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Foreword

The purpose of the five papers in this RECORD is to present a picture of the "state of the art" concerning the strength of concrete—strength meaning not only compressive, but also the other strengths normally considered in designing portland cement concrete pavements and structures. The authors in presenting and comparing their data with those of other investigators have pointed out possible gaps in our knowledge. In addition, inadequacies in existing strength-test methods are indicated. As design and construction methods become more refined, even small inadequacies assume greater importance. These papers should be of interest to materials engineers as well as to those involved in research on, and design of, concrete structures.

In the first paper, Mather illustrates how strong, strong-concrete can be and in so doing presents some "do's and don'ts" for those who would like to make their weak concrete stronger. This paper points out that routine production of concretes having 90-day compressive strengths of 10, 000 psi, or more, is practical.

Smith and Tiede present a world-wide review of accelerated strength-testing procedures which have been used to estimate the 28-day compressive strength at early ages. They also describe a new procedure, using autogeneous curing, which shows promise. The authors take issue with the whole concept of using 28-day strengths as a basis for judging concrete. In a discussion of this paper, Wilson, Zoldners and Malhotra have filled a gap in the data on accelerated strength-testing by presenting data obtained with lightweight aggregate concretes using a boiling water method.

Popovics explores the world literature in an attempt to determine whether acceptable approximations can be made of the relations between the various types of concrete strengths. He concludes that limited use of approximations is possible. However, full use of approximations is not possible at this time because of inadequacies in test procedures and in the assumptions used in interpreting test results. In a discussion of this paper, Malhotra emphasizes the difficulty of determining the "true" tensile strength of concrete.

Antrim discusses his investigation into the fatigue behavior of cement pastes and of the same pastes diluted with various aggregates. Data developed in this study are analyzed in an attempt to explain the mechanism of failure under cyclic loads. The author found that the fatigue characteristics of plain concrete are apparently governed by the fatigue characteristics of the cement paste.

Another step in the attempt to explain the mechanism of failure in plain concrete is undertaken by Scholer. He presents the data from an investigation designed to indicate the role of mortar-aggregate bond in the strength of concrete. He concludes that the amount of microcracking is dependent on the mortar-aggregate bond, but that initiation of the microcracking is primarily dependent on the properties of the mortar.

—T. W. Reichard

Contents

Stronger Concrete

BRYANT MATHER, U. S. Army Engineer Waterways Experiment Station, Jackson, Miss.

The most frequently encountered difficulty with any particular quantity of concrete is that the test specimens representing it fail to develop a high enough apparent compressive strength. In many cases the concrete represented by the specimens is actually strong enough for its intended use, since the test results are incorrect or the specified strength is in excess of that needed. In other cases the concrete represented is actually not strong enough.

There are many factors that affect concrete strength and many ways of making stronger concrete. For example, the practical routine production of portland cement concrete having a compressive strength consistently above 10,000 psi after 90 days moist-curing can be accomplished by careful selection of materials; mixture proportions; and mixing, placing, consolidating, and curing procedures. The most important single factor in the successful production of strong concrete is the maintenance consistently of an adequately low water-cement ratio. Concrete with a strength of 10,000 psi cannot be obtained from mixtures having water-cement ratios higher than 0.45 by weight; and in order to provide adequate workability, it will usually be necessary to use at least 7 bags of cement per cubic yard of mixed concrete. Similar methods to those that may be used to make strong concrete stronger may also be used to make weak concrete stronger.

OUR understanding of compressive strength of concrete would be facilitated if a theory of strength or a theory of failure existed that would adequately deal with this phenomenon. Siess (48) stated in 1958 that the most important need appeared to be for theories based on mechanical models which in turn are based on fundamental knowledge regarding the character and properties of the cement paste. He noted that phenomenological theories of failure are not capable of predicting deformation characteristics and suggested that the goal should be the development of hypotheses or theories capable of predicting both deformation and failure.

Three years later the question of the origin and nature of strength of concrete was again considered. The unsatisfactoriness of theories based on the assumption that concrete can be considered as ideal, continuous, and isotropic was noted, and consideration was directed to those theories based on the assumption that matter consists of particles held at certain spacings by fields of force. It appeared that knowledge of the kinds and numbers of bonds, their strength, and the time and condition of their formation is requisite to an understanding of the nature and origin of strength and the mechanism of fracture and failure of concrete (30).

In the absence of a theoretical background, including either an adequate theory of strength or an adequate theory of failure, and also a developed technology for the production and utilization of high-strength concrete, it is necessary to assume both that high-strength concrete may represent simple extrapolation from concretes of normal strength, for which there are abundant data, and that unusually high-strength concrete may involve relations that are other than simple extrapolations. The available data can be interpreted either way. Some of the data suggest that attempts to produce unusually high-strength concrete will involve encountering a

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"ceiling" which cannot be exceeded by progressive changes in proportions and materials of the same sort that have been found to be effective in raising strengths from low levels to medium levels. Other data suggest that no such ceiling exists. It has been suggested by some investigators that an apparent ceiling can be encountered either because of the properties of the aggregates or the specific nature of the aggregate-paste interfaces, particularly when the aggregate particles have surface textures and shapes that are not conducive to the development of high bond strengths. It has also been suggested that an apparent ceiling is frequently encountered in compression testing at high stress levels due to the flexibility of testing machines. Taking all available data into account, however, it is concluded that, for strengths up to at least 10,000 psi, no new principles and no new procedures will be required.

FACTORS AFFECTING COMPRESSIVE STRENGTH OF CONCRETE

If, from a given concrete mixture, a large number of test specimens are made, using molds of different sizes and shapes in which the concrete is consolidated to differing degrees, and these specimens are cured under different conditions for different lengths of time, and finally tested by different testing procedures and in different testing machines, a wide range of values for compressive strength will be obtained. A consideration of the effects of such factors has often been described as "factors affecting the strength of concrete." These, however, are merely the factors that affect the degree to which the strength indicated by the testing of a given specimen approaches the potential maximum strength that could be attained by a specimen of the given concrete mixtures under study.

Here we are concerned primarily with the more fundamental factors that affect what the potential strength of a concrete mixture can be. These factors depend on the inherent properties of the materials used in the mixture, on the proportions in which they are combined, and the manner in which they interact.

The factors affecting potential compressive strength of concrete will be considered in the following order: (a) characteristics of the cementing medium; (b) characteristics of the aggregate; (c) proportions of the paste; (d) paste-aggregate interaction; (e) mixing, consolidation, and curing; and (f) methods of measuring the strength.

Price (41), reviewing factors affecting concrete strength, presented data showing the effects on strength of age; type (composition) of portland cement; fineness of cement; brand of cement; air content of concrete; cement content of concrete; water-cement ratio; temperature, moisture conditions, and duration of curing; initial temperature of concrete; use of chemical admixtures; size of test specimens; proportions of test specimens; duration of load; and deterioration of concrete.

Wygant (60) stated:

> The strength of hydraulically bonded bodies can be higher than is usually realized. Concrete used for construction generally falls in the range 2000- to 4000-psi compressive strength. However, at least one concrete within my knowledge has achieved a strength of 15,000 psi, and strengths of 9000-12,000 psi are not uncommon. These are obtained using portland, high-alumina, or pure calcium aluminate cements. High strengths depend upon a balance of several properties. A low water-cement ratio, excellent compaction of a mix designed for density, a sufficient cement content, and careful curing are very important. The particle size distribution for such strengths is not simply arrived at. Its nature differs with the fracture characteristics or particle shapes of the aggregates and with the size of the larger aggregate.... For the highest strength, uniformly graded aggregates will not do.... Additions of cement above about 25% by volume usually are of no benefit.

Alexander (3) summarized work done at the (Australian) Commonwealth Scientific and Industrial Research Organization on factors controlling the strength and shrinkage of concrete. In this summary concrete was regarded as a chain of three links: cement paste, cement-aggregate bond, and aggregate.

Wig et al (59) reported results of 20,000 tests, involving 240 sands and 60 coarse aggregates. Their discussion of factors affecting strength, written on March 3, 1915, is as valid and pertinent today as it was then; these factors are as follows:

a. Aggregate. No type of gravel or stone aggregate can be said to be generally superior to any other type used as coarse aggregate in concrete. The range of qualities in any type is very great.

b. Workmanship. The method of mixing is of little importance so long as it results in a homogeneous mass. Differences in manipulation of the mixture by the workmen of several experienced concrete contractors may cause a maximum average variation in the compressive strength of the resulting concrete of 70% or more.

c. Consistency. The quantity of water added affects materially the strength at all ages. With the proper quantity of water the strength may be several hundred percent greater than that obtained with a large excess of water.

d. Density. With the same aggregates and the same proportion of cement to total volume of aggregate, the mixture having the greatest density will usually have the highest compressive strength.

e. Exposure. If the original water is permitted to evaporate and water is subsequently excluded, the strength may be reduced 40% or more. Concrete should be kept wet for several days or weeks if the maximum strength is to be attained.

f. Grading of aggregate. There is no definite relation between grading of aggregate and compressive strength which is applicable to any considerable number of different aggregates.

Characteristics of the Cementing Medium

Portland Cements—Powers (40) gave values of compressive strength for cements of differing composition and noted that these "indicate that cement gels low in C_3A are stronger than those higher in C_3A." He further commented: ". . . it seems unlikely that the strength of cement gel is due exclusively to physical forces. . . it seems probable that there are many points of chemical bonding between the particles. . . . As to the relative importance of the two sources of strength, one can only speculate."

Lerch (28) reported work on the effect of gypsum content of portland cement on the properties of cement pastes. His data include a number of 90-day mortar compressive strength values in excess of 10,000 psi. Figure 1 shows data on two of these.

A Waterways Experiment Station investigation, which has been reported in part (29), involved strength tests on specimens made using five portland cements. The trends of strength development of concrete mixtures all containing the same aggregates and a water-cement ratio of 0.5 are shown in Figures 2 and 3.

Alexander (3) reported that, when a mineral admixture is used, and the ratio of water to blend is constant, the strength change of paste depends not only on the change in water-portland cement ratio but also on the amount of admixture used. He found that when a siliceous material was ground to produce a very high surface area (>40,000 sq cm/g) many materials not regarded as pozzolans showed strong pozzolanic activity, but he noted that the high water requirements made such materials unattractive from a practical standpoint.

High-Alumina Cements. Lea (27, p. 450) gave data, credited to Robson (44), showing strength development of 1:2:3 concrete of 0.55 water-cement ratio, using high-alumina cement showing a 28-day strength of 10,200 psi. Tomita and Well (53) reported cube strengths of up to 12,700 for type II portland cement at 29 days and up to 15,200 psi for high-alumina cement at 8 days. Neville (34) reported that concrete made with aluminous cement and aluminous-cement clinker aggregate, having a water-cement ratio of 0.5, can give a compressive strength of 14,000 psi in 24 hr and 18,000 psi in 28 days when tested as cubes.

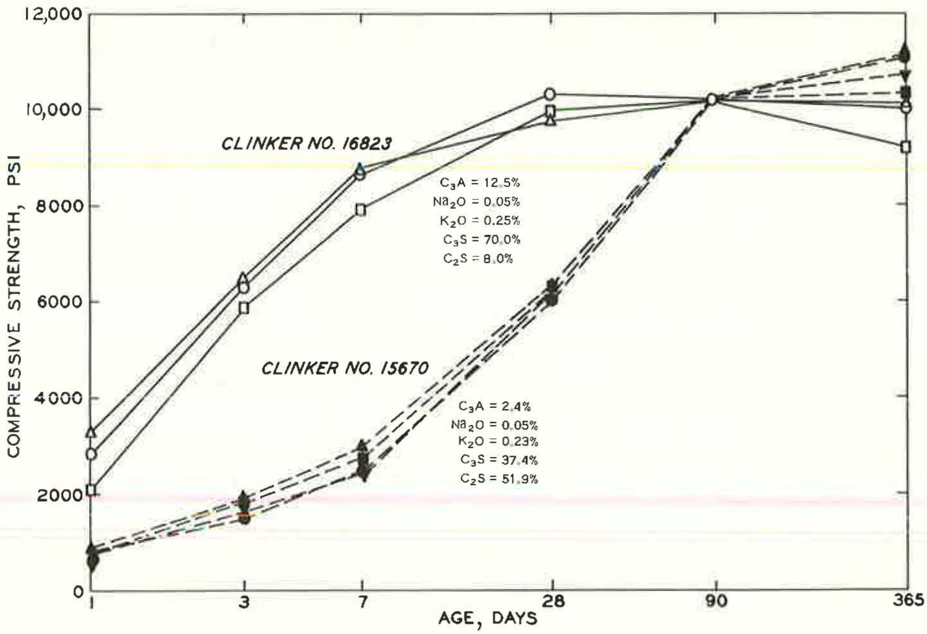

Figure 1. Effect of cement composition on rate of strength development (from Lerch, 28, Table VIII).

"Plastic" Cements. Engineering News-Record (49) referred to Soviet reports of a concrete that had attained compressive strengths of 14,000 to 17,000 psi.

Expanding Cements. Raymond E. Davis of Berkeley, California, informally advised that Alexander Klein of the Engineering Materials Laboratories, University of California, in connection with research on expanding cements (22) had produced concrete having compressive strengths in the range of 14,000 psi using such cements and periclase (MgO) as aggregates.

Characteristics of the Aggregate

Kaplan (18) reported that the use of different types of coarse aggregate in a given concrete mixture resulted in variation in the compressive strength of as much as 29 percent. He used three basic mixtures in evaluating 13 coarse aggregates. These included crushed and uncrushed gravels of three types and four types of crushed stone. Using mixture I, of "ordinary portland," a cement-aggregate ratio of 1:3.08 (average cement factor 9½ bags per cu yd), and a water-cement ratio of 0.35, he obtained 28-day compressive strengths in the neighborhood of 10,000 psi.

The TVA (50) compared compressive strengths of concrete made at a number of its projects for a range of water-cement ratios. The results at 1 yr reveal the marked effect of type of aggregate on strength (Fig. 4).

Collins (10) reported on work of the research organization of the Cement and Concrete Association (Great Britain) and stated that the empirical approach to selection of proportions for concrete mixtures had been found to be much more successful than any fundamental or theoretical approach. For concretes having strengths of 7,000 psi or more, both the aggregate and the workability play increasingly important roles and both set limits on the strength obtainable. The limit set by the aggregate generally lies in the range from 12,000 to 15,000 psi. The limit set by the workability depends on the means available for consolidation, and even when vibration is combined with pressure, no further gain in strength can be obtained by making the mixture richer than about 1:2.5 or 1:3 by weight. Collins also reported that a procedure had been developed to allow for these factors and permit mixtures to be proportioned to give average strengths of 8,000 to 10,000 psi at 28 days with ordinary portland cement and normal moist-curing.

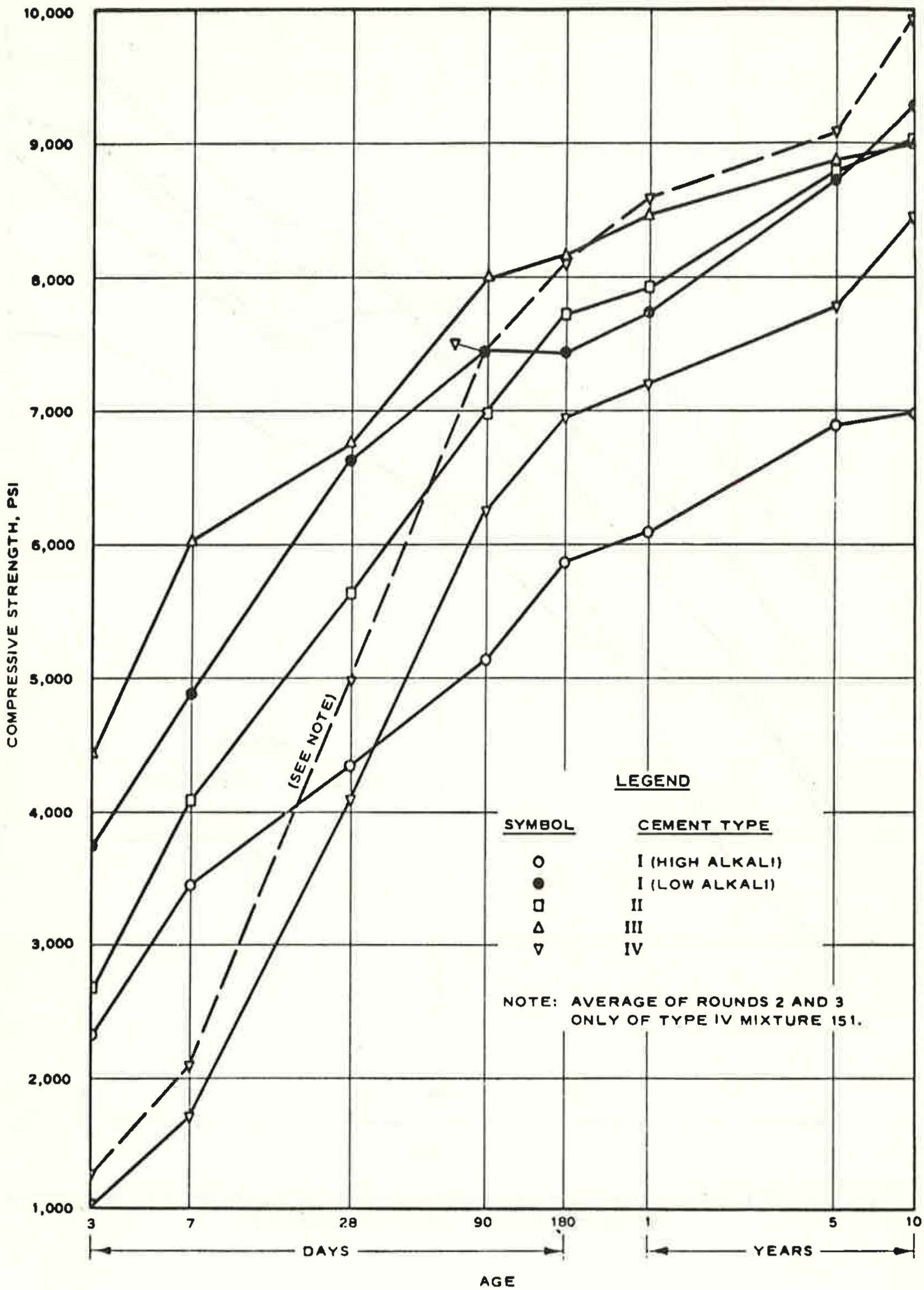

Figure 2. Effect of cement characteristics on strength development to 10 years age of 0.5 water-cement ratio concrete with no entrained air.

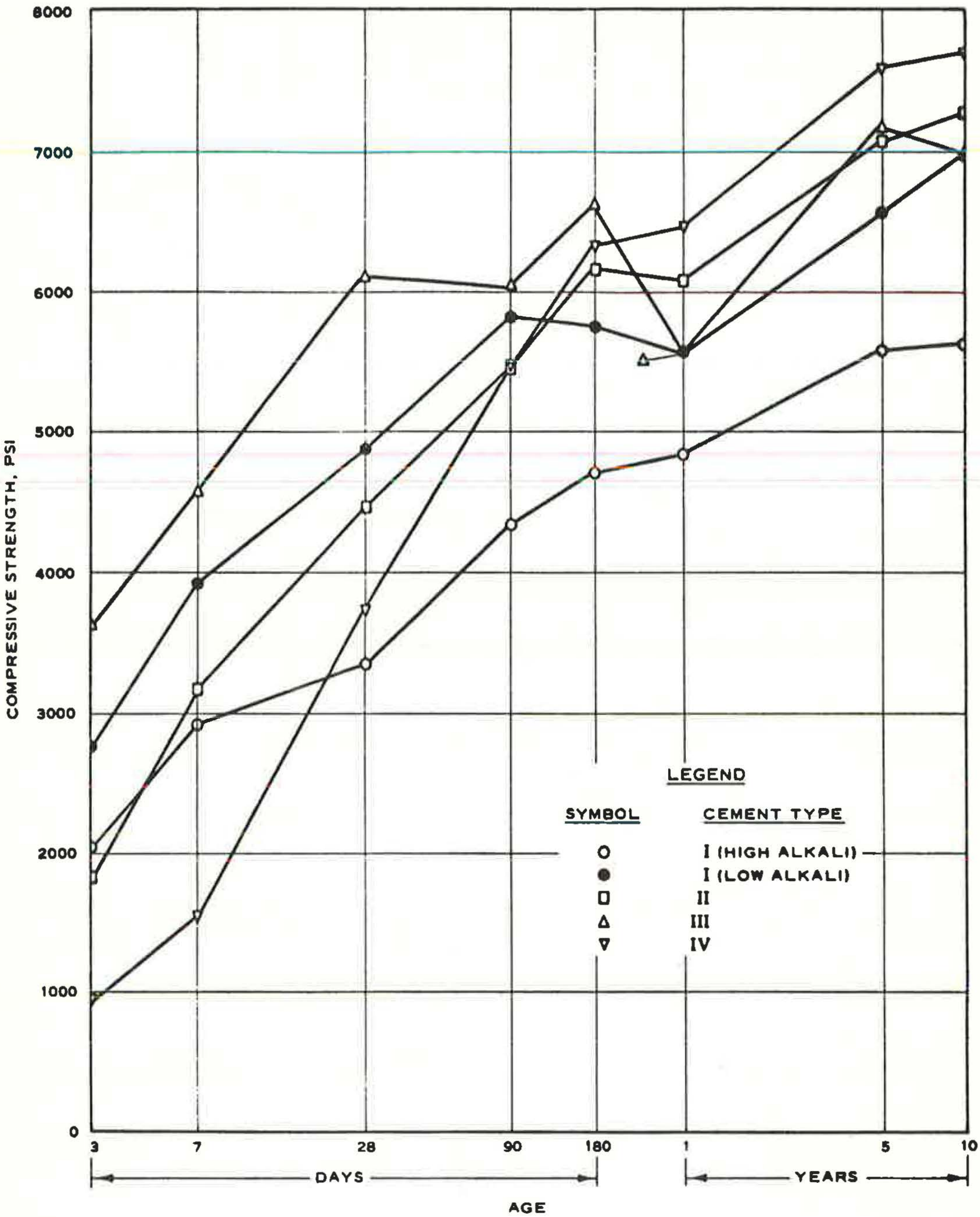

Figure 3. Effect of cement characteristics on strength development to 10 years age of 0.5 water-cement ratio concrete with 6 ± 0.5 percent entrained air.

Proportions of the Paste

The most important single factor affecting the compressive strength of concrete is the water-cement ratio of the cement paste. Cement gel has a density of about 2.15 g/cc

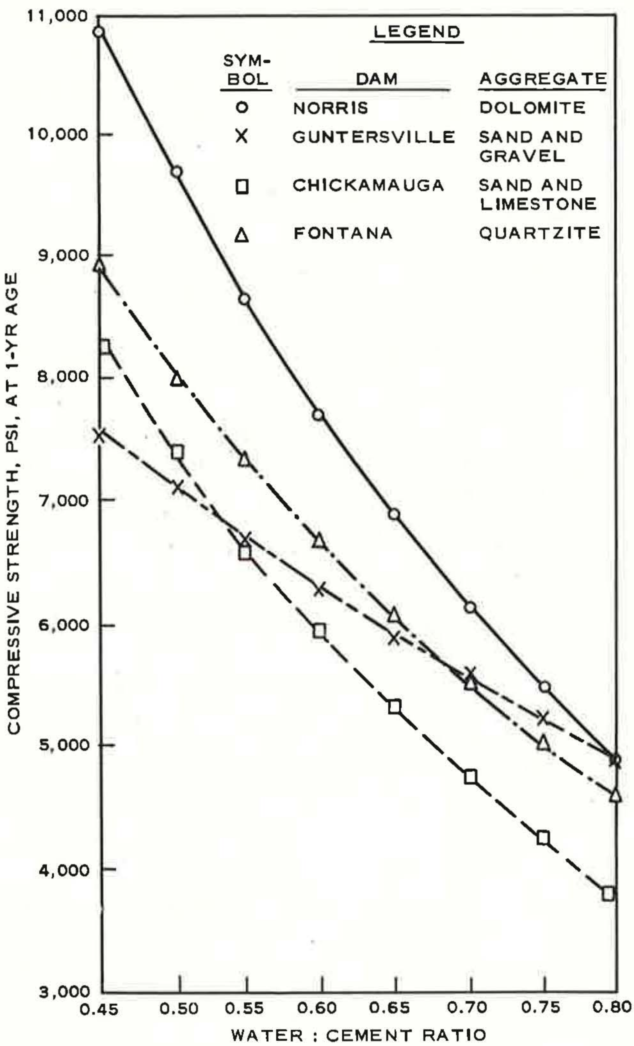

Figure 4. Relation of water-cement ratio to 1-yr compressive strength for various aggregate combinations used by TVA.

and a porosity of about 26 percent. The space in hardened paste that is not filled with gel is referred to as capillary pore space (39).

Powers (39) stated:

> We have no adequate theory about the strength of cement gel .... However, it is unlikely that physical forces account for all the strength.... We assume...that inter-particle chemical bonds exist that cannot be severed by the spreading pressure of water. Accordingly, we assume that strength is derived from inter-particle physical forces and chemical bonds, the chemical bonds affecting a relatively small part of the cross-sectional area. A given cement gel should be expected to have a characteristic strength, but the strength of the structure built of it, that is, the paste as a whole, should depend on the amount of gel in the space available to it.... The amount of space available to gel is equal to the sum of the volume of water-filled space originally present plus the space made vacant by the hydration of cement... [Fig. 5] is an example of the relation between compressive strength and gel/space ratio. The points plotted represent specimens from three different mixes... at 6 different ages ranging from 7 days to 2 years....

Compressive strength is directly proportional to the cube of the gel/space ratio. The proportionality factor, 34,000 for this particular cement and type of test specimen, might be considered a measure of the intrinsic strength of the gel itself, it being the strength when the gel/space ratio is unity.

Powers (38) also reported a private communication from M. A. Swayze concerning materials prepared under the direction of Duff A. Abrams in the laboratories of the Lone Star Cement Corporation which were regarded as "The strongest materials yet made with portland cement (which) were neat cement cylinders, molded under pressure, containing about 0.08 g of water per g of cement. . . (from which) strengths as high as 40,000 psi were obtained." Powers noted that "Very little of the cement in these cylinders could have become hydrated," and that the strength obtained "is equal to the strength of natural stones such as granite, dolomite, or basalt." It is a natural consequence of the implications of the dependence of compressive strength on water-cement ratio (or gel/space ratio) that for strengths above some given level, there will be insufficient water to hydrate all the cement that is used, i.e., an insufficient volume of originally water-filled space to accommodate the potential hydration products if all the cement were to become hydrated. It therefore follows that in the high-strength concrete strength ranges, the higher the strength desired, the lower the proportion of the cement that can become hydrated.

Plowman (35) showed the linearity of the relation of strength to time (log scale) for ages of 1 day to 1 yr and concretes of water-cement ratio of 0.3 to 0.9 (only those for 0.3 to 0.7 are shown in Fig. 6).

Tests made in 1931 at the Lone Star Cement Research Laboratory under the direction of Duff A. Abrams were reported in 1956 by Ernst Gruenwald (15). The results (Figs. 7 and 8) cover mixtures made using natural sand and gravel graded to $1\frac{1}{2}$ in., and a sand-gravel ratio of 1:1.86. The concrete was machine-mixed. Two 6 by 12-in. cylinders were made from each mixture, moist-cured, and tested at 28 days. Values given are the average of the testing of the two cylinders.

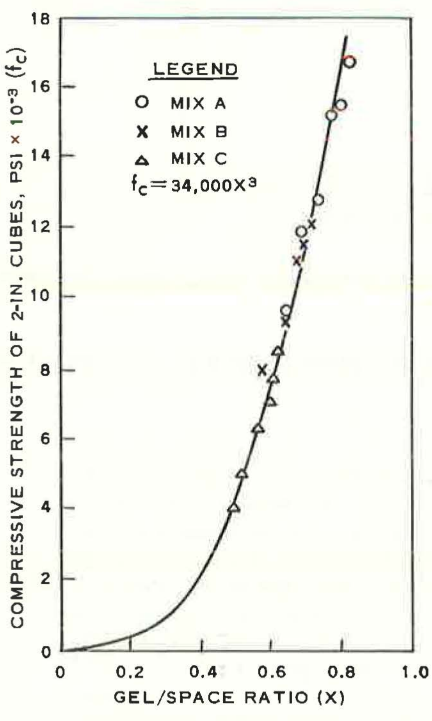

Figure 5. Relation of gel/space ratio to strength (from Powers, 39).

Klieger (24) indicated that concrete of water-cement ratios as low as 0.29 by weight could be produced and fully compacted. If the concrete was to be consolidated by hand, the slump needed to be about $1\frac{1}{2}$ in. and the mixture required about 20 percent fine aggregate. When external vibration (4,900 vpm) was used, the mixture could be proportioned to have zero slump and about 18 percent fine aggregate. The hand-placed mixture of 0.29 water-cement ratio required 12 bags of cement per cu yd; the vibrated mixture only required $9\frac{1}{2}$ bags per cu yd at the same water-cement ratio.

Collins (9) stated that optimum high strength is obtained at that water-cement ratio which is as low as possible and still permits full consolidation. The mixture proportions and type of aggregate are much less important than water-cement ratio. Crushed granite and natural sand at a water-cement ratio of 0.40 give a compressive strength of about 9,000 psi at 28 days, whereas natural gravel and sand give a compressive strength of only about 6,700 psi at the same water-cement ratio.

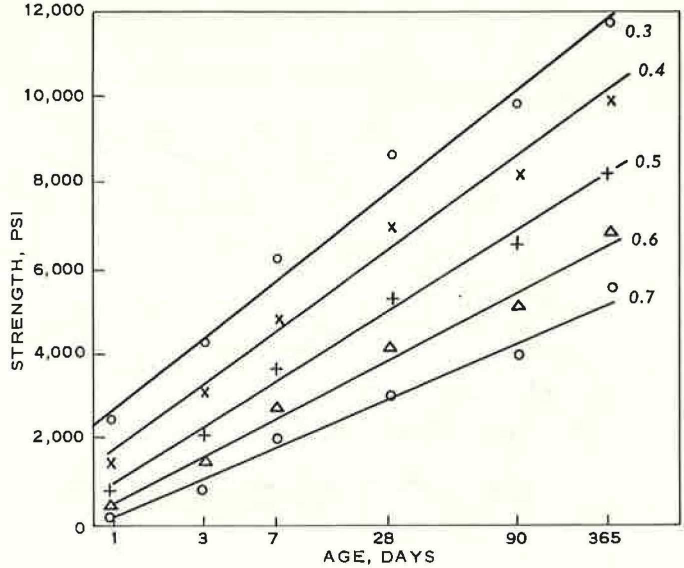

Figure 6. Age-strength relation for various water-cement ratios (from Plowman, 35, Fig. 6).

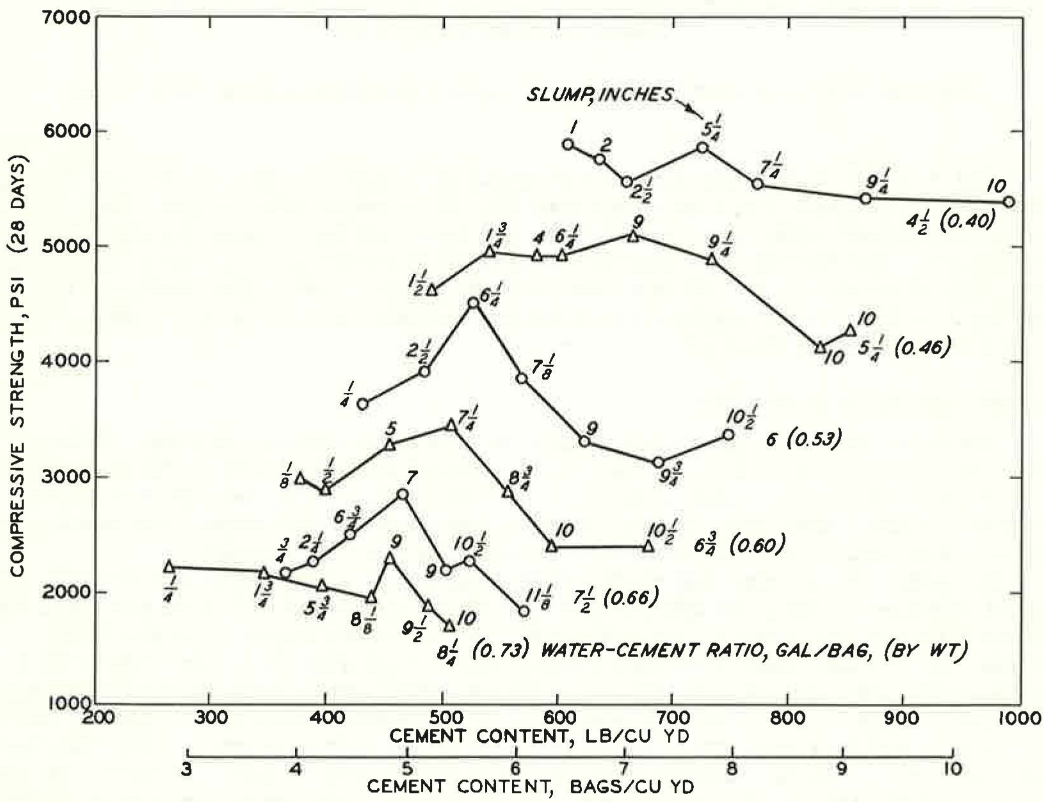

Figure 7. Relation of cement content and strength, "normal" portland cement.

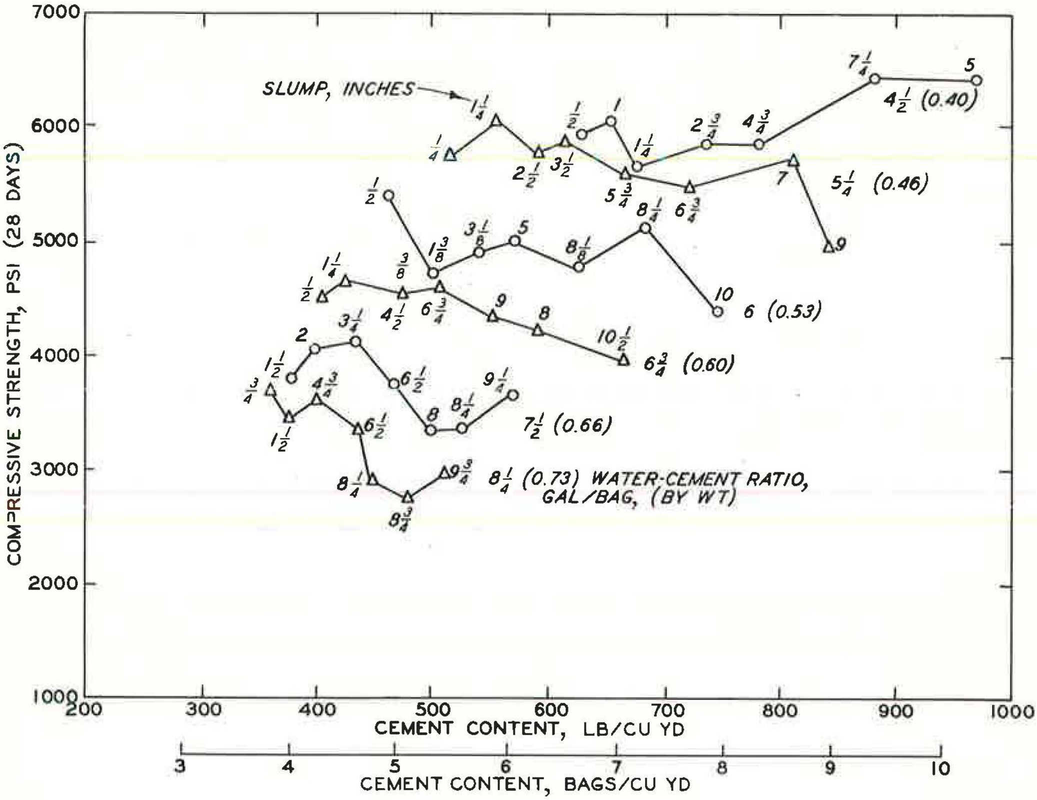

Figure 8. Relation of cement content and strength, high-early strength portland cement.

TVA correlated (50, Appendix H) strength data relating to over 10,000,000 cu yd of concrete in 12 major dams for which over 26,000 cylinders were tested. The relations of water-cement ratio, age, cement type, and aggregate to compressive strength were developed. The Norris Dam aggregates with type II cement gave a 1-yr compressive strength of nearly 11,000 psi at a water-cement ratio of 0.45. The relation of the strength of this cement-aggregate combination to water-cement ratio at 7 days, 28 days, and 1 yr is shown in Figure 9.

Paste-Aggregate Interaction

The term "paste-aggregate interaction" is used to include consideration of those factors affecting strength of concrete that have been described both as "paste-aggregate bond" and "cement-aggregate reaction," since there is, as yet, very little basis on which to make a distinction between physical and chemical effects that may develop and affect paste-aggregate interaction and consequently concrete strength.

Alexander (3) reported that results of tests at 7 days indicate that the strength of the bond between cement paste and plane rock surfaces is normally 50 to 60 percent of the paste strength, but some exceptional rocks can give much higher or much lower strengths. He also found that bond forming ability appeared not to vary widely for a given rock type regardless of locality of origin. Both cement-aggregate bond strength and paste strength decrease with increasing water-cement ratio; it is suggested that if, at a given water-cement ratio, fracture occurs at the aggregate-paste bond, this mode of failure is not likely to change with change in water-cement ratio. However, the mode of rupture is likely to change with age, failure of paste being more likely at later ages. Cement-aggregate bond strength was found to decrease rapidly with increasing aggregate

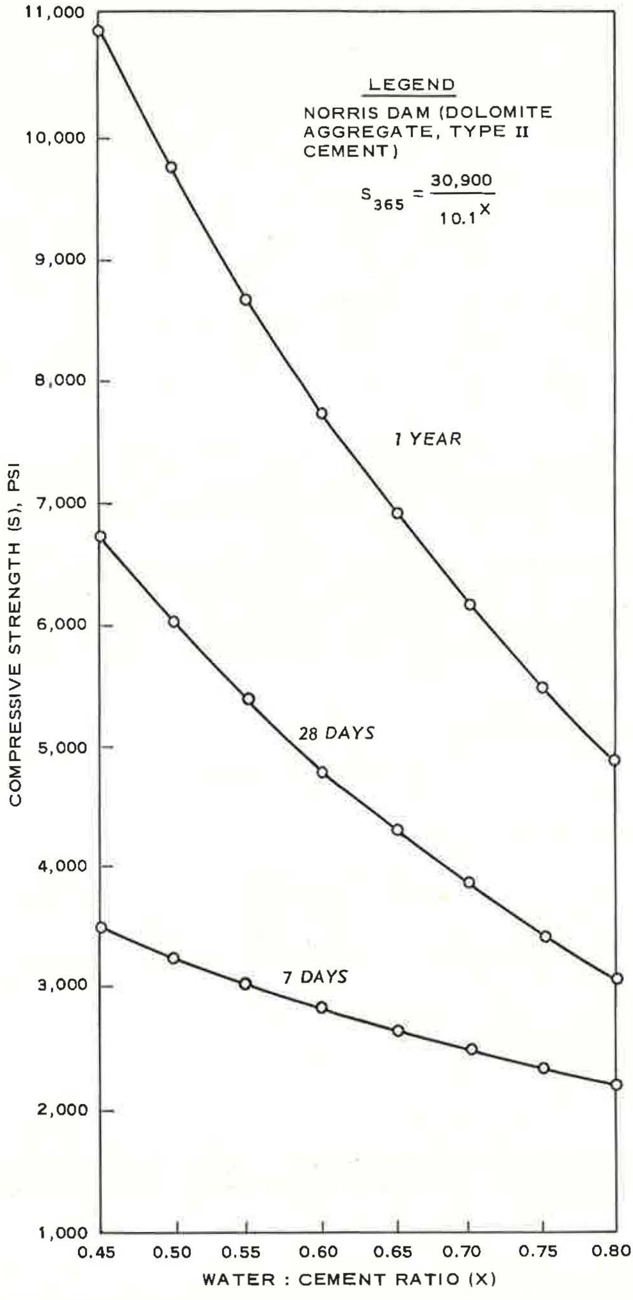

Figure 9. Relation of water-cement ratio to strength at three ages.

size, bond to a 3-in. particle being only about one-tenth that to a corresponding ½-in. particle.

Alexander and Wardlaw (4) discussed data that tend to suggest that there are likely to be specific zones of weakness in concrete, and that these zones are located at the junction between the paste matrix and the largest aggregate particles present.

Alexander (2) reported results of approximately 1,000 determinations of cement-aggregate bond strength and paste strength. He concludes that a given paste develops

TABLE 1

COMPARISON OF EXPERIMENTALLY DETERMINED AND CALCULATED VALUES

Aggregate	E_{agg} (kg/sq cm × 10^{-3})	E_{mortar} (kg/sq cm × 10^{-3})	V_{agg} (%)	E_{conc} (kg/sq cm × 10^{-3})	E_{calc} (kg/sq cm × 10^{-3})	
					Soft	Hard
Diorite	1041	346	0.50	550	520	694
Glass	742	313	0.55	467	458	549
Steel	2200	313	0.55	600	592	1350
Limestone	795	374	0.52	472	519	592
Limestone	632	374	0.52	465	478	507
Limestone	473	374	0.52	399	422	425
Limestone	131	374	0.52	284	192	248

quite different degrees of bond strength at 7 days age with different, strong, uniform aggregates; that the 28-day bond strength is in general about 50 percent greater than the 7-day strength. He also found that all bond strengths were lower than the strength of the paste.

Hansen (16) considered the relations of two kinds of combined materials: (a) combined hard material in which the continuous phase has a high modulus and the discontinuous phase a lower modulus; and (b) combined soft material consisting of elastic particles of high modulus in a matrix of lower modulus. The moduli of elasticity of such combined materials are as follows:

for the combined hard material

$$E = V_s E_s + V_h E_h$$

and for the combined soft material

$$E = \frac{1}{\frac{V_s}{E_s} + \frac{V_h}{E_h}}$$

where

- V = volume concentration of component,
- E = modulus of elasticity,
- s = soft, and
- h = hard.

Hansen employed these formulas and compared the results with experimentally determined values for the modulus of a number of combined materials, including those given in Table 1.

Mixing, Consolidation, and Curing

The production of high-compressive-strength concrete will normally require the use of lower water-cement ratios than are generally used in construction. Concrete proportioned using such lower-than-average water-cement ratios will be "drier," "harsher," "less workable," and "less plastic" than mixtures generally used. Consequently, the mixing, placing, and consolidation procedures employed will generally need to be modified from those customarily used. Since the high compressive strengths of interest are to be evaluated when the concrete is older than the ages at which concrete strengths are normally specified in construction, advantage should be taken of the opportunity to develop strength over a longer period of curing than is usually provided. Hence curing procedures will also properly be modified when high compressive strengths are desired.

Mixing—Bloem (7) reported studies of the effect of mixing on strength. He noted that slurry mixing, a process involving the advance preparation of a cement-water mixture which is then blended with aggregate to produce concrete, had aroused some interest. Benefits thus obtained are presumably attributable to more efficient hydration of the cement resulting from the more intimate contact between cement particles and water achieved in the vigorous blending of cement paste.

Mixing the slurry at very high speed produced a slight increase in fluidity, a slight increase in the average weight of the cubes which corresponded to a reduction in air of about 0.5 percent, and an increase of 13 percent in compressive strength.

Prolonged Mixing—A limited examination was made by Bloem (7) of the effect on concrete strength of mixing for an excessive period of time. The batch was mixed initially for 6 min. Thereafter, the mixing was continued for 8 hr during which the concrete was sampled and tested at intervals of approximately 1 hr. At each testing period, one set of tests was made without adjusting the mixture and one set after mixing with added water to restore slump to the design range.

The test results show that on the average the concrete lost approximately 2 in. of slump during each 1-hr period of mixing. The amount of water required to restore the slump averaged approximately 4 gal/cu yd or, very roughly, 2 gal/in. of slump. The additions increased progressively from 2 gal/cu yd/in. of slump after the first hour to 9 gal after the fourth hour, and then became less. The temperature of the concrete rose from 82 F initially to about 100 F when the test was discontinued after 8 hr. The strength was detrimentally affected only when water was added to restore the slump lost during prolonged mixing. The magnitude of the reduction in each case was about what would be expected to result from the increase in water-cement ratio.

Ray et al (42) reported that when 2 to 3 hr were allowed to elapse between mixing the concrete and preparing the test cylinders, the strength was higher than if the specimens were made either earlier or later. The increase over those made soonest was approximately 20 percent. The authors attributed this to decrease in water-cement ratio due to evaporation losses and absorption of water by the aggregates.

Consolidation—Davies (13) investigated effects of consolidating concrete by vibration concluding that:

> The effectiveness of vibration depends mainly on its acceleration, and not on the individual values of its frequency and amplitude; but very small amplitudes, of the order of 0.002 in., are comparatively ineffective. There is little to choose between vertical linear and horizontal circular vibration; vertical circular vibration is definitely better than either. The effectiveness of vibration increases with its acceleration, probably up to at least 20 g; but above about 12 g the increase is slow.

The Joint Committee of the Institutions of Structural and Civil Engineers (17) emphasized the importance of consolidation in the attainment of the compressive strength for which the constituents and proportions of a mixture were selected. Figure 10 shows that, for the data plotted, the presence of 5 percent voids reduced the strength by 30 percent. The report points out that with a properly proportioned concrete mixture "extended vibration will be wasteful of effort but not harmful to the concrete." If, on extended vibration, segregation of the concrete mixture occurs, this indicates that the mixture is not properly proportioned.

Studies at the Road Research Laboratory (21) resulted in the conclusion that for a given acceleration, best consolidation would be obtained using a vibrator of large amplitude and low frequency.

Curing—Hansen (16) showed (Fig. 11) that between 6 hr and 28 days on a log-log scale plot of compressive strength versus age there will be three different straight lines, one for the first 30 hr, another from 30 to 72 hr, and a third from 3 to 28 days. He noted that these correspond to the observations of Grudemo of three stages of hydration as observed by electron microscopy.

Antill (6) reported results of tests on 1,800 specimens made with six types of cement manufactured in New South Wales. He concluded that the equation for the relation of compressive strengths after 7 and 28 days of curing was best represented by $S_{28} = 1.60\ S_7$.

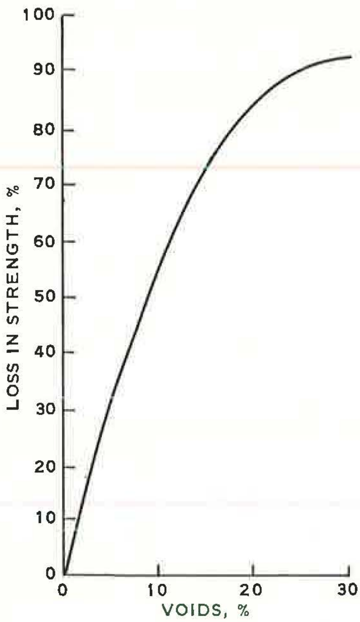

Figure 10. Relation of percentage of voids to consequent loss in strength (from reference 17).

Creskoff (11) developed a procedure by the use of which, from a limited number of pilot tests at 3 or 7 days age and at 28 days, a relation could be developed from which 28-day strengths of a given mixture could be predicted from earlier-age strength test results with a known probable error. The procedure involves making at least 6 and preferably 10 sets of 3 or 7-day and 28-day specimens, varying the slump from ½ to 8 in., and plotting the relation of earlier strengths and 28-day strengths. From these a relation is established and a probable error computed. In using this method for controlling strength of concrete, Creskoff recommended that for every 100 specimens made for test at earlier ages, 10 be made for test at 28 days to check on the continuing accuracy of the relation and the probable error.

Plowman (35) established that concrete cured at 11 F did not gain strength. He therefore concluded that the maturity of a concrete should be based on the age in hours multiplied by the number of degrees above 11 F at which it had been cured. Thus for 28 days at 64 F the maturity is 35,600 F-hr. He then showed that the percentage of the strength at 35,600 F-hr could be calculated from:

$$A + B \log_{10} \frac{\text{maturity}}{10_3}$$

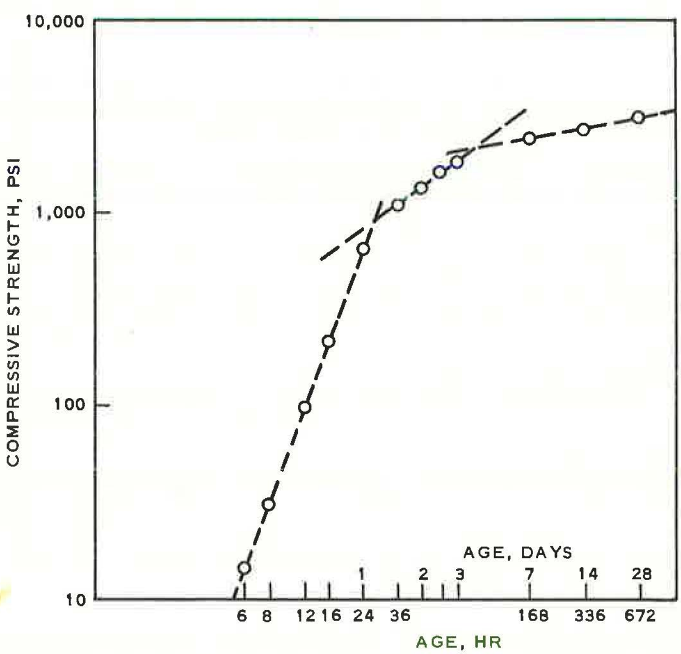

Figure 11. Compressive strength versus age showing three stages of hydration (from Hansen, 16, Fig. 3.2.1, p. 35).

TABLE 2

VALUES FOR CONSTANTS A AND B

Strength (psi) After					Constants		
35,600 F-hr	26,700 F-hr	17,800 F-hr	8900 F-hr	3800 F-hr	A	B	Zone
0- 2,500	0-2300	0-2000	0-1600	0-1000	-7	68	I
2,500- 5,000	2300-4800	2000-4000	1600-3200	1000-2200	6	61	II
5,000- 7,500	4800-7100	4000-6300	3200-5200	2200-3800	18	54	III
7,500-10,000	7100-9300	6300-8500	5200-7000	3800-6000	30	46.5	IV

This he referred to as a "law." He gave a tabulation of values (Table 2) for the constants A and B, pointing out that the use of these values throughout the range of each "zone" did not introduce errors in excess of 2 percent over using exact values computed for particular points within each zone. Plowman (35) concluded that: (a) concrete made with portland cement obeys the law; (b) the law is independent of quality of the cement, water-cement ratio, aggregate-cement ratio, curing temperature below 100 F, and the shape of the test specimen; (c) the datum temperature for maturity calculation is 11 F. Maturity is a summation of the integrals of time-temperature of the concrete above 11 F, negative values being disregarded; (d) the values of A and B are related linearly to the strength at any age and may be predicted; (e) values of the constants for the four "zones" specified are sufficiently accurate for all normal work; and (f) given the strength at any maturity, the strength of that grade of concrete at any other maturity may be calculated.

In a discussion of Plowman's paper, Klieger (23) noted that other data would fit this law only if (a) the relation between the logarithm of maturity and strength is linear, (b) the initial temperature of the concrete is in the range 60 to 80 F, and (c) no loss of moisture by drying occurs during the curing period. He observed that concretes now made in the United States do not show such a linear relation. He also observed that "ultimate strength is influenced by the rate of hydration during the early stages of hydration." A high early rate generally results in lower ultimate strength.

Klieger reported elsewhere (24) that for low water-cement ratio concretes it is more necessary to supply additional water during curing than is the case with high water-cement ratio concretes. For concretes of 0.29 water-cement ratio, the strength of specimens made with saturated aggregates and cured by ponding water on top of the specimen was 850 to 1,000 psi greater at 28 days than that of comparable specimens made with dry aggregates and cured under damp burlap. He also noted that although early strength is increased by elevated temperatures of mixing and curing, later strengths are reduced by such temperatures.

Price (41) showed that at 180 days age, concrete specimens that had been continuously moist-cured showed higher compressive strength than any whose curing had been interrupted. However, in tests of specimens at 90 days age, he found higher strength for specimens moist-cured to 28 days and thereafter stored in laboratory air. These results were for concrete of 0.5 water-cement ratio, $3\frac{1}{2}$-in. slump, cement content of 556 lb per cu yd, 36 percent sand, and 4 percent air. He also showed that the lower the initial temperature of concrete in the range 40 to 115 F, the higher the strength at all ages from 28 to 180 days when the specimens were molded, sealed, and stored at the specified initial temperature, maintained at that temperature for 2 hr, and then stored at 70 F until tested. These results were based on concrete of 0.53 water-cement ratio, cement content of 606 lb per cu yd, 40 percent sand, 0 percent air, and type II cement.

Methods of Measuring Strength

Specimen Size—Neville (33) pointed out that the strength of a brittle material can be defined as the critical state of stress at which fracture occurs. He noted that the actual stress at fracture is considerably lower than the theoretical strength estimated from

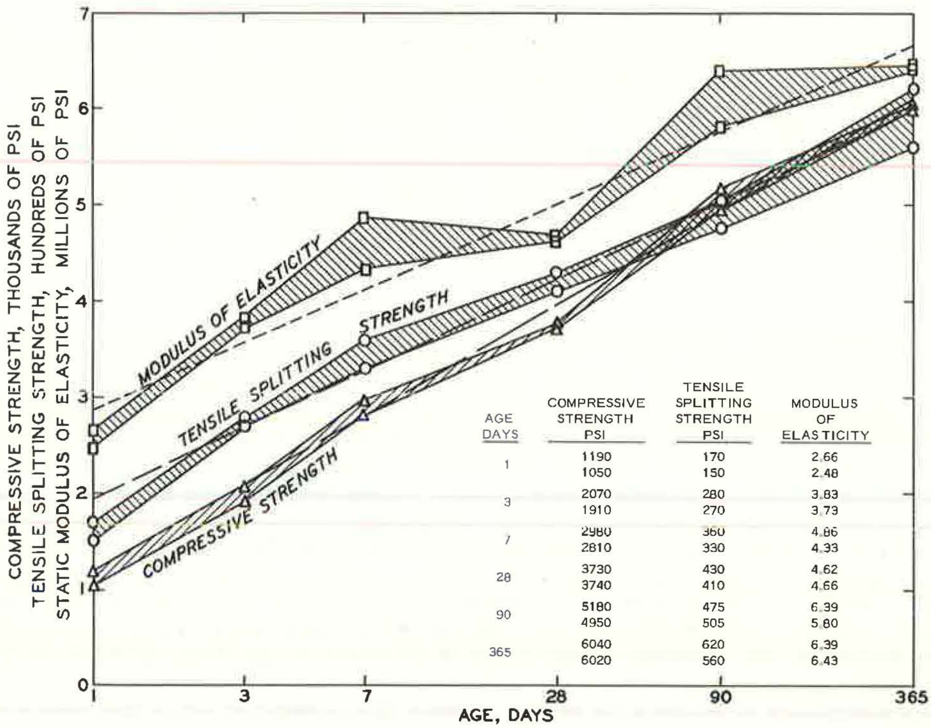

Figure 12. Relation of compressive strength, tensile splitting strength, and modulus of elasticity at six ages.

molecular cohesion and calculated from surface energy. He suggested that the Griffith theory relating to cracks and flaws which lead to high stress concentrations and local fracturing while the average stress is low can be invoked not only to account for the low actual strength of such materials, but also for the observed effects of specimen size on indicated strength of concrete. He suggested that as specimen size increases, the probability of the presence of a critical flaw of critical location and orientation likewise increases. In a review of data on flexural strength he found "convincing proof" that the effect of specimen size was as follows: as the area of the bottom surface of the beam subjected to critical stress increases, the probability of a critical flaw being present increases; hence the lower average strength and the lower the coefficient of variation.

TABLE 3

RELATIONSHIPS OF STRENGTH TEST RESULTS

Compressive Strength (psi)		Tensile Strength (psi)	Flexural Strength (psi)
Cylinders	Cubes		
2000	2670- 3000	200	375
3000	4000- 4500	275	485
4000	5340- 6000	340	580
5000	6670- 7500	400	675
6000	8000- 9000	460	765
7000	9340-10,500	520	855
8000	10,670-12,000	580	930

Testing Methods—Anderson (5) presented data on the use of the Schmidt concrete test hammer for evaluating the compressive strength of hardened concrete when the concrete had compressive strengths up to 11,000 psi when tested as cubes. He concluded that the accuracy obtained approaches that attainable in tests of carefully prepared cylinders.

Tests of gravel aggregate concrete made at the Waterways Experiment Station in 1961 revealed the relation

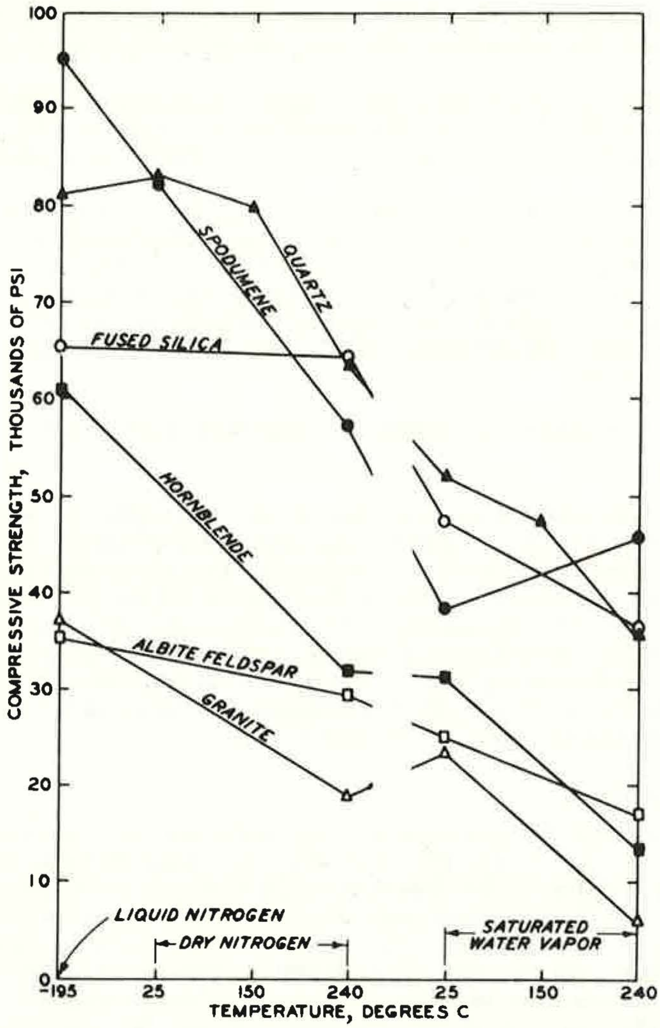

Figure 13. Effect of environment of testing on indicated compressive strengths of silica and silicates (from Charles, 8).

shown in Figure 12 for compressive strength, tensile splitting strength, and "static" modulus of elasticity.

Abeles (1) gave the approximate relations (Table 3) between results of various types of tests for strength.

Kesler (19) reported results of over 1, 400 tests of concrete made with the same aggregates and tested as beams in flexure and as modified cubes and cylinders in compression at different mixture proportions and ages. He concluded that:

a. The modulus of rupture varies from about 22% of the cylinder compressive strength for low-strength concrete to about 12% for high-strength concrete.

b. The cylinder strength varies from about 7.5 to 6.5 times the modulus of rupture for low- and high-strength concrete, respectively.

c. The modulus of rupture varies from about 18% to about 12% of the modified cube strength for low- and high-strength concrete, respectively.

d. The cylinder strength was about 80% of the modified cube strength for low-strength concrete, but about the same as the modified cube strength for high-strength concrete.

Kesler also found the standard error of estimate is as follows: modulus of rupture from cylinder strength, 75 psi; cylinder strength from modulus of rupture, 630 psi; modulus of rupture from cube strength, 75 psi; and cylinder strength from cube strength, 420 psi.

Testing Conditions—The environment of the test specimen during testing can markedly affect the results of the test. Data given by Charles (8) are plotted in Figure 13. It is indicated that, in general, strengths in compression are markedly reduced by either raising the ambient temperature during testing or raising the moisture content of the atmosphere in which testing is done, or both. Quartz was found to have a compressive strength of 35,800 psi at 240 C in saturated water vapor, but 83,000 psi in dry nitrogen at 25 C.

STRENGTH DESIRED—STRENGTH ATTAINED

Specified Values

The uses to which portland cement concrete are commonly put require that the concrete, when cured, develop compressive strength to some specified degree or to some degree that was assumed in the structural design. One summary (56) of the strength properties of plain concrete published by the Corps of Engineers, U.S. Army, stated, "The strength of concrete depends upon many factors, including water-cement ratio, age, cement strength, proportions of cement, sand, and aggregate; and the methods used for the placing and curing of the concrete." This reference then stated, "Concrete having values of f'_c from 3,000 to 5,000 psi, as measured by standard 28-day cylinder tests, is generally used in construction. . . ."

High Strengths Attained

The 55-year index to the publications of the American Concrete Institute (58) listed five references (12, 43, 51, 52, 54) under the entry "High-Strength Concrete." The earliest of these is Hardy Cross' discussion (12) of design of columns. He noted that "increased concrete strength increases column strength very little even where the moment is fixed. In the case where the moment varies with the column stiffness, the use of high-strength concrete may, if we follow the code, actually weaken the column, so that a larger column is needed the greater the concrete strength." The index did not refer to the 1910 paper by Porter (37) which includes the following:

> "...investigations [were] made in connection with one of the bridges recently constructed by the City of New York to ascertain how the strength of concrete might be increased.... It was found by the addition of a certain number of cut nails or spikes per cu ft of concrete, the crushing and tensile strength were increased enormously. One series of tests giving results as high as 18,000 psi or nearly eight times the ordinary strength." It is not stated whether these tests were made on cylinders or cubes.

Gonnerman and Lerch (14) pointed out that between 1904 and 1950 the principal changes in portland cements were "an increase in the average computed C_3S content and increase in fineness, each of which has contributed to higher concrete strengths at all ages up to 10 yr. . . ." They showed results of compression tests to an age of 10 yr for laboratory cements procured in 5-yr periods from 1916 to 1950 (see Fig. 14).

One group of data reported by Gonnerman and Lerch (14, Tables XX and XXI) concerned tests of a blend of four type I cements at a water-cement ratio of 4 gal per bag using $1\frac{1}{2}$-in. sand and gravel. Data from this series of tests are given in Table 4. These data indicate the effect of water-cement ratio by giving essentially equal strength for concrete having equal water-cement ratios in spite of a range in slump from less than 2 to more than 6 in. and a range in cement content from less than 7 to more than 9 bags per cu yd.

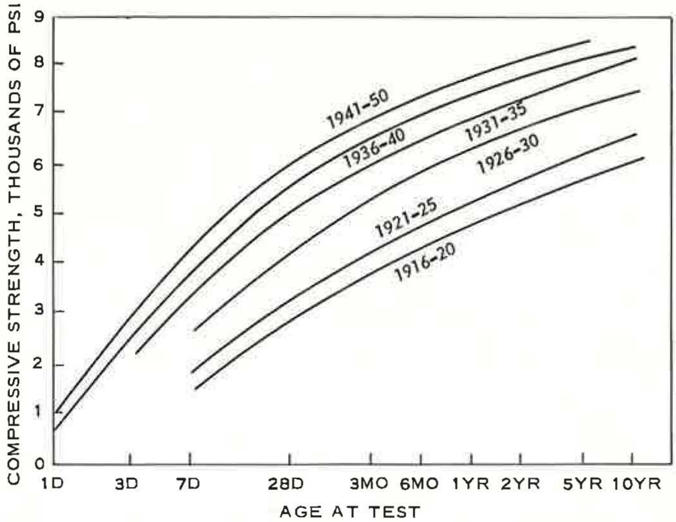

Figure 14. Age-strength relations for concrete made with laboratory cement (from Gonnerman and Lerch, 14, 1951, Fig. 5).

Klieger (25) reported results of strength tests on specimens of concrete containing 6 bags of cement per cu yd and having a 2 to 3-in. slump up to 3 yr of age. One mixture, at 3 yr, developed compressive strength in excess of 10,000 psi. Data on this mixture are given in Table 5.

Shideler (47) reported tests in which compressive strengths in excess of 10,000 psi were obtained at 90 days and in excess of 11,000 psi at one year using sand and gravel aggregate (Fig. 15). The aggregate was uncrushed and well rounded. The mixture contained 330 lb of water, 950 lb (10.11 bags) of cement, 727 lb of sand (25%), and 2,177 lb of gravel. The slump was ¼ in.

U. S. Bureau of Reclamation Studies

J. E. Backstrom of the Engineering Laboratories, U. S. Bureau of Reclamation, Denver, informally provided the following information regarding exceptionally high-strength concretes encountered in that laboratory. These data relate to six groups of tests; permission for their inclusion in this paper was provided by the Chief Engineer, U. S. Bureau of Reclamation.

TABLE 4

TEST RESULTS ON BLEND OF FOUR TYPE I CEMENTS

	Mix 1	Mix 5	Mix 9
Sand, %	33	33	33
Average slump, in.	1.8	3.7	6.2
Cement, absolute volume, %	12.1	14.0	16.0
Sand, absolute volume, %	24.0	22.9	21.5
Gravel, absolute volume, %	48.2	45.8	43.2
Water, absolute volume, %	13.9	16.0	18.2
Air, absolute volume, %	1.8	1.3	1.1
Unit weight, pcf	153.0	152.2	150.8
Cement, bags/cu yd	6.84	7.90	9.01
Compressive strength, psi:			
16 hr	990	990	990
1 day	1,770	1,740	1,710
3 days	4,340	3,810	3,890
7 days	5,810	5,850	5,870
28 days	7,600	6,970	6,990
3 months	8,780	8,310	8,040
1 yr	9,150	9,250	9,420
5 yr	10,250	10,410	10,450

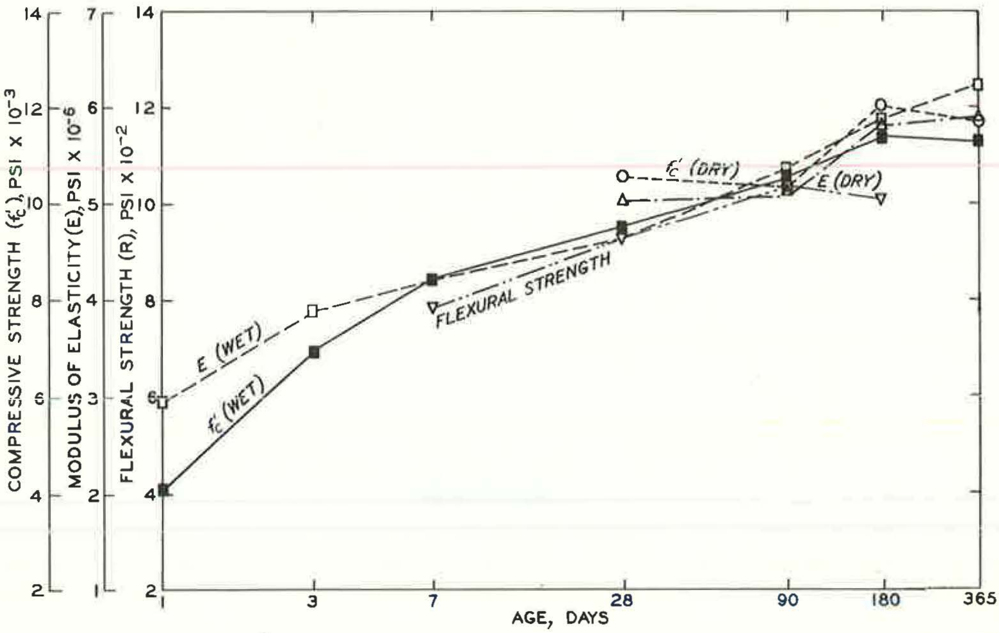

Figure 15. Effect of moisture conditions on strength (from Shideler, 47, Tables 3 and 5).

The highest concrete strength was 16,650 psi on a single 6 by 12-in. cylinder made using 9 bags of high-early-strength cement per cubic yard, a water-cement ratio of 0.37, ⅜-in. crushed stone coarse aggregate, and a well-graded, washed, screened river sand as fine aggregate. The mixture had a slump of 0 to ½ in. The concrete was made in Costa Rica, for use in prestressed concrete construction. The cylinder was fog-cured for 12 months and then shipped to a university in the United States for testing. The strength of the concrete exceeded the capacity of the available testing equipment, and therefore the specimen was shipped to the U. S. Bureau of Reclamation laboratory. It was capped for testing at a total age between 15 and 18 months (having been allowed to air-dry after 12 months age), and was tested first for modulus of elasticity. Modulus in the air-dry condition was 2,310,000; Poisson's ratio was 0.19.

The specimen was then loaded to a total of 397,000 lb at which time the caps failed without damaging the specimen. Twelve days later, on June 14, 1960, after the specimens had again been prepared for testing by grinding the ends plane, it was tested and found to have a strength of 16,650 psi.

TABLE 5

RESULTS OF STRENGTH TESTS

Cement No.	16
Type	I
Cement content, bags/cu yd	6.01
Water-cement ratio, gal/bag	4.63
Slump, in.	1.9
Air content, %	1.38
Compressive strength, psi (modified cubes)	
1 day	1,820
7 days	6,040
28 days	7,230
3 months	8,080
1 yr	9,540
3 yr	10,160

Waterways Experiment Station Studies

As was previously mentioned (31), the strongest concrete I ever saw, and the strongest yet made at the Waterways Experiment Station, was some made for and reported on by K. Mather (32). An average strength of 13,200 psi was obtained by testing after 90 days moist curing three 6 by 12-in. cylinders molded from a batch using ½-in. magnetite aggregate, 10.3 bags of type I cement, a carboxylic-acid water-reducing admixture, having an 0.30 water-cement ratio, a ¾-in. slump, and a unit weight of 230 pcf.

TABLE 6

RESULTS OF UNIVERSITY OF COLORADO STUDIES

Coarse Aggregate	W/C by Wt	Cement (bags/cu yd)	Slump (in.)	Compressive Strength (psi)	
				28 days	90 days
Red sandstone	0.50	6.7	1.0	4,550	5,570
Red sandstone	0.40	8.4	0.6	6,500	7,710
Red sandstone	0.33	10.1	0.6	8,710	9,990
Red sandstone	0.29	11.8	0.2	9,720	10,470
White sandstone	0.50	6.7	1.0	3,940	5,530
White sandstone	0.40	8.4	0.8	6,070	7,630
White sandstone	0.33	10.1	0.8	8,350	9,990
White sandstone	0.29	11.8	0.2	9,560	11,080
Gravel	0.33	10.1	3.0	8,000	9,070
Basalt	0.33	10.1	0.8	10,150	12,000

University of Colorado Studies

Thoman and Raeder (52) noted in 1934 that "published data on concrete of an ultimate strength greater than 6000 psi are meager." They reported studies begun in 1932 at the University of Colorado, results of which are given in Table 6. Test specimens were 3 by 6-in. cylinders and were moist cured. Thoman and Raeder commented ". . . it is reasonable to suspect . . . that for very high strength concrete the ultimate strength attained depends to some extent upon the kind of coarse aggregate used. . . . Further research to study the part played by the coarse aggregate. . . should prove highly interesting."

Cast Stone

Walker (57) noted that tests made on samples of cast stone selected at random from actual jobs, tested as 2-in. cubes, dried at 105 C before test, showed strengths (in psi) as follows:

9490	7,380	8900
9130	10,860	9000
9540	10,950	9700
9760	9,100	9240

Concrete Slabs for Fort Peck Dam

Strengths of over 18,000 psi were reported (55) to have been obtained in connection with the manufacture of mesh-reinforced precast concrete slabs for powerhouse wall facing at Fort Peck Dam, Montana. The mixture was composed of 1 bag of type I cement, 64 lb of silica sand passing a No. 50 sieve, 220 lb of $^3/_8$- to $^1/_4$-in. crushed limestone, 0.9 lb of a water-reducing retarding admixture, and 4 gal of water. The specifications called for a minimum compressive strength of 9,000 psi. The average slab compressive strength was "well over" 12,000 psi, many exceeded 15,000 psi, and the maximum was 18,147 psi. The slabs had an average absorption of 4 percent. They were cast on a vibrating table, and the forms were removed after 1 day. The slabs were moist-cured using quilts. The wooden pallet on which the slabs were cast was covered with an absorptive form lining. The placing and consolidating operation was completed in $1^1/_2$ min.

Concrete in Paris Exhibition Hall

A report in Engineering (26) described a new exhibition hall in Paris as follows:

> The new exhibition hall of the Centre National des Industries et Techniques in Paris, just beyond the Pont de Neuilly, is built wholly of reinforced and prestressed concrete. In plan it is an equilateral triangle with sides 738 ft long. The crown of the roof is 150 ft above ground level. The roof covers a main exhibition

floor at first storey level of 220,000 sq ft with another of 270,000 sq ft below it. To meet the structural requirements of the roof a particular dry mix concrete was used (to give 17,000 lb per sq in.) and to place the concrete in the shells a high-frequency low-amplitude vibrator was used in conjunction with a light float for surface tamping. The roof was concreted in bays, each supported by a close system of tubular scaffolding. All the concrete was mixed in a central batch-weighing plant located 2½ miles from the site.

The roof is believed to contain the longest span yet built as a thin shell vaulted structure and the three sections are the largest surfaces supported at a single point.

The foregoing is the only published account that was located during the course of this review in which it was reported that job-mixed concrete was produced to have a compressive strength in excess of 10, 000 psi.

APPLICATIONS, ECONOMICS, AND PROCUREMENT OF HIGH-COMPRESSIVE-STRENGTH CONCRETE

Applications

While advantage cannot economically be taken of very high-strength concrete in ordinary reinforced concrete construction using normal-strength reinforcing steel, the situation is quite different in shell structures below ground. Such structures derive their advantages from soil-structure interaction as a result of which the loading is distributed advantageously and the dynamic strength of the soil is utilized in resisting the effects of dynamic loading. In order for the soil strength to be economically utilized and to mobilize the dynamic energy of distortion of the soil, the shell must be flexible. Even the thinnest reinforced concrete arches fail in compression at high overpressures long before critical buckling stresses are reached. The lowest possible section modulus is conducive to maximum flexibility. Hence partially or wholly underground reinforced concrete arches designed to withstand dynamic loading should be as thin as possible; such design makes appropriate the use of materials of the highest available strength. Asymmetric loading induced progressively higher bending stresses the thicker the section becomes. A thin section backed by a firm soil can absorb the entire load in diaphragm compressive stresses.

Economics

Richart (43) in 1936 gave comparative costs for concrete in place in structures; and he concluded that, giving principal weight to the values for tied columns, the saving in relation to the cost of 2, 000-psi concrete would be as indicated in Table 7.

Towles (54) in 1932 discussed the advantages of the use of "high-strength concrete." His analysis was based on the "assumption that it would be possible to manufacture satisfactorily concrete of a 28-day strength of 7000 psi." He further assumed that "the extreme fibre stress in compression is limited to 37. 5 per cent . . . or 2600 psi . . . and concentric compression to 25 per cent or 1750 psi." His first discussion dealt with "arch construction in which concrete fulfills most completely its function of carrying load by compression of the structural members." He considers, for simplicity, "arches of open spandrel construction in which the load is carried vertically at equal intervals to the arch ring or rib." He stated: "It will be found that the following ratios of weight will hold approximately for arches with solid ribs of the same type and proportions with equal load per ft of length imposed upon the arch rings (exclusive of the dead load of the arch rings themselves), and for ordinary ratios of rise to span (say

TABLE 7

COMPARATIVE COSTS AND SAVINGS

Compressive Strength (psi)	Price per Cubic Yard ($)	Saving (%)
2000	7.20	0
3000	7.50	20
4000	7.90	33
5000	8.40	42
6000	9.05	50

from 1/5 to 1/8)." Comparing concrete of strength f_{cc} = 750 psi as given in most then-current codes with concrete of a strength of 1750 psi, he found:

Span (ft)	W_{750}/W_{1750}
50	2.5±
100	2.7±
150	2.7 to 3.1
200	2.8 to 3.6
250	3.0 to 4.0

This comparison, he noted, revealed that "for the same proportions in the arch ring and for the same loading per ft imposed on the arch rib, it would be possible with 1750-lb concrete to construct a span with the same weight per ft in the arch rib which would be 2⅓ times as long as with 750-lb. . . concrete." He adds ". . . in the future when we have surmounted practical difficulties of producing high-strength concretes we may expect arch spans of unprecedented length." He assumed that 7,000-psi concrete would cost 60 percent more than 3,000-psi concrete and calculated that "on the basis of a proportion of span weights of 1.6 to 1. . . a 100-ft span of 750-lb concrete corresponds approximately in this ratio with a 125-ft span of 1750-lb concrete." He concluded: ". . . from the standpoint of cost, there would be marked advantages in the use of high-strength concretes, particularly for designs involving multiple arch spans, for single long span crossings, and for arches in which it is desired to keep the ratio of rise to span at a minimum."

In discussion of Towles' paper, Rosov (46) reported that by combined vibration and pressure Freyssinet had obtained compressive strengths as high as 8,500 psi. Rosov also reported (45): "In daily manufacturing of precast concrete the strength of 14,000 psi is attained."

Procurement

The procurement of high-strength concrete implies that there will be a compressive strength specification that test specimens of the concrete must meet. For the purpose of this paper, the strength level desired has been taken as 10,000 psi as indicated by tests after 90 days moist-curing of standard molded cylinders having a height equal to twice their diameter.

For a coefficient of variation of 15 percent and a specified design strength of 10,000 psi, the required average strength for an allowable probability of 1 failure in 10 will be 12,400 psi, which may well be unattainable under specific concrete producing conditions. It may therefore normally be necessary, in order consistently to produce high-strength concrete, to achieve a lower variance of test results than is now regarded as good field control.

It is possible that the characteristic distribution of compressive-strength test results for high-strength concrete will not be normal since a skew distribution frequently characterizes data whose mean is close to a limit. If this were to be found to be the case, values of required average strength might well need to be higher than those computed on the normal assumptions.

Wig et al (59) in 1915 came to the following conclusions regarding the production of concrete of desired compressive strength. These, with only very slight modifications, are still entirely valid:

> 1. No standard of compressive strength can be assumed or guaranteed for concrete of any particular proportions made with any aggregate unless all the factors entering into its fabrication are controlled.
>
> 2. A concrete having a desired compressive strength is not necessarily guaranteed by a specification requiring only the use of certain types of materials in stated proportions. Only a fractional part of the desired strength may be obtained unless other factors are controlled.

3. The compressive strength of a concrete is just as much dependent upon other factors, such as careful workmanship and the use of the proper quantity of water in mixing the concrete, as it is upon the proper quantity of cement.

4. The compressive strength of concrete may be reduced by the use of an excess of mixing water to a fractional part of that which it should attain with the same materials. Too much emphasis cannot be placed upon the injurious effect of the use of excessive quantities of water in mixing concrete.

5. The compressive strength of concrete may be greatly reduced if, after fabrication, it is exposed to the sun and wind or in any relatively dry atmosphere in which it loses its moisture rapidly....

6. The relative compressive strength of concretes to be obtained from any given materials can be determined only by an actual test of those materials combined in a concrete.

7. ...[T] he relative value of several fine aggregates to be used in concrete cannot be determined by testing them in mortar mixtures. They must be tested in the combined state with the coarse aggregate.

8. ...[T] he relative value of several coarse aggregates to be used in concrete cannot be determined by testing them with a given sand in one arbitrarily selected proportion....

9. No type of aggregate such as granite, gravel, or limestone can be said to be generally superior to all other types. There are good and poor aggregates of each type.

10. By proper attention to methods of fabrication and curing, aggregates which appear inferior and may be available at the site of the work may give as high compressive strength in concrete as the best selected materials brought from a distance, when the latter are carelessly or improperly used.

11. Density is a good measure of the relative compressive strength of several different mixtures of the same aggregates with the same proportion of cement to total aggregate....

12. Two concretes having the same density but composed of different aggregates may have widely different compressive strength.

13. There is no definite relation between the gradation of the aggregates and the compressive strength of the concrete which is applicable to any considerable number of different aggregates.

14. The gradation curve for maximum compressive strength, which is usually the same as for the maximum density, differs for each aggregate.

15. With the relative volumes of fine and coarse aggregate fixed, the compressive strength of a concrete increases directly, but not in a proportionate ratio, as the cement content. An increase in the ratio of cement to total fine and coarse aggregates when the relative proportions of the latter are not fixed does not necessarily result in an increase in strength, but may give even a lower strength.

16. The compressive strength of concrete composed of given materials, combined in definite proportions and fabricated and exposed under given conditions, can be determined only by testing the concrete actually prepared and treated in the prescribed manner.

17. ...[T] he compressive strength of most concretes, as commercially made, can be increased 25 to 100 percent or more by employing rigid inspection which will insure proper methods of fabrication of the materials.

SUMMARY

It has long been known that it is possible to use portland cement in ways that produce products having much higher compressive strengths than are generally required. Cement pastes have been made which have developed compressive strengths of 40,000 psi. Portland cement concrete specimens representing concrete made for specific engineering use in construction have developed compressive strengths as high as 18,000 psi. In precast concrete work compressive strengths of 14,000 psi have, in some cases, been obtained regularly.

The practical, routine production of portland cement concrete having a compressive strength consistently above 10,000 psi after 90 days moist-curing will require careful selection of (a) materials; (b) mixture proportions; and (c) mixing, placing, consolidating, and curing procedures. It does not appear that extraordinary materials or procedures are demanded. The most important single factor affecting the producibility of high-compressive-strength concrete in any given situation is, and will be, the achievement of an adequately low ratio of weight of mixing water to weight of cement in the mixture. With normally available strong aggregates, processed under close control to a specified grading, virtually any commercial portland cement meeting current specifications can be used to produce concrete having a compressive strength of 10,000 psi, provided means are available and are properly employed to consolidate concrete of the particular water-cement ratio that will be required to attain this strength with the particular materials selected. For some materials the required water-cement ratio will be lower than for others. The lower the required water-cement ratio, the greater will be the need for special procedures to obtain effective consolidation.

In those cases where materials that can readily be used to obtain high-compressive-strength concrete are not locally and economically available, it may prove more economical to resort to more vigorous methods of consolidation in order to permit utilization of local materials, the effective use of which requires unusually low water-cement ratios and consequently unusually dry mixtures. In other cases, where the available materials approach those having optimum properties for production of high-compressive-strength concrete, the required water-cement ratio will not be so low, the concrete will be more workable, and more customary methods of consolidation will be effective. The choice, therefore, between low-cost materials and high-cost compaction procedures versus high-cost materials and lower-cost consolidation procedures will be a matter of engineering analysis in any specific situation.

The available data indicate it to be unlikely that 10,000-psi compressive-strength concrete will be obtained from mixtures having water-cement ratios higher than 0.45 by weight. They also indicate that in order to maintain a mixture of sufficient workability to be consolidated, it will in most instances be necessary to use at least 7 bags of cement per cu yd of mixed concrete. With cements, aggregates, and admixtures of the most desirable types it will be possible to use higher water-cement ratios, lower cement contents, and more workable mixtures than with materials of less desirable types. At a given water-cement ratio, it will often be possible to develop equivalent strengths at a variety of combinations of workability and cement content, increases in the latter being resorted to to provide increased workability at greater materials cost.

The production of concrete consistently having a high compressive strength will, in nearly all cases, involve achieving a low variance in test results, since it will, in most such cases, not be possible to produce concrete having an average strength very much higher than the specified strength in order to compensate for large variance in test results. The achievement of such low variance requires closer production control than is normally obtained on most construction projects. An essential feature in obtaining such close control will be an effective inspection organization composed of experienced inspectors who receive support from all echelons of project management.

High-compressive-strength concrete has been produced. In 1934, it was reported that workable concrete with slump of 0.8 in., a 28-day compressive strength of 10,150 psi, and a 90-day strength of 12,000 psi using basalt aggregate, 10.1 bags of cement per cu yd, and a water-cement ratio of 0.33 by weight had been made. In 1960, a 6 by

12-in. cylinder made in Costa Rica from concrete containing 9 bags of high-early-strength cement per cubic yard, a water-cement ratio of 0.37 by weight, and having a slump of 0 to ½ in. was tested at an age between 15 and 18 months, and had a strength of 16,650 psi. These and similar instances must, in the present state of the art, be considered exceptional since there has been little or no demand for high-compressive-strength concrete and consequently normal concrete production practices have not been established for routine production of such concrete.

High-compressive-strength concrete may have little if any economic advantage in conventional reinforced concrete construction in which steel of normal working-stress limits is used. However, high-compressive-strength concrete may be expected to have very important economic advantages in arch and dome construction, especially when the design requirements include resistance to dynamic loadings of high intensity and particularly where the structure is underground.

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Earlier Determination of Concrete Strength Potential

P. SMITH and H. TIEDE, Department of Highways, Ontario, Canada

A world-wide review is presented of accelerated strength-testing procedures that have been, or are being used, to obtain an early estimate of the 28-day compressive strength of concrete.

A new procedure, autogenous curing, is described in which the strength development of a concrete test cylinder is accelerated by curing in a well-insulated container that retains part of the heat of hydration.

The whole concept of using 28-day compressive strengths as a basis for judging the acceptability or strength potential of concrete is questioned. It is argued that an accelerated strength test could provide information equally as valid, at least 26 days sooner.

•IT would be very useful if the strength potential of portland cement concrete could be determined substantially sooner than is possible with the conventional procedure which requires 28 days of standard moist curing.

The main objection to the present approach is that it takes too long to obtain information on the compressive strength for it to be of real value for either concrete construction control or acceptance purposes. If low-strength concrete is not detected until 28 days after it is placed, replacement is usually only undertaken if a conclusive inquest shows that the safety and load-carrying capacity of the structure would be in jeopardy if the defective work remained. Furthermore, evaluated strength data are not available in time to influence product or quality control. More timely information on strength would help to achieve greater uniformity with resulting improvements in concrete performance and savings in cost.

A reliable accelerated strength test used for concrete acceptance or control purposes would also increase the confidence of designers, especially when one of their more daring designs was being built. Early assurance would be available to at least confirm that the concrete would accommodate the allowable design stresses and would have the necessary ultimate strength provided proper curing was given to the structure.

A researcher conducting strength-dependent experiments is currently forced to wait 28 days while the cylinders cure in a fog room. If an accelerated strength test could indicate the ultimate strength potential of concrete at least as well as the present 28-day test does, considerable savings in laboratory time and space would result.

The purpose of this paper is to discuss what has been done, is being done, and might be done to develop and use an accelerated strength test to meet these needs. One limitation placed on the subject area is that the accelerated curing procedures discussed ultimately require a test specimen to be broken to determine its strength. Only passing reference is made to inferences of strength which may be drawn from tests on plastic concrete or from ultrasonic or other nondestructive tests on hardened concrete.

The first part of the paper is devoted to a review of the historical and present state of the art of accelerated strength testing. Most of the accelerating procedures, so far

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advocated and used, require the external application of heat at a specific temperature for a given time. This may cause problems where the construction site is far from the testing laboratory, or where the optimum curing cycle does not fit a normal working day. The second part presents a new procedure, developed by the authors, known as "autogenous curing," which holds promise as a more convenient and simple method of accelerated curing that, at the same time, could offer greater uniformity in the handling, storage and shipping of field cylinders in general. The third aspect of concrete strength discussed is the whole concept of using 28-day compressive strengths as the measure of acceptability or ultimate strength of concrete, when evidence is mounting, that this could be replaced with a determination of strength at an earlier age.

REVIEW OF ACCELERATED STRENGTH-TESTING PROCEDURES

The first significant use of accelerated curing to obtain early estimates of concrete strength appears to have been by Patch (4) during the construction of the Hoover Dam in the early 1930's. By curing standard cylinders for 7 hours in boiling water almost immediately after they were made, he was able to obtain results within 8 hours, in time for the next placing shift to make necessary adjustments.

A similar test remained in use for over ten years by the Bureau of Reclamation (7). The ratio of 28-day to accelerated strengths was, however, found to vary from about 2.8 to 5.6 with different materials on different projects. This test was eventually abandoned because it did not prove sufficiently reliable. Time has shown, however, that the basic idea was sound. The inherent defect in this particular procedure is also common to others using high curing temperatures without an appropriate delay time. As will be discussed, the reason is now understood from later work, especially that of Smith and Chojnacki (53).

Developments Outside North America

Revival of interest in accelerated testing occurred during the early 1950's. Two procedures were developed in England, one by King and his co-workers (17, 24, 27, 28, 29, 31, 37, 43) which used dry heat in an oven, and the other by Akroyd and his co-worker (20, 40), which used hot water as the accelerating medium.

The dry heat procedure involved heating the specimens in an oven while they were covered and still within their molds. Usually the concrete cubes were placed in a cool oven one-half hour after mixing. The oven was then brought to 200 F (93 C) within 2 hours and maintained at this temperature for a total heating time of 6 hours. The cubes were then tested one-half hour after removal from the oven. This gave a testing time of 7 hours. Modifications of this basic procedure offered a range of testing cycles more convenient to fit the working day.

The original work of Akroyd and Smith-Gander (20) was based on 8½ hours of total testing time. One-half hour after mixing, the cubes were placed in water at 140 F (60 C) and then brought to a boil. After 7 hours in the water, the cubes were removed, cooled, and tested 1 hour later. Akroyd's later work (40) modified this testing cycle to a more convenient one of 24 hours' standard curing followed by 3½ hours' boiling and testing 1 hour later. Probably because of its inherent simplicity and convenience in fitting into the working day Akroyd's modified boiling method, with minor variations, has found considerable favor in England (40) and elsewhere (61, 71, 80). The accelerated to 28-day strength relationship established using this procedure is shown in Figure 1.

In discussing Akroyd's work, Thompson (43) presented a procedure which was successfully and widely used by a large contractor and in which the concrete specimens were heated in hot water at 95 F (35 C) for 24 hours. This gave results comparable to those obtained by King, yet was more convenient and simple. The regression line established by Thompson is shown in Figure 1. The latter's experimental work is also of interest for his use of the sonoscope to monitor the specimens during the curing cycle. Since then work in England has largely been in connection with the cooperative testing program which is discussed later in this paper.

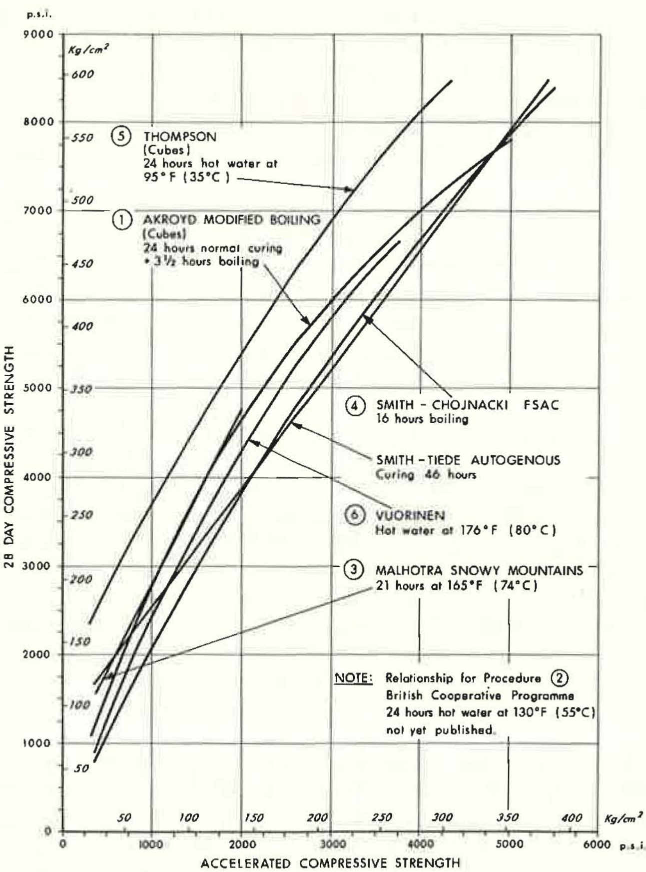


Figure 1. Comparison of some of the more promising or widely used accelerated strength-testing procedures.

In 1963, RILEM announced a symposium by correspondence under the chairmanship of Professor A. Berio. Its scope was indicated by its title: "Accelerated Hardening of Concrete with a View to Rapid Control Tests." The nine papers elicited, together with discussion and an excellent general report by the Chairman, were published in RILEM Bulletin No. 31, June 1966. The general report is in both English and French and where a paper is in French an adequate English summary is given. Reference annotations (59 - 67) give an outline of each paper. Since the publication is recent and accessible, further discussion can be limited to the disappointing comment that significant new information was sparse and discussion was brief. The publication of the symposium should, however, bring the possibilities of accelerated strength testing and details of several potentially useful procedures to the attention of a wider audience.

Two procedures that contain features of special interest are in use in Europe. The first, developed by Lichtenberg (73) and used in Denmark since 1961, appears to be the shortest procedure investigated to date. In this procedure the test cubes, within their molds, were placed in water at 150 F (65 C) immediately after casting, and the water was then brought to a boil in 20 minutes. When tested after only 2 hours' boiling an empirical relationship of $R_{28} = 37\sqrt{Ra}$ was claimed for normal concretes with an accuracy in the order of an 8 percent coefficient of variation. The second, reported by Vuorinen (44, 62) is notable because, other than for King's early work, it appears to be the only one using hot air as the accelerative medium. It has been in use in

Finland since 1961 for concrete control on dams and other projects. Heating has usually been by hot air at about 180 F (82 C) within an oven with forced-fan circulation. After an initial delay of 1 hour, the cylinders were brought up to temperature in 4 hours; after 20 hours' heating a 3-hour cooling period prior to testing was allowed. Upon occasion a similar cycle, but using hot water as the accelerating medium, has been used with similar results. The regression line for this procedure is also shown in Figure 1.

Dams which involve large amounts of concrete appear favorite projects for experiments with accelerated strength testing. In Australia, Cornwell (19), Malhotra (34, 60), and Nichol (69) have all reported on the value of such tests, which have been in use since 1955. The procedure reported by Malhotra involved testing cylinders which were cured for 21½ hours in hot water at 165 F (74 C). Allowing for an initial delay of one-half hour, testing could be completed within 24 hours. An accuracy of prediction of 28-day strengths of between 15.2 and 23.6 percent was claimed for a range of both mass and structural concretes used in the Snowy Mountains hydroelectric projects. The regression line for this test (Fig. 1) reflects that Type II cement was used.

Australia has also produced two other interesting procedures which, however, use steam curing. Mercer (15) in 1952 reported that by testing standard cylinders autoclaved at 350 F (177 C) for 21 hours, a relationship with 7-day standard cured strengths could be established. His results, however, did not correlate with strengths at later ages and were dependent on differences in water/cement ratio. In 1964, Boundy and Hondros (68) suggested that by steam curing concrete cubes for 6 hours at 190 F (88 C) and then testing them after 1 hour's cooling with a Schmidt impact hammer, a useful field test that did not require such facilities as a compression testing machine could be developed. For practical use the relationship derived between 28-day normal cured strength R_{28} and the rebound number R_S obtained from the steam-cured specimens was $R_{28} = 19\,R_S^{1.8}$.

Mention should be made of two reports from Europe of procedures where steam curing has been used to accelerate the strength development of mortars to obtain an estimate of the 28-day strength of normal cured concrete or for the estimation of the strength-producing ability of cements prior to use. Autoclaving has been used by both Meyer (79) in Germany and by the Swiss Federal Laboratory for Testing Materials and Research (23). The former's work is of interest because of the very small size of test specimen used. The mortar test cylinders were only 7/16 in. (11.3 mm) high and 3/16 sq in. (1 sq cm) in cross-sectional area. An initial 3-hour delay moist curing at 104 F (40 C) was used and when tested at 5 hours after autoclaving, their strength was reported to be approximately equal to the 28-day strength of concrete containing the same paste. The Swiss test appears to have been used mainly as an accelerated cement strength acceptance test on concrete dam construction. Its most noteworthy difference from other procedures is that final cooling to room temperature, after withdrawl from the autoclave, is in a hot-water bath. The accelerated strengths obtained were almost equal to the corresponding 28-day mortar strengths and the study also included comparisons up to one year.

The idea of using only the mortar fraction for accelerated curing is interesting, and recalls that some of the earliest work (2) on accelerated testing was done on mortars.

Other procedures for the accelerated strength testing of concrete have been reported from the USSR (16), Poland (6, 64), Belgium (67), France (10, 63), Italy (32, 81), India (47, 56), Germany (46), Venezuela (55), Holland (82) and Romania (65). They are not discussed in detail since they are similar to those already described, have not been carried beyond the exploratory state, or involve low-pressure steam curing. A general discussion of steam curing will be found in the next section of this paper. It is of interest to note that some workers, for example, Dutron (67) or Jarocki (64), included unusual materials such as blast furnace slag cements in their tests with no adverse effects.

An appropriate conclusion to this world-wide survey is an account of what is probably the greatest practical use being made of accelerated strength testing. Grant (72) has reported the development of a procedure in which concrete cubes made earlier in the day are placed at the end of the afternoon into a water tank at 65 F (18 C). The temperature is then raised to 180 F (82 C) in approximately two hours, and curing at this temperature is continued for 14 hours. The cubes are tested in compression immediately upon withdrawal from the tank. Though the results show greater scatter than those obtained by some of the more sophisticated procedures, they are considered sufficiently accurate to be the basis of day-to-day control. Initially the test was used simply to give an early indication of 28-day strengths. After some years' experience, a correlation history has been built up so that by now, 28-day testing is used only as a check test to confirm that the correlation is still valid. The general relationship found was linear with $R_{28} = 1.2233 R_a + 1238$. This system is being used in twenty central laboratories covering more than 100 plants in England that produce well over 20,000 cu yd of concrete a day from 500 different mixes. The method has also spread to France, Israel, and Australia, and as Grant has commented (72), "it enables us to adjust our cement contents, up for technical reasons or down for more profit, 27 days ahead of our previous system."

Developments in North America

In 1958, the Ontario Department of Highways, initiated development work on accelerated strength testing as part of a program to improve concrete quality control. The results of this work have already been published (53). The main feature of the procedure ultimately adopted was the introduction of a measured delay time based on the degree of set, prior to the start of acceleration. It was demonstrated that, provided the concrete had reached a fixed degree of set, the ratio of accelerated to 28-day strength was independent of many variables, such as type of cement and presence of admixtures, that had led to erratic results when earlier procedures were used. This procedure, known as Fixed Set Accelerated Curing (FSAC) uses boiling water and a heating time of 16 hours. With the measured delay time and 1 hour for cooling and capping, this gives a total time of just over 24 hours for an average concrete. The procedure has been in use for five years and has proved both useful and reliable as a quality control measure. The accelerated strength of concrete so determined (Fig. 1) has been included as a specification requirement for concrete acceptability.

Malhotra, Zoldners and others (61, 71) have developed Akroyd's modified boiling method for day-to-day quality control at both ready-mix plants and at hydroelectric projects involving concrete dam construction. Included in their work has been extension of the boiling time to attain more reliable results where Type II cements are used. These results almost exactly match Akroyd's (40); the regression line is shown in Figure 1.

Thompson's simple method of heating for 24 hours in hot water at 95 F (35 C) has been modified by Abdun-Nur (83) by casting cylinders vertically and then turning them horizontal to avoid the need for capping. This procedure is currently being used with good success on a number of projects as an aid to control.

One of the earliest reports of accelerated strength testing was by Gerend (1) in 1927, who used steam curing. In recent years there has been considerable active research in North America (and, of course, corresponding work elsewhere) into the effects of steam curing because of its increasing importance as a manufacturing process. A report of ACI Committee 517 (58) has presented a comprehensive review of the subject from which the significance of pre-steaming delay periods, rate of temperature rise or cooling, and the temperature and duration of steam-curing test cycles, required to obtain optimum results, can be readily appreciated. Papers by Merritt and Johnson (48), Hanson (52), Higginson (45), Brown (75), and other references cited in the ACI report expand upon the details. Explanations of the effects of steam curing based on the difference in cement hydration may be found in the work of Nurse (12), Rey (13), Verbeck (74), and Mironov (85).

Though no accelerated strength-testing procedure comparable, for example, to that of Dutron (67) or Mihail (65) has resulted from such work in North America, the acceptability of concrete units for their readiness to accept transfer of stress, is often judged on the basis of cylinders steam cured along with the units.

Limited accelerated testing of steam-cured cement mortar cubes has been undertaken by Wagner (41), among others, to give an early determination of the influence of cements on concrete strength.

Limitations Inherent in Accelerated Strength Testing Procedures

Curing Temperature, Initial Delay—Most workers have found that, while their acceleration procedures worked well for a given set of materials on one job or in one laboratory, as soon as significant variables were introduced such as different cements, aggregates, or sometimes even different water/cement ratios or variations in number of specimens within the accelerating container, greater dispersion in results occurred. These problems have appeared to be most serious in those procedures which involved little or no delay time between mixing and the commencement of acceleration, or where the accelerating cycle was short, at high temperature, and where the temperature rise was rapid, or where ovens rather than water baths were used.

Though such behavior is not altogether surprising since green concrete is somewhat delicate, the reasons for it remained unexplained until the importance of the delay prior to acceleration was specifically demonstrated by Smith and Chojnacki (53) and Tiede (77). Confirmatory deductions can be made from the work on steam curing previously cited and the investigations by Nurse (12), Saul (14), Plowman (21) and Narayanan (35, 38), that led to the maturity concept of concrete strength. During investigations by the former researchers, both physical expansion and evidence of thermal shock have been observed, accompanied by erratic results at temperatures of 140 F (60 C) or above where a significant delay time prior to acceleration was not allowed. This has been especially true of concretes that have contained cements slower than average to set or where water-reducing admixtures have been used. At 190 F (88 C) a 6-hr delay proved insufficient and even at 140 F (60 C) a 5-hr delay was inadequate. However, when the initial water temperature was 100 F (38 C) and a 1-hr delay was used, these problems were not encountered.

The apparent success of many procedures, in spite of inherent weaknesses because delay time and temperature effects have been ignored, is probably due to the fact that unusual cements or admixtures have not been used in the concretes tested. This may be especially true outside North America, though Lichtenberg (73) commented that where a well-known water-reducing admixture was included in the concrete he boiled for two hours without a delay time, no measurable strength was obtained.

Very high temperatures such as those used in high-pressure steam autoclaving produce different hydration products from those given by normal or low-pressure steam curing (85) and might also promote pozzolanic reactions with lime or silica (or other pozzolans such as fly ash), if present. For these reasons and the complicated equipment needed, procedures involving temperatures above 212 F (100 C) are unlikely to be of value as standard accelerated strength tests of concrete.

Accelerative Medium, Temperature Control and Duration of Heating—Much of the initial work in England used dry heat, within an oven, as the accelerating medium. Akroyd (40) questioned this practice on the grounds of lack of convenience and uniformity especially since each oven he examined had different characteristics. This appears to have been confirmed since the oven test was eliminated from the final round of the British cooperative testing procedure which is discussed later. As far as can be ascertained this then leaves only one procedure, that of Vuorinen (44, 62), which uses an oven. Other than in the autogenous curing procedure, hot water or steam appears to have become the universal means of heating the test specimens. The accelerated curing chamber must, of course, be designed to provide even circulation to achieve uniform heat distribution with close thermostatic control of temperature. Temperature control to approximately ± 3 F (2 C) appears to have been achieved in most hot-water procedures. One problem with the boiling water procedure is, of course, that the boiling point varies with atmospheric pressure. The effect is not significant at normal altitudes. At 5,000 ft, corresponding to the altitude of Denver, water boils at 202 F (95 C) and a minor correction might then be needed.

Variation in duration of heating has less effect than might be expected (20, 37, 53, 71). It appears that test specimens may be removed from the accelerated curing tank within ±10 percent of the total heating time without invalidating the test. It is, of course, desirable to stay as close to the specified time as possible.

In order for a procedure to be universally applicable, without first correlating the accelerated strength results to normal 28-day results for the specific concrete mixture in use, it is almost certain that it must contain one of the following essential features:

1. A long unmeasured delay time, as is featured by Akroyd's modified boiling method (40). There is evidence (53, 77) that a long initial delay period has disadvantage in that the ratio of accelerated strength to 28-day strength will be lower and will be only slightly higher than that given by normal curing of the same duration as the accelerated curing. The reason for this may be because the basic gel structure has become well established prior to commencement of acceleration (85).
2. The delay time required for a particular mixture must be measured. The required delay time can be determined from the degree of set of the concrete using ASTM Method C 403 as suggested by Smith and Chojnacki (53). Recent work indicates that for below boiling point accelerated curing temperatures a suitable delay time is 20 minutes after a Proctor needle penetration resistance of 1,000 psi has been attained.
3. A low rate of temperature rise must be used and the maximum accelerated curing temperature must be below 140 F (60 C). The precise limit below this temperature is not known. The 95 F (35 C) temperature used in Thompson's procedure (43) is, however, known to be safe. This is one of the principal advantages of his method.

Overtime Work—The work of many investigators shows that at some stage they have modified the optimum procedure for acceleration that they originally developed to produce a cycle fitting a normal working day. This approach is sensible because a test is unlikely to gain popular acceptance if it involves the cost and inconvenience of overtime or shift work.

Of the promising methods in use, the one most affected by the disadvantage of overtime work is the FSAC procedure developed by Smith and Chojnacki (53). The essential determining measurement of the set of the concrete and transfer of the cylinders into the water tank is likely to occur somewhere between 6 and 12 hours after mixing. However, it is one of the few procedures that has demonstrated the independence of results obtained by it, from variables in the concrete mixture. Suggestions have been made by Mather (57) that the hot water might be run into the curing tank through an automatic time valve, and by Vellines (76) that readiness of the concrete to receive acceleration might be determined when the temperature had risen 5 F (3 C) (because hydration had started) above that of the fresh concrete. These two ideas might be combined so that the rise in temperature actuates the valve and the curing procedure becomes automatic.

Capping and Delay Before Breaking—The wisdom of capping hot or warm cylinders has been questioned; however, no real problem has been reported with the normal type of sulfur-granular cap. Cubes, as used in England, offer advantages in this respect since capping is not required. However, introduction of cubes for accelerated strength-testing purposes would not appear sufficiently advantageous to warrant the disturbance that would be involved. Abdun Nur (83) has tried to get around the problem in the case of cylinders by turning them horizontally to get true plane ends which do not require capping. One thing against this is that during the setting process incipient weaknesses might develop on one side of the cylinder leading to eccentric loadings and spurious results on testing. It is also not feasible with light metal (tin) molds. General acceptance of accelerated strength testing is more likely if it is compatible with present accepted specimen fabrication and testing procedures.

One of the few investigations into the effect of delay time between the end of acceleration and testing in compression was by Malhotra and Zoldners (61), who found little effect. It may be that problems after acceleration are more imaginary than real, providing reasonably standard techniques are used and timing and cooling is always kept essentially the same. It would, however, be more satisfactory if this area was more thoroughly investigated.

Level of Accelerated Strength Developed and the Weak Aggregate Problem—Figure 1 shows that accelerated strengths of the more common types of concrete are in the order of 40 to 60 percent of the corresponding 28-day strengths. Low-temperature or short-time accelerating procedures are on the low side of this range. Long, high-temperature, hot-water curing procedures are at the high end of the range.

Unless a procedure is found that gives considerably higher levels of accelerated strength, which is unlikely judging from the evidence of work on steam curing, two problems exist:

1. The accuracy of prediction of 28-day strengths is reduced because the effect of errors is magnified; and
2. The concrete might contain some feature (most likely a poor aggregate) which, while not affecting the lower accelerated strength, would prevent the higher predicted 28-day strength from ever being reached.

The first problem may be mitigated by selecting a procedure which is simple and the least susceptible to operator, equipment, or other error. The second cannot be solved directly. If a possibility of low-strength aggregates exists, some assurance can be obtained from physical tests on the aggregate. Alternatively, when time permits, a correlation can be established at the mixture design stage by making both normal and accelerated cured cylinders. When time is really "tight," an additional trial mix can provide the answer if an increased cement factor is used that would bring the strength of the accelerated cylinders at least up to the strength level anticipated from the normal cured cylinders made with the lower cement factor specified for the work.

The influence of aggregates probably needs further investigation, since there is conflict in published reports as to whether they do or do not affect the results. This conflict, however, may be caused by the nature of the accelerating procedure used rather than a fundamental effect. When a procedure is designed to take account of concrete variables (for example, Smith and Chojnacki's FSAC, 53) no disturbing influence due to normal concrete aggregates has been reported. Unusual aggregates (i.e., lightweight aggregates) have not generally been investigated. Thompson (42) however, reported that with one such aggregate "Lytag," although a relationship existed between accelerated and 28-day strengths, the curve was displaced from that determined for normal concretes.

Other Possible Methods of Accelerating Strength Development—The reactions involved in hydration of cement are chemical and physical; such reactions may be accelerated by three means: heat, pressure, or catalysis. The application of external heat has been the basis of most procedures investigated in the past. Erlin (70) has tried a pressure of 5,000 psi for 24 hours by placing cylinders in a high-pressure air meter, but the accelerated strengths developed were too low to be of practical value. The work was discontinued, though a combination of heat and pressure might still be tried. The addition of a catalyst or chemical accelerator might also be investigated. However, experience with the obvious choice, calcium chloride, and deduction from some of the results presented subsequently in this paper, suggest that results would be unreliable since the addition rate could not be related to a known cement content and different behavior occurs with different cements.

The only accelerating refinement in sight would appear to be the use of internal rather than external heat. One such procedure, autogenous curing, is described in this paper. Other procedures might be based on dielectric heating, which is beginning to receive attention as a curing method (78). (This is, of course, distinct from electrical resistance methods of heating which have also received some attention 11.)

While it is only fair to point out weaknesses that may exist in accelerated strength testing, the process contains built-in safeguards; for example, on large projects or in plants using known materials, an individual correlation history is soon established if both accelerated and 28-day cylinders are made. Furthermore, and if nothing else, strengths that are going to be low in later normal cured cylinders will undoubtedly show up in the corresponding accelerated cured cylinders in time for corrective action to be taken.

AUTOGENOUS ACCELERATED STRENGTH TESTING

The hydration of portland cement involves exothermic chemical reactions. The heat generated by normal Type I cements, over the first three days, is in the order of 75 calories per gram. Darey and Fox (5) showed that given adiabatic curing, the temperature rise in concrete is substantial. For example, with an average 1:2:4 concrete at a water/cement ratio of 0.60, they found a temperature rise after 24 hours of about 55 F (31 C) above the starting temperature; at 48 hours the temperature rise was 72 F (40 C) and after 72 hours it was 80 F (44 C). The successful use of insulated forms for the protection of concrete placed in winter provides practical confirmation that conditions need not be adiabatic for a substantial temperature rise of the concrete to occur and strength to develop.

Seeking for a way to overcome some of the shortcomings of accelerated strength-testing procedures, the authors postulated that curing a concrete cylinder autogenously inside a well-insulated container might alone be sufficient to provide accelerated strength development comparable to that achieved by applying external heat. If this proved so, an autogenous curing procedure would have many advantages. For one thing, since the specimens would be gradually heated from within, the process should be self-regulating and this would obviate waiting out a delay time before commencing acceleration. All that would then be needed to make the procedure fit the working day would be the selection of a convenient end-time. Another apparent advantage would be in the simplicity of the equipment. The insulated container used would also provide a uniform and safe means for handling, shipping and storing cylinders before testing without the need for elaborate water curing tanks or moist rooms.

An investigation was therefore initiated into autogenous curing to determine its potential as an accelerated strength-testing procedure either alone or as the first stage of a hot-water procedure.

Initial Experiments

Experiments were made to devise a suitable insulated curing container and to make a preliminary screening of the following possible testing cycles to determine which afforded the most promise for detailed investigation:

1. Autogenous curing for 22 hours.
2. Autogenous curing for 22 hours plus additional hot-water curing: (a) 24 hours at 100 F (38 C); (b) 24 hours at 140 F (60 C); (c) 24 hours at 212 F (38 C); (d) 3 hours at 175 F (80 C); and (e) 3 hours at 212 F (100 C).
3. Autogenous curing for 46 hours.

Companion cylinders, moist cured for the same total time, were also included to check if useful acceleration was being achieved; comparisons were also made with normal 28-day moist-cured cylinders.

In this initial work the range of concrete mixture variables was limited to one cement at three cement factors, two water/cement ratios, three air contents and three dosage rates of one or more of three admixtures. Three cylinders were made for each variable examined in each test cycle.

The time of testing the accelerated cylinders was always 1 hour after completion of the curing, and allowing 1 hour from the time of mixing to placing the cylinders in the insulated containers gave testing cycles of either 24 or 48 hours, except for procedures 2(d) and (e), where it was 28 hours.

Full details of this work are contained in a thesis by Tiede (77). Features of importance to the procedures selected for detailed examination are brought out later, and at this stage it is only necessary to comment on the following points to justify the selection made:

1. For 22 hours' autogenous curing alone, the relationship between 24-hour accelerated strength and 28-day or normal cured strengths showed considerable scatter, but the relationship was very promising when the autogenous curing time was extended to 46 hours.

2. The combination of 22 hours' autogenous curing with 24 hours' hot-water curing appeared to produce results comparable to those obtained by 46 hours of autogenous curing alone. Of the three different temperatures examined, 212 F (100 C), 140 F (60 C), and 100 F (38 C), the latter appeared to offer slight advantage, as far as the level of accelerated strength development was concerned.

3. Where only a short additional hot-water curing period (3 hours) was used there was little additional strength development over that obtained by autogenous curing alone for 22 hours or wet-burlap curing for 23 hours.

4. All companion cylinders, moist cured for either 23 hours or 47 hours, showed lower strengths than those subjected to one of the accelerated curing procedures. They were approximately 25 percent lower; when compared with the 28-day moist-cured strengths, they showed much wider dispersion.

5. Variations in both the starting temperature of the plastic concrete and ambient temperatures outside the container affected the results. Several measures, such as the use of correction factors, additional insulation or bringing the concrete to a standard starting temperature, appeared to offer promise for mitigation, and it did not appear that this problem nullified the concept.

As a result of these experiments, the two most promising methods were further investigated for a wide range of concrete variables, using the equipment and testing procedures developed during this phase of the work. The two methods were the following:

1. Autogenous curing for 46 hours.
2. Autogenous curing for 22 hours, followed by hot-water curing for 24 hours at 100 F (38 C), or 140 F (60 C), or 212 F (100 C).

Equipment

Figure 2 shows details of the insulated autogenous curing container. The container holds one standard 6 by 12-in. cylinder which is cast in a light metal (tin) mold. The protruberances on heavy metal molds prevent the insulation fitting closely; therefore, they are not suitable. The insulated container can easily be made from a standard plastic garbage can and foamed-in-place polyurethane. The free air space around the cylinder should be as small as possible. (In certain earlier experiments wooden boxes having equivalent insulating properties were used.) A plastic bag or a plastic cylinder lid was used to retain moisture in the cylinder during curing. Figure 3 shows the complete autogenous curing container receiving a test cylinder.

In the development experiments, temperatures during the curing cycle were continuously recorded from embedded thermocouples. For general field use, a maximum-minimum thermometer within the container air space may be used to record peak and final curing temperatures. (The starting temperature, that of the plastic concrete, should also be measured with a normal thermometer.)

Test Procedures

The timing of the test procedures was designed, with convenience of fitting the working day in mind, to give either a 24-hr or 48-hr total testing time. The test procedures used in all experiments were as follows:

1. All specimens were standard 6 by 12-in. cylinders made in accordance with ASTM Method C 192, and tested in compression in accordance with ASTM Method C 39.
2. Cylinders for autogenous curing were covered with steel plates and wet burlap immediately after making, and so maintained until 1 hour after time zero ("time zero" is the time at which the mixing water was added).
3. Cylinders for autogenous curing were sealed in a plastic bag or covered with a plastic lid and placed in the insulated container 1 hour after time zero.
4. Autogenous cured cylinders were removed from the containers either 22 or 46 hours later (23 or 47 hours after time zero) depending on the particular total timing of the test cycle in use.

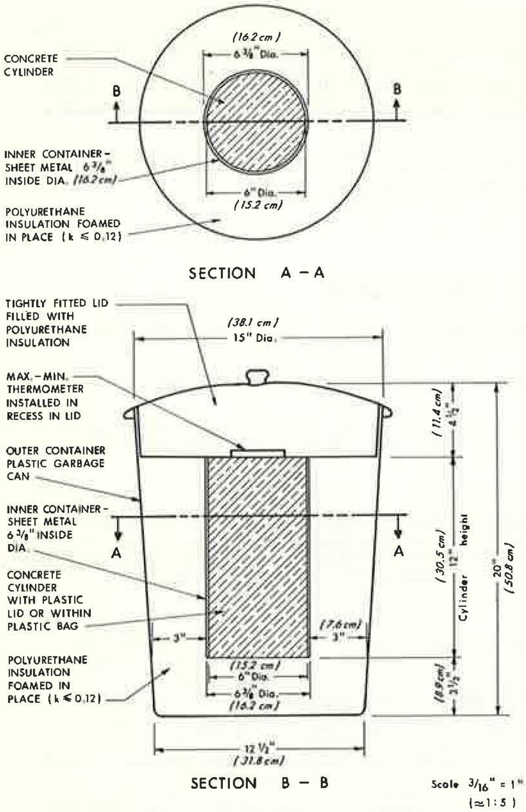


Figure 2. Autogenous curing container.

Figure 3. Complete autogenous curing container receiving test cylinder.

5. Cylinders were then demolded and allowed to stand in room temperature for 30 minutes. They were then capped and broken in compression at 24 or 48 hours after time zero.

6. Companion cylinders, for testing after 24 or 48 hours normal curing, were stored under wet burlap until one hour before testing.

7. Comparison 28-day cylinders were cured normally in accordance with ASTM Method C 192.

8. Where tests involved additional hot-water curing, after step 4 at 23 hours after time zero, the cylinders were immediately transferred to a hot-water tank (53). They were then maintained at the specified temperature 100 F (38 C), 140 F (60 C), or 212 F (100 C) for 24 hours before proceeding with step 5.

9. Temperatures during curing cycles were determined by embedded thermocouples.

10. All other tests, such as those for the properties of the plastic concrete, were performed in accordance with standard methods.

11. The starting temperature of the plastic concrete, and the ambient storage temperature of the insulated containers was 70 to 75 F (21 to 23 C), except when these were the variables under examination.

INVESTIGATION OF 46-HR AUTOGENOUS CURING AND 22-HR AUTOGENOUS CURING COMBINED WITH 24-HR HOT-WATER CURING

The results of the initial experiments were considered sufficiently encouraging for a further series of tests to be undertaken to examine the effects of concrete variables.

Two procedures were used: Series I, autogenous curing for 46 hours, and Series II, autogenous curing for 22 hours, followed by hot-water curing at 100 F (38 C), or 140 F (60 C), or 212 F (100 C) for a further 24 hours. Allowing one hour delay at the beginning, and one hour for cooling and capping, the total time for both procedures was 48 hours.

In all respects, the experiments used the equipment and procedures already described. All strength results are the average of 3 cylinders.

Series I: Autogenous Curing for 46 Hours

Concrete variables included in Series I were the following:

1. Cement: 6 Type I cements, each from different mills; 1 of each Types II, III, IV, and V.
2. Cement contents: 350 lb/cu yd (227 kg/m^3), 525 lb/cu yd (340 kg/m^3), 700 lb/cu yd (454 kg/m^3), with each type of cement.
3. Water/cement ratio: 0.4, 0.5, 0.6, 0.7 (with one Type I and one Type III, cement).
4. Air contents: 4, 6 and 8 percent.
5. Admixtures: 3 water-reducing admixtures designated A, B and C (two of which, B and C, were set retarding and were used at increased doses), 1 percent and 2 percent calcium chloride; the effects of these admixtures were examined with 3 different Type I cements at one cement factor.

Temperature-Time Results—Figures 4, 5, 6, and 7 show the temperature-time relationships obtained. Generally speaking, peak temperatures were between 30 and 50 F (17 and 28 C) above the starting temperature and occurred between 18 and 24 hours. There appeared to be no direct or immediate correlation between either peak temperature or net heat input area under the curve and accelerated strengths. This point is discussed in detail later when the strength results are considered.

The effect of cement content and different cement types is apparent from Figure 4. As might be expected, higher cement contents have greater temperature increases while the peak temperatures with the Type II, IV, and V cements are somewhat lower. One Type I cement (1-2) is a maverick both in respect to its lower peak temperature and the lateness at which this occurs. This will assume considerable significance when the effect of admixtures is discussed, and strengths are considered. As Figure 5 shows, the effect of water/cement ratio on the temperature curve was small, peak temperatures at the highest water/cement ratio tested were less than 10 F (6 C) lower than the highest. Even less effect was observed with differences in air content (Fig. 6).

Figure 7 shows the effect of admixtures. Compared with the same concretes without retarders, attainment of the peak temperature is delayed. With cements of "normal behavior" the delay may amount to 5 to 10 hours. With the maverick Type I (1-2) cement, however, the delay was such that the peak temperature did not occur, in one case, within the 46-hr test period, and in other cases was delayed until 40 hours. The reason for this is not entirely understood; the cement has a long initial (over 4 hours) and final (almost 6 hours) Vicat setting time. It is considered an excellent summertime cement for these reasons. However, there is obviously a danger in using this cement without due regard when concrete is being protected by insulated forms in winter. With this same cement, a striking effect occurred when calcium chloride was added. The peak temperature then occurred at the same time as when other Type I cements were used, and the temperature rise was about the same. Surprisingly, however, with the other two Type I cements tested, the addition of calcium chloride had little or no effect on the peak temperature and only slightly, if any, advanced the time at which this occurred.

Similar time-temperature measurements, although not developed for the purpose, might offer possibilities for the general study of anomalous admixture-cement behavior. For example, with Type I cement (1-1), admixture A, which contains an accelerator, gives a peak temperature both lower and later than admixture B which is the same material without the accelerator, whereas with another Type I cement (1-3), the position is reversed.

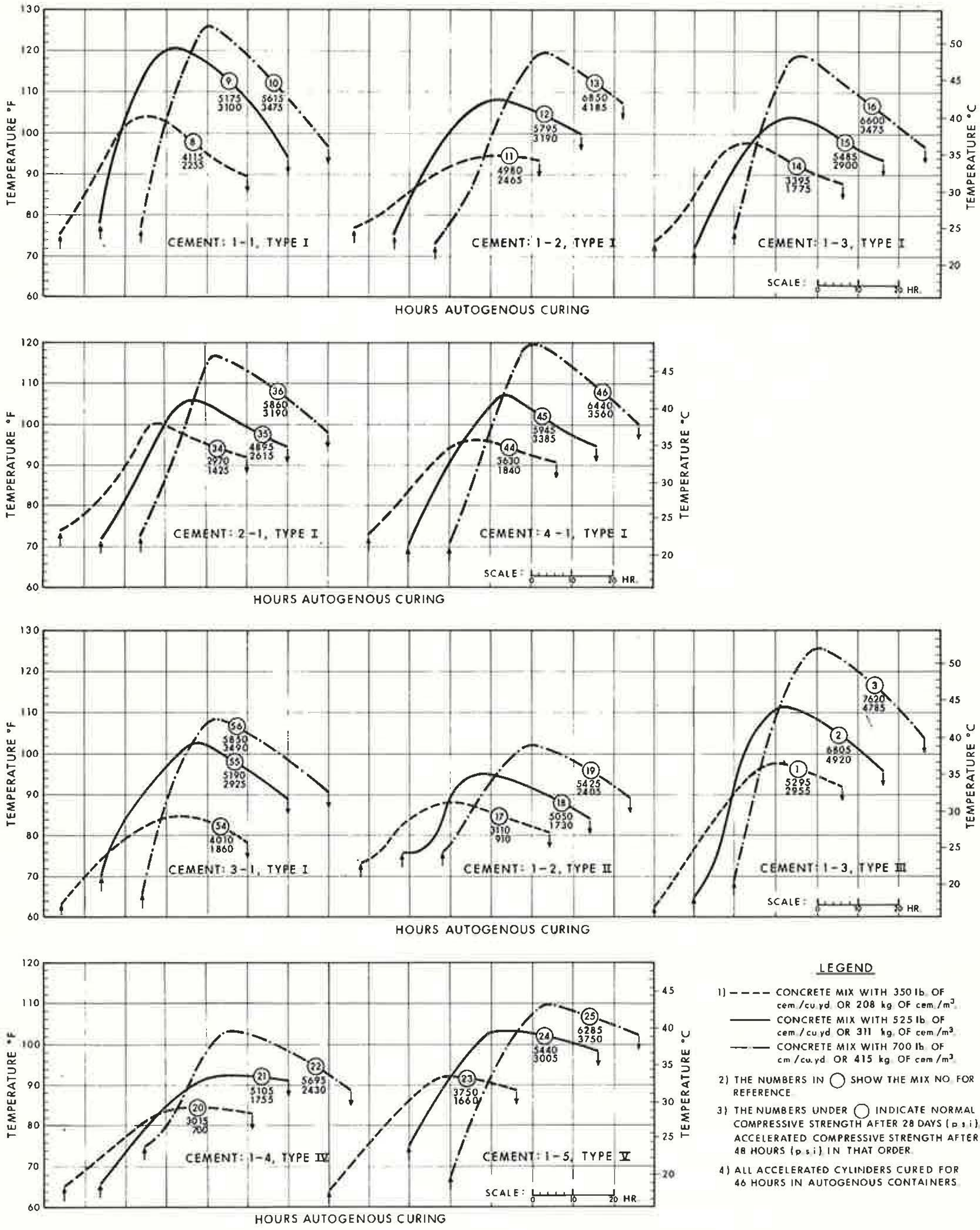
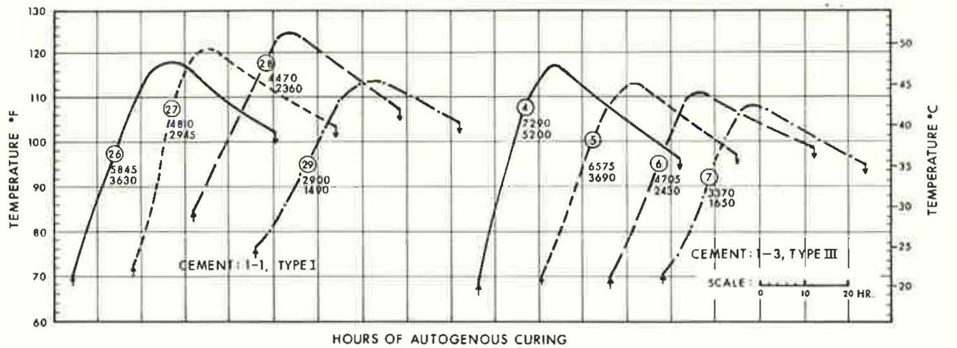


Figure 4. Series I-effect of cement type and factor on temperature rise.

48-Hour Autogenous to 28-Day Strength Relationship—Figure 8 shows the combined data from all the tests. (For convenience, the individual strength results have been shown in the figures of the corresponding temperature-time curves. In each case the upper number shown is the 28-day strength and the lower is the corresponding accelerated strength.) A linear or very close to linear relationship is apparent. There are evident and explainable differences in the population (see Figs. 9, 10, 11, and 12).



NOTE:
ALL MIXES WITH 525 lb. CEMENT/cu.yd. OR 311 kg. CEMENT/m^3.

LEGEND

1) ——— W/C 0.4
- - - - - W/C 0.5
— — — W/C 0.6
—·—·— W/C 0.7

2) THE NUMBERS IN ◯ SHOW THE MIX NO. FOR REFERENCE.
3) THE NUMBERS UNDER ◯ INDICATE NORMAL COMPRESSIVE STRENGTH AFTER 28 DAYS (p.s.i.) ACCELERATED COMPRESSIVE STRENGTH AFTER 48 HOURS (p.s.i.) IN THAT ORDER.
4) ALL ACCELERATED CYLINDERS CURED FOR 46 HOURS IN AUTOGENOUS CONTAINERS.

Figure 5. Series I-effect of water/cement ratio on temperature rise.

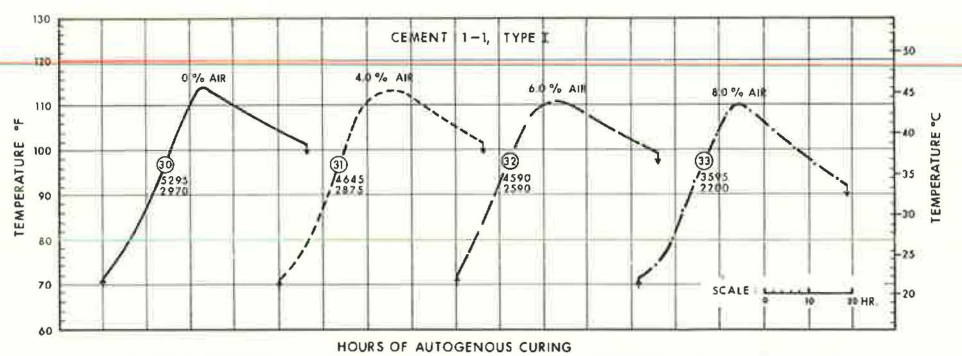
A regression line from the data in Figure 8, excluding that for Types II and IV cements and considering only the results in the normal working range of 28-day concrete strengths below 6,500 psi (457 kg/cm^2), has been established by the method of least squares. The relationship between accelerated strength R_A and 28-day strength R_{28} is

$$R_{28} = 1.35\ R_A + 1,180 \text{ psi}$$

or

$$R_{28} = 1.35\ R_A + 82.97 \text{ kg/cm}^2$$

with a standard deviation of 301 psi (21.2 kg/cm^2). This regression line and the 2σ limits are shown in Figure 8. Further statistical consideration shows that the same



NOTE:
ALL MIXES WITH 525 lb. CEMENT/cu.yd. OR 311 kg. CEMENT/m^3.

LEGEND

1) ——— NO AEA USED
- - - - - 4 % ENTRAINED AIR
— — — 6 % ENTRAINED AIR
—·—·— 8 % ENTRAINED AIR

2) THE NUMBERS IN ◯ SHOW THE MIX NO. FOR REFERENCE.
3) THE NUMBERS UNDER ◯ INDICATE NORMAL COMPRESSIVE STRENGTH AFTER 28 DAYS (p.s.i.) ACCELERATED COMPRESSIVE STRENGTH AFTER 48 HOURS (p.s.i.) IN THAT ORDER.
4) ALL ACCELERATED CYLINDERS CURED FOR 46 HOURS IN AUTOGENOUS CONTAINERS.

Figure 6. Series I-effect of air content on temperature rise.

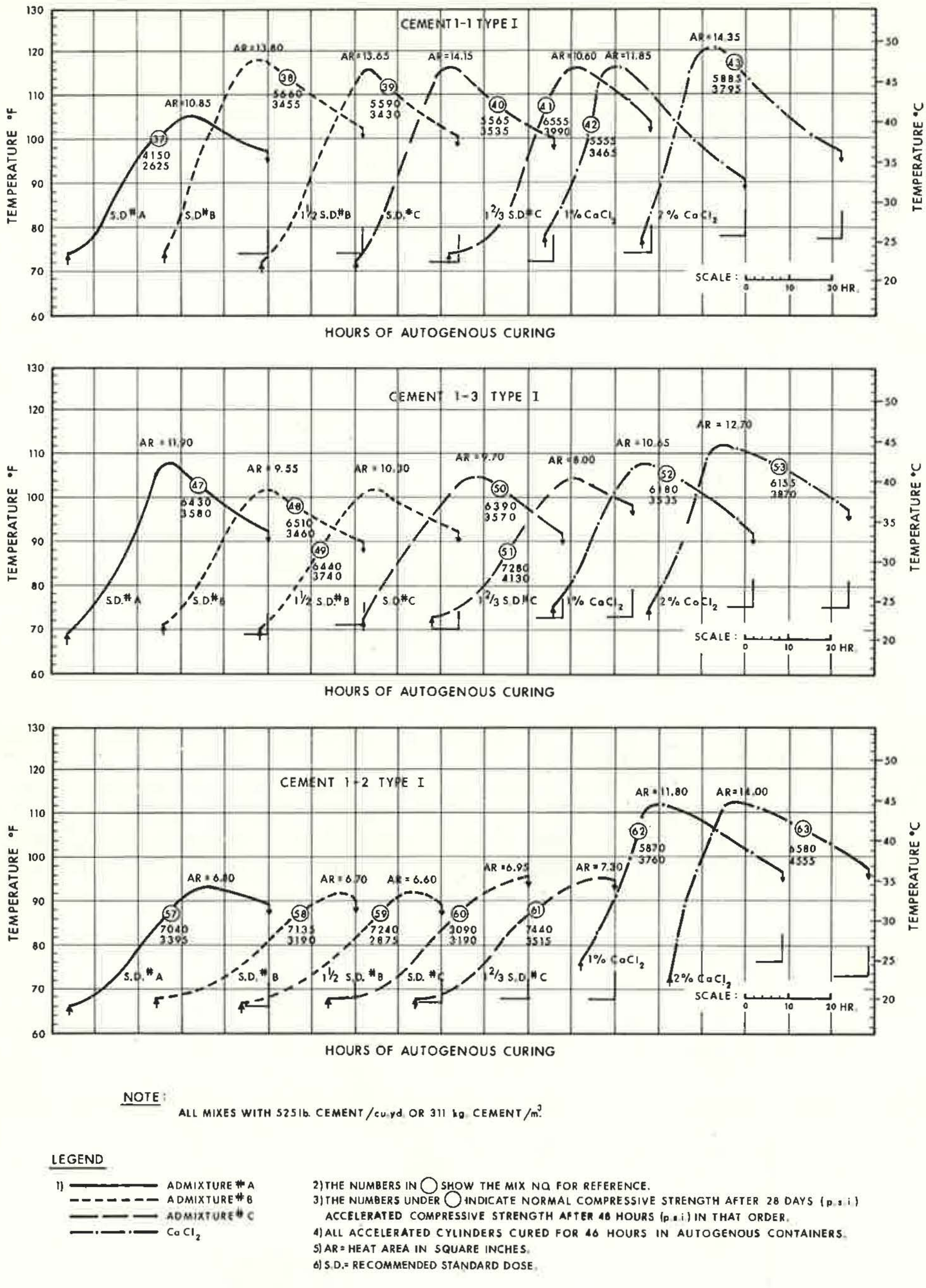


Figure 7. Series I-effect of admixtures on temperature rise.

relationship is essentially valid above 6,500 psi, but inclusion of these results would increase the standard deviation to 602 psi (42.4 kg/cm^2).

The accuracy of the autogenous curing procedure would appear to compare closely with those of other accelerated curing procedures. The range of standard deviations reported by the more promising procedures is from 250 to 500 psi (17.6 to 35.2

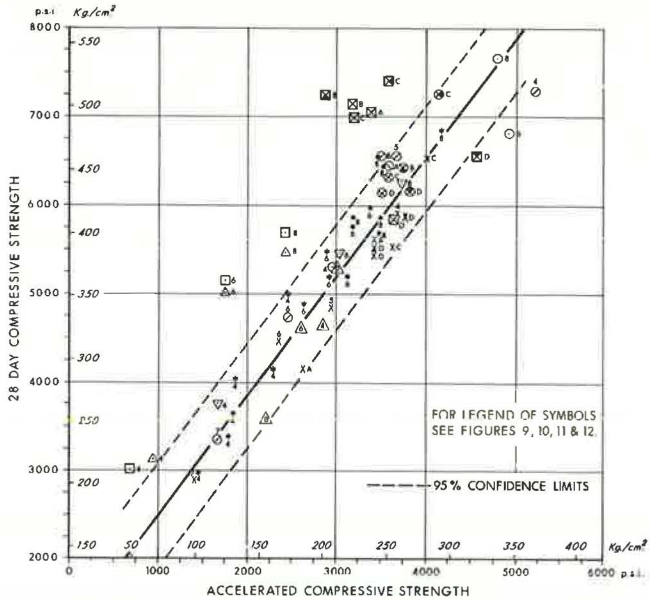


Figure 8. Series I-combined results.

kg/cm²), for example, with Smith and Chojnacki (53) FSAC procedure the standard deviation was 345 psi (24.4 kg/cm²).

Figures 9, 10, 11, and 12 break down the strength data into component parts. From Figure 9 it is clear that all the Type I cements tested, including maverick cement (1-2) and the Types III and V cements, show a similar and good relationship between accelerated and 28-day strengths. Figure 10 shows that different water/cement ratios have

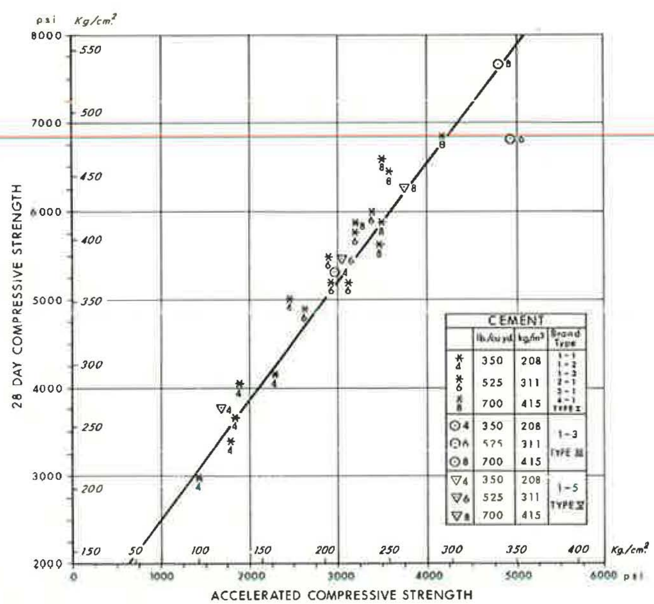


Figure 9. Series I-effect of cement Types I, III, V, at varying contents.

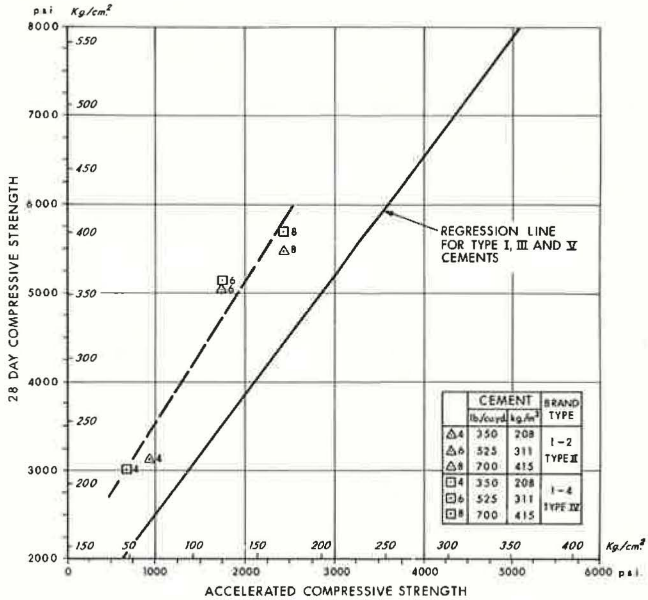
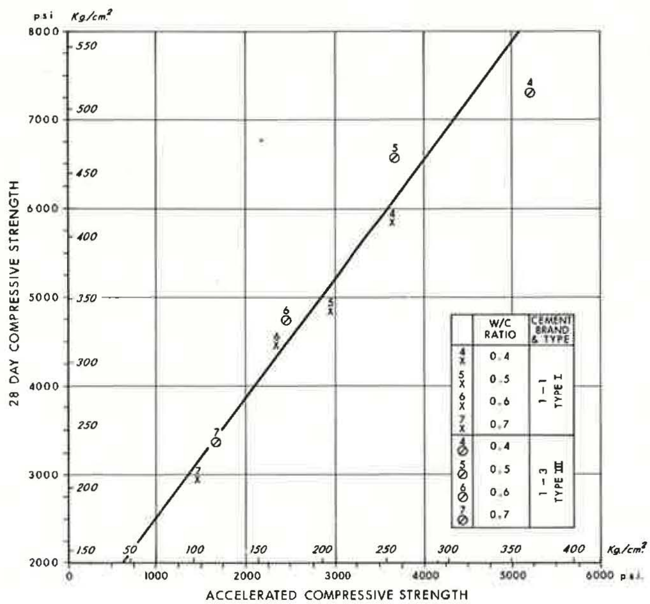


Figure 10. Series I-effect of cement Types II and IV at varying contents.

no effect on the relationship. As might be expected, the Types II and IV cements, because of their lower heats of hydration, exhibited the lower almost parallel relationship between accelerated and 28-day strengths shown in Figure 11, viz:

$$R_{28} = 1.58 R_A + 1,960 \text{ psi}$$



NOTE: ALL MIXES WITH 525 lb. CEMENT/cu.yd. OR 311 kg. CEMENT/m³.

Figure 11. Series I-effect of water/cement ratio.

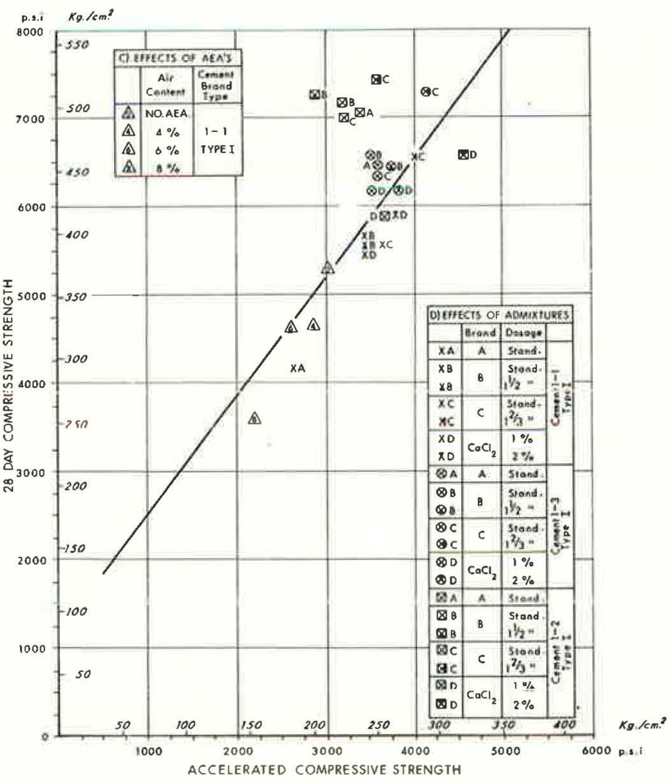
or

$$R_{28} = 1.58 \, R_A + 137.81 \text{ kg/cm}^2$$

with a standard deviation of 344 psi (24.2 kg/cm^2).

Figure 12 shows the effect of admixtures. The anomalous results with cement (1-2) stand out just as they did in the temperature results. The normal 28-day strengths are considerably higher than the average, the ratio of accelerated to 28-day strength is lower. The results with cement (1-1) are also displaced from the established relationship, but in this case, the accelerated strengths are higher than expected and normal strengths are lower.

An explanation may be sought from the corresponding time-temperature curves. In general terms, the temperature increases with cement (1-2) were below average, while those with cement (1-1) were above average. The area under the time-temperature curve (noted against each curve as AR) is a measure of the net heat input into the system. Comparing the order of "heat input" areas, for cement (1-1) they were from 10.6 to 14.3, for cement (1-3) 8.0 to 12.7, for cement (1-2) 6.6 to 7.3. This is also of the order in which the strengths are above or below the regression line. So far a correction factor derived from temperature data has eluded the authors. It is hoped that further data from additional experiments and examination of the composition of the cements may clarify the matter.



NOTE: ALL MIXES WITH 525lb. CEMENT/cu. yd. OR 311 kg. CEMENT / m³.

Figure 12. Series II-effect of admixtures and air content.

Series II—Autogenous Curing Combined with Hot-Water Curing

Concrete variables included in Series II were the following:

1. Cement: 1 Type I, 1 Type II, 1 Type III, and 1 Type IV.
2. Cement contents: 350 lb/cu yd (227 kg/m^3), 525 lb/cu yd (340 kg/m^3), 700 lb/cu yd (454 kg/m^3).
3. Admixtures: Three water-reducing admixtures designated A, B, and C (two of which, B and C, were set retarding and were used at increased doses), 1 percent and 2 percent calcium chloride. The effects of admixtures were examined with 2 different Type I cements at one cement factor. These were cements (1-1) and (1-2), which showed the anomalous behavior in Series I.

Equipment, procedures and test specimens were as previously described.

Temperature-Time Results—The temperature curves for the autogenous part of the cycle are similar to those described in Series I. At the conclusion of the 22-hr autogenous curing, the temperature of the concrete cylinders at the time of transfer to the hot water tank ranged from 85 F (29 C) to 120 F (49 C), representing a rise of between 20 F (11 C) and 50 F (28 C), depending on the type of cement and the cement factor. With Type III cements, especially at the higher cement factors, the temperature had peaked shortly before the start of hot-water curing and the temperature of the cylinders was up to 8 F (4 C) below the peak temperature reached during autogenous curing. When considered in conjunction with the strength results, it is apparent that temperatures

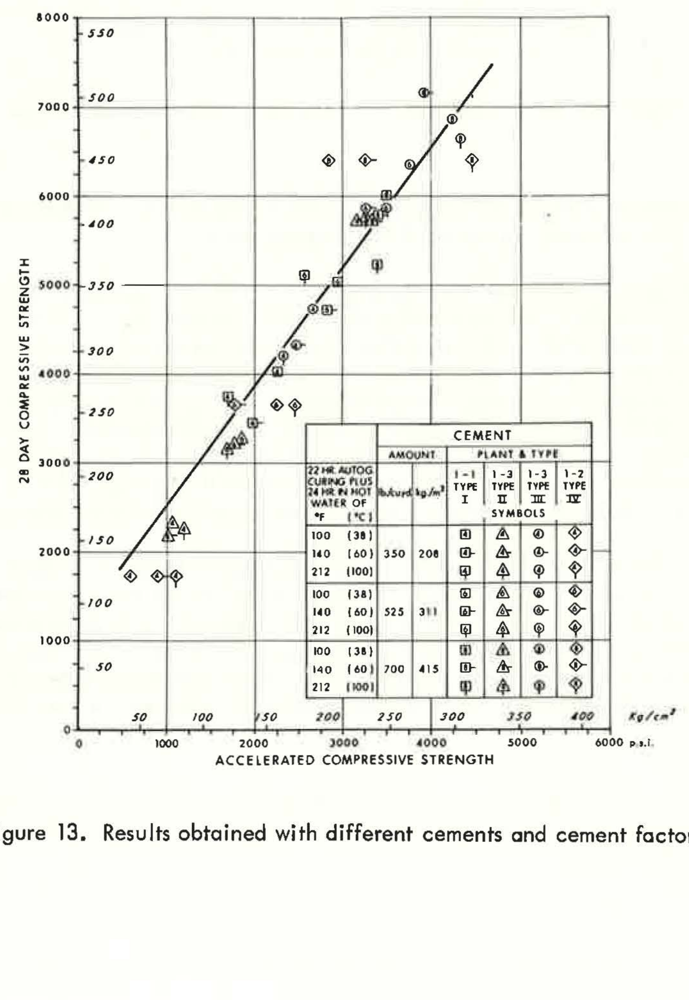


Figure 13. Results obtained with different cements and cement factors.

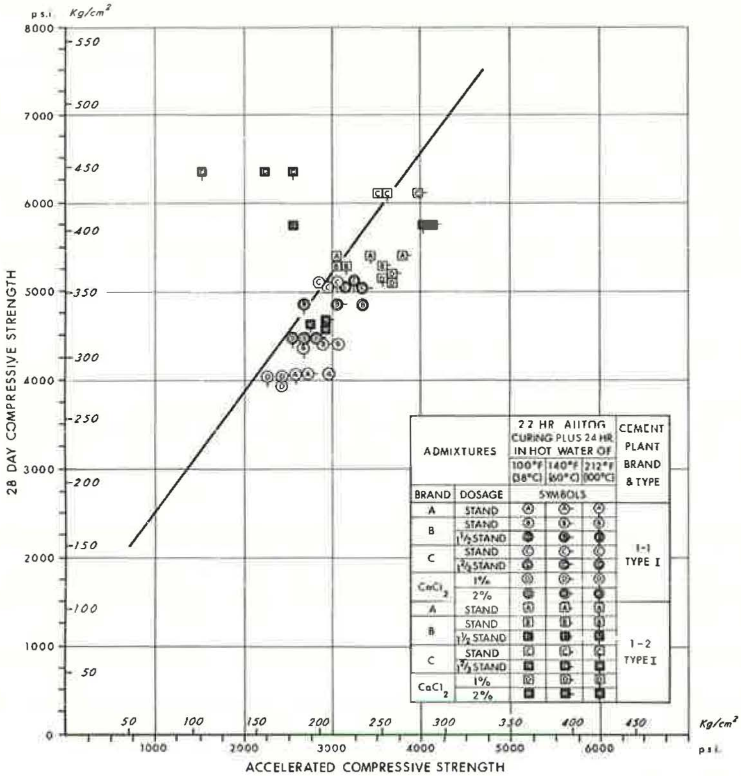


Figure 14. Results obtained from concretes containing admixtures.

during the autogenous part of the cycle had little or no effect. Temperatures during each hot-water curing were the same for all specimens.

22-Hour Autogenous Plus 24-Hour Hot-Water to 28-Day Strength Relationship—Figure 13 shows the results obtained with different cements and cement factors. These results should be compared with those shown for autogenous curing alone in Figure 8. The results for Types I and III cements are generally similar and for these cements there is thus no benefit in dividing the accelerated curing cycle between two methods. However, with the Types II and IV cements tested, the accelerated concrete strengths given by the combined procedure are considerably higher and the results fit the regression line for the other types of cements. There is no clear superiority among the different hot-water temperature regimes used. For ease of comparison the regression line developed in Figure 8 is repeated in Figures 13 and 14.

The results obtained from concretes containing admixtures are shown in Figure 14. The admixtures and cements investigated were those whose behavior was shown as anomalous in Figure 12. Again, the results with cement (1-2) and increased doses of a retarding admixture do not conform with the pattern of the other results. The combinations examined of autogenous curing and the hot-water curing procedures showed, both in this case and generally, no advantage over autogenous curing alone.

Conclusions

The reported work undertaken so far was intended only to validate the principle of autogenous curing as an accelerated strength-testing procedure. Within this limitation it may be concluded that:

The idea is valid and for the normal range of concretes and cements examined, a good relationship exists between the normal cured 28-day strengths R_{28} and the accelerated strength R_A after 48 hours. The relationship is $R_{28} = 1.35\ R_A + 1180$ psi ($R_{28} = 1.35\ R_A + 82.97\ kg/cm^2$). No reliable relationship exists where the autogenous curing period is only 22 hours. A convenient test procedure based on autogenous curing would require 48 hours' total time.

While a 24-hr test cycle would be desirable, 48 hours has the advantage of permitting time for cylinders to be shipped from almost any site to a central laboratory for testing. Limited field trials with both pavement and structural concrete and with initial concrete temperatures above 54 F (12 C) have, so far, shown no problems with the practical use of autogenous curing. Results to date have all conformed to the regression line and have been within the 95 percent confidence limits established in the laboratory.

Additional work should be undertaken into anomalous behavior of certain cements, including Type II and Type IV, and the effect of admixtures, with the hope of determining a more reliable procedure or a correction factor that can be applied to results obtained by the present procedure. Effect of variations in starting and ambient temperatures also requires further investigation to the same end.

A 48-hr accelerated curing procedure involving initial autogenous curing for 22 hours, followed by hot-water curing for 24 hours, gives similar results to autogenous curing alone for the same total time. Unless it is advantageous on a particular project for the results with Types II and IV cements to fit the regression line for other types of cement, there is little to be gained by switching procedures to place the test cylinders into hot water after 22 hours. Further investigation is required into the behavior of retarding admixtures at increased dosage with certain cements and into other combinations of autogenous and hot-water curing in particular to establish the optimum time of the separate phases.

The autogenous curing procedure and equipment is simple and easy to use. While accelerated curing is proceeding, field cylinders may be safely handled, shipped, and stored in their insulated containers until tested in the laboratory.

No work outside the normal day is involved. Overtime work is required, however, to break cylinders made on a Thursday or Friday. These could probably be tested first thing Monday and a correction factor applied. This aspect has yet to be examined.

THE FUTURE OF ACCELERATED STRENGTH TESTING

Standardization of Accelerated Strength Testing

This paper has shown that several accelerated strength-testing procedures have been developed and are being used with apparently a satisfactory degree of reliability to provide early information on concrete strength. As yet no one test appears to have been adopted anywhere as a standard test.

Berio, in conclusion to his general report on the RILEM symposium (84), stated: "Finally, the test cycles extended over 24 hours, in use in the laboratories that have given information as to these, differ among themselves only in non-essential details. It therefore appears possible to unify all these methods in a preliminary standardization. The preparation of these standards could well be confided to the RILEM Concrete Committee."

A similar desire to have a standard test has also been expressed in both England and North America, although in both these cases the approach has differed in that it has first been considered essential to undertake a cooperative testing program to further evaluate promising procedures.

Authentication Plans Outside North America—In England, following controversy over the merits of various procedures, as typified by the divergent discussion of Akroyd's paper (43), a special subcommittee of The Institution of Civil Engineers under the chairmanship of Professor King was formed to examine the subject of accelerated strength testing.

Over the last 5 years this subcommittee has carried out an extensive cooperative testing program. Initial testing was undertaken by six laboratories of a variety of procedures covering dry heat in ovens at 200 F (93 C) for 6 hours, and hot water at temperatures ranging from 95 F (35 C) to 212 F (100 C) for periods of from 3 hours to 24 hours with, in some cases, a delay time of up to 24 hours of normal curing prior to commencing acceleration. From the results of this program the range was narrowed down to eleven of the most promising procedures. The oven test was still included, while the hot-water tests were limited to temperatures of 95 F (35 C) and 130 F (55 C) for periods of 6, 18, 24, or 30 hours, and 176 F (80 C) for 3 hours after 3 hours at 130 F (55 C) or 18 hours or 24 hours of normal curing. The program was concluded by additional tests at one temperature 130 F (55 C) in hot water. Two heating periods were used, 24 hours and 48 hours with, in the latter case, the specimens demolded for the last 24 hours. One-half hour's delay was allowed before heating commenced and one-half hour was allowed for cooling before testing. Among the variables included in this comprehensive program were 3 different cements, 4 water/cement ratios, and 4 aggregate/cement ratios. The specimens used were 4-in. cubes with nominal maximum aggregate size of $^3/_4$ in.

At the time of writing, the tests have been completed and publication of the results, their analysis, and the recommendations of the subcommittee are eagerly awaited. The ASTM subcommittee considering the same subject has been most fortunate to have had Professor King as a contributing member, and thus to have benefited from knowledge of this work.

Authentication Plans in North America—In 1964, Subcommittee II-i of ASTM Committee C-9 was charged with the following scope: "To develop information concerning accelerated strength testing of concrete and to devise one or more procedures to determine the potential strength of concrete in a meaningful and reproducible manner at a significantly earlier time than is conventional, and to study the use of such procedures in assessing the acceptability or ultimate quality of hydraulic cement concretes."

This subcommittee has reviewed the state of the art as reflected by the work discussed previously in this paper. Those procedures which appear most promising (refer to the regression lines between the accelerated and 28-day strength in Fig. 1), are the basis for initiating a cooperative testing program.

The three procedures to be fully evaluated are: (a) Akroyd's (40) modified boiling method (line 1, Fig. 1); (b) Smith and Chojnacki (53), fixed set accelerated curing procedure (line 4, Fig. 1); and (c) Thompson (43), (line 5, Fig. 1).

In addition, certain laboratories will evaluate four other procedures for which further basic information is sought: (d) autogenous curing for 46 hours (as described in this paper); (e) 130 F (55 C) hot-water curing for 24 hours; (f) 168 F (75 C) hot-water curing for 24 hours; and (g) 194 F (90 C) hot-water curing for 24 hours.

Procedures e, f, g are being included in order that the full range of below boiling temperatures will have been investigated. Specifically included is procedure e, which is the most favorable procedure found in the British cooperative program. Procedures f and g cover regimes similar to those used by Malhotra (60) (Snowy Mountains), line 3, Figure 1, and Vuorinen (62) (hot-water alternative), line 6, Figure 1.

Additionally, for procedures e, f, g, a measured delay time will be incorporated, and for procedure d the temperature-time curve will be recorded.

The program will be undertaken by a wide range of laboratories in the United States and Canada. Variables included are two types of cement, Types I and III at three cement factors and two different dose rates of admixtures. All mixes will be at a constant slump of $2^1/_2 \pm {}^1/_2$ in., air content of 5.5 ± 0.5 percent with one size of coarse aggregate (1 in.) and both coarse and fine aggregates of standard grading. Because each participating laboratory will be using locally available materials, a wide range of material variations will be covered. Using two standard 6 by 12-in. cylinders for each variable and duplicate batches, comparison will be made with cylinders normally cured for the same total time as the accelerated test, and for 28 days, 91 days and one year.

This program is just moving from the planning to the execution stage, and it will be some time before recommendations based on the results obtained can be made to ASTM Committee C-9 for their consideration.

Meanwhile, a number of procedures are in day-to-day use, and individual agencies will be continuing research and development of their own procedures. This is all to the good, since the development of experience and confidence in the idea of accelerated strength testing, irrespective of the procedure used, is a prime requisite to adoption of a standard test.

Why Wait 28 Days?

The odds appear good that before long one or more accelerated strength tests will receive the official stamp of approval. This probability raises as far-reaching a question as concrete technology has faced for some time: viz., Can an accelerated strength test stand in its own right as the accepted measure of concrete strength potential and quality, or should it be used only as a quick control aid to estimate the likely 28-day strength?

Part of the answer will lie in the repeatability and reproducibility of the method selected, especially in relation to strengths at later ages than 28 days. It is for this important reason that 91 and 365-day comparison strengths are being included in the ASTM cooperative testing program. A greater deciding influence may be our ingrained resistance to change.

There are some quite pressing reasons why serious thought should be given to upsetting the established order; the current breathing spell while awaiting the evaluation of accelerative procedures might be used to advantage in considering them. First, of course, there is the question of need. There are serious doubts that the 28-day strength test is of value for concrete control and acceptance purposes. Second, there is promise that an accelerated strength test will prove to be reliable and capable of producing significant results. These results could be converted to the equivalent and familiar 28-day values prior to use. However, this would appear to be an unnecessary step for both design and specification purposes, since 28-day strengths are not a measure of an ultimate concrete property. The last and most important matter requiring consideration is, therefore, the whole concept and use of 28-day results as the measure of concrete strength, quality and acceptability.

The Past, Present and Future of the 28-Day Compressive Strength Test

The origins and acceptance of the 28-day test as a good measure of both ultimate strength potential and overall quality go back to the earliest days of the scientific study of concrete. The conception was completely valid, since it was relatively easy to break concrete, and thereby learn something of its properties. Strength was an important consideration, and with the type of cement then in use, 28 days was probably the first convenient age by which one could be certain that the strength gain-time curve had started to flatten out. Since then, every generation has had the sacrosanctity of 28 days hammered home from the pages of every concrete text book, and the lips of every lecturer.

As a result, the 28-day strength test has probably become the most firmly entrenched test applied to concrete. The one thing everybody seems to know about concrete is that it gets hard and you wait 28 days to find out if it is strong enough.

Up to a point they are right; the basis of the test still holds true. Strength is a good indicator of other desirable qualities, and when measured at 28 days it is certainly on the flatter part of the strength gain-time curve. Most of us know that modern cements gain strength faster than did old-time cements, and that their strength gain-time curve has started to flatten out considerably before 28 days. A first step in the right direction might be to substitute a strength test at 14 days for the conventional 28-day test.

Fourteen days is, however, still too long to wait for strength results in face of the present pace of concrete construction. Experience has shown that the degree of confidence that can be placed in strength results after shorter periods of normal curing, such as 7 days, is not adequate. Indeed, there is little point in seriously considering normal curing for less than 28 days when there is already strong evidence that any of a number of accelerated strength tests, requiring only one or two days to complete, will provide at least as good, if not better, predictions of later strengths. Using these procedures, 28-day strengths over the range of 2,000 to 7,000 psi can be predicted within about 500 psi, and often even closer. Since no structure is so precisely designed that it will collapse solely because the 28-day strength determined on a concrete cylinder, "representative" of the work, is 500 psi less than specified, this order of accuracy is probably acceptable for most control and acceptance purposes. Statisticians will no doubt be able to refine the parameters applicable to the evaluation of compressive strength results in the manner of the recommendations of ACI Committee 214 when they are based on accelerated curing. Faster methods of data processing will also be required (87).

An important point to consider in evaluating strength results, especially when they are low, is how representative they are of the concrete in the structure. Apart from errors due to sampling, making, handling, curing and testing cylinders, there are significant differences in the curing conditions. In fact, rarely does the concrete in the structure receive curing as adequate as the cylinders upon which its acceptance is based. As has been previously stressed, there is little control value in 28-day cylinders. By the time the results are available it is too late for effective corrective action to be taken and the concrete by then is probably already deeply buried and being subjected to design loads and sometimes abnormal construction loads. The only purpose of 28-day strength tests is therefore to satisfy specification acceptance requirements that are endeavoring to safeguard against inferior concrete. This can become a costly item in relation to the benefits obtained.

The Department of Highways in Ontario uses about 500,000 cu yd of concrete a year, from which it makes and tests about 20,000 cylinders to insure that the concrete is of the quality specified. The total cost of this operation probably approaches $100,000 per year. The amount of inferior concrete detected and replaced on this basis is negligible, and results are too late to be of real value for quality control. The experience of other concrete users, large and small, is probably similar. If the money were spent on accelerated strength testing (and somewhat better inspection and testing of plastic concrete), this should provide at least equal, and probably greater, assurance of quality and more meaningful day-to-day control of uniformity and faster acceptance of the work. A useful development of this approach might be to take cores after 24 hours to check pavement slab thickness and compaction, and then accelerate their curing prior to testing for strength.

The limitations apparent in the 28-day strength concept appear to justify an all-out effort to authenticate an accelerated strength test for use as a control during construction, to accept the quality of concrete, and as a yardstick for strength-dependent experiments.

In the natural order of events an accelerated strength test would initially only supplement a 28-day test. As confidence is gained, it might then gradually supplant the 28-day test. The evidence and arguments presented are hopefully intended to show that, if and when a suitable accelerated test is put on the books, it should be to replace the 28-day compressive strength test.

OVERALL CONCLUSIONS

There is a real need for early information on concrete strength. In many parts of the world "reliable" accelerated strength-testing procedures are in day-to-day use for concrete control purposes. Authentication of one or more of these procedures as a standard test is urgent and plans to this end are actively under way, principally in England and North America.

Full value will only be gained from the adoption of accelerated strength testing if, at the same time, our whole thinking about the significance of concrete strength and the time at which it should be determined are clarified.

This would be easy if portland cement concrete was a brand new space-age material, the properties of which we were looking into for the first time. If this were the case, based on need and without our present preconceptions, the strength test developed and adopted would undoubtedly be based, if at all possible, on accelerated rather than normal curing.

The heart of the matter is that a simple, quick, repeatable and reproducible standard accelerated strength test will probably be available in the not too distant future. Meanwhile, there is time to think out what this could do. It will at least provide numbers (psi) that categorize the general quality of concrete and place it in its appropriate ultimate strength range. Any problems are not likely to arise from the test method. The greatest difficulty may be to forget the old numbers (28-day psi) and think in terms of the new ones.

The authors firmly believe that numbers such as 1000, 2000, 3000, 4000, derived from an accelerated strength test (in this example, autogenous curing), will be as well suited to the future needs of concrete technology as the corresponding numbers 2530, 3880, 5230, and 6580 have been to past practice. The new numbers will, however, be available at least 26 days sooner.

ACKNOWLEDGMENTS

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(Note: Both papers based on a lecture delivered to the Society of Chemical Industry, Road and Building Materials Group, London, Dec. 1959. Annotation also covers Refs. 17, 24, 27, 28, 29, and 33.)
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Reported accelerated test procedures used for field control in variety of concretes used in dam projects. Cylinders cured in hot water at 165 F (74 C) for 21½ hr after ½-hr delay. When tested at 24 hr, relationships were established with 28-day strengths for Type II cements in structural and mass concrete.

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[2]References 59-67 with discussion all published in RILEM Bull. 31, June 1966.

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66. Yokomichi, H., and Hayashi, M. Influence of High Temperature Curing in Early Ages on Strength of Concrete. RILEM Symposium, 1964.[2]
Concrete cylinders were cured in hot water at 104 F (40 C), 140 F (60 C), 176 F (80 C), for periods of 1, 2, and 3 days, after delay times up to 24 hr and then normal cured for up to 91 days. Concluded that in order to attain 80 percent of 28-day strength in 3 or 4 days, and not to adversely affect strengths at 28 or 91 days, early curing should not exceed 104 F (40 C), should commence at between 4 and 8 hr and should be maintained for 2 days.

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TABLE 1

PHYSICAL PROPERTIES OF LIGHTWEIGHT AGGREGATES

Property	Size Fractions				
	Aggrite			Haydite	
	-1½ in. +½ in.	-½ in. +4 mesh	-4 mesh	-1 in. +4 mesh	-4 mesh
Unit weight, pcf	43.4	44.7	56.3	46.5	53.2
Bulk specific gravity, SSD[a]	1.60	1.61	1.89	1.50	1.75
Absorption by weight, %	10.5	10.7	11.2	8.7	9.4

[a]Saturated, surface-dry basis.

The average mix proportions and characteristics of the fresh concrete are given in Table 2. The weights of aggregates recorded in this table are computed on a saturated, surface-dry basis (SSD). The free water shown is the total water used less the amount absorbed by the aggregate.

Molding of Test Cylinders

Eight 6 by 12-in. test specimens were prepared from each mix by filling cylindrical steel molds in two approximately equal layers. Each layer was compacted with a 1⅛-in. diameter internal vibrator by a single insertion for 4 to 6 seconds. Watertight steel covers were placed on two of the molds. All test specimens, still in their molds, were transferred immediately to a moist-curing room at a temperature of 23 ± 1 C and 100 percent relative humidity. This type of mold with cover plate is now available commercially.

Accelerated Curing and Testing

The modified boiling method was used for accelerated curing, and because the equipment and curing are somewhat different from that given in reference (89), they are described here in some detail.

The equipment consists of a covered, steel water tank, 36 in. by 12 in. and 22 in. high, in which a 230 v, 5000 w, thermostatically controlled immersion heater is located horizontally about 3½ in. from the bottom, and a metal rack on which the specimens are placed about 7 in. from the bottom. The tank was originally designed to accommodate four 6 by 12-in. cylinder molds, but in this investigation only two molds were treated at one time.

The tank was filled with water to a depth of 17½ in. so that when the specimens were placed in the tank the water level rose to the top of the molds. Approximately

TABLE 2

CONCRETE MIX DATA

Type of Mix	Aggregate	Mix Proportions (per cu yd)						Mix Characteristics		
		Cement (lb)	Coarse Aggregate (lb)	Fine Aggregate (lb)	Natural Sand (lb)	Free Water (lb)	AEA Darex (oz)	Slump (in.)	Air[a] (%)	Unit Weight (pcf)
Low strength	Aggrite (A)	385	980	765	285	245	2.0	2	8.5	98
	Haydite (B)	430	915	700	255	270	1.2	1.5	11.0	95
Medium strength	Aggrite (A)	585	880	775	290	270	2.0	2	7.5	104
	Haydite (B)	625	850	705	260	290	1.2	2	8.0	101
High strength	Aggrite (A)	785	780	785	290	320	2.0	2	6.5	110
	Haydite (B)	785	780	705	260	310	1.2	2	7.5	105

[a]Due to lack of equipment for the volumetric method (ASTM C173-66), the pressure method (ASTM C231-62) was used.

TABLE 3

COMPRESSION TEST RESULTS[a]

Type of Mix	Mix No.	Acceler. Cured, 28 hr (psi)	Standard Cured, 28 hr (psi)	Standard Cured, 7 day (psi)	Standard Cured,[b] 28 day (psi)	Ratio, 7d:28d (%)	Ratio Acc/7d (%)	Ratio, Acc/28d (%)
Low-strength mixes	1A	830	350	1,340	2,220	60	62	37
	2A	1,030	420	1,280	2,410	53	80	43
	3A	890	460	1,550	2,440	64	57	37
	1B	1,040	490	1,620	2,370	68	64	44
	2B	720	320	1,200	1,970	61	60	37
	3B	810	370	1,430	2,200	65	57	37
Medium-strength mixes	1A	2,190	1,260	3,060	4,540	67	72	48
	2A	1,980	1,270	3,180	4,090	78	62	48
	3A	1,940	1,240	3,160	4,530	70	61	43
	1B	1,350	880	2,230	3,180	70	61	42
	2B	1,870	840	3,010	4,250	71	62	44
	3B	2,000	1,240	3,370	4,490	75	59	44
High-strength mixes	1A	3,000	2,060	4,560	5,990	76	66	50
	2A	3,010	2,210	4,480	5,720	78	67	53
	3A	2,650	1,930	4,200	5,500	76	63	48
	1B	2,290	1,720	3,760	4,740	79	61	48
	2B	2,820	1,620	4,070	5,240	78	69	54
	3B	2,990	1,950	4,540	5,480	83	66	55

[a]Each result is an average of tests on two cylinders.
[b]Moist cured 7 days, air dried 21 days (ASTM Specification C 330-64T).

one hour was required to raise the temperature of the 27 gal of water to boiling (100 C).

Procedure

The accelerated curing method consists of the following steps:

1. After 23 hours of standard curing, the two cylinders fitted with watertight steel covers were removed from the curing room and placed, complete with their molds, in the boiling water. The temperature dropped to 98 C in about 2 minutes but returned to 100 C in 15 to 17 minutes, after placement of the cylinders.
2. After 3½ hours, the specimens were removed from the boiling water, demolded, and allowed to cool to room temperature which required about 1¾ hours.
3. The specimens were capped and tested in compression 15 minutes later.

The total elapsed time between molding and testing was 28½ hours.

Standard Curing and Testing

Simultaneously with the testing of the accelerated cured specimens, for comparison, two standard cured specimens were tested in compression.

The four remaining specimens were left in the moist-curing room for 7 days, after which two were tested. The final two were removed from the moist-curing room and stored at a temperature of 23 ± 1 C and a relative humidity of 50 ± 2 percent for an additional 21 days and then tested (ASTM Standard C 330-64T).

The results of compressive strength, tests of accelerated and standard cured test specimens at different ages for concretes made with both aggregates A and B are compiled in Table 3. Also shown are the relationships between the average 7-day and 28-day strengths, and between the average accelerated and 28-day strengths.

Analysis of Test Results

A total of 72 test results (each an average of two cylinders) was available for analyses. The analyses have been confined to the relationship between the accelerated and standard cured, 7 and 28-day compressive strengths (Figs. 15 and 16).

The results of this investigation have been compared with those obtained for normal weight concrete using the same test method but using a slightly different

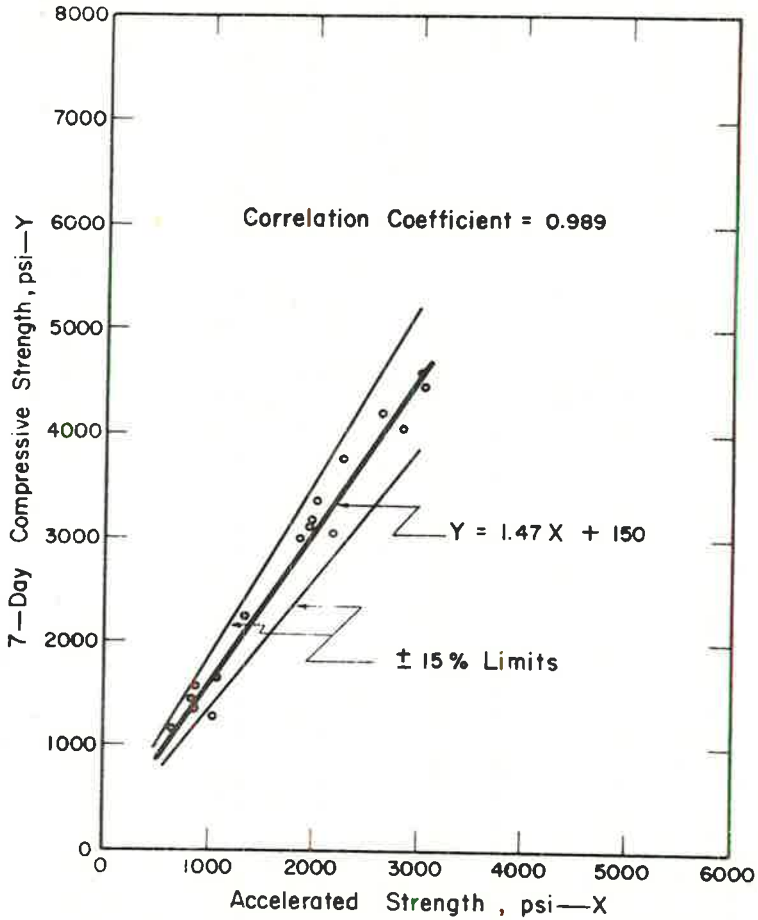

Figure 15. Relationship of accelerated to 7-day strength (combined data for both Aggrite and Haydite concretes).

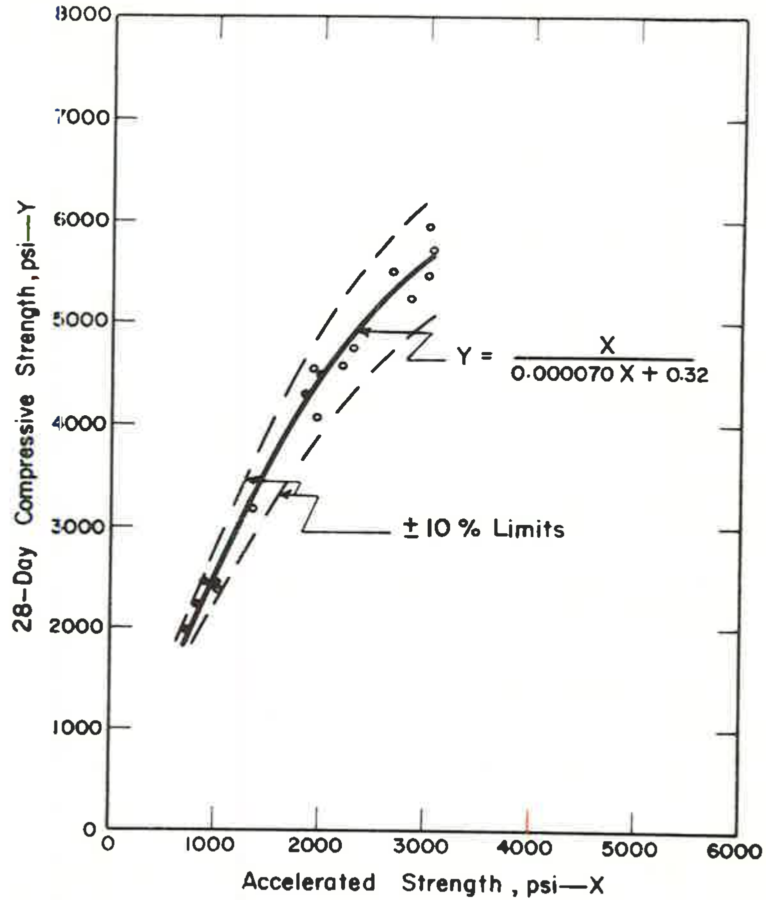

Figure 16. Relationship of accelerated to 28-day strength (combined data for both Aggrite and Haydite concretes).

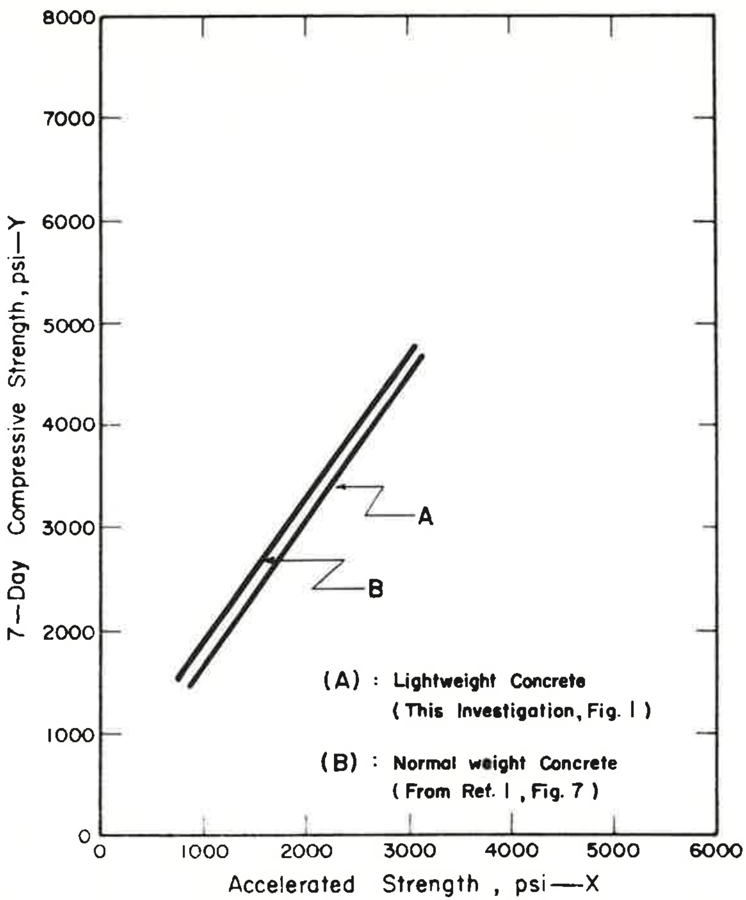

Figure 17. Comparison of results of normal and lightweight concretes using the same test method.

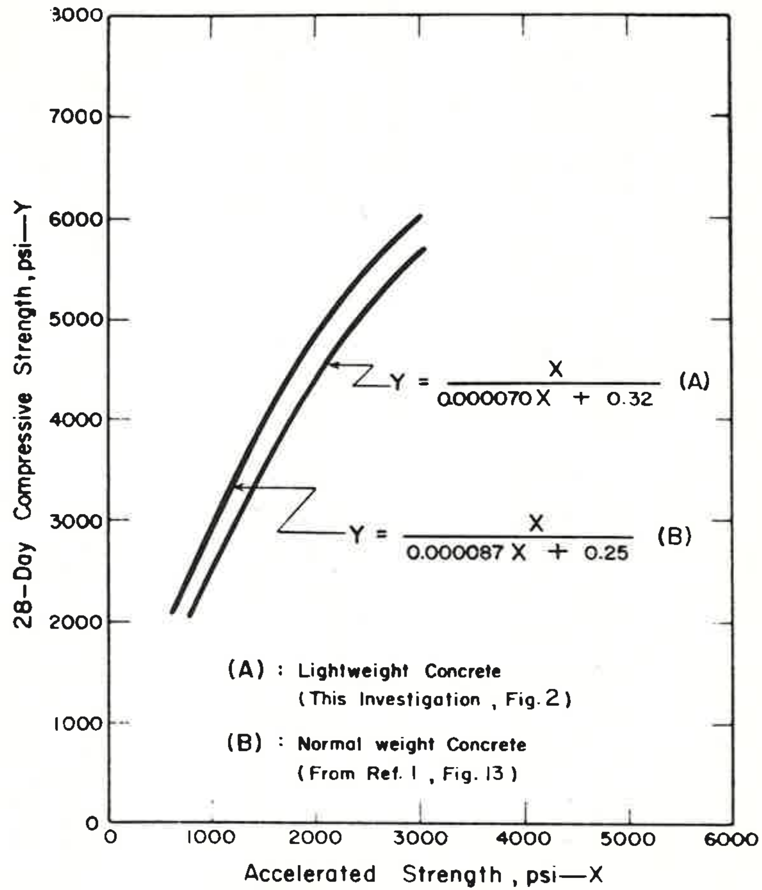

Figure 18. Comparison of results of normal and lightweight concretes using the same test method.

accelerated curing cycle: moist curing = 24 hours; boiling = $3\frac{1}{2}$ hours; and testing = at the end of $28\frac{1}{2}$ hours (Figs. 17 and 18).

Concluding Remarks

The modified boiling method appears to be a satisfactory means of accelerating the strength development of lightweight aggregate concretes in order to predict 7 and 28-day compressive strengths at $28\frac{1}{2}$ hours. If the same lot of cement and the same accelerated curing cycle were used for both lightweight and normal weight concretes, the relationship between accelerated and 7 and 28-day strengths might be more nearly identical for the two types of concrete.

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P. SMITH and H. TIEDE, Closure—The authors wish to thank Messrs. Wilson, Zoldners and Malhotra for their significant contribution to one of the areas in which knowledge of accelerated strength testing is lacking. Similar studies into the effect of variables due to the use of different concrete materials with both the modified boiling method and other procedures would be of great value in unraveling more of the unknowns of accelerated strength testing.

Relations Between Various Strengths of Concrete

SÁNDOR POPOVICS, Professor, Department of Civil Engineering, Auburn University

Analysis of published experimental data shows certain correlations between cylinder, cube, and modified cube strength; between compressive, flexural, direct tensile, and splitting strengths; and between torsion, shear, and other strengths. Formulas related to these correlations are discussed. However, their general application requires caution because they would not necessarily hold for tests carried out under different conditions.

•FACTORS affecting the compressive strength of concrete, such as the water-cement ratio, air content, type of mineral aggregate, age, and methods of testing and curing, also affect the flexural and other strengths of concrete but, as a rule, to a different degree. Also, various simplifying assumptions are used in the procedures of different test methods to convert a measured load to a calculated failure stress. Some of these procedures require more doubtful assumptions than others, which may considerably influence the relationships between various concrete strengths. For instance, the results of a compressive strength test are less sensitive to the fundamental assumptions (zero eccentricity, homogeneity, etc.) than the results of the direct tensile test.

However, the direct tensile test provides more realistic values for the tensile strength of concrete than the flexural test because the usual manner of computation of flexural stresses requires Hook's law to hold. Thus, empirical relations exist between the various types of concrete strength, but the approximation of any of these relations is acceptable only for a limited range of concrete composition as well as for specified curing and testing conditions.

The establishment of reliable relations between the various concrete strengths is important from both the theoretical and practical standpoints. It helps the theorist understand better the mechanism of concrete failure and the internal structure of concrete, and it is also important for practical engineers because it may simplify the quality control of concrete. For instance, with the knowledge of a reliable relationship, the flexural strength of a concrete can be estimated from the compressive strength, or vice versa, without additional experimental work.

SEVERAL TYPES OF COMPRESSIVE STRENGTH OF CONCRETE

Without discussing the effect of the specimen size in detail, the relationships between the usual types of determination of compressive strength of concrete should be mentioned, namely, the relationships between the cube, cylinder, and prism strengths. In most cases the compressive strength is determined by 6 x 12-in. cylinders in the United States, whereas 20-cm (8-in.) cubes or prisms are used for the same purpose in many European laboratories. Thus, Americans need to know the relationships between these compressive strengths for the evaluation of European strength results.

The difference between f_c (cylinder compressive strength) and f_{cu} (cube strength) has been recognized for a long time. Wig and his coworkers, for instance, published pertinent data as early as 1916. Their data show that the f_c/f_{cu} ratio, determined on 8 x 16-in. cylinders and 6-in. cubes, respectively, varied within 0.73 and 0.81 at the

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TABLE 1

COMPRESSIVE STRENGTH OF 6 × 12-IN. CYLINDERS vs COMPRESSIVE STRENGTH OF CUBES

f_c/f_{cu}	Authority	Remarks
$0.76 + 0.20 (\log f_{cu}/200)$	L'Hermite (2, pp. 117-133)	15-cm cubes; f_{cu} in kg/cm^2
0.75	British Standard B.S. 1881:52 (3)	6-in. cubes
0.62 to 0.865	L'Hermite (3)	15-cm (= 6-in.) cubes
0.88	Hummel (3)	20-cm cubes
0.82 to 0.88	Beton Kalender (3)	20-cm cubes
0.71 to 0.95	Gonnerman (3)	6-in. cubes
0.77 to 1.04	Gonnerman (3)	8-in. cubes
0.80 to 0.90	Graf (3)	Cube side equals cylinder diameter
0.64 to 0.94	Bignoli (3)	8-in. cubes, overall
0.75 to 0.80	Bignoli (3)	8-in. cubes, high slump concrete
$0.66 + 0.20f_{cu}/500$	Australian Standard A.S. A-78 (3)	15-cm cubes; f_{cu} in kg/cm^2
$0.68 + 0.22f_{cu}/500$	Australian Standard A.S. A-78 (3)	20-cm cubes; f_{cu} in kg/cm^2
$0.82 - 21.1/f_{cu}$	Hernandez (3)	Rounded siliceous aggregate; 15 or 20-cm cubes; f_{cu} in kg/cm^2
$0.87 - 17/f_{cu}$	Hernandez (3)	Crushed limestone aggregate; 15 or 20-cm cubes; f_{cu} in kg/cm^2
0.65 to 0.84	Gyengo (3)	20-cm cubes
0.81 to 0.89	Walz (4)	20-cm cubes
$50/(77.5 - \sqrt{f_{cu}})$	Vuorinen (5)	20-cm cubes; f_{cu} in kg/cm^2
0.86	Lyse and Johansen (6)	10-cm or 20-cm cubes
$0.85 - 0.21f_{cu}/1,000$	Petersons (7, pp. 115-117)	15-cm cubes; f_{cu} in kg/cm^2
$0.85 - 12/f_{cu}$	Poijarvi and Syrjala (8)	20-cm cubes; f_{cu} in kg/cm^2
$0.85 \pm 50/f_{cu}$	Campus et al (9)	20-cm cubes; f_{cu} in kg/cm^2

TABLE 2

COMPRESSIVE STRENGTH OF PRISMS vs COMPRESSIVE STRENGTH OF CUBES

f_p/f_{cu}	Authority	Remarks
0.8 to 1.0	Lazard (3)	Height-width ratio not given
0.85 to 0.95	Hummel (3)	Height-width ratio: $h/w = 2$
0.75 to 0.85	Hummel (3)	$h/w = 4$
0.7	Beton Kalender (3)	$h/w = 4$
0.84 to 0.95	Deutsche Bauzeitung (3)	h/w varied from 12 to 2
0.65 to 0.85	Graf (3)	$h/w = 4$, strength about 160 kg/cm^2
0.60 to 0.80	Graf (3)	$h/w = 4$, strength about 300 kg/cm^2
0.74 to 0.81	Morsch (3)	$h/w = 5$
0.8	Ros (3, 16, pp. 1-9)	Strengths lower than 350 kg/cm^2
0.77 to 0.88	Graf (3)	$h/w = 4$, strength between 167 to 870 kg/cm^2
0.70 to 0.89	Graf (3)	$h/w = 4$, with different aggregates
0.73 to 1.04	Rusch (3)	$h/w = 10.6$
0.66	Mesnager (3)	$h/w = 3$
0.72 to 0.80	Dutron (3)	$h/w = 3$, aggregate: crush porphory
0.62 to 0.71	Dutron (3)	$h/w = 3$, aggregate: Rhine River gravel and sand
0.56 to 0.74	Thomas (7, par. 2)	10-cm cubes
Independent of h/w	Crawford and Fry (7, par. 2)	h/w varied from 3 to 12
0.61	Hajnal-Konyi (7, par. 2)	6-in. cubes
0.56 to 0.92	Evans and Lawson (7, par. 2)	6-in. cubes
$0.66 + 7/f_{cu}$	Petersons (7, par. 2)	Valid from f_{cu} = 120 to 480 kg/cm^2
$0.85 - f_{cu}/1,720$	Graf (7, par. 2)	$h/w = 4.3$, valid from f_{cu} = 150 to 470 kg/cm^2
$0.86 + 9.5/f_{cu}$	Poijarvi and Syrjala (8)	$h/w = 2$, valid from f_p = 100 to 450 kg/cm^2
0.86	Kuczynski (17)	$h/w = 3$, 20-cm cubes, f_{cu} is between 50 and 350 kg/cm^2

age of 28 days, and within 0.88 and 0.93 at the age of one year, depending on the type of the mineral aggregate (1).

The recommendations of other investigators for the f_c/f_{cu} ratio are summarized in Table 1. The first half of the table was taken from a RILEM Report of 1957 (3). All these recommendations agree, in that the cylinder strength is lower than the cube strength; however, the f_c/f_{cu} ratio varies within 0.62 and 0.95, i.e., there is about a ± 20 percent scattering in the recommended values. Also, in certain cases this ratio increased with increasing concrete strength; in other cases it decreased; and again in others it appeared independent of the concrete strength. Thus, for lack of a more accurate solution, one may simply assume that the cylinder strength of a concrete is slightly under 80 percent of the cube strength, provided that the 6 × 12-in. cylinders are made and tested according to the American standards, the 20-cm cubes are made and tested according to the German standards, and the composition and strength of the concrete is within the structural range. In this case the approximate value of the cylinder strength can be obtained in psi by multiplying the cube strength in kg/cm² by 10.

The relationship between the f_{cu} and f_p (prism compressive strength) of a concrete has also been tested by many investigators (Table 2). Again, prism strength is lower than cube strength; however, the values obtained for the f_p/f_{cu} ratio vary within 0.56 and 1.0. However, there are further uncertainties. For instance, Tucker (10) showed

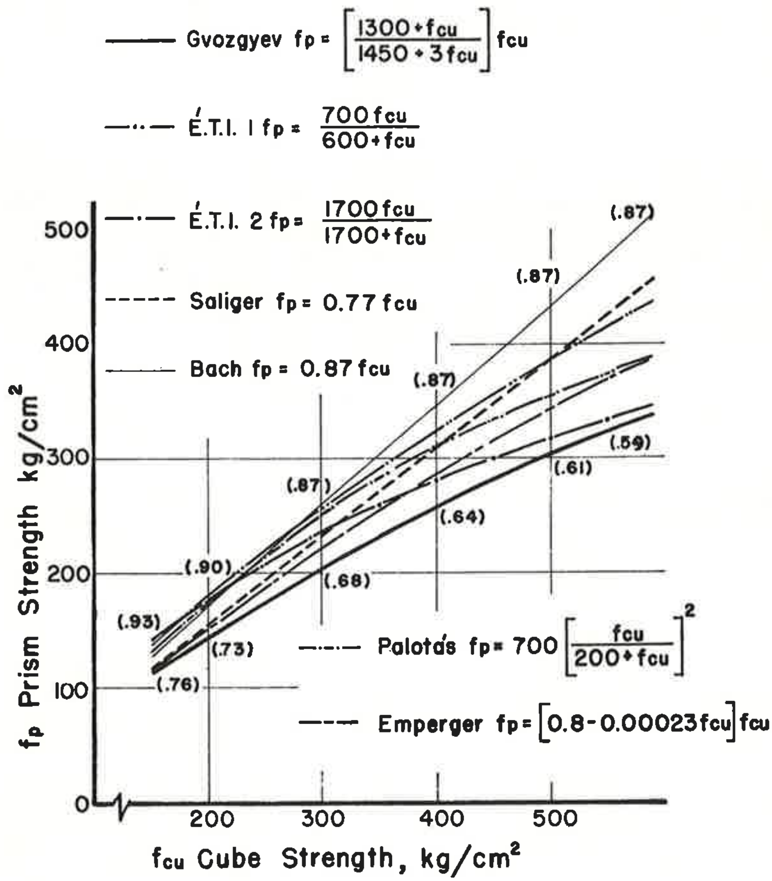

Figure 1. Recommended relationships between the prism strength and cube strength of concretes (12).

by the application of a statistical theory of strength that the f_p/f_{cu} ratio increases both with the increase of concrete strength and with the reduction of the slenderness of prism. However, certain experiments in Table 2 indicate that the f_p/f_{cu} ratio is practically independent of the strength, or slenderness, whereas according to other experiments, the prism strength is even relatively higher at low cube strengths than at high strengths. For instance, experimental data by Rusch (11) indicated that the prism strength of certain low-strength lightweight aggregate concretes was practically the same as their cube strength.

In addition, Palotas (12) presents several empirical formulas recommended by various investigators for the average relationship between cube strength and prism strength of a concrete (Fig. 1). In general, the height-width (h/w) ratio of the prisms was 3 for these formulas. The numbers in parentheses (Fig. 1) show the upper and lower limits for the calculated ratio of f_p/f_{cu}. Even these average ratios vary within 0.59 and 0.93, depending on the concrete strength and applied formula, and the f_p/f_{cu} ratio decreases with increasing strength. Experimental evidence appears to show that this latter statement also holds in cases where the h/w ratio is less than one (13, 14), although the numerical values are rather scattered. A further comparison of compressive strengths obtained by various investigators with specimens of different slenderness also resulted in fluctuating strength ratios (15).

The relationship between the f_c and the f_{mc} (modified cube strength) of a concrete, determined on portions of specimens remaining from flexural test, is also uncertain. Certain test series do not show noticeable difference between the two strengths for structural concretes, whereas others do but in conflicting directions (Table 3). For the relationship between cube strength and modified cube strength of a concrete the following formula was recommended (8):

$$f_{cu} = 0.82 f_{mc} + 70 \tag{1}$$

Eq. 1 is valid from f_{mc} = 300 to 700 kg/cm² and for 10-cm modified cubes. It was also found (12) that the compressive strength obtained by 20-cm modified cubes was lower than the comparable compressive strengths of the same concrete obtained by 20-cm cubes, but higher than the prism strength. However, a pertinent British specification (B.S. 1881: 1952) suggests a value between 1.0 and 0.95 for the f_{cu}/f_{mc} ratio when 6-in. cubes are used. Results obtained on Ottawa sand cement mortars also show that the f_{cu}/f_{mc} ratio is close to unity (18, 19).

TABLE 3

COMPRESSIVE STRENGTH OF CYLINDERS vs COMPRESSIVE STRENGTH OF MODIFIED CUBES

f_c/f_{mc}	Authority	Remarks
1.0	Koenitzer (3)	6-in. modified cubes
0.85 to 0.97	Withey (3)	6-in. modified cubes
1.0	Klieger (21)	6-in. modified cubes
$0.9 - 100/f_{mc}$	Kesler (22)	6-in. modified cubes; f_{mc} in psi
0.88	Klieger (23)	6-in. modified cubes
1.0 to 1.16	Mather (24)	f_c for cores; f_{mc} for specimens cut from slabs; ratio is about 1 up to 5,000 psi, above this it increases
1.2	Mather (24)	6¼, 8, 10, and 12-in. modified cubes; specimens casted in laboratory
1.0	Shideler (25)	6-in. modified cubes; sand and gravel concrete
$1.2 - 485/f_{mc}$	Shideler (25)	6-in. modified cubes; structural lightweight concrete; f_{mc} in psi
0.79 to 1.39	Sen and Bharara (26)	6-in. modified cubes; ratios larger than 1 belong to strengths lower than 2,000 psi

Very few experimental data are available for the relationship between cylinder strength and prism strength. Although it is generally assumed that these two compressive strengths are the same when the specimen height and the h/w ratio are the same, experimental evidence appears to indicate that the cylinder strength is somewhat lower than the prism strength (17, 20).

The main reason for the difference between the various types of compressive strength is the differing effect of the friction between the steel compression plates of the testing machine and the concrete specimen. This friction is caused by the lateral elongation of the specimen subjected to longitudinal compression. The frictional forces counteract the lateral elongation, creating lateral compressive stresses in the concrete adjacent to the compression plates, and these stresses may artificially increase the measured compressive strength of the specimen. This mechanism implies the following:

1. The typical failure of a specimen under uniaxial compression takes place in the middle of the height because the restricting effect of the frictional forces is minimun here.
2. The more slender the specimen, the lower the measured compressive strength because the smaller become the frictional stresses in the middle sections. However, when the specimen is slender enough, a further increase in the h/w ratio does not cause further significant reduction in the strength.
3. If the friction is reduced, for instance by applying interlayers between the loading surfaces, this not only reduces the measured value of the compressive strength but also reduces the variation of strength caused by the variation in slenderness of the specimen.

On this basis, one would also expect that modified cubes provide higher compressive strength than comparable normal cubes because the unloaded portion of the modified cube specimen restrains the lateral deformations of the loaded core. However, experimental data do not support this expectation.

A theoretical distribution of the combined stresses of the uniaxial compression and friction was presented by Ros (16, pp. 1-9), but there are several additional discussions of this problem, including deformation measurements and photoelastic studies (2, pp. 117-133; 15, 17, 27, 28, 29, 30, 31). These theoretical or semi-theoretical discussions as well as experimental data clearly show that under usual circumstances the effect of a change in the h/w ratio on the measured compressive strength is strong when this ratio is less than 1. 5, but the effect becomes slight when h/w is larger than 2 (13, 14, 15, 32). The effect of the h/w ratio is less pronounced when the friction is decreased by applying friction reducing interlayers between the loading surfaces (28, 30, 32, 33).

Thus, the assumed role of the friction is well supported by experimental results. However, this mechanism alone cannot explain the disagreement among the strength relations indicated in Tables 1, 2, and 3. There should be additional factors that influence the relations between various concrete strengths. The age of the concrete as well as the composition (the type of mineral aggregate, water-cement ratio, air content of the concrete, etc.) and the type of compaction may influence these strength ratios, perhaps through their effects on the modulus of elasticity, Poisson's ratio, and ratio of the tensile strength to the compressive strength of the concrete (34).

Kesler noticed, for instance, that the compressive strength of structural lightweight concretes changes less, and the strength of dried or autoclaved concretes changes more, with a change in the h/w ratio than occurs with comparable normal concrete specimens. The strength can also be a direct factor (Fig. 1) because a high pressure on the specimen may cause a sliding between the steel compression plates and the concrete. This, in turn, reduces the result of the strength test (2, pp. 117-133; 28). A possible difference between the cylinder and prism strengths might be attributed to the fact that the perimeter/area ratio is smaller for a circle than for a square. However, the most important reason seems to be that the compression is not uniformly distributed on the specimen during the test. This affects the strength results to differing degrees, depending on the size and shape of the specimens.

There are several factors that can cause an uneven load distribution. An obvious one is an unsatisfactory condition of the specimen ends (30, 35). Also, both L'Hermite (2, pp. 117-133) and Rusch (36) have pointed out that the usual compression plates of the testing machines are not sufficiently rigid; they bend during testing; therefore, the load will not be distributed uniformly on the specimens. Accordingly, testing machines of different rigidity would provide different compressive strengths (37) and different compressive strength ratios under otherwise identical conditions. This effect increases as the strength and the modulus of elasticity of the concrete increases. An eccentricity of the load also causes an uneven stress distribution on the specimen. The eccentricity may come either from a geometric eccentricity, or from the heterogeneity of the specimen, or from both. The compressive strength of slender specimens is much more reduced by an eccentricity than the strength of a cube (2, pp. 117-133; 16, pp. 1-9). This effect also increases with the increase of the strength and modulus of elasticity of the concrete. In view of these considerations, the strength correction factors given in the ASTM C 42-64 should be considered as being of an approximate nature because they express the change of cylinder strength solely in terms of the h/w ratio.

Several authors believe that the fineness modulus of the mineral aggregate also influences the relations between various concrete strengths. This opinion, however, might be the result of a misinterpretation of the experimental data. In the case of constant consistency and cement content, a change in the aggregate grading changes the water requirement of the mixture, and the concrete strength changes accordingly. Therefore, it is quite conceivable that it is not the change in the fineness modulus directly, but a change in the concrete strength caused by the change in the fineness modulus which influences the strength relations.

Finally, several simple empirical formulas are recommended by Neville (38) for the relation of compressive strengths of concrete specimens of different shapes and sizes, i.e., for cube, cylinder, and prism strengths. One of his expressions can be written in the following form (39):

$$P/P_6 = 1/d^{0.095} \cong \tag{2}$$

$$\cong 0.85(6/d)^{0.095} \tag{2a}$$

where

d = maximum lateral dimension, in.,

P = compressive strength of a specimen of d lateral size, and

P_6 = compressive strength of a 6-in. cube.

These formulas are very simple but their precision is inadequate for many practical cases. This can be seen from the fact that the slenderness of the specimen is not taken into consideration in the formulas, and that they provide the value of 0.85 rather than 1.00 when the formulas are applied to the 6-in. cube.

RELATIONSHIP BETWEEN COMPRESSIVE AND FLEXURAL STRENGTHS

The relationship between the flexural strength and compressive strength of a concrete is less reliable as a rule than the relationships between the various types of compressive strength. In other words, values obtained for the ratio of flexural strength to compressive strength under various conditions have relatively larger scattering than, for instance, the ratio of f_c to f_{cu}. Figure 2 shows several empirical relationships which support this statement. The curves are based on experimental data by Kesler (22), Graf (40, par. 16), and Walker and Bloem (41). For the sake of clarity, the experimental values were omitted; only curves representing the average values and estimated tolerance limits were plotted. Within a single test series of a concrete the

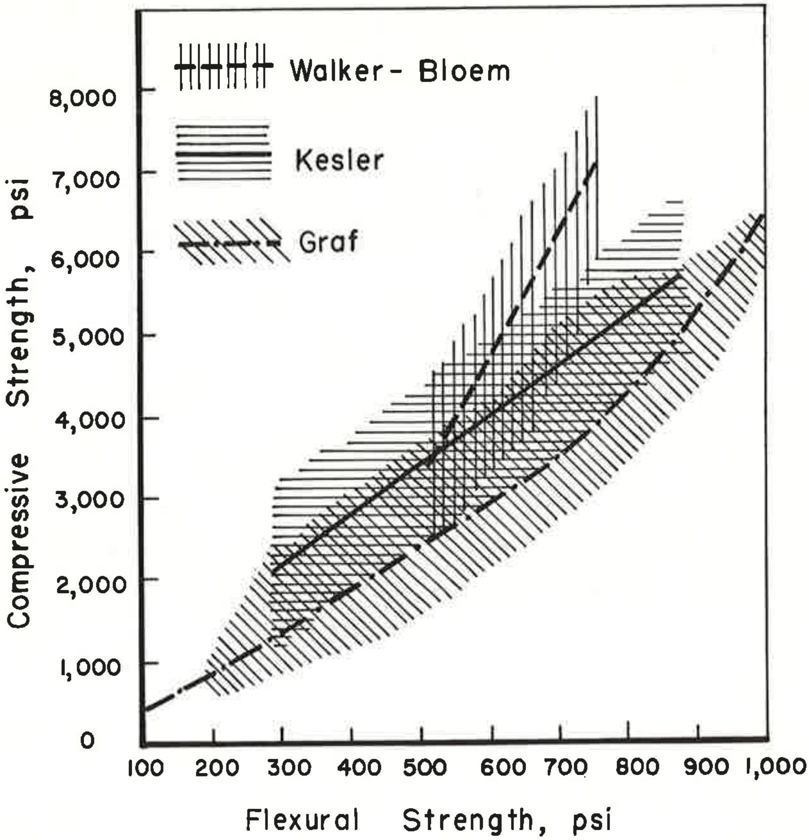

Figure 2. Empirical relations between compressive and flexural strengths of concretes determined by various investigators.

compressive strength can be estimated from the result of a flexural test with an accuracy of about ± 1,000 psi. The reliability of a general, overall relationship is poorer.

Similar conclusions can be drawn from a report by Saul (42). Further related data are given in Table 4 which gives a summary of the findings of a group of investigators for the ratio of f_{fl} (flexural strength) of a concrete to its f_{co} (compressive strength). These data show that the flexural strength of a high-strength concrete is about 10 percent of its compressive strength, but it may increase up to 30 percent with the decrease of the strength. However, the f_{fl}/f_{co} ratio is also influenced to a high degree by the age and composition of the concrete as well as by the curing and testing conditions. Walz (43) pointed out, for instance, that the flexural strength of structural lightweight concretes is only about 70 to 90 percent of the flexural strength of normal concretes of the same compressive strength.

The correlation is better when compressive and flexural strengths of standard cement mortars are compared because here the number of variables is reduced by the standardized circumstances. This has been demonstrated both for American standard cement mortars (18) and for German mortars (57, 58, 59).

Abrams was probably the first to propose a formula, a parabola, for the relationship between the compressive and flexural strengths (50). Since then numerous formulas have been recommended by other investigators (60), as indicated in Table 4. The simplest is a linear approach as follows:

$$f_{co} = af_{fl} - b \tag{3}$$

where a and b are empirical factors which are independent of the strength but depend on the applied units as well as on the composition of concrete, curing and testing conditions. Such a linear relationship is supported by experimental data within restricted limits, such as the range of structural concretes (Fig. 2). In addition, Table 4 indicates the well-documented fact (18, 48, 61, 62, 63, 64, 65, 66, 67) that the f_{fl}/f_{co} ratio decreases as the concrete strength increases, because from Eq. 3:

$$f_{fl}/f_{co} = 1/a + b/af_{co} \tag{3a}$$

The main reason for the increased uncertainty of the f_{fl} vs f_{co} relations seems to be that the flexural strength is affected by the concrete composition—i. e., the type of cement, water-cement ratio, air content, and type of mineral aggregate—to differing degrees than the compressive strength (41, 48, 60, 68, 69, 70). Also, the size and shape of the specimen and the methods of curing and testing have considerably more effect on the values obtained for the flexural strength than on those obtained for the compressive strength. It is enough to mention here the considerable reduction in the flexural strength caused by drying or sudden cooling of the concrete specimen (40, sect. 16; 70, 71). There is also a decrease in the f_{fl}/f_{co} ratio with an increase in the age of concrete, because the development of the tensile strength of a concrete, and thus

TABLE 4

FLEXURAL vs COMPRESSIVE STRENGTHS

f_{fl}/f_{co}	Authority	Remarks
0.125 to 0.20[a]	Ros (16, pp. 1-9)	$f_{co} = f_p$
0.12 to 0.22[a]	Kesler (22)	$f_{co} = f_c$
0.125 to 0.20[a]	Graf (40, par. 16)	$f_{co} = f_{cu}$
0.09 to 0.12[a]	Walz and Wischers (43)	$f_{co} = f_c$, structural lightweight concrete
0.112 to 0.23[a]	Gonnerman and Shuman (44)	$f_{co} = f_c$
0.13 to 0.25[a]	Kenis (45)	$f_{co} = f_c$
0.154 to 0.298[a]	Akazawa (46)	$f_{co} = f_c$
0.1 to 0.3[a]	Kaplan (47)	$f_{co} = f_{mc}$
0.13 to 0.17	Walz (48)	f_{cu} = about 460 kg/cm² at the age of 28 days with different coarse aggregates
0.11 to 0.20[a]	Bonzel (48)	f_{cu} varies from 600 to 100 kg/cm²; aggregate is sand and gravel
0.13 to 0.25[a]	Bonzel (48)	f_{cu} varies from 600 to 100 kg/cm²; aggregate is crushed stone
0.13 to 0.20[a]	Walker and Bloem (49)	f_c varies from 6,500 to 1,500 psi; with aggregates of different maximum sizes
$2.793/f_{mc}^{0.37}$	Sen (26)	f_{mc} is in psi
$0.29 - 0.000032f_c$	Abrams (50)	f_c is in psi
$9.2/\sqrt{f_{cu}}$	Road Research Lab. (51)	f_{cu} is in psi
$8/\sqrt{f_{cu}}$	Short and Kinniburgh (52, Chap. 10)	f_{cu} is in psi; structural lightweight concrete
$1/f_{cu}^{0.30}$ to $1/f_{cu}^{0.40}$	Hummel (53)	Ratio increases with the angularity of the particles
$1.15/f_{cu}$	Williams (54)	f_{cu} is in psi
$0.09 + 50/f_{cu}$	Willaims (54)	f_{cu} is in psi
$7.5/\sqrt{f_c}$ to $12/\sqrt{f_c}$	ACI Committee 435 (55)	f_c is in psi
$5/\sqrt{f_c}$ to $11/\sqrt{f_c}$	ACI Committee 435 (55)	f_c is in psi; lightweight concrete in a drying condition
$3000/(4f_c + 12,000)$	Sozen et al (56)	f_c is in psi

[a]The higher the concrete strength, the lower becomes the ratio.

that of the flexural strength, stops much sooner than the development of the compressive strength (72, 73, 74, 75). The complete stesss-strain curves for concretes of different strengths differ (76), accounting for the fact that the higher the concrete strength, the lower the f_{fl}/f_{co} ratio. The difference in the flexural strengths of normal and lightweight concretes of identical compressive strength probably lies in their differing mode of failure in flexure. In a normal concrete this failure occurs primarily as a result of a breakdown of the bond between the hardened cement paste and the surface of aggregate, whereas in a lightweight concrete the fracture is caused mainly by the weakness of the aggregate particles (77).

Thus, flexural test of plain concrete beams may be used for checking compressive strength on the construction project only if the same concrete materials are used continuously and the entire test procedure, including pouring, curing, and age, is kept unchanged (73).

RELATIONS CONCERNING TENSILE AND SPLITTING STRENGTHS

Experiments have been performed for the comparison of the f_{sp} (splitting strength) and f_t (direct tensile strength). Table 5 summarizes the results of several investigators. The data clearly indicate that the splitting strength is greater than the direct tensile strength of the same concrete, but otherwise a correlation exists between the two tensile strengths. The goodness of the correlation within a test series is shown in Figure 3, which gives results obtained by Ledbetter and Thompson (79) on structural lightweight concretes. Bonzel, based on his analysis of the experiments of several investigators, concluded that the f_t/f_{sp} ratio appears to decrease somewhat with the increase of the tensile strength (81). However, according to experiments by Kaplan, the relationship between f_t and f_{sp} is dependent on the concrete composition, and the difference between these strengths tends to disappear at higher strength levels (86). The proportionality of splitting and tensile strengths has also been demonstrated for ceramics (87).

As mentioned, the measured value of tensile strength is reduced more by loading eccentricity, stress peaks, etc., than the measured value of splitting strength. But apart from this, there are several more possible reasons why the f_{sp} calculated by the usual method may be greater than f_t, although the failure of concrete in the splitting

TABLE 5

DIRECT TENSILE vs SPLITTING STRENGTHS

f_t/f_{sp}	Authority	Remarks
0.86 to 0.94	Malhotra and Zoldners (65)	Max. particle size is 3/8 in.
0.68	Wright (74)	f_{sp} = 405 psi at the age of 28 days
0.85	Laboratorio Central de Ensayo de Materiales de Construction (78)	f_{sp} = 14.1 kg/cm² at the age of 7 days
0.62 to 0.68	Ledbetter and Thompson (79)	Structural lightweight concrete; f_{sp} varies from 100 to 500 psi
0.9	Pincus and Gesund (80)	f_{sp} varies from 150 to 450 psi
0.41 to 0.75	Ramesh and Chopra (81, 82)	Cement mortar; f_{sp} varies from 65 to 14 kg/cm²
0.72 to 1.0	Rusch and Hilsdorf (81)	f_{sp} varies from 14 to 33 kg/cm²
0.75	Bonzel (81)	Average based on the findings of several investigators
0.69 to 0.96	Ali et al (83)	Cement mortars of 1:1 and 1:2 mix proportions; f_{sp} varies from 316 to 618 psi
0.55 to 0.93	Campus et al (84)	Aggregates with different surface textures
0.69	Baus and Campus (85)	Cores taken from a 14-yr-old concrete beam

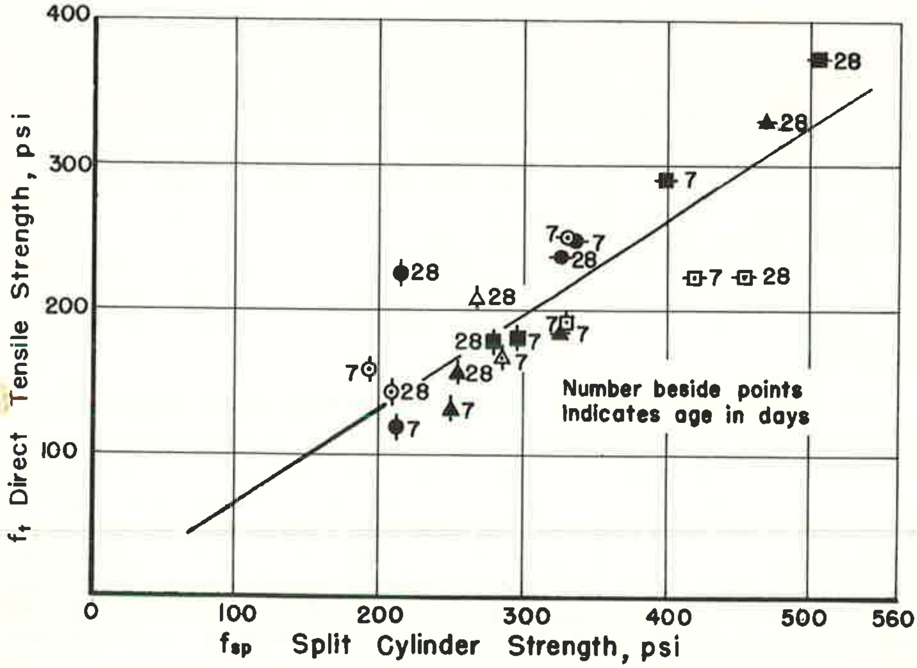

Figure 3. Relationship between direct tensile and splitting strengths of lightweight concretes (79).

test is controlled by the tensile strength. It should be recalled that the concrete does not follow Hook's law; therefore, the use of this assumption in the calculation of the splitting strength increases the apparent value of f_{sp}. Also, in the direct tensile test the failure of the concrete can occur anywhere in the length of the specimen. In the splitting test, however, the failure has been observed in or near the diametral plane of the specimen, which strongly reduces the number of weak links in the zone of maximun stresses. Finally, restraint due to friction between the splitting specimen and the plates of the testing machine may also increase the apparent value of f_{sp}. However, one would expect from the biaxial stress condition in the splitting specimen that the value of f_{sp} as calculated by the usual method is somewhat smaller than f_t (2, pp. 134-143).

Results by several investigators concerning the relationship of the flexural strength to the direct tensile strength and splitting strength, respectively, are summarized in Tables 6 and 7. A group of pertinent experimental results is also shown in Figure 4 (81). These data show that the f_{fl} flexural strength calculated in the usual way, i.e., by the application of Hook's law, is higher than either the comparable direct tensile strength or the splitting strength of the same concrete. Otherwise, experimental data indicate good correlation between the flexural strength and direct tensile strength or splitting strength within a single test series. More specifically, it is a fair approximation that the f_t vs f_{fl} relationship is linear within practical limits (44, 80, 86). The f_{sp} vs f_{fl} relationship can also be considered as linear (26, 46, 67, 88, 89, 90). This is also shown in Figure 5 by experimental data of Kaplan (86), Popovics (91), and

TABLE 6

DIRECT TENSILE vs FLEXURAL STRENGTHS

f_t/f_{fl}	Authority	Remarks
0.48 to 0.63	Price (3, 62)	Valid from f_t = 110 to 630 psi
0.6	L'Hermite (3)	
0.52 to 0.77	Hummel (3)	
0.6	Hamada (3)	
0.55 to 0.70	Dutron (3)	With different aggregates
0.6	Ros (16b)	
0.48	Graf (40, par. 16)	Wet curing
0.48 to 0.63	Gonnerman and Shuman (44)	Valid from f_{fl} = 230 to 1,010 psi
0.37 to 0.56	Walz (48)	
0.48 to 0.60	Malhotra and Zoldners (45)	Max. particle size is 3/8 in.
0.70	Pincus and Gesund (80)	Average value; f_{fl} varies from 200 to 500 psi
0.38 to 0.57	Campus et al (84)	Aggregates with different surface textures
0.52 to 0.70	Kaplan (86)	f_{fl} = varies from 550 to 850 psi
0.45	Wright (94)	f_{fl} = 605 psi at the age of 28 days

Fowler (92). By using these experimental values, the following approximate formula can be obtained for the f_{fl} vs f_{sp} relationship:

$$f_{fl} = 1.2f_{sp} + 100 \tag{4}$$

TABLE 7

SPLITTING vs FLEXURAL STRENGTHS

f_{sp}/f_{fl}	Authority	Remarks
0.8	Sen and Bharara (26)	f_{fl} varies from 214 to 630 psi; tested at various ages
0.39 to 0.74	Akazawa (46)	Recommended average: 0.47
0.45 to 0.53	Ramesh and Chopra (48, 82)	f_{fl} varies from 34 to 68 kg/cm²
0.67 to 0.91	Efsen and Glarbo (48)	f_{fl} varies from 16 to 42 kg/cm²; ratio decreases with decreasing strength
0.62 to 0.90	Walker and Bloem (49)	f_{fl} varies from 800 to 300 psi; with aggregates of different maximum sizes
0.63 to 0.83	Rusch and Vigerust (63)	f_{fl} = about 45 kg/cm²; ratio decreases with decreasing strength
0.72 to 0.77	Kaplan (86)	f_{fl} varies from 850 to 550 psi
0.55 to 0.71	Narrow and Ullberg (88)	f_{fl} varies from 550 to 850 psi; with different aggregates
0.65 to 0.89	Grieb and Werner (89)	f_{fl} varies from 640 to 840 psi; crushed stone with 1-in. maximum size
0.51 to 0.78	Grieb and Werner (89)	f_{fl} varies from 350 to 955 psi; crushed stone with 1½-in. maximum size
0.50 to 0.80	Grieb and Werner (89)	f_{fl} varies from 250 to 790 psi; sand and gravel with 1½-in. maximum size
0.57 to 0.88	Grieb and Werner (89)	f_{fl} varies from 430 to 750 psi; lightweight aggregate concrete
$0.6 + 100/f_{fl}$	Popovics (91)	f_{fl} varies from 490 to 750 psi; with different aggregates
0.67	Wright (94)	f_{sp} = 405 psi at the age of 28 days
0.63	Sell (95)	f_{fl} = about 45 kg/cm² at the age of 14 days
0.66	McNeely and Lash (96)	f_{fl} = 690 psi; coarse aggregate is crushed gravel

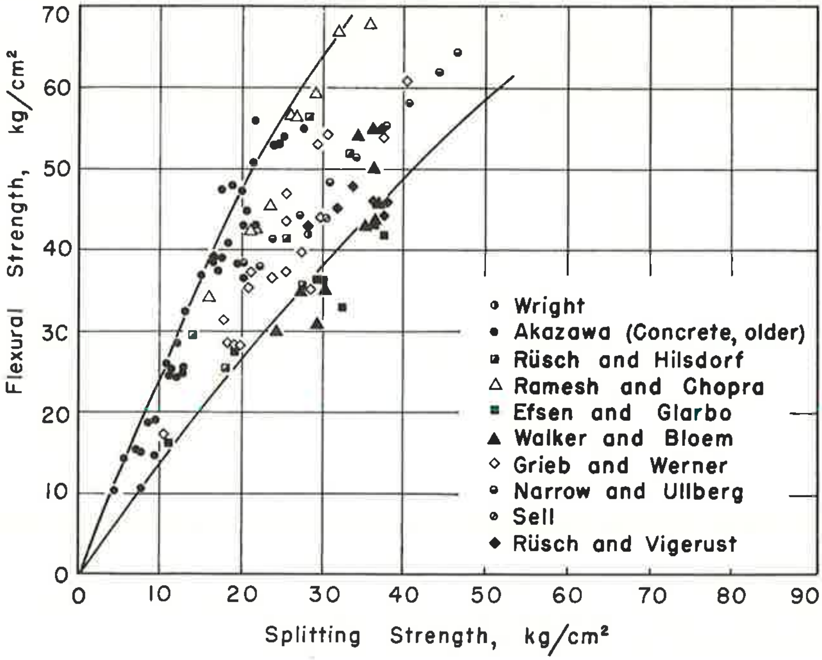

Figure 4. Experimental results obtained by various investigators for the relationship between splitting strength and flexural strength of concrete (after Bonzel 81).

As shown by Figure 4, Eq. 4 would not necessarily hold for tests carried out under different conditions. Nevertheless, the formula shows again that the ratio of f_{sp}/f_{fl} increases as the concrete strength increases. As Table 7 indicates, this is in accordance with the observations of other authors (48, 65, 81, 93).

The failure of a plain concrete beam in the bending test is controlled by the tensile strength of the concrete. Nevertheless, there are several possible reasons again why the flexural strength of a concrete is greater than its splitting and tensile strengths. L'Hermite attacked this problem from two different theoretical directions (2, pp. 134-143). When he applied the difference between the elastic energy of the fissure propagation in the direct tensile test and that in the bending test with third-point loading, he obtained a limit value of 0.575 for the ratio of f_t/f_{fl}. However, when he used the difference between the average deformations in the two tests, he obtained 0.5 for the limit of this ratio. Tucker (97) showed that at the failure of nonuniformly stressed specimens local stresses may be present which may exceed the tensile strength of the same material as determined by uniformly distributed loading. He found that, according to a statistical theory of strength, the flexural strength may be as much as 90 percent greater than the direct tensile strength.

Another, and perhaps the main reason for the difference between f_{fl} and f_t seems to be the application of Hook's law in the usual calculation of the flexural strength of concrete. Concrete deformations do not follow Hook's law, particularly in the tensile zone of the concrete beam; therefore, the calculation based on a linear stress distribution throughout the depth of the beam results in fictitious tensile stress values which are higher than the actual stresses in the beam. For instance, the flexural strength calculated from the hypothetical stress distribution that the material is "rigid-plastic in tension and proportional in compression" is two-thirds of the flexural strength, when calculated from the assumption of a linear stress distribution (80). Pincus and Gesund also quote that Blakey and Beresford reported (98) a similar factor of 0.735 using second and third-degree parabolas for the stress distribution in the beam as

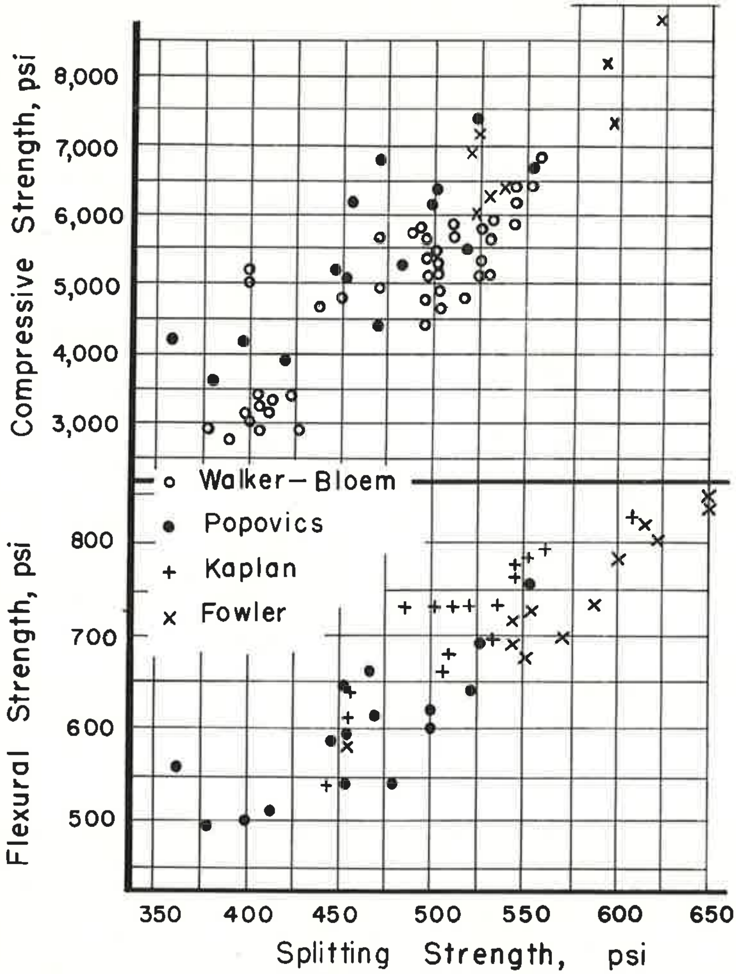

Figure 5. Empirical relations for compressive and flexural strengths vs splitting strength.

compared to linear distribution. The importance of the stress distribution with respect to the flexural strength of concrete has also been demonstrated experimentally by Blackman et al (99).

Certain correlations also exist between the compressive strength and direct tensile strength of cement mortars and concretes (42, 44, 65, 79, 100; 101, pp. 620 and 1292; 102, 103). Similarly, there is correlation between the compressive and splitting strengths of concretes, as shown in Figure 5 by using experimental data of several investigators (49, 91, 92). Other data (Fig. 6) also support this correlation (26, 46; 52, Chap. 10; 65, 81, 82, 84, 89, 90, 104-113). Bonzel, for instance, presents empirical formulas by several investigators for the relationship between the compressive and splitting strengths of cement mortars and concretes (81). The general form of these formulas is

$$f_{sp} = Cf_{co}^{n} \tag{5}$$

In the case of cylindrical specimens the value of C varies from 0.34 to 0.88, and n varies from 0.71 to 0.74 depending on the composition of the specimen and the test

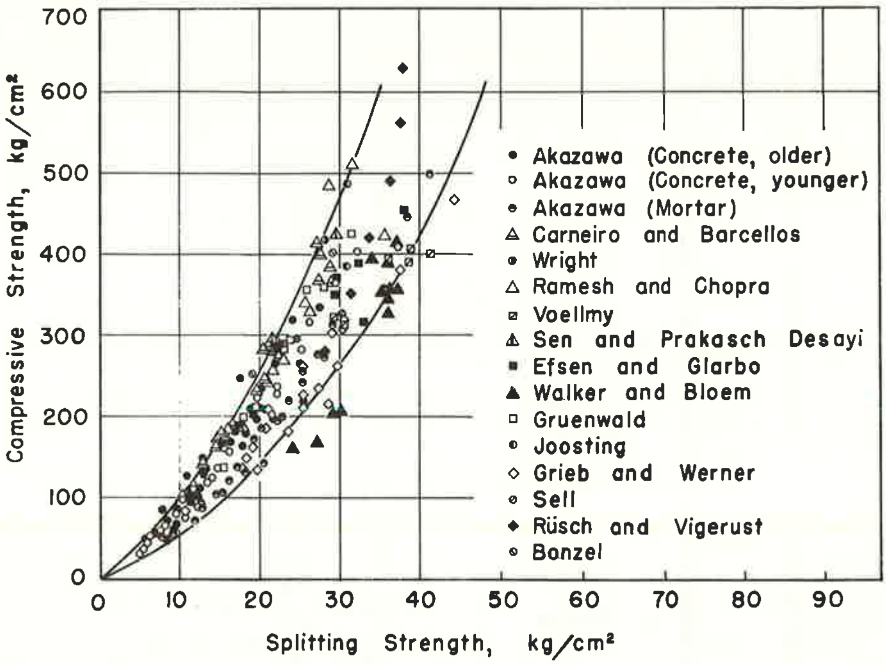

Figure 6. Experimental results obtained by various investigators for the relationship between splitting strength and compressive strength of concrete (after Bonzel 81).

method applied. These formulas, as well as numerous other experimental data, show that this relationship is curvilinear, thus the ratio of tensile strength to compressive strength decreases with increasing concrete strength (62, 63, 89; 101, p. 1292; 114, 115, 116). It may be mentioned that the f_{sp}/f_{co} ratio is less for structural lightweight concretes than for normal-weight concretes of identical compressive strength (90). A similar phenomenon was observed in connection with the f_{fl}/f_{co} ratio.

OTHER CONCRETE STRENGTHS

As has been emphasized, other concrete strengths are affected by the same factors as compressive or tensile strengths, but the degree of the effects is usually different. Therefore, there again exist correlations between torsion, shear, etc., strengths and compressive, tensile, etc., strengths of a concrete but these correlations are significantly influenced by the applied test methods and other circumstances of the testing. For instance, a relationship is expected between the torsion strength and tensile strength of a concrete because the failure in torsion is actually a tensile failure. However, this relationship is strongly dependent on the size and shape of the torsion specimens as indicated in Table 8, and on the method used to calculate torsion strength (117, 118). The torsion strength determined on specimens of circular section and calculated on the basis of rigid-plastic stress distribution—plastic theory—is 75 percent of the torsion strength calculated from a linear stress distribution—elastic theory (119). Therefore, the magnitude of the pure tensile strength of a concrete is expected to be less than its comparable torsion strength which was calculated from a linear stress distribution (80). Experimental data conclusively support this statement (Table 8). Incidentally, the torsion strength of concrete has an increasingly practical significance because the failure of prestressed concrete in pure torsion is essentially similar to that of plain concrete (118).

Table 8 summarizes experimental data for the torsional strength of concrete by several investigators. In addition, two other related experiments can be mentioned.

TABLE 8

COMPARISON BETWEEN TORSION AND DIRECT TENSILE STRENGTHS

Authority	Section Tested	Torsion Strength (psi)		Direct Tensile Strength (psi)
		Plastic Theory	Elastic Theory	
Bach and Bauman (40, par. 16)	Circle	269	364	264
	Circular ring	269	244	264
	Square	269	433	264
	2:1 rectangle	269	463	264
Graf (40, par. 16)	Circle	Not given	265	164
	Circular ring	Not given	196	164
Turner and Davies (121)	Square	247 and 264	395 and 422	301
	2:1 rectangle	309 and 223	528 and 381	219
Bach and Graf (121)	Circle	261	350	Not given
	Square	265	425	Not given
	2:1 rectangle	263	450	Not given
Young, Sagar and Hughes (121)	Square	336	539	Not given
	1.5:1 rectangle	278	467	Not given
	2:1 rectangle	352	602	Not given
Anderson (121)	Square	336	539	Not given
	1.25:1 rectangle	306	505	Not given
	1.5:1 rectangle	342	575	Not given
Anderson (121)	Square	400	640	Not given
	1.25:1 rectangle	369	609	Not given
	1.5:1 rectangle	392	658	Not given
Graf and Morsch (121)	Circle	151	202	164
Marshall and Tombo (121)	Circle	243	325	282
	Square	318	508	282
	1.5:1 rectangle	287	472	282
	1.7:1 rectangle	313	530	282
	2:1 rectangle	302	516	282
	3:1 rectangle	349	580	282

Iyengar et al (120) that torsion strengths of various concretes were only a little higher than their comparable splitting strengths which varied from 174 to 488 psi in the test series. Unfortunately, it is not reported how the torsion strength was calculated. Rudeloff's experimental data are also quoted by Graf (40, pp. 215-217). Numerous specimens of square cross sections and of various compositions were tested at various ages; these resulted in a good approximation of a value of 0.43 for the ratio of direct tensile strength to torsion strength where the latter was calculated presumably by the elastic theory. The values of direct tensile strength varied from 100 to 240 psi in this test series.

Marshall (121) has the opinion that the elastic approach is not the correct one for the calculation of torsion strength of concrete, because the shear stresses obtained from this theory are in some cases more than twice the value of the direct tensile strengths. This opinion is not unexpected because concrete cannot be considered an elastic material, especially at a high level. Moreover, if concrete behaved elastically under torsion, then failure would begin at the midpoint of the longer side of a specimen with a rectangular section and gradually spread to the remainder of the section. However, such failure never occurred, the method of failing being by sudden cracking of the whole section tending to show a uniform stress distribution (121). Nevertheless, it is conceivable that the elastic theory is more applicable for the calculation of torsional stresses in high-strength concretes, especially when the specimen is a hollow cylinder, because the plasticity of concretes decreases with increasing strength.

There are several test methods for the determination of the shear strength of a concrete. In one such test a plain concrete beam or slab of a short span is subjected to a load applied very close to the supports. This method does not provide pure shear because certain normal stresses are produced in the specimen. Nevertheless, the f_{dsh} (direct shear) strength of the concrete can be calculated from the ultimate load and the dimensions of the specimen in the usual way. Graf (40, pp. 215-217) applied this method to concretes with f_{cu} cube compressive strengths varying from 130 to 310 kg/cm², and in another series, to concretes with f_{fl} flexural strengths varying from 18 to 43 kg/cm². He found that within these strength limits the f_{dsh}/f_{cu} ratio was 0.23, and the f_{dsh}/f_{fl} ratio was 1.6 with good approximation. The following empirical formula was recommended by Palotas (12) for the relationship between the direct shear strength and cube strength of a concrete:

$$f_{dsh}/f_{cu} = 0.25 - f_{cu}/8,000 \tag{6}$$

where f_{cu} is expressed in kg/cm².

Another popular test method for the determination of the shear strength of a concrete is to load reinforced concrete beams without web reinforcement for bending up to shear failure; the f_{sh} shear resistance of the beam at diagonal cracking, i.e., the shear cracking strength of the concrete, can be calculated from the ultimate load and the characteristics, such as dimensions and reinforcement, of the specimen in the usual way. This shear strength is not identical to the direct shear strength, and it is not pure shear either because here the failures are actually of tensile nature on a different plane from that on which the maximum shear stress occurs.

Viest and others (122, 123, 124) found a correlation between the f_{sh} shear cracking strength and $\sqrt{f'_c}$ and Hanson (90) found a correlation between f_{sh} and f_{sp}. In both cases the proportionality factor is a function of the characteristics of the beam. In accordance with this, the 1963 ACI Building Code contains a formula (Eq. 17.2) that provides the f_{sh} shear resistance of a concrete as a linear function of $\sqrt{f_c}$ (125). However, recent experimental evidence appears to indicate that the f_{sh} shear strength varies just slightly with the compressive strength of the concrete, at least for under-reinforced beams. A typical example is the recommendation by Evans and Dongre (126), who presented empirical formulas for the relationship between the f_{sh} shear strength and f_c cylinder strength of a concrete, as follows:

For normal concrete:

$$f_{sh} = 0.04f_c + 100 \tag{7}$$

For structural lightweight concrete:

$$f_{sh} = 0.03f_c + 75 \tag{8}$$

where all strengths are expressed in psi. Although these formulas are valid, strictly speaking, only for a certain type of beam, other investigators (127, 128) have also published experimental results and/or formulas that show a similar tendency for normal concrete, structural lightweight concrete (129), or for both (52, Chap. 13; 130).

Theoretically, the f_{psh} strength of concrete in pure shear—that is, when no normal stresses are present on the plane of failure—can be obtained only from tests under combined stresses. On this basis, Kesler (131) estimated that the pure shear strength is approximately 20 percent of the compressive strength of the concrete. On a similar basis, Sen and Desay (112) recommended the following relation:

$$f_{psh} = 0.608\, f_{cu}^{0.737} \tag{9}$$

and

$$f_{psh} = 0.689\, f_{c}^{0.741} \tag{10}$$

Comparative experiments show again that the f_{sh} shear strength of a structural lightweight concrete is less in general than that of a normal concrete of identical compressive strength. This difference can be as high as 40 percent depending on the type of the lightweight aggregate used (52, Chap. 13; 126, 130, 132).

The impact strength of concrete is again strongly dependent on the applied method of testing (133). In addition, however, other factors, such as the concrete composition, curing conditions, and age, also influence the relationship between impact strength and other concrete strengths (134). Consequently, various investigators arrived at conflicting conclusions concerning these relations. Glanville et al concluded that the impact strength of a concrete may be assumed between one-half and two-thirds of its static cube strength (135). Others also found a good correlation between impact and compressive strengths (68). However, Green (136) quotes that Dutron believed the impact strength of a concrete to be connected with its tensile strength, and that Passov and Framm found no relationship at all. Results of impact tests by Walz are strongly influenced by the angularity and surface texture of the coarse aggregate used, but they do not show good relations with either the compressive or the flexural strengths of the concrete (137).

More recent Belgian experiments with various cements indicate certain correlations between the impact strength and the compressive and flexural strengths respectively, although not for the splitting strength. This correlation exists up to the age of one year, but only when the specimens were cured in 100 percent relative humidity until the testing. Comparable specimens cured in 50 percent relative humidity do not show any of these correlations because the impact strength of these specimens declined at later ages whereas the other strengths increased (71).

A possible explanation for the foregoing conflict in opinions is that under certain circumstances, such as dry curing and brittle or smooth aggregate, the concrete becomes disproportionally more brittle with increasing static strengths, and thus the resistance to impact decreases. It also appears probable that the magnitude of bond between the cement paste and aggregate has a more pronounced effect on the impact strength of a concrete than on its static strengths. Therefore, one may expect correlation between the impact strength and a static strength of a concrete only if the specimens were wet cured until the testing, and if the type of mineral aggregate does not vary strongly in the comparable test series.

THEORETICAL RELATIONSHIPS

The relationships presented so far between the various strengths of a concrete are empirical and were obtained at best by fitting a curve to experimental data. It is also possible, however, to obtain formulas for the relations between various types of strength from certain theoretical or semi-theoretical considerations. For instance, by the assumption of a straight line for the Mohr's envelope, which is equivalent to the Coulomb theory of failure, the f_{psh} and the f_{tor} (torsion strength), respectively, can be expressed in terms of the f_c compressive and f_t tensile strengths.

$$f_{psh} = 0.5\sqrt{f_c \cdot f_t} \tag{11}$$

and

$$1/f_{tor} = 1/f_t + 1/f_c \tag{12}$$

The pure shear strength is expected to be half the geometric average of the tensile and compressive strengths, and the torsion strength to be half the harmonic average of the same two strengths. By using the rule of thumb that the compressive strength of a concrete is ten times as high as the tensile strength, Eqs. 11 and 12 provide the following approximate values for the shear strength and the torsion strength of the concrete:

$$f_{psh} = 0.16f_c = 1.6f_t$$

and

$$f_{tor} = 0.09f_c = 0.91f_t$$

Even this oversimplified discussion reveals that the torsion strength of a concrete is almost the same as its tensile strength. As Eq. 12 shows, this is generally characteristic of the materials that have much higher compressive than tensile strength.

One can also write formulas for various strengths with the help of a given ϕ angle of inclination of the straight line for the Mohr's envelope; for instance:

$$f_{tor} = f_c \frac{1 - \sin\phi}{2} \tag{13}$$

$$f_{psh} = f_{tor}/\cos\phi = f_c \frac{1 - \sin\phi}{2\cos\phi} \tag{14}$$

Despite their academic interest, formulas such as Eqs. 11 through 14 are not reliable enough for the determination of a concrete strength at the present stage of concrete technology. The reason for this may be either the inadequacy of the applied theory, or the amply demonstrated fact that there is no way to measure the "true" compressive, tensile and other strengths because the results of concrete strength tests are decisively dependent on the method of testing applied.

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Discussion

V.M. MALHOTRA, Concrete Engineer, Department of Energy, Mines and Resources, Ottawa, Canada—The writer wants to commend the author for his excellent review of the entire field of concrete strengths. However, the writer wishes to offer some comments on the relations concerning direct tensile and splitting tensile strengths.

Prof. Popovics has correctly pointed out that because of the biaxial nature of stresses in the split-cylinder test, the splitting tensile strength should be lower than the direct tensile strength. However, the summary of results of various investigators (Table 5) indicates that the reverse is true. In his explanation of this paradoxical situation, Prof. Popovics has failed to mention the inherent inadequacies of the various types of direct-tension test methods.

All the test methods reported by various investigators with the exception of Todd's method (used by Ledbetter and Thompson) are burdened with misalignment and/or

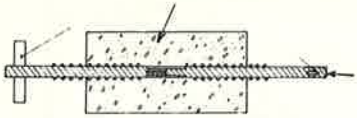

Figure 7. Line drawing of test sample after Peltier for direct-tension test (138).

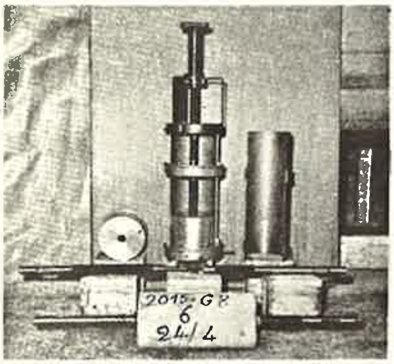

Figure 8. View of cylindrical mold for Peltier test (138).

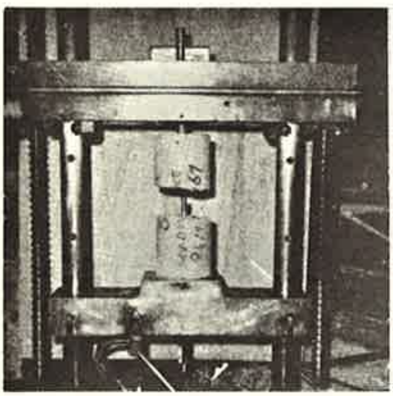

Figure 9. View of test sample after break (138).

eccentricity stresses, and this explains the relatively low tensile strengths obtained compared to the "true" tensile strength of concrete. The data reported for Todd's method relate only to lightweight concrete and therefore should not be used for comparison involving normal weight concretes.

Prof. Popovics in his literature survey has not referred to two very important studies dealing with the subject under discussion. The first, by Halabi (138), is the only published report to date which shows that the direct tensile strength of concrete is greater than its splitting tensile strength. Halabi used the Peltier test in his investigations. In this direct-tension test method, the test cylinders are cast in metallic molds, 18 cm in diameter and 36 cm in height, around a central notched steel rod 32 mm in diameter. The steel rod consists of two independent parts, but these remain in alignment owing to a grooving device in the middle zone. Furthermore, a small spring maintains the two parts slightly apart to avoid interior stresses which develop during the setting of concrete. Also, the central parts of the steel rods are polished and are covered by a waterproof linen sockette to avoid adherence of concrete and to insure failure at this reduced section.

Figures 7 to 9 show the various stages of this test method. A summary of the data reported by Halabi is given in Table 9.

The second study that merits consideration is by Ward (139), and indicates that within certain limitations, the ratio of the direct to splitting tensile strength is unity (Fig. 10). The direct-tension test method employed a lateral gripping device (Fig. 11).

TABLE 9

DIRECT TENSILE vs SPLITTING TENSILE STRENGTHS[a]

(18 × 36-Cm Cylinder)

No. of Test Results	Direct Tensile Strength (psi)	Splitting Tensile Strength (psi)	Ratio 1:2
5	440	250	1.76
16	425	325	1.31
3	485	410	1.18

[a]After A.K. Halabi (138). Direct tensile test carried out using Peltier test method; all strengths at 28 days; w/c ratio (by weight) varied from 0.42 to 0.53.

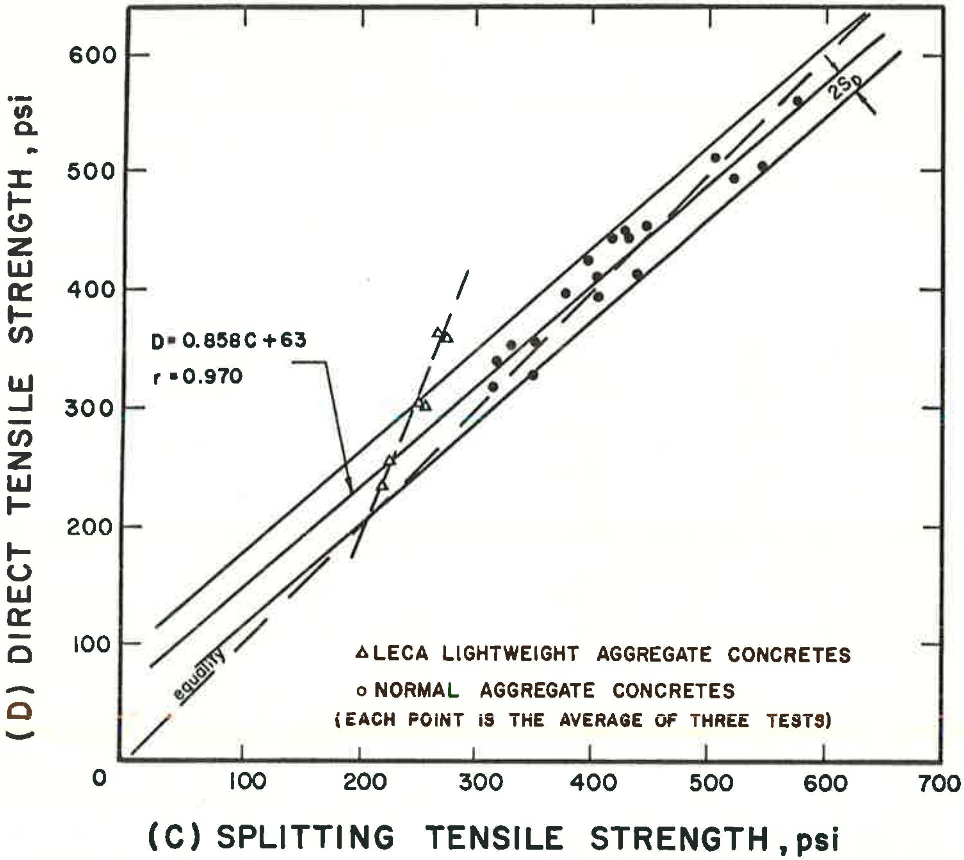

Figure 10. Relationship between direct tensile and splitting strengths (139) for concrete containing three different normal weight and one lightweight coarse aggregate.

Figure 11. Direct-tension test method using lateral grips (139).

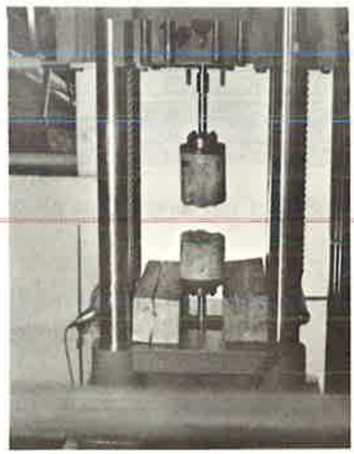

Figure 12. Direct-tension test method using thick steel plates glued to the ends of a concrete cylinder with epoxies (140).

TABLE 10

DIRECT TENSILE vs SPLITTING TENSILE STRENGTHS[a]
(6 × 12-In. Cylinders)

Mix No.	Compressive Strength (psi)	Direct Tensile Strength (psi)	Splitting Tensile Strength (psi)
1	2110	290	275
2	3320	405	355
3	3760	385	395
4	4370	450	425
5	6050	445	545
6	7585	540	685

[a]Canada Mines Branch results. Each result is a mean of three tests; all strengths at 28 days; direct tensile test employed specimens fabricated by gluing thick steel plates to the ends of concrete specimens with epoxies.

Recent studies carried out at Canada Mines Branch in conjunction with Ecole Polytechnique, University of Montreal, confirm that, at least for low strength concretes, the splitting tensile strength is lower than the direct-tensile strength (Table 10). This is true in spite of the direct-tension test employed (140), which was one in which thick steel plates are glued by epoxy to the ends of concrete specimens that are then pulled apart in tension (Fig. 12). For high strength concretes, this test does not appear satisfactory because of the high stress concentration near the end plates, which causes the specimen to break near the plates.

The writer believes that no meaningful comparisons can be made between the splitting tensile strength and direct tensile strength results because almost none of the currently available direct-tension test methods give the "true" tensile strength of concrete. Further research work to develop satisfactory tests is strongly urged.

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SÁNDOR POPOVICS, Closure—I an grateful to Mr. Malhotra for his presentation of further references and experimental data. He concluded from this new material that the usual negative difference between the measured direct tensile strength of a concrete and the corresponding splitting strength reported by various investigators is due to inherent inadequacies of the applied methods for testing the direct tensile strength.

The strongest support for this contention appears to be that any inadequacy in the direct tensile test, such as eccentricity, additional and internal stresses, should lead to a reduction in the measured tensile strength. Thus, one is inclined to attribute extra significance to the relatively high direct tensile strengths presented by Mr. Malhotra. Nevertheless, I feel that the available experimental evidence is not enough to form a final opinion concerning the relationship between direct tensile and splitting strengths. In any case, his presentation focuses the attention to the fact that further investigation is definitely needed in this direction.

The Mechanism of Fatigue in Cement Paste and Plain Concrete

JOHN D. ANTRIM, Associate Professor, Clemson University

An investigation was conducted to determine the fatigue behavior of cement paste loaded in axial compression and the fatigue behavior of the paste when it is diluted with aggregate.

Cement pastes were made with water-cement ratios of 0.70 and 0.45 by weight and these same water-cement ratios were used in concretes containing natural aggregates and concretes containing synthetic aggregates. Cylindrical specimens, 2 in. in diameter by 4 in. in height, were used for evaluating the cement pastes and cylindrical specimens, 3 in. in diameter by 6 in. in height, were used for evaluating the concretes. Specimens were tested in a saturated condition and at moisture contents less than that at saturation. Over 500 specimens were tested statically in unconfined compression and over 150 specimens were tested dynamically in unconfined compression at specific stress levels at a speed of 1,000 cpm.

It was found that the fatigue behavior of cement paste is sensitive to changes in the water-cement ratio of the paste and to changes in the moisture content of the paste. It was also found that within the limits of the investigation, the fatigue characteristics of plain concrete are apparently governed primarily by the fatigue characteristics of the cement paste.

The fatigue mechanism proposed for cement paste and plain concrete is basically the same and it is that fatigue failure occurs because small cracks form and propagate in the cement paste under repeated applications of loads less than the static failure load. The resulting crack pattern weakens the section to the point where it cannot maintain the applied load. The development of this damaging crack pattern depends primarily on the water-cement ratio of the cement paste and the presence of shrinkage stresses in the cement paste.

•ONE area of portland cement concrete technology that has been lacking in knowledge of basic mechanisms is the area of fatigue of concrete. The general behavior of concrete under repeated loadings has been reasonably well established by numerous investigations, but it has only been during the past ten years that the mechanism of fatigue has received attention. It is the purpose of this paper to review only the work done towards determining the mechanism of fatigue in plain concrete. For a comprehensive review of published information on the fatigue properties of plain concrete, it is suggested that Nordby's (1) or Murdock's (2) review be consulted.

Murdock and Kesler (3) studied the effect of coarse aggregate by inserting single, preshaped limestone aggregates in the tension zone of mortar flexure specimens. Each coarse aggregate piece extended the full width of the test specimen so as to make the problem essentially a two-dimensional one and the orientation of the aggregate cross section, as well as the proximity of its outer surface to the free tension surface of the specimen, was varied from one test series to another. Static and fatigue flexure tests of plain mortar specimens and mortar specimens containing coarse aggregate inclusions gave sufficient data to form an hypothesis that attributes the initiation of fatigue failure

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to the progressive deterioration of the bond between coarse aggregate and the binding matrix.

Doyle, Kung, Murdock, and Kesler (4) studied the flexural behavior of the cement-fine aggregate matrix with the inclusion of various preplaced coarse aggregates and also the behavior of the matrix with a preformed void. Specimen condition, size, etc., were the same as in the Murdock and Kesler study, however, this time the aggregates were cylindrical pieces of aluminum or granite and the preformed voids were dimensionally the same as the natural aggregate cylinders. They concluded that (a) residual stresses due to shrinkage of mortar around a single, comparatively rigid, unbonded inclusion have no significant effects on either static or fatigue strength, and (b) static and fatigue failures initiate in the bond between the coarse aggregate and the mortar matrix when the modulus of elasticity of the aggregate is greater than that of the matrix.

Neal, Kung, and Kesler (5) increased the complexity of the models used by Doyle et al, so that they could examine the effects of more than one particle of coarse aggregate in close proximity to each other. Specimen condition, size, etc., were the same as the prevously mentioned studies except that all the testing was conducted on saturated specimens (previous specimens were air-dried). Aggregate pieces were again cylindrical and were either of granite or limestone. Their conclusions were (a) fatigue failure begins in the bond between the coarse aggregate and the mortar matrix and apparently is not influenced by the elastic modulus of the coarse aggregate, and (b) residual stresses due to shrinkage of the mortar matrix around particles of coarse aggregate do not occur to an extent great enough to influence either the static or fatigue strength of the specimen, or to influence the plane of failure.

Glucklich (6) investigated the mechanics of the propagation of fatigue cracks in mortar beam specimens. Specimen size and the water-cement ratio were the same as used in the previously mentioned studies. Notched and unnotched beams were used to determine stress-strain characteristics during fatigue life. Measured strains were interpreted in terms of crack lengths, the correlation between compliance and crack length having been predetermined in static tests. The product of the critical crack length and the square of the maximum stress proved to be a constant number for either unnotched beams or notched beams with approximately equal notch depths. Glucklich noted that the material had the same criterion of fracture in both fatigue and static loadings, namely, the strain-energy release rate.

The author (7) conducted an investigation which was concerned first with the fatigue behavior of cement paste loaded in axial compression, and second, with the fatigue behavior of cement paste diluted with aggregate.

SCOPE

The fatigue behavior of cement paste was investigated with respect to the effect of the structure of the hardened paste and the effect of the moisture content of the paste. To obtain a paste with an open capillary structure, a water-cement ratio of 0.70 by weight was selected and to obtain one with a dense structure, a water-cement ratio of 0.45 by weight was selected. Moisture contents selected were 100 percent of saturation and approximately 95 percent of saturation. Saturation moisture content was taken as the moisture content of the specimen upon removal of the specimen from the curing water. Moisture contents below saturation are average moisture contents estimated from specimen weight changes occurring after removal from the curing water. Moisture contents below 90 percent for the low water-cement ratio cement paste specimens were not practical because of specimen cracking.

The influence of the aggregate on the fatigue characteristics of the cement paste was investigated by testing concrete made by diluting the high and low water-cement ratio pastes with identical amounts of the same aggregate. The resulting concretes were then tested at moisture contents similar to those selected for the undiluted pastes. Because concrete is less susceptible to major shrinkage cracking, additional moisture content levels were obtained by air-drying specimens for five weeks and by oven-drying specimens. In addition, the effect of the aggregate strength was investigated by combining each of the two water-cement ratio pastes with aggregates whose strengths were either weaker or stronger than the strengths of the undiluted pastes. Since aggregate strength was the variable under consideration, it was essential to control the other characteristics of the aggregates used and this was accomplished by manufacturing the aggregates in the laboratory. The manufacturing process was simply one of crushing hardened cement pastes of known strengths, sieving, and then recombining to give an aggregate with a predetermined gradation.

SPECIMEN MANUFACTURE

A procedure was developed for making one specimen at a time and which allowed successive specimens to be made in an identical manner, the result being a low mix to mix variation for a given mix design. The procedure used to make cement paste specimens, 2 in. in diameter by 4 in. in height, was to make two specimens each day, allow them to wet cure for 28 days and then immediately perform the required static or fatigue test. In those cases where a specimen was to be tested at a moisture content of less than the saturation moisture content, drying was started at the age of 28 days and immediately after a specified drying time, the specimen was tested statically or in fatigue. Saturated specimens tested in fatigue were kept wet by wrapping specimen with wet absorbent cotton followed by polyethylene film. Air-dried specimens were wrapped with polyethylene film to prevent further loss of moisture during the fatigue testing.

Test specimens for the paste-aggregate combinations were prepared one at a time and in an identical manner although upwards to 12 specimens, 3 in. in diameter by 6 in. in height, were made each day. As with the cement paste specimens, the concrete specimens were cured for 28 days in a saturated lime solution. Testing format was

TABLE 1

AGGREGATE CHARACTERISTICS

Physical Properties	
Limestone coarse aggregate	
Dry rodded unit weight	94 pcf
Bulk specific gravity	2.66
Absorption	0.72%
River terrace sand fine aggregate	
Bulk specific gravity	2.66
Absorption	1.28%
A-1 Aggregate (w/c = 0.85)	
Dry rodded unit weight	55 pcf
Bulk specific gravity	1.17
Absorption	43.7%
Approximate compressive strength	2900 psi
A-2 Aggregate (w/c = 0.52)	
Dry rodded unit weight	65 pcf
Bulk specific gravity	1.48
Absorption	27.8%
Approximate compressive strength	8000 psi
A-3 Aggregate (w/c = 0.36)	
Dry rodded unit weight	76 pcf
Bulk specific gravity	1.77
Absorption	18.6%
Approximate compressive strength	13,000 psi

Gradation
(Natural and Synthetic Aggregate)

Coarse Aggregate Fraction		Finer Aggregate Fraction	
Sieve Size	Percent Finer	Sieve Size	Percent Finer
½ in.	100	No. 4	100
⅜ in.	55	No. 8	90
No. 4	0	No. 16	67
		No. 30	42
		No. 50	20
		No. 100	6
		Fineness modulus	2.75

TABLE 2

STATIC STRENGTH CHARACTERISTICS OF CEMENT PASTE TEST SPECIMENS

Uncorrected w/c (lb/lb)	Operator	n	$\overline{Y}$ (psi)	C (%)	S (%)	Air Drying Time
0.70	1, 2	40	3671	7.7	100	
	3	14	3248	3.1	100	
	3	6	3538	4.3	96	4½ hr
0.45	3	17	9630	3.7	100	
	3	6	10878	2.6	91	3 days

where n = the number of test specimens,
$\overline{Y}$ = the average compressive strength of the test specimens,
C = the coefficient of variation for the test specimen strengths,
S = the approximate average degree of saturation (at time of removal from the curing water, specimens were assumed to be in a saturated condition).

the same as that used for the cement paste specimens, that is, saturated specimens were tested at the end of the curing period and air-dried specimens were tested right after completion of the air-drying period. The paste-aggregate combination that used natural aggregates had limestone as the coarse aggregate and sand as the fine aggregate. The three synthetic aggregates were similar in particle size, particle shape, and gradation (fine through coarse fraction); and they were different in strength and porosity. The physical properties and gradation of these three aggregates, which have been designated A-1, A-2, and A-3 as a means of identifying them, are given in Table 1 along with the characteristics of the natural aggregates.

All cement paste specimens and concrete specimens were made with a Type I portland cement from a single clinker batch.

CEMENT PASTE TESTS

Static Compression Tests

The purpose of the static compression tests was to establish a reliable estimate of the strengths of the two cement paste mixes. The strength data for the two cement paste mixes are summarized in Table 2. Two sets of data are shown for the 0.70 w/c cement paste at 100 percent of saturation because a statistical analysis showed that it was necessary to treat the two groupings separately. The difference was attributed to operator technique in making the specimens since all operations, other than making the specimens, were performed by operator 3.

Fatigue Tests

Since the objective of this part of the investigation was the determination of the fatigue characteristics of cement paste with respect to the effect of certain variables rather than the establishment of an S-N curve, it was not deemed necessary to conduct the testing at numerous load levels.

All the fatigue tests were started at the age of 28 days for the saturated specimens and immediately after the completion of drying for the air-dried specimens. If a test was interrupted for any reason, it was started over again with a new specimen. In those cases where a specimen had not failed by the time the fatigue machine had to be shut down, it was immediately tested statically.

All fatigue specimens were tested in the Krouse-Purdue axial-load fatigue machine which is of the constant deflection type and derives its force from hydraulic pressure acting on a large piston directly connected to the test piece through a piston rod. The machine has a capacity of ±60,000 lb and operates at 1,000 cpm. Loads are measured to within ±100 lb by an electronic system that is actuated by a Baldwin-Lima-Hamilton type U-1, SR-4 load cell which is an integral part of the load screw which holds the test specimen in place.

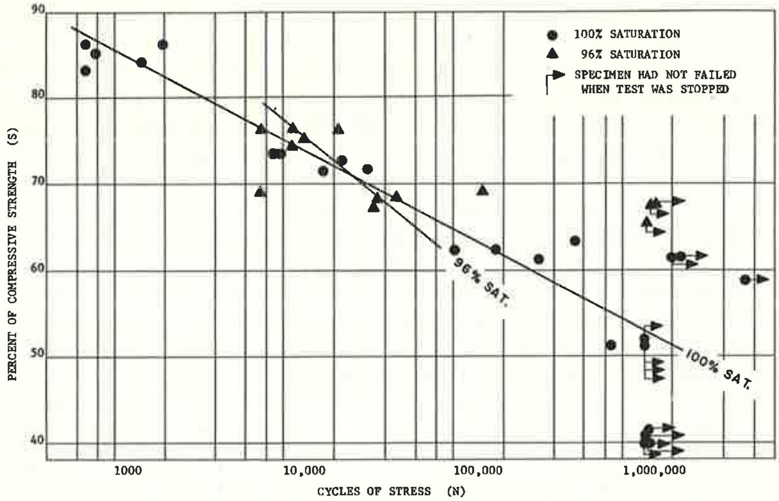

Figure 1. S-N diagram for 0.70 w/c cement paste.

The fatigue test data for both cement paste mixes are shown in Figures 1 and 2. The fitting of a straight line to the data of each grouping was accomplished by the method of least squares on the assumption that the S-N relationship within the range of stress levels studied is linear and that the transformed cycles to failure ($\log_{10}N$) are normally distributed (8). The regression lines shown do not include the effect of those specimens which had not failed when the fatigue test was stopped. The equations of these lines are not shown because the curves should not be used to predict the fatigue performance of cement paste. Prediction curves would require considerably more data than were obtained. There is, however, sufficient data to justify the use of these lines to show the fatigue behavior of cement paste when it is subjected to changes in its water-cement ratio and its moisture content.

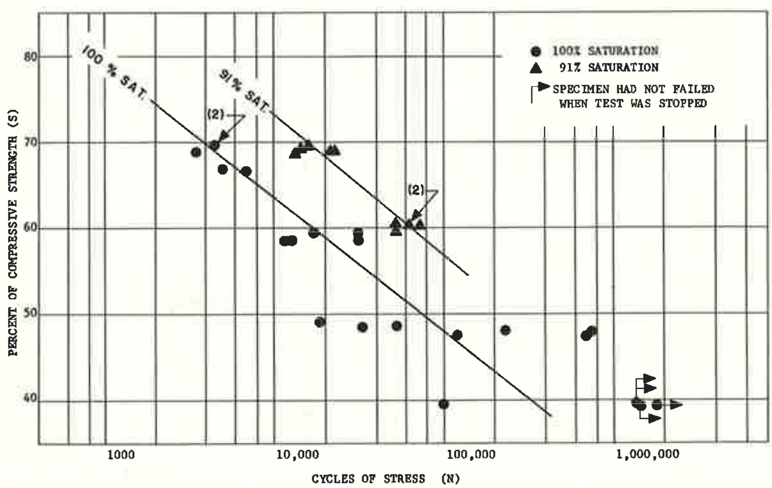

Figure 2. S-N diagram for 0.45 w/c cement paste.

Results

If the results of the static compression tests are considered in light of what is known or proposed concerning the behavior of cement paste, one finds that the results of this first part of the investigation are in agreement with existing knowledge and theory.

The presence of capillary pores reduces the load-carrying area per unit of gross area; thus a cement paste with an open capillary structure will be weaker than an equivalent section of cement paste with a dense structure. This is borne out in this investigation, the 0. 45 water-cement ratio paste specimens being approximately three times as strong as the 0. 70 water-cement ratio specimens.

The change in strength caused by reducing the moisture content of the hardened paste can be explained by the presence of shrinkage stresses in the cement paste. Powers (9) explains the shrinkage process as one which essentially occurs in the gel, the amount of shrinkage depending on the amount of water withdrawn from the gel, thus a paste with few capillary pores (low water-cement ratio) will undergo shrinkage which is directly proportional to water loss. On the other hand, a paste with a considerable quantity of capillary pores (high water-cement ratio) will undergo very little shrinkage while loosing water; however, as the drying continues a point is reached when the shrinkage becomes directly proportional to the water loss.

Although no shrinkage was measured this relationship of water loss to theoretical amount of shrinkage and hence the development of shrinkage forces was clearly evident in this investigation. The presence of shrinkage stresses in the air-dried 0. 45 w/c paste specimens is indicated by the need to apply an additional compressive load (Table 2) to overcome the tensile stresses which develop as the water is withdrawn. The small, if any, gain in strength of the air-dried 0. 70 w/c paste specimens is due to the lack of shrinkage stresses since the water loss was capillary water. That this water loss was capillary water can also be explained by what is known about the action of water in capillary tubes; that is, water in a large diameter capillary tube will evaporate much faster at a given relative humidity than water in a small diameter capillary tube. In this investigation, equivalent amounts of water were lost in considerably different time periods, the 0. 70 w/c paste specimens loosing their water in $4\frac{1}{2}$ hours and the 0. 45 w/c paste specimens loosing their water in 3 days. The only conclusion possible here is that the 0. 70 w/c paste specimens definitely had a much more extensive capillary pore system.

The curves of Figures 1 and 2 have been reproduced in Figure 3 better to illustrate the fatigue behavior of the cement pastes. The S-N curve for the saturated 0. 70 w/c

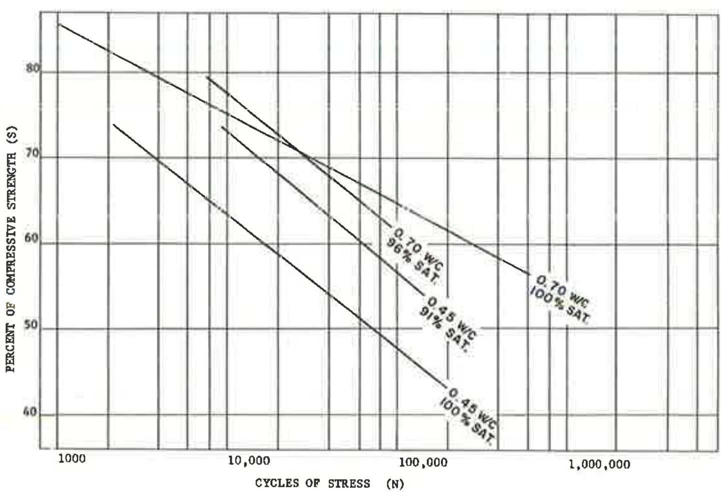

Figure 3. Graphical representation of cement paste fatigue data.

TABLE 3

PHYSICAL PROPERTIES OF THE NATURAL AGGREGATE CONCRETES

w/c (lb/lb)	Degree of Saturation (%)	Air Drying Time	Number of Specimens	Average Compressive Strength (psi)	Strength Coefficient of Variation (%)	Average Air Content** (%)	Air Content Coefficient of Variation (%)
0.70	100		9	2,875	3.1	11.0	8.5
	95	3 hr	9	2,931	4.5	10.6	5.9
	24	5 wk	9	2,900	4.1	11.3	6.6
	0	—*	11	2,754	7.5	10.3	6.1
0.45	100		13	4,437	7.8	9.3	8.7
	95	3 hr	12	4,383	8.8	9.0	12.1
	47	5 wk	11	6,113	7.3	7.2	11.1
	0	—*	8	4,550	11.9	8.4	11.8

*Oven dried at 220 F to constant weight.

**The natural aggregate concrete was an air-entrained concrete because earlier work (8) had indicated that entrained air would decrese the variability of the fatigue data.

paste is above that for the saturated 0.45 w/c paste. That is, at equivalent percentages of compressive strength, the 0.70 w/c paste withstands considerably more cycles of stress before failure than does the 0.45 w/c paste. A possible mechanism explaining this behavior is as follows: the 0.70 w/c paste is less brittle than the 0.45 w/c paste; thus it is capable of readjusting its structure, and stress concentrations are slower to build up. Accordingly, crack initiation is delayed, crack propagation is not as rapid as in the 0.45 w/c paste, and the material is capable of withstanding more cycles to failure than its more brittle counterpart. It should be noted that the paste structure is the critical element because the load-carrying gel particles are basically the same in both pastes.

Figure 3 also shows that the S-N curve for the air-dried 0.70 w/c paste is essentially superimposed on the curve for the saturated 0.70 w/c paste, but the curve for the air-dried 0.45 w/c paste lies above that for the saturated 0.45 w/c paste. These relative positions suggest that the change in fatigue properties of cement paste with air drying might be a result of the same factors that are considered to affect changes in static compressive strength. Table 2 indicates that the compressive strengths of the air-dried and saturated 0.70 w/c pastes were approximately the same, but that the compressive strength of the air-dried 0.45 w/c paste was greater than that for the saturated 0.45 w/c paste. It is suggested that shrinkage stresses play a greater role in the fatigue strength then they do in the static strength because they serve to restrain crack propagation.

TABLE 4

PHYSICAL PROPERTIES OF THE SYNTHETIC AGGREGATE CONCRETES

w/c (lb/lb)	Degree of Saturation (%)	Number of Specimens	Average Compressive Strength (psi)	Strength Coefficient of Variation (%)	Aggregate Designation	Approx. Aggregate Strength (psi)	Approx. Cement Paste Strength (psi)	Failure Type[a]
0.70	100	5	1,944	2.1	A-1	2,900	3,200	Bond[b]
	100	7	2,853	3.8	A-2	8,000	3,200	Bond
0.45	100	5	3,614	2.6	A-1	2,900	9,600	Bond & Aggregate[c]
	100	7	4,993	3.0	A-2	8,000	9,600	Mostly Aggregate
	100	8	6,014	5.2	A-3	13,000	9,600	Mostly Aggregate

[a]Deduced from the appearance of the fractured specimens.

[b]The bond failure referred to is at the coarse aggregate interface.

[c]The aggregate failure referred to is failure through the aggregate.

Crack propagation is an important factor in the fatigue failure of any material since final fracture is dependent on the extent of the crack pattern. Fatigue fracture is somewhat similar to brittle fracture in that the ability of the material to resist failure is dependent upon the random distribution of imperfections or weak spots. Brittle fracture occurs when the stress at one or more of these points reaches the strength of the material and little, if any, yielding precedes the failure (10).

Although a material that fails in fatigue does so because of brittle fracture (even ductile steel shows a zone of brittle fracture), the process leading up to failure is different from that in the static situation in that the whole chain of events preceding fatigue fracture depends on a series of random processes. This is the reason for the pronounced scatter which is characteristic of all fatigue data. The important point is that fatigue failure starts at a few weak spots and if the propagation of the resulting cracks can be delayed, failure will be delayed. It is proposed that such a situation exists in the air-dried 0.45 w/c paste, that is the shrinkage stresses in the vicinity of a newly formed crack are of sufficient magnitude to stop the propagation of the crack, at least temporarily. Once the section is weakened by an accumulation of these tiny cracks, the crack propagation proceeds as it does in the saturated 0.45 w/c paste.

Consideration was also given to alternate reasons for the observed pattern of fatigue behavior. At first glance the data suggest that the generation of hydraulic pressure in the pore water might account for the observed difference. However, the change in fatigue behavior brought about in the 0.45 w/c paste by a small amount of drying seems to eliminate this as a primary consideration.

Based on the preceding discussion, the following hypothesis is proposed for the mechanism of fatigue in cement paste:

> Fatigue failure in cement paste occurs because small cracks form and propagate under repeated applications of loads less than the static failure load. The resulting crack pattern weakens the section to the point where it cannot maintain the applied load. The development of this damaging crack pattern depends primarily (if not entirely) on the water-cement ratio of the cement paste and the presence of shrinkage stresses in the cement paste.
>
> The crack pattern is slower to develop in an open capillary structure cement paste than in a dense structure cement paste because the high water-cement ratio paste is less brittle and it can readjust its structure, thus delaying the buildup of stress concentrations.

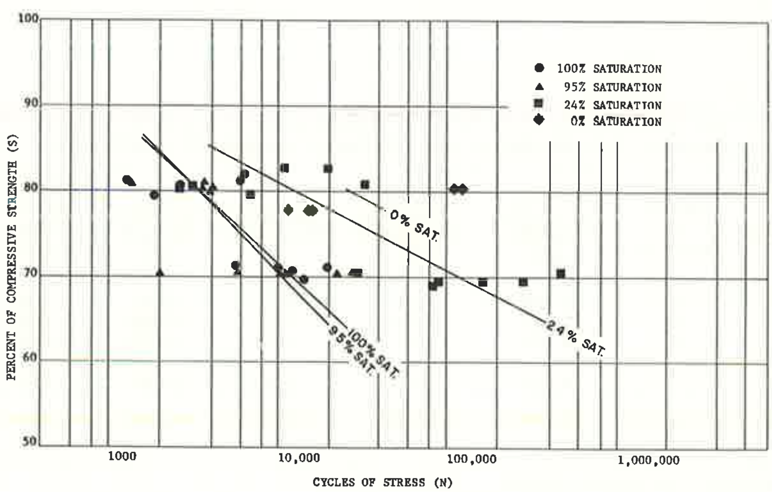

Figure 4. S-N diagram for 0.70 w/c natural aggregate concrete.

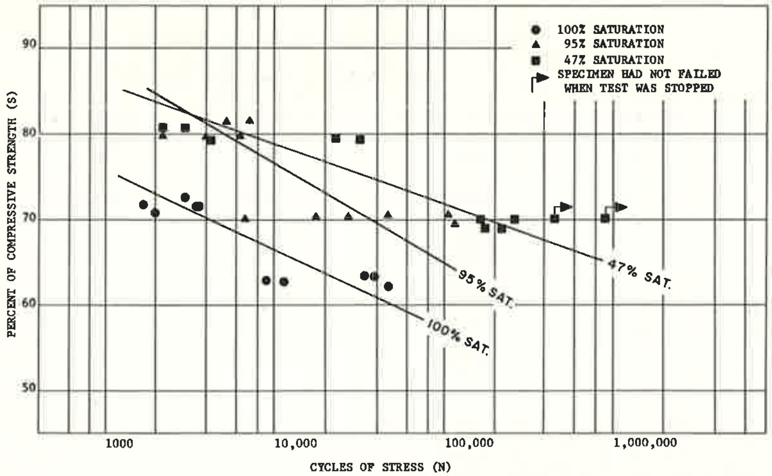

Figure 5. S-N diagram for 0.45 w/c natural aggregate concrete.

In addition, the crack pattern is slower to develop in a cement paste which has undergone a loss of gel water because the shrinkage stresses that develop effectively restrain crack propagation and materially delay fatigue failure. However, the beneficial effect of the shrinkage stresses is reduced as shrinkage cracks are formed since they contribute to a reduction in the load carrying section.

PASTE-AGGREGATE COMBINATION TESTS

Static Compression Tests

The physical properties of the natural aggregate concretes are summarized in Table 3; those for the synthetic aggregate concretes are summarized in Table 4.

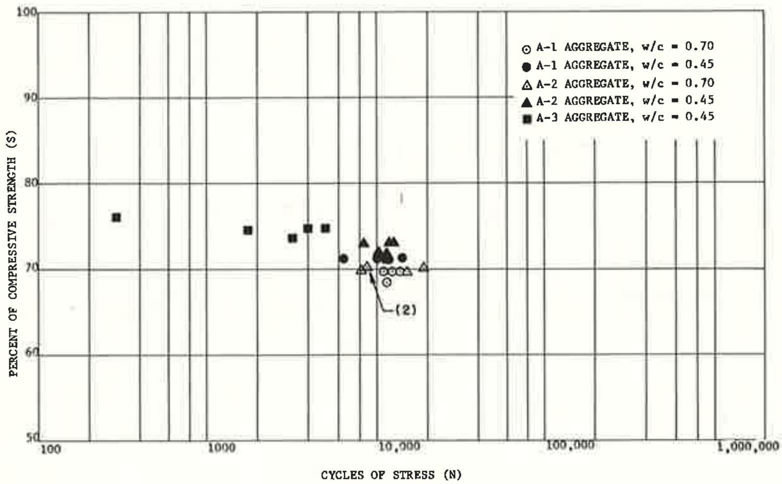

Figure 6. S-N diagram for the synthetic aggregate concretes.

TABLE 5

CEMENT PASTE AND CONCRETE STATIC STRENGTHS

w/c (lb/lb)	Degree of Saturation (%)	Approximate Compressive Strength				
		Cement Paste (psi)	Natural Aggregate Air-Entrained Concrete[a] (psi)	A-1 Aggregate Concrete[b] (psi)	A-2 Aggregate Concrete[c] (psi)	A-3 Aggregate Concrete[d] (psi)
0.70	100	3,200	2,900	1,900	2,900	
	96	3,500				
	95		2,900			
	24		2,900			
	0		2,800			
0.45	100	9,600	4,400	3,600	5,000	6,000
	95		4,400			
	91	10,900				
	47		6,100			
	0		4,600			

[a]Approximate compressive strength of limestone aggregate: 26,000 psi.
[b]Approximate compressive strength of A-1 aggregate: 2,900 psi.
[c]Approximate compressive strength of A-2 aggregate: 8,000 psi.
[d]Approximate compressive strength of A-3 aggregate: 13,000 psi.

Fatigue Tests

All the fatigue tests in this part of the investigation were conducted in the Krouse-Purdue fatigue testing machine. The various procedures necessitated by the operating characteristics of the machine and the objective of the investigation, which were discussed with respect to the fatigue testing of the cement paste specimens, are also applicable to the testing of the concrete specimens.

The fatigue test data for the natural aggregate concretes and the synthetic aggregate concretes are shown in Figures 4, 5, and 6. The method of least squares was used to fit curves to those groupings where sufficient data were available. This procedure was identical to that used for the cement paste fatigue data and the same limitations on the use of the least square lines that applied then, apply to this collection of data.

The oven-dried 0.70 w/c natural aggregate concrete was tested at only one stress level, therefore, the least squares method could not be applied to these data. A short line in Figure 4 helps locate the position of the oven-dried data. This line was positioned by plotting the mean of the five stress level values and the mean of the logarithms of the five stress cycles to failure values. No data are shown in Figure 5 for oven-dried

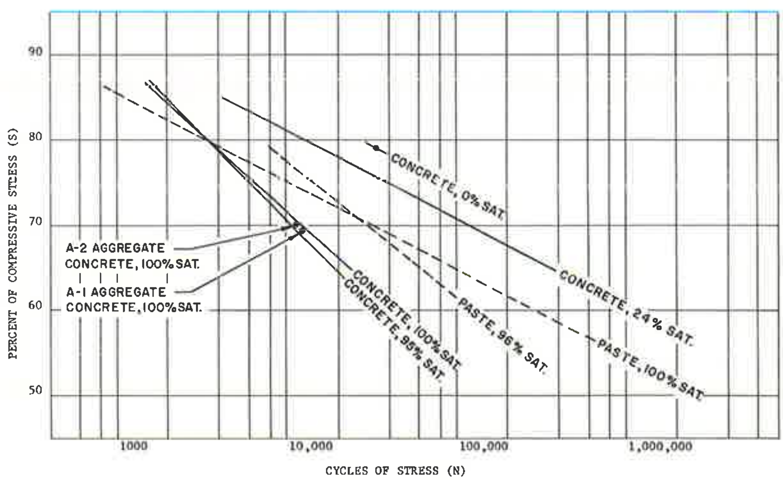

Figure 7. Graphical representation of 0.70 w/c cement paste and concrete fatigue data.

0. 45 w/c natural aggregate concrete because the supply of specimens was depleted while trying to find a stress level that would permit the accumulation of a few thousand stress cycles. There was some indication that a stress level of about 65 percent would have worked.

Results

In Table 5, it is apparent that the addition of aggregate to cement paste influences the static strength of the paste by introducing new weak links. One such weak link can be the bond at the aggregate interface, another can be the strength of the aggregate, and yet another can be the restraining influence of the aggregate when the paste shrinks upon loss of gel water. An explanation of the effect of the aggregate on the static compressive strength of the concrete is not in order for this paper; however, it has been discussed by others (11, 12).

Figures 7 and 8 indicate that the natural aggregate concrete responded to changes in the water-cement ratio of its paste in a manner similar to that of the pure cement paste. That is, the S-N curve for the saturated 0. 70 w/c concrete lies above the curve for the saturated 0. 45 w/c concrete, this being the case for the 0. 70 and 0. 45 w/c pastes (Fig. 3).

The natural aggregate concrete also responded to changes in moisture content in a manner similar to that of the pure cement paste. A small reduction in moisture content from 100 percent of saturation did not materially change the position of the S-N curve for the 0. 70 w/c concrete (Fig. 7). This same behavior was characteristic of the 0. 70 w/c paste. Figure 8 indicates that a small reduction in moisture content from 100 percent of saturation shifted the curves for the 0. 45 w/c concrete to a higher position just as a similar reduction in moisture content shifted the curve for 0. 45 w/c paste upwards.

This behavior of the 0. 70 and 0. 45 w/c natural aggregate concretes suggests that the mechanism hypothesized for cement paste is operative in the concrete, that is, the fatigue behavior of the cement paste is a major factor in the fatigue behavior of concrete.

Further support for this mechanism is seen in the behavior of the natural aggregate concretes as their moisture contents were reduced below 90 percent of saturation. Figure 7 indicates that reducing the moisture content of the 0. 70 w/c concrete from 95 percent of saturation to 24 percent of saturation shifted the S-N curve upward and that a further reduction in moisture content to 0 percent of saturation resulted in an additional upwards shift of the S-N curve. Figure 8 indicates that reducing the moisture content of the 0. 45 w/c concrete from 95 percent of saturation to 47 percent of

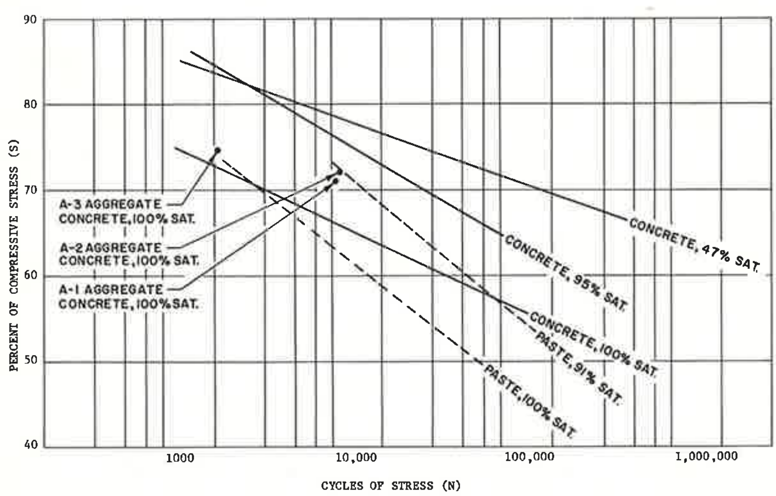

Figure 8. Graphical representation of 0.45 w/c cement paste and concrete fatigue data.

saturation resulted in an upwards shift of the S-N curve. Reducing the moisture content of the 0.45 w/c concrete to 0 percent of saturation did not result in an additional upwards shift of the S-N curve, but rather a pronounced downward shift of undetermined magnitude. This behavior of the two concretes is directly related to the shrinkage forces present in the cement paste. As more and more gel water is lost, the shrinkage forces increase and shrinkage cracks come into existence. The shrinkage cracks that develop are readily propagated by the fatigue action and hence the upwards shift of the S-N curve becomes less and less as the moisture content is continually reduced. In the case of the 0.45 w/c concrete at 0 percent of saturation, the shrinkage cracks are very numerous and little propagation is needed to produce a section which cannot sustain the applied load.

Figures 7 and 8 also indicate that the aggregate, or at least its presence, has an influence on the fatigue behavior of the concrete. Figure 7 indicates that the S-N curves for the saturated and slightly dried 0.70 w/c concrete lie below the curves for the comparable 0.70 w/c pastes. The explanation proposed for this behavior is that the existing bond cracks in the concrete merely have to propagate, while the pure cement paste must first develop a crack system before propagation can proceed. Figure 8 indicates that the S-N curves for the saturated and slightly dried 0.45 w/c concrete lies above the S-N curves for the comparable 0.45 w/c pastes. It is suggested that this behavior, which is the opposite of that noted for the 0.70 w/c paste and concrete, is caused by the paste in the immediate vicinity of the aggregate having a less brittle structure than that of the main body of the paste in the concrete. In other words, the cracks are slower to initiate and thus the reason for the higher position of the S-N curves for the 0.45 w/c concrete. Once a crack pattern is established, crack propagation proceeds in the concrete as it does in the pure cement paste.

The fatigue behavior of the saturated synthetic aggregate concrete (Figs. 7 and 8) does not appear to be consistent with the fatigue behavior pattern established by the saturated cement pastes and the saturated natural aggregate concretes. However, it should be realized that for each specimen tested, a situation existed which involved the degree of bond between paste and aggregate, a different modulus of elasticity for paste and aggregate, and the volume ratio of paste to aggregate. Since the number of cycles to failure is dependent on the stress level, it is suggested that it is necessary to determine the stress level of the critical component by taking into account the load carrying area of the component and its modulus of elasticity. Unfortunately the data obtained in this investigation are not of the type which permits an analysis to be made along the lines suggested. (It is hoped that an explanation of this behavior will be one of the results of an investigation the author is currently conducting.)

Based on the preceding discussion, the following hypothesis is proposed for the mechanism of fatigue in concrete:

> Fatigue failure in plain concrete occurs because small cracks form and propagate in the cement paste under repeated applications of loads less than the static failure load. The resulting crack pattern weakens the section to the point where it cannot maintain the applied load. The development of this damaging crack pattern depends primarily, if not entirely, on the water-cement ratio of the cement paste and the presence of shrinkage stresses in the cement paste.
>
> Concretes of different water-cement ratios develop a crack pattern in their cement paste in the same manner as does pure cement paste, that is, the crack pattern is slower to develop in the cement paste that has an open capillary structure than in the cement paste that has a dense structure and, in addition, the crack pattern is slower to develop in a cement paste that has undergone a loss of gel water.

SUMMARY AND CONCLUSIONS

Analysis of the results of some 150 specimens tested in fatigue indicate that the fatigue behavior of cement paste is sensitive to changes in the water-cement ratio of the paste and to changes in the moisture content of the paste. It was also found that the fatigue characteristics of plain concrete are apparently governed primarily by the fatigue characteristics of the cement paste. The aggregate was found to have some influence on the fatigue behavior of the concrete and the explanation offered is that there are bond cracks present at the aggregate interface and that the paste surrounding the aggregate is not necessarily the same as the main body of the paste.

The fatigue behavior of the synthetic aggregate concrete suggests that several factors influence the number of cycles to failure. Apparently either the paste or the aggregate becomes the critical component in accordance with its modulus of elasticity and the volume of the component present in the concrete.

The proposed mechanism for plain concrete is different from the fatigue mechanism proposed by Murdock and Kesler (3). They hypothesized that crack propagation is dependent upon the deterioration of the paste-fine aggregate bond whereas the present proposal hypothesizes that crack propagation is through the cement paste. The work reported in this paper and the work conducted at the University of Illinois have indicated that the aggregate has a role in the fatigue characteristics of plain concrete but as yet, the extent of its role is not clear.

A previous paper (8) revealed that entrained air does not affect the fatigue behavior of concrete, and the results of this study give no cause for disputing that conclusion. This earlier study is not directly comparable with the present study because of slightly different water-cement ratios and different proportions of paste and aggregate. The concretes, however, were oven-dried, and when their S-N curves are compared with the curves of this study, one finds that their locations are in keeping with the findings of this study.

ACKNOWLEDGMENTS

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The Role of Mortar-Aggregate Bond in The Strength of Concrete

CHARLES F. SCHOLER, Assistant Professor of Civil Engineering, Joint Highway Research Project, Purdue University

Mortar-aggregate bond in concrete is defined and factors which influence the bond are discussed. The discussion is built around original work by the author and work published by others.

The role which mortar-aggregate bond appears to play in the mechanism of concrete failure is discussed. Particular emphasis is given to the relationship of microcracks and mortar-aggregate bond.

The necessity of measuring bond with saturated specimens, to prevent drying shrinkage stresses, produces results which may not be representative of the bonds in concrete. Consideration of what influences bonds in normal concrete is given.

The effect of different measured mortar-aggregate bond strength on the strength of concrete is discussed with data provided.

•THE concept of bond has been widely used to explain many observed properties of concrete (1, 2, 3). Its use has been at least partially based on the intuitive concept that in a heterogeneous material the adherence of the various components to each other is important to the behavior of the material. No doubt intuition is correct, but it is deficient, at least for most people, in being quantitative and enlightening with regard to the mechanism involved. Before discussing recent findings that help to make up for the deficiencies of intuition, one must first carefully define the mortar-aggregate bond.

For this discussion bond may be thought of as the force required to separate two solid components in concrete at their interface. The components may be the paste and aggregate, or mortar and coarse aggregate. It so happens that either may be used without great changes in the interpretation of the results. Bond can be thought of as the effective adhesive force holding two surfaces together. It is derived from two sources, one being chemical and the other mechanical.

Chemical bonds appear to develop with all aggregates since it is difficult to find a material to which cement paste does not, to some extent, adhere. These are aggregates commonly described as being inert. Relative to cement they seem to be inert, but in a strict sense a chemical bond appears to develop. It is difficult to isolate the chemical bonds due to the coexistence of mechanical bonds.

Mechanical bonds may be thought of as those forces required to fracture paste interlocked in irregularities of aggregate and of shear forces which are developed along an irregular interface.

No practical means has been found to measure one type of bonding while excluding the other. Results of investigations of bonds reported in the literature are in terms of combined effects of both chemical and mechanical bond. This will, henceforth, be true of the term bond in this paper.

Measurement of bond has been made by a variety of methods each designed to measure the strength of an interface between the mortar (or paste) and the coarse

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aggregate. These methods include use of a "mortar briquet" specimen (4, 5), cast prisms (1, 6) and the author's method of cores (7).

The author's method used a slab of rock, with one of its surfaces given a desired finish, on which was placed a plastic cement mortar. After a period of curing, this mortar-covered slab was cored with a diamond core drill and numerous individual specimens, approximately 0.6 in. in diameter, were obtained. This method enables a large number of specimens to be obtained with a reasonable effort. These specimens were tested in a cantilever loading with the aggregate end gripped in a vice arrangement and the mortar portion of the core loaded by a rolling frame. The distance from the mortar-aggregate interface to the point of loading was constant, 0.523 ± 0.001 in. The bond values for the investigation were used relative to each other. Therefore, the values reported are loads in pounds and not convereted to modulus of rupture. For quantitative comparison to other investigations these bond values must be converted to modulus of rupture. In each of the above methods failure occurs along the mortar-aggregate interface of the specimen whether the test be one of tension, shear, flexure or torsion. The results are generally comparable for similar types of loading regardless of the form of the specimen. Preference as to the type of specimen is therefore dictated by economy and versatility.

An important and essential requirement in making such bond tests is never to allow the specimen to dry. General practice is to keep specimens in a saturated condition, otherwise great reductions in bond strength are noted. This reduction, 50 percent and more, is attributed to drying shrinkage and to surface conditions at the time of test. To the author's knowledge, no quantitative method has been developed to measure the effect of the surface and other bond factors in a less than saturated condition.

The bond test results obtained by the author (7) and by others (1) frequently have a coefficient of variation as high as 20 percent. This makes it essential that a sufficient number of tests be made in order to have reliable results. For example, with a difference in sample mean from population means of 5 lb considered significant at a 0.05-α level, with a calculated mean of 50 lb, the "t" distribution shows that a sample size of 18 is needed. The high coefficient of variation appears to be inherent in the nature of the mortar-aggregate bond.

FACTORS AFFECTING BOND

Most factors that affect strength in a hardened cement paste or mortar affect bond strength in a similar way. These factors include water-cement ratio, cement, age, and cement factor. Alexander and Taplin have reported that bond strength at early ages does not increase as rapidly as the strength of cement paste under elevated curing temperatures. At later ages, however, when pastes cured at normal temperatures approach the strengths of those cured at elevated temperatures, bond once again has increased comparable to the paste strengths (8).

The aggregate has considerable effect on the mortar-aggregate bond with a significant difference being discernible among a great many different aggregates. The author investigated a wide variety of carbonate rocks used for concrete aggregate in Indiana, as well as several other types of aggregate (Tables 1 and 2). The results from tests on the carbonate rocks are all relatively close together. However, an analysis of variance showed that differences in carbonate rocks were statistically significant. It should be noted in Figure 1 that the highest bond strengths were obtained with quartzite. This agrees with the recently published work by Alexander (9) which suggests that silica content of the aggregate is an important factor. The general reputation of a quartzitic material in concrete, especially quartz gravels, is that it does not have especially high bond strength. This may not be caused by a low bond development but instead caused by quartz being more susceptible to having the bond destroyed by shrinkage stresses.

The effect of different surface conditions for the aggregate was also investigated by the author. Aggregate surfaces, smoothly ground with No. 1200 abrasive grit or roughly ground with No. 60 abrasive grit, did not give consistent results. Many carbonate aggregates tested did not give significantly different results with the two surface

TABLE 1

PETROGRAPHIC AND CHEMICAL PROPERTIES OF AGGREGATES

Aggregate No.	Aggregate Name	Petrographic Description: Average Grain Size (mm)	Petrographic Description: Lithologic Properties	Chemical Analysis[a]: Percent by Weight Dolomite	Chemical Analysis[a]: Percent by Weight Calcite	Chemical Analysis[a]: Insoluble Residue
2	Jeffersonville (dolomite)	0.005	Carbonate, very fine grained, crystalline	98	1	1
6	Liston Creek (dolomite)	0.02	Carbonate, very porous, rombic	97		3
7	Baraboo (quartzite)	0.16	Quartz, interlocking structure			
9	St. Genevieve (limestone)	0.01	Carbonate, fossiliferous detrital quartz, opaque metallics	10	88	2
10	St. Genevieve (limestone)	0.08	Carbonate, fossiliferous, opaque metallics	2	92	5
11	Lincoln "quartzite" (sandstone)	0.17	Angular quartz grains in a carbonate matrix			
12	Diorite					
13	Harrodsburg (limestone)	0.40	Carbonate, fossiliferous, detrital quartz and opaque metallics	6	95	1
14	Geneva (dolomite)	0.04	Carbonate, very porous, rombic	99		1
15	Louisville (dolomitic limestone)	0.03	Carbonate, some large fossils, rombic crystals, opaque metallics	60	37	3

[a]Calculated from laboratory determinations of calcium, magnesium and insoluble residue contents.

TABLE 2

PHYSICAL PROPERTIES OF AGGREGATES[a]

Aggregate No.	Aggregate Name	E x 10^{6}*	Bulk Spec. Gravity	Absorption (%)	Particle Shape: L/W**	Particle Shape: W/T**
2	Jeffersonville (dolomite)		2.67	1.29	1.33	1.81
6	Liston Creek (dolomite)		2.58	1.78	1.41	1.52
7	Baraboo (quartzite)	13.8		0.20	1.42	1.43
9	St. Genevieve (limestone)	11.0	2.69	0.42	1.39	1.74
10	St. Genevieve (limestone)	12.1	2.68	0.46	1.34	1.53
11	Lincoln "quartzite" (sandstone)					
12	Diorite					
13	Harrodsburg (limestone)	10.1	2.65	0.66	1.36	1.69
14	Geneva (dolomite)	10.4	2.43	3.43	1.37	1.55
15	Louisville (dolomitic limestone)	15.1	2.76	0.62	1.38	1.59
16	Gravel (quartzite)		2.62	0.22	1.30	4.42

[a]Rounded and washed, passing 1 in., retained 3/4 in.

*Modulus of elasticity, from stress-strain relationships determined with a Tuckerman optical strain gage. The average strain was used for the reported values.

**The mean greatest dimension of pieces of aggregate.

W = the mean intermediate dimension of pieces of aggregate.

T = the mean smallest dimension of pieces of aggregate.

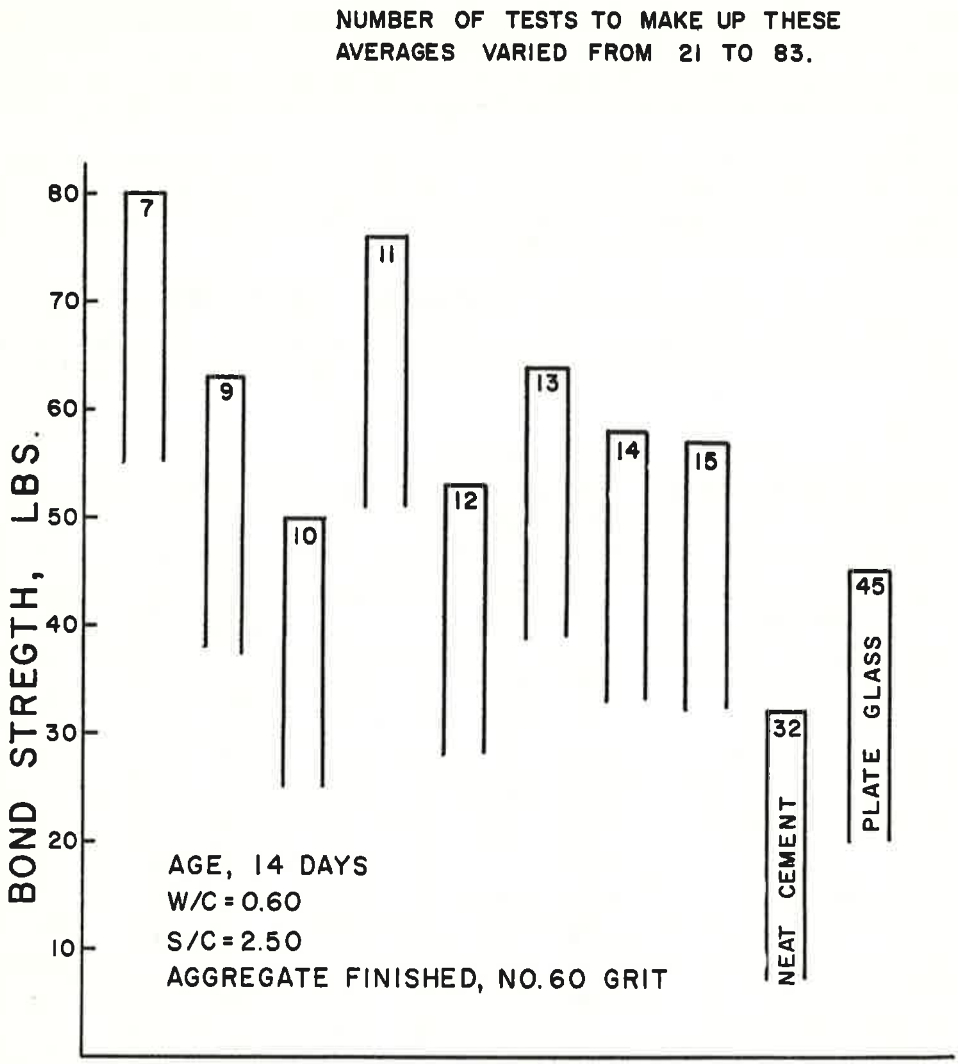

Figure 1. Mortar-aggregate bond variation with aggregate.

conditions. Several, however, were influenced by the surface in opposite ways. No statement as to the effect of surface finish on the mortar-aggregate bond of carbonate rocks can be made. Results on several rocks other than carbonates, notably the sandstones, indicated that surface finish was of greater importance to them. Other investigators have found a greater significance in the surface conditions under direct tensile tests (5).

It is the author's belief that these bond tests belie the true effect of surface in a practical condition of concrete because they cannot allow drying to take place in or on the test specimen. Aggregate surface may have considerable influence on the bond between mortar and aggregate in a condition where drying occurs. The rougher surfaces may well provide strong mechanical restraints upon shrinkage and its buildup of stresses at the mortar-aggregate interface.

The size of the mineral grains in the rock composing the aggregate was considered by the author in his investigations. Although a fine grained aggregate might be expected to have better bond due to the greater number of edges involved, the grain size of carbonate aggregates did not influence the bond strength. This is based upon an investigation of three limestones of similar lithologic and chemical composition but with an average grain size of 0.01, 0.08 and 0.40 mm. Although significant difference in mortar-bond strength did exist, the low strength was with the intermediate grain size aggregate.

Insofar as the carbonate aggregates are concerned, no single property is easily relatable to mortar-aggregate bond strength. Evidence indicates that bond strength is influenced by the amounts of silica in the aggregate with a stronger chemical bond developing with the more siliceous aggregates.

EFFECT OF BOND ON THE MECHANISM OF COMPRESSIVE FAILURE

The results of the compression tests on 3 by 6-in. cylinders (Table 3) have been plotted versus the average mortar-aggregate bond strength of the coarse aggregate in

TABLE 3

RESULTS OF COMPRESSION TESTS WITH ROUNDED, ONE-SIZE, COARSE AGGREGATE

(3 x 6-Inch Cylinders, Age: 14 days)

Aggregate		L/W	W/T	Cyl. No.	S_c (psi)	Inflection		S_c (psi)	S_I (psi)
No.	Description					Stress S_I (psi)	Ratio (%)		
2	Jeffersonville (dolomite)	1.330	1.811	128	3,126	1,272	40.7		
		1.330	1.811	129	3,281	1,697	51.7	3250	1410
		1.330	1.811	130	3,338	1,272	38.1		
6	Liston Creek (dolomite)	1.409	1.518	115	3,055	2,744	78.7		
		1.409	1.518	116	3,253	1,666	51.2	3230	1940
		1.409	1.518	118	3,380	1,413	41.8		
9	St. Genevieve (limestone)	1.386	1.735	114	3,649	1,900	73.6		
		1.386	1.735	117	2,730	1,305	47.8	3140	1520
		1.386	1.735	119	3,027	1,344	44.4		
10	St. Genevieve (limestone)	1.337	1.533	134	3,479	1,131	32.5		
		1.337	1.533	135	3,352	1,415	42.2	3420	1460
		1.337	1.533	136	3,423	1,838	53.7		
13	Harrodsburg (limestone)	1.558	1.690	126	3,536	2,263	64.0	3250	2260
		1.558	1.690	127	2,956	2,261	76.5		
14	Geneva (dolomite)	1.372	1.553	131	2,999	1,697	56.6		
		1.372	1.553	132	3,395	2,403	70.8	3150	1930
		1.372	1.553	133	3,677	1,699	46.2		
15	Louisville (dolomite-limestone)	1.385	1.590	120	3,182	1,699	53.4		
		1.385	1.590	121	3,005	2,353	78.3	3150	1870
		1.385	1.590	122	3,253	1,555	47.8		
	Marbles (smooth)	1.00	1.00	104	1,641			1770	1340
		1.00	1.00	105	1,711	1,414	82.6		
		1.00	1.00	106	1,966	1,273	64.7		
	Marbles (rough No. 60)	1.00	1.00	107	2,546	1,980	78.0	2550	1760
		1.00	1.00	108	2,829				
		1.00	1.00	109	2,829	1,545	56.0		
		1.00	1.00	137	1,981				
	Mortar			110	4,653	1,697	36.5	4310	2810
				111	4,993	2,150	43.1		
				112	—*				
				113	—*				
				123	3,791	1,781	47.0		
				124	3,890	1,186	30.5		
				125	4,215	—**			
	Plexiglas			138	1,584			1420	
				139	1,259				

*Discarded due to defective molds.
**Impulse counting apparatus failed.

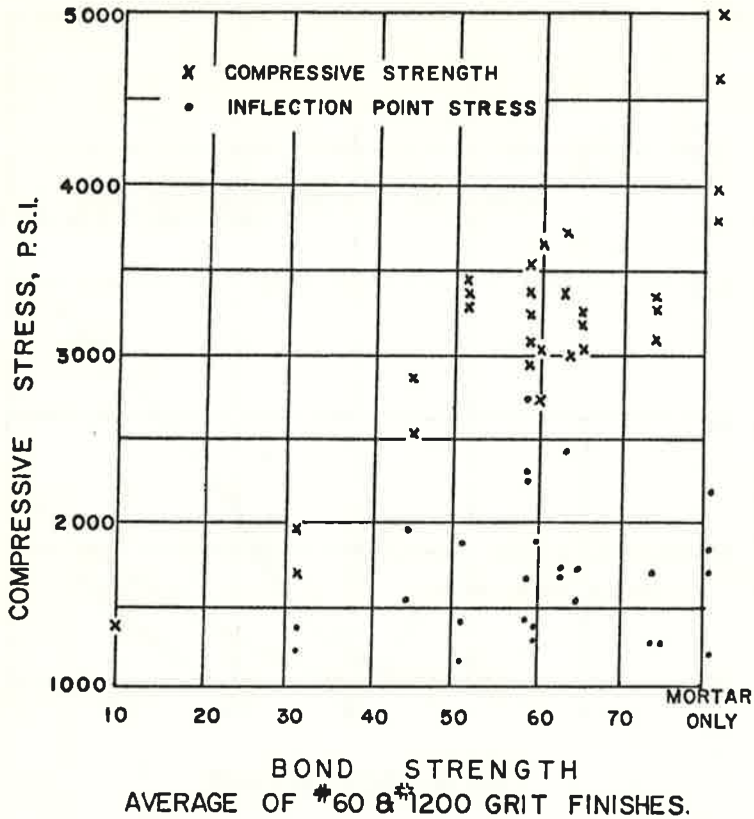

Figure 2. Compressive stresses vs bond strength at failure and inflection points.

Figure 2. Both glass marbles and pieces of plexiglas rod were used for aggregate to provide a range in bond strengths.

The values of bond strength for both plexiglas and mortar were not determined by bond tests for such tests could not be performed using these materials. The plexiglas formed too weak a bond to permit measurement although some bond did exist hence a value of 10 lb was selected as an estimate.

Due to difficulties in obtaining a perfectly true core when coring mortar, a problem not experienced in mineral aggregates, cores of mortar with no coarse aggregate did not perform satisfactorily in the test apparatus. The term bond strength is used for mortar although it is not a bond developed at an interface between two separate mortars. The bond strength for mortar without aggregate was selected in a manner similar to that used for the plexiglas. It was known that the mortar had a greater bond strength than most of the bonds. Some mortar failures occurred with aggregates having the higher bond strengths, hence it was assumed that the mortar's bond strength was not a great deal more, probably 80 lb or slightly higher.

The plot of the compressive test results vs the mortar-aggregate bond strength of the coarse aggregate shows that the influence of bond strength of the coarse aggregate on compressive strength is similar to its influence on flexural strength, which is discussed later in this paper. The carbonate aggregates produced concrete whose mean compressive strengths varied between 3,150 and 3,420 psi. These concretes are closely grouped compared to the mean compressive strength obtained for concretes made with marbles, 2,550 psi for roughened and 1,773 psi for the smooth marbles. The influence of bond is apparent, over the range of values included. The mortar cylinders, without coarse aggregate, are included in Figure 2 with the estimated bond strength for the

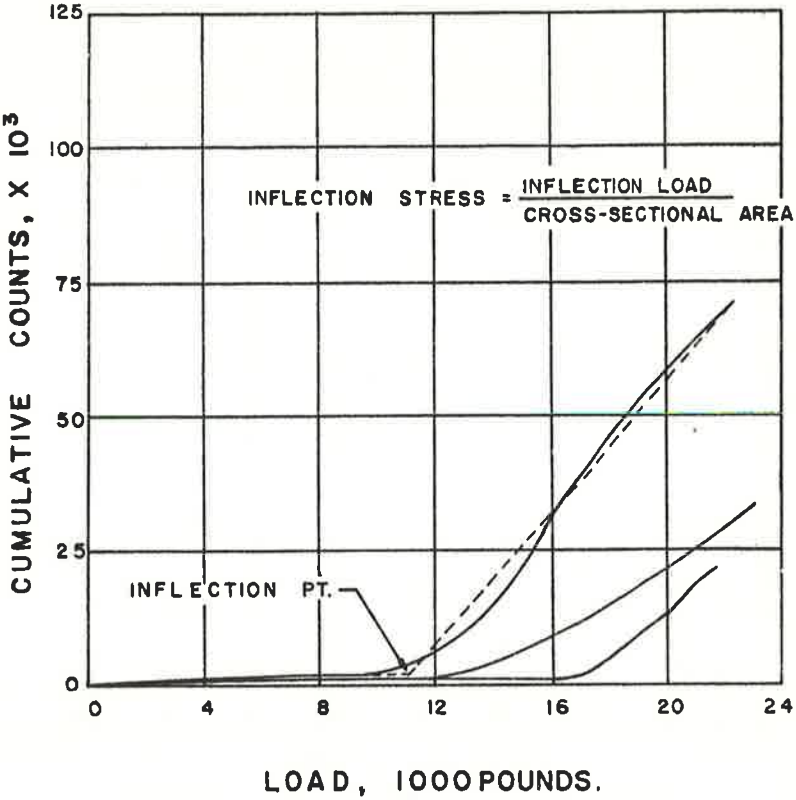

Figure 3. Cumulative impulse counts vs load in the compression of concrete cylinders.

mortar. The compressive strength was greater for concrete produced with a coarse aggregate having a greater mortar-aggregate bond strength if other factors remain unchanged.

MICROCRACKS RELATED TO BOND

The occurrence of microcracks in concrete is believed to occur most frequently at an interface between coarse aggregate and the mortar in a concrete (5). The location of these cracks, whether they exist before loading or form after load is applied, illustrates that the mortar-aggregate bond has broken at a load well below the ultimate load which will be sustained by the concrete. If intuition may be followed, it may be presumed that a stronger mortar-aggregate bond would delay the occurrence of these cracks, hence the concrete would be capable of sustaining heavier loads.

Investigations, by the author, of concretes made with rounded, one-size aggregates to reduce the effect of size, gradation, particle shape and particle texture indicate that low aggregate bond strengths do not necessarily mean a low stress level for the initiation of microcracks under compression loading.

Figure 3 illustrates typical results obtained using a vibration pickup to detect the occurrence of cracking impulses within a concrete specimen during a compression test. An electronic counter analyzer recorded the number of detectable impulses received. An oscilloscope was used in conjunction to obtain visibly a relative measure of the impulse picked up. Using this equipment, the initiation of cracking and its relative rate could be determined during a compression test of a specimen.

The plot of the inflection stresses obtained from the cracking impulse count data did not appear to be greatly influenced by the bond strength between the coarse aggregate and the mortar. The inflection ratio or inflection stress/ultimate stress was generally greater for concretes whose coarse aggregate had the lower mortar-aggregate bond strength.

Data were not sufficient to be conclusive, but did suggest the following hypothesis for concrete failure.

> When the mortar-aggregate bond strength characteristic is the only variable among aggregates, i.e., the aggregates have identical size, shapes, physical properties, etc., concrete made with these coarse aggregates and a mortar will form microcracks at a compressive stress that is primarily dependent upon the properties of the mortar. The amount of microcracking, as reflected by the increase in number of microcracks to reach incipient failure, is dependent upon the mortar-aggregate bond of the coarse aggregate.

The microcracks (separations) occurring in the concrete, as reflected by the impulse counts, take place at approximately the same rate (Fig. 3), once they commence. These microcracks occur in order to relieve stresses developing within the concrete. The initiation of cracking is dependent upon the formation of sufficient stress concentrations to cause microcracking or separations. If an increment of stress will be relieved by one crack, regardless of the mortar-aggregate bond involved, it becomes apparent that the area of separation required to relieve the increment of stress will be greater for concrete having the low mortar-aggregate bond. As the load on the concrete continues to be increased, the number of stress increments requiring relief will remain the same regardless of the mortar-aggregate bond. The total cumulative area of separation, however, will be greater for the concrete with the lower bond strength thus the condition in which the overall strength of the specimen is insufficient to withstand the load is reached at lower applied load for the concrete with the lower bond strength.

This hypothesis cannot be applied to the work of Jones and Kaplan (3) inasmuch as their aggregates were intentionally different in variables other than the bonding characteristics. One of their observations is nevertheless worthy of note and appears to be in agreement with the above hypothesis: "The relation between the flexural strength of concrete and the stress at which cracks first appear in compression was independent of the type of coarse aggregate." This suggests that other aggregate variables such as shape and physical properties may not greatly influence the initiation of microcracking.

Regardless of the mechanism by which mortar-aggregate bond contributes to strength, it has been shown that considerable variations in the bond strength will affect the compressive strength of concrete. The influence, however, is greatly overshadowed in concrete by the effect of the paste. Alexander related the compressive strength of concrete to paste strength and paste-aggregate bond strength. The compressive strength was a function of the expression, B+2P, where B equals the modulus of rupture of the cement aggregate bond and P equals the modulus of rupture of paste. Using the lowest bond strength, 700 psi, reported by Alexander (10) and his highest, 1,500 psi, with a constant paste strength would result in computed compressive strengths of 5,200 psi and 6,000 psi.

The author's investigation showed no appreciable difference in compressive strength for concrete made with the variety of carbonate aggregates. However, for special aggregates with lower bond strength a compressive strength change was noticeable. Relatively speaking, these results agree with those reported by Alexander, although the author emphasizes that what may be a statistically significant difference in aggregate bond strength will be of consequence in practical concrete only if the bond difference is very large.

The effect of bond on flexural strength is shown in Figure 4. The distribution of points indicated that the bond strength was indicative of the flexural strength. Some inconsistencies occurred but they did not disturb the overall trend.

Insofar as feasible, bond strength was the only variable, however, many other factors pertaining to the coarse aggregate may have influenced the strength of the concretes. If, for a given type of material, differences in bond strength of that material to a mortar

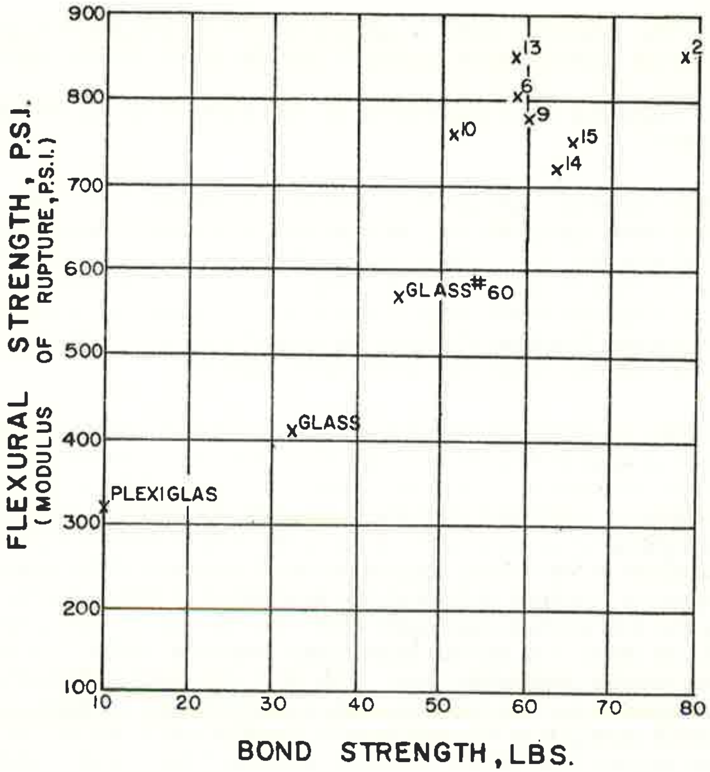

Figure 4. Bond strength vs flexural strength.

of a given composition is primarily dependent on surface finish, one would expect such differences to be reflected in the strength of concretes if bond affects strength. The glass marble aggregates were used for two different concretes with the only variable being the surface finish. It was gratifying to see that the relative positions for these concretes (Fig. 4) agreed well with the other results. Other factors being the same, the coarse aggregate with the greatest mortar-aggregate bond strength will result in the greater flexure and compressive strength for concrete.

If the bond strength values obtained in this investigation are converted to the modulus of rupture, the results of tests on concrete may be compared with those obtained by Alexander et al (10). The results agree reasonably well even though the type of bond test specimen was not the same.

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