tested as options.

If a selected zone satisfies all of the preceding requirements, commercial land corresponding to this size commercial center is then assumed to exist in that zone. This zone has therefore become more competitive—its ability to attract trips is greater. The surface of competition must then be changed to account for this new land. Also, the new trips attracted to this zone are added to the initial trip surface. After a new commercial center is located and the competition surface is revised, the entire process is repeated. A new trip potential surface is obtained, again assuming that new land exists in each zone.

Three commercial center sizes are permitted; each size may have a distinct trip sufficiency rate and travel-time exponent. In this phase, the model will locate all possible commercial units which satisfy the trip- and land-sufficiency criteria. It operates iteratively and will continue until no additional satisfactory sites can be found. Because we are dealing with present conditions, it is possible that new commercial land cannot be supported anywhere in the region. The model determines if this situation exists; if it does, the final or "forecast" phase is begun immediately. Commercial centers which are located in the initialization phase are "permanently" added to the present land-use pattern before the forecast phase is executed.

¹The model does not attempt to distinguish between persons traveling to the new center in a zone and those traveling to previously existing land in the zone. Conceivably, new commercial development in a particular zone could induce more trips to existing land. Furthermore, some new commercial developments might consist of additions to previous commercial centers. For these reasons, this model is not referred to as a "shopping-center" model.



Figure 3. Trip-potential surface, 1962 (each dot equals 150 trips).

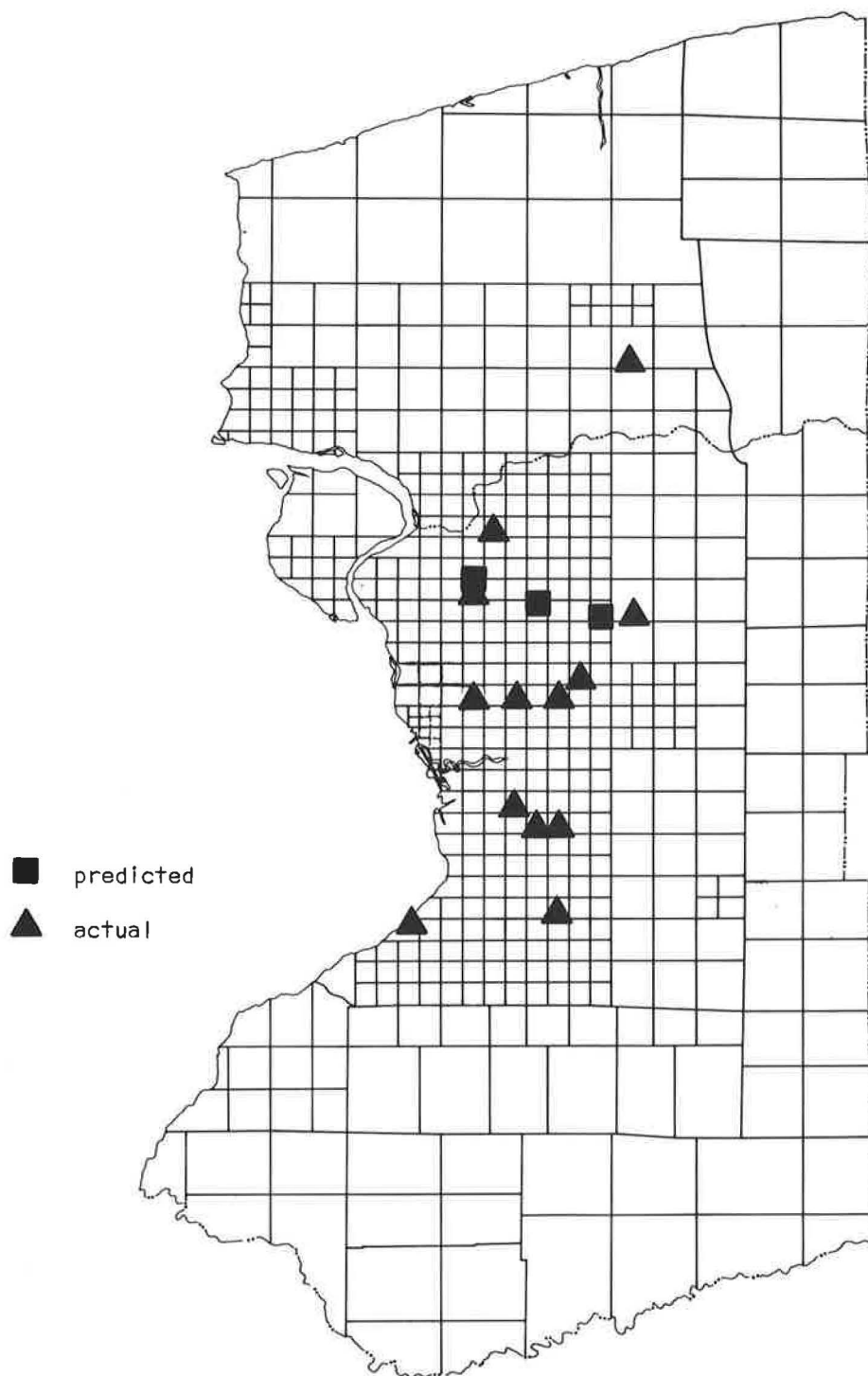


Figure 4. Model-allocated commercial units, 1962, shown with post-survey locations of planned commercial development.

TABLE 1
SEQUENCE OF THIRTY BEST ZONES FOR
COMMERCIAL UNIT LOCATION, 1962 CONDITIONS

Rank	Zone	Trip Potential	Sufficient Land
1	57	3,004	No
2	115	2,565	Yes
3	200	2,556	Yes
4	64	2,502	No
5	104	2,445	Yes
6	109	2,254	Yes
7	66	2,145	Yes
8	103	2,129	No
9	54	2,073	No
10	106	2,061	Yes
11	170	2,052	Yes
12	63	2,045	Yes
13	169	2,026	Yes
14	62	2,014	No
15	116	1,966	Yes
16	105	1,894	Yes
17	52	1,892	No
18	102	1,881	No
19	111	1,778	Yes
20	123	1,732	No
21	26	1,674	No
22	117	1,658	Yes
23	53	1,642	No
24	110	1,634	Yes
25	122	1,623	Yes
26	27	1,596	No
27	87	1,543	Yes
28	98	1,506	Yes
29	118	1,481	Yes
30	171	1,471	Yes

Application

The initialization phase was run for the Niagara Frontier Area (for 1962) using the trip-class, land-use combination, exponent, etc., chosen from the results of the calibration phase. A commercial center size of 500,000 sq ft (not including parking) was selected. Twelve study area traffic analysis zones, known to contain major commercial activity clusters, were examined to obtain the trip sufficiency rate for this center size. This rate was 7.3 trips (nonwork trips to convenience goods land) per 1000 sq ft of land (convenience goods land plus specialty goods land). Under this assumption, no satisfactory zones for additional commercial land could be found. Table 1 shows the 30 zones having highest trip potentials for this run and Figure 3 shows the trip potential for all zones within the Niagara Frontier corridor area (2). It is apparent from Figure 3 that the downtown area possesses little potential for further commercial development.

Since the surveys for this study area were completed, several major commercial developments have been planned. As

a further test of the model, it was decided to reduce the trip sufficiency rate to a level which would permit the location of new centers. This rate was 4.0 trips per 1000 sq ft, which is approximately the study area average rate for the class of trips in question. Using this rate, three commercial centers of 500,000 sq ft were located. Figure 4 shows the locations of these centers and also the locations of the actual planned developments. The results are quite reasonable; one unit is within a zone where development is planned and another is located in a zone adjacent to planned development.

THE FORECAST PHASE

Description

In the forecast phase it is assumed that a certain time period has elapsed and that some regional growth has occurred. Thus, the pattern of demand for commercial goods and services will have changed. Because commercial demand is measured in units of person-trip origins, this phase requires an independent forecast of this variable by zone. The changes in traffic network travel times and zone household income factors should also be predicted, but these are optional.

The magnitude of growth in commercial demand (trip origins) controls the number of new commercial centers that can be located in this phase. For example, if there are 300,000 trip origins in the present year and if 500,000 trip origins are forecast for the future year, then 200,000 trips are available to be distributed to new commercial centers.

The allocation technique for these future trip origins is the same as is used in the initialization phase. Again, three commercial center sizes are permitted, each having a distinct trip sufficiency rate and travel-time exponent. The first center size is selected, and a trip potential surface is obtained for this size (the final trip surface obtained from the initialization phase is used as the base trip surface in determining trip potential in this phase). A best zone is chosen and is tested for trip sufficiency and vacant land availability. The process of locating new commercial units is then repeated as before. Moreover, each time a new unit is located, the new trips attracted to its zone are removed from the pool of available trips.

TABLE 2
ZONES RECEIVING COMMERCIAL LAND, 1985

Zone	Land Area (000's sq ft)	Added Trips
204	500	14,720
234	500	6,236
137	500	10,059
372	500	5,340
163	500	6,604
203	500	7,189
180	250	6,710
119	250	3,388
183	250	6,126
117	250	3,200
248	250	3,964
Total	4,250	73,535

This phase operates until one of the following conditions occurs: (a) the supply of trip origins to be distributed is exhausted; (b) any additional commercial centers will not attract a sufficient number of trips; or (c) an independent estimate of the number of new centers to be located during the forecast period has been made, and this number of centers has been located. In practice, condition (a) is very unlikely to occur since some of the growth in trips will probably be absorbed by existing commercial land.

One important option available in this phase is known as the "planned centers option." If it is known that a certain amount of land is committed for commercial development at some future date, this information may be communicated to the model before it locates any additional future units. In this way, the competitive effect of the planned center will be a factor in any subsequent choice of commercial center location (by the usual model procedure). This option also provides a measure of the trip potential of these planned centers and may therefore be used to test the feasibility of such locations. The option may be used in conjunction with the usual forecast, or independently of it.

Another option in this phase locates "neighborhood" commercial centers in high-potential zones which did not receive new centers during the standard run. The purpose

TABLE 3
1985 FORECAST SUMMARY BY DISTRICT

District	Commercial Land		Trip Destinations Allocated				Trip Density	
	1962	1985	1962	1985	Diff.	Rate	1962	1985
0	1,880	1,880	4,047	3,605	-443	0.89	2.2	2.0
10	7,761	7,761	17,707	15,435	-2,273	0.87	2.3	2.0
20	1,838	1,838	5,579	5,144	-435	0.92	3.0	2.8
21	1,758	1,758	8,444	9,976	1,532	1.18	4.8	5.7
22	3,097	3,097	11,727	11,531	-196	0.98	3.8	3.7
23	3,179	3,179	7,013	7,058	45	1.01	2.2	2.2
24	2,121	2,121	8,614	7,976	-639	0.93	4.1	3.8
25	227	227	1,277	1,692	416	1.33	5.6	7.5
30	2,521	2,521	7,630	8,260	631	1.08	3.0	3.3
31	3,283	3,283	26,654	25,658	-996	0.96	8.1	7.8
32	2,696	2,696	24,090	31,396	7,306	1.30	8.9	11.6
33	728	728	4,257	8,273	4,016	1.94	5.8	11.4
34	1,324	1,324	4,269	16,453	9,185	2.26	5.5	12.4
35	1,362	1,362	6,433	8,513	2,080	1.32	4.7	6.3
40	647	647	4,064	9,345	5,281	2.30	6.3	14.4
41	3,415	3,415	32,274	37,443	5,169	1.16	9.5	11.0
42	1,692	2,192	13,749	30,227	16,478	2.20	8.1	13.8
43	834	834	2,308	8,613	6,304	3.73	2.8	10.3
44	1,526	2,026	5,825	24,734	18,909	4.25	3.8	12.2
45	800	800	2,934	10,104	7,170	3.44	3.7	12.6
50	279	779	1,525	10,977	9,452	7.20	5.5	14.1
51	3,540	4,040	17,924	52,093	34,168	2.91	5.1	12.9
52	2,939	3,939	8,833	31,737	22,904	3.59	3.0	8.1
53	1,514	1,514	8,761	24,013	15,252	2.74	5.8	15.9
54	230	230	766	7,263	6,497	9.49	3.3	31.6
55	1,145	1,895	4,467	19,478	15,011	4.36	3.9	10.3
60	7,790	7,790	34,768	53,055	18,287	1.53	4.5	6.8
61	1,154	1,154	5,104	22,260	17,156	4.36	4.4	19.3
62	1,628	1,628	3,235	13,457	10,222	4.16	2.0	8.3
63	153	153	555	5,013	4,458	9.03	3.6	32.8
64	1,521	1,521	6,012	21,631	15,620	3.60	4.0	14.2
65	1,042	1,042	5,406	17,288	11,883	3.20	5.2	16.6
66	216	716	1,043	5,435	4,391	5.21	4.8	7.6
70	1,111	1,111	3,653	11,399	7,746	3.12	3.3	10.3
71	5,875	5,875	13,655	36,761	23,106	2.89	2.4	6.5
Total	72,626	76,876	317,602	613,294	295,692	1.93	4.4	8.0

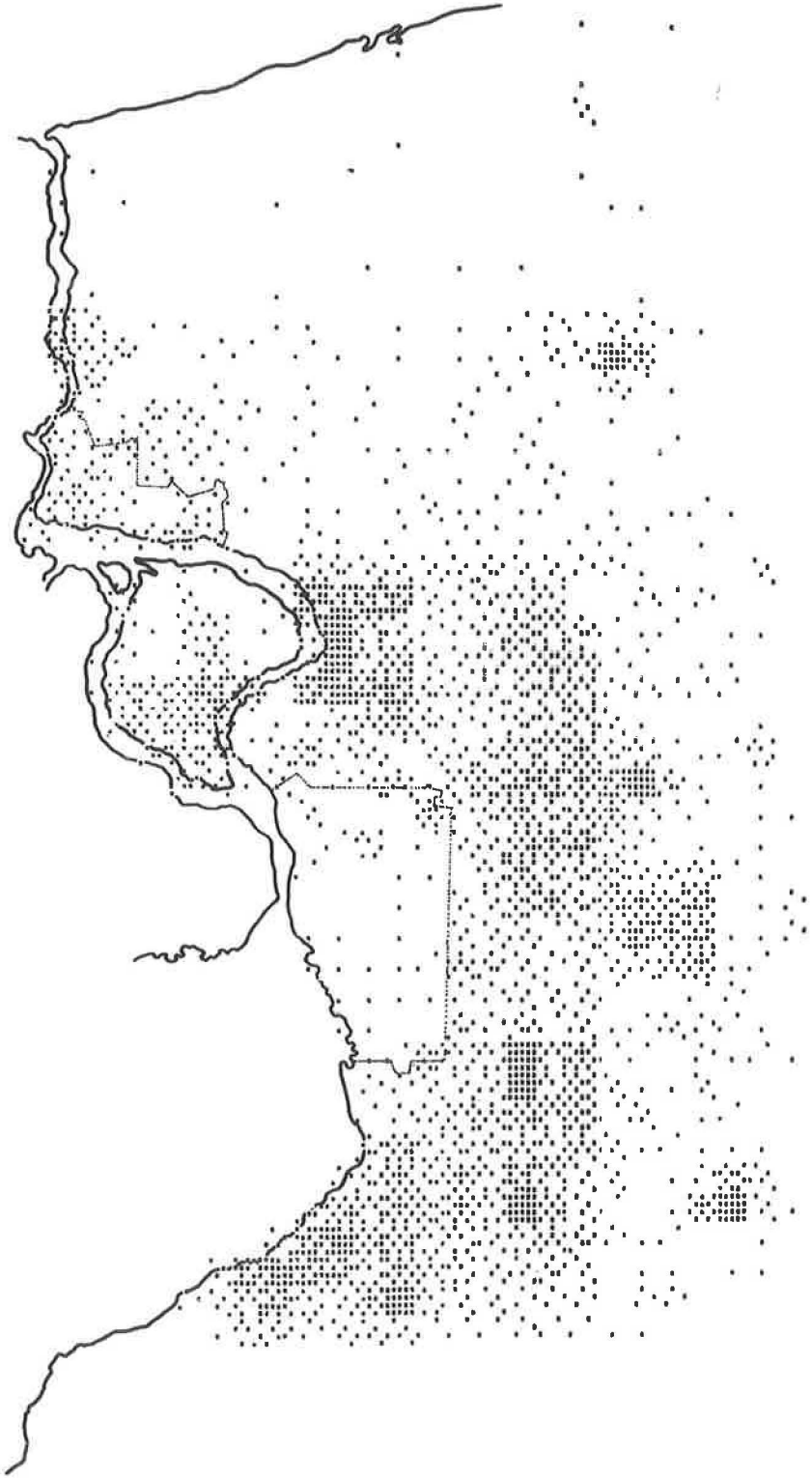


Figure 5. Trip-potential surface, 1985 (each dot equals 300 trips).

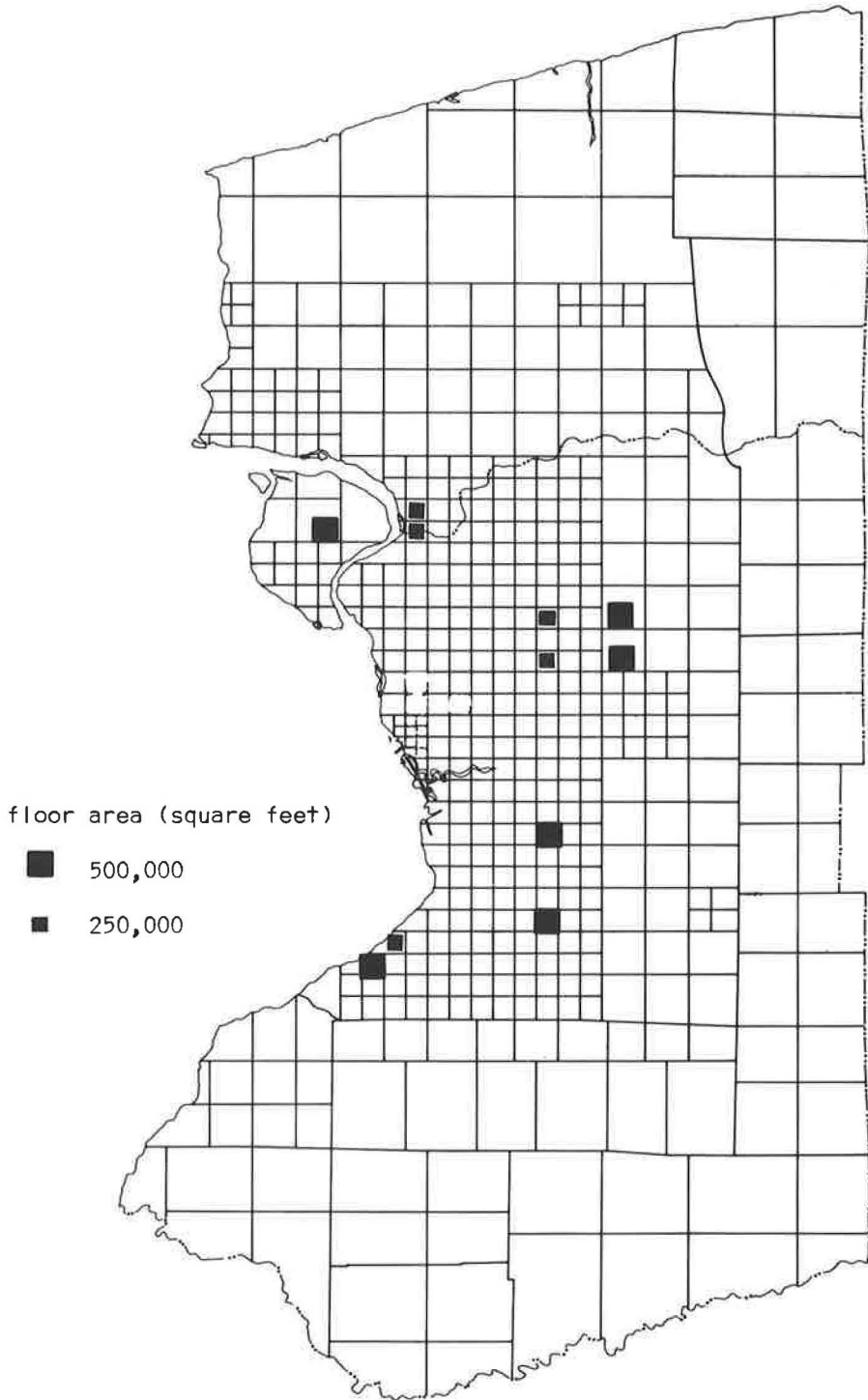


Figure 6. New commercial unit locations.

of this option is to allocate a token amount of land to zones having considerable "unused" potential. A trip sufficiency rate is also required for these centers, but the updating procedure of the model is not used.

After the forecast for one time period has been completed, this phase may be repeated for another time period—provided that the required set of inputs is available. Thus a 20-year forecast could be produced in as many cycles as desired.

The primary output of the model (all phases) is a list of traffic zones receiving new commercial land, the amount of additional land in these zones, and the relative trip-attracting potential of each. Other intermediate outputs are available, as well as the option to produce study area maps of commercial land and trips in both present and future periods.

Application

This phase was also applied to the Niagara Frontier Area to produce a forecast for 1985. The change in the pattern of commercial demand is measured by the change in the distribution of person-trip origins. This variable was forecast by using the regional-growth model developed by the staff of the Subdivision of Transportation Planning and Programming (3). Appropriate trip-origin growth rates were obtained on a district basis and were then applied by zone to the special class of 1962 commercial trip origins used in this particular forecast (nonwork trips to convenience goods land). This technique produced an increment of 296,000 trip origins.

An approximation of 1985 median family incomes by zone was also prepared by performing a district trend analysis on U. S. Census data. These income factors were then used to weight the preceding trip origins. Travel-time data for this forecast were again obtained from the traffic assignment program, using the 1962 highway network plus committed additions or improvements. The three centers located as a test of the initialization phase of the model were not included in this run.

Two commercial center sizes were defined—500,000 and 250,000 sq ft. Figure 5 shows the zone-trip potentials for the first iteration. The trip sufficiency rates assumed for these sizes were 7.5 and 8.5 trips per 1000 sq ft.

Eleven centers were located. Table 2 gives a list of the zone locations of the centers; Figure 6 shows their location within the study area. It will be observed that seven of these eleven centers are within or adjacent to zones in which known commercial development has been planned since survey time (Fig. 4). Also, a comparison of the 1962 and 1985 trip potential in Figures 3 and 5 reveals the apparent shift of commercial development potential as the region grows.

Table 3 gives the final district summary of this forecast. Figures 7 and 8 are computer-printed maps of commercial land and commercial trip destinations as of 1985. The planned centers and neighborhood centers options were not exercised in this forecast.

SUMMARY AND CONCLUSIONS

This paper describes a model to predict future locations of commercial activity in any urban area. Figure 9 shows the model components in block diagram form. The model uses a gravity formula to simulate the movement of persons to commercial land. Consumer demand for commercial goods and services is defined as a special class of person-trip origins; person-trip destinations represent satisfied consumer demand. The model allocates person-trip origins to their commercial destinations, using an existing distribution of commercial land and highway network travel times.

The calibration phase compares model-estimated trip destinations with actual trip destinations for the purpose of measuring predictive accuracy. This phase has also been used to demonstrate the reasonable behavior of the model to the following variables: travel-time exponent, land-use type, and family income.

The initialization and forecast phases predict future zone locations of commercial activity. In the initialization phase, commercial units are added to the present land-use pattern—assuming that no regional growth has occurred. This phase answers the



Figure 7. Commercial land, 1985 (each dot equals 100,000 sq ft).



Figure 8. Commercial trip destinations, 1985 (each dot equals 500 trips).

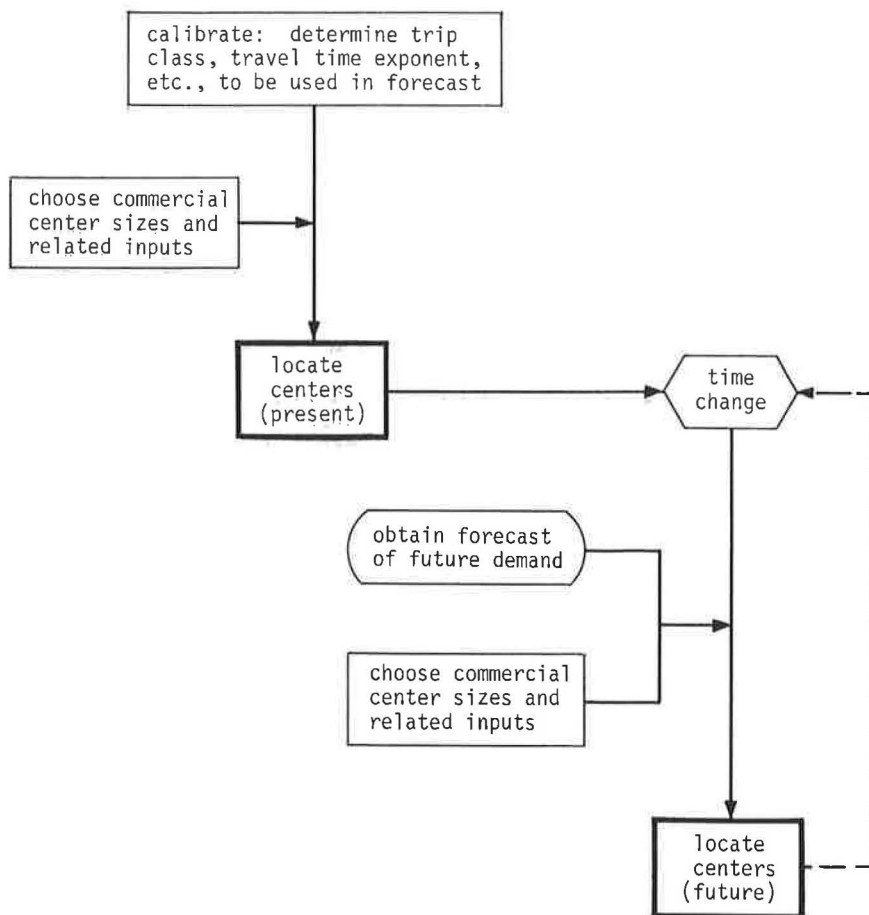


Figure 9. Diagram of commercial activity location model.

question: Can additional commercial development be supported in the region, given the current distribution of commercial activities?

In the forecast phase, it is assumed that a certain time period has elapsed and that the distribution of the demand for commercial goods will have changed. This change, specified in terms of future person-trip origins, is an input to this phase.

In the latter two phases, three sizes of commercial centers (in land area) are permitted. Each size requires a trip sufficiency rate to determine the adequacy of selected sites. The examination of selected sites for available vacant land and zoning or land cost restraints is also a feature of the model. In the forecast phase, plans for commercial sites which develop after study area land-use data are obtained can be used as inputs. This phase may be repeated for successive time periods.

Initial applications of the model have been encouraging. Sites selected for future commercial unit locations in the initialization phase have been very near to known commercial developments in the one study area tested. The sites selected by the forecast phase also compare favorably with known developments and are in areas where regional growth is expected to be intense. The model appears to be a very useful transportation planning tool.

ACKNOWLEDGMENT

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