Embankment Analysis and Field Correlation

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•ANALYSIS of earth embankments has been a problem of long standing. Considerable interest in this area arises naturally from its practical applications in the construction of highway fills and earth dams.

Earlier approaches to this problem area have necessarily focused attention on slope stability. The development of techniques to determine possible states of plastic equilibrium permits estimating the ultimate performance and safety of the earth structure. Unitl recently, however, little effort has been devoted to evaluation of stress and deformation states in embankments during construction and prior to failure.

The development of computer techniques makes possible the solution of geometrically complicated boundary-value problems. Articles appearing in recent works (1, 2) survey the field of numerical methods of structural and stress analyses. These methods include the techniques of finite differences and finite elements. The finite element method (FEM) has recently attracted the attention of many structural analysts. This is primarily due to its inherent versatility and to the ease with which the technique can be applied to problems heretofore considered intractable. Extension of the FEM to include the effects of nonlinear materials has been made as well (3, 4). This clearly indicates the possibility of a quantitatively meaningful stress and deformation analysis of earth embankments.

Such analyses of earth embankments have been attempted by Goodman and Brown (5) and by Clough and Woodward (3). These analyses model the embankment as a continuum constructed by a "placement in layers" process. In addition, Clough and Woodward include the possibility of nonlinear material behavior where the nonlinearity is taken as a piece-wise linear function. While these analyses give results that are at least qualitatively correct, it still remains to determine the accuracy of the continuum material model.

In a recent construction program, the Materials and Research Department of the California Division of Highways placed and instrumented some high earth embankments. It was a program intended to develop accurate and reliable instrumentations for stress and deformation measurements and to acquire technical experience in placing these gages. Assuming the integrity of these field measurements (within reasonable experimental scatter), these experimental results afford an excellent opportunity to assess the validity of the mathematical model.

It is the purpose of this paper to perform the necessary numerical analyses for a particular embankment and compare the results with the field measurements. This comparison then permits an assessment of the accuracy of the mathematical model and prediction technique.

MATHEMATICAL MODEL AND THE NUMERICAL TECHNIQUE

Due to the complexities attendant to a stress and deformation analysis of a highway fill resting on natural terrain, a number of simplifying assumptions must be introduced to render the problem tractable. One important assumption is that the actual three-dimensional system can be represented as a two-dimensional plane strain problem. Thus, the cross section of the embankment and flexible foundation is considered to be a typical section taken from the long roadway and fill. Complete continuity is assumed

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to exist between the fill and the foundation. Another assumption is that both embankment and foundation materials are linearly elastic, but each may possess different mechanical properties. The effect of incremental construction of the embankment is necessarily accounted for, as all embankments are built by an incremental process where the fill is gradually placed. At least one previous analysis (8) was made under the hypothesis that stresses and deformations in the embankment were obtained by direct application of the gravitational body forces on the complete structure (single-lift analysis). Obviously, this is generally not valid, inasmuch as actual construction of the fill does not follow this procedure. It has been shown (3) that the values of the displacements from an incremental analysis differ appreciably from a single-lift analysis. Recapitulating the description of the mathematical model, the present analysis is based on a plane strain system with linearly elastic isotropic materials that is constructed and loaded incrementally.

The finite element method is employed to obtain stresses and displacements in this typical cross section. In this method, the continuous system is idealized as an assemblage of smaller, but finite, elements connected at a discrete number of points called nodes. The basis for the method is a consistently derived stiffness for each of these elements, the stiffness being a relationship between the generalized forces and the displacements at the nodes of each element. Stiffness of the elements used in the present analysis is predicated on a material possessing linear elastic properties. When the stiffnesses of all elements have been defined, the analysis of the stresses and deformations in the element assemblage resulting from any given loading condition is a standard structural analysis problem, which requires the simultaneous satisfaction of (a) equilibrium—the generalized force at each nodal point must equilibrate the externally applied nodal force—and (b) compatibility—deformations must be continuous from one element to another. Further information concerning this method will be found in the works mentioned previously (1, 2). It should be noted that the aforementioned description of FEM is one way of regarding the method. The same technique can be thought of as a minimization problem (Rayleigh-Ritz) in the calculus of variations, where the approximate coordinate functions used to represent the displacement patterns are automatically generated from the values of the nodes (6).

In FEM, the incremental construction as well as the complicated geometrical configuration of the cross section can be simulated without any additional difficulties. The present analysis is predicated on a model where 9 layers, each approximately 25 ft in height, were used. Although the 25-ft height of each layer appears large compared to the actual construction procedure where normally less than one foot of material is placed at a time, it has been shown (3) that this size of layer in comparison to the total depth of fill is adequate for representing the incremental construction. To further substantiate that this incremental lift size is reasonable in the present analysis, a typical standard embankment was analyzed (Fig. 1). Various cases were calculated with 4, 8, 16, and 32 lifts. The horizontal and vertical displacements and the vertical stress at a particular point (see Fig. 1 for its location) are plotted in Figure 2. It is seen that the results converge sufficiently with the use of 8 lifts. Beyond 8 lifts, the results were not altered appreciably, thus indicating a diminishing return for the additional computational labor. The relative size of lift to total height in the actual embankment is in the neighborhood of the 8-lift analysis in this convergence study. Consequently, the convergence of the analytical results with respect to lift size for the present model is supported, thus permitting this mathematical simulation of the incremental construction process.

The underlying assumptions of plane strain and time-independent linearly elastic mechanical properties were made only to facilitate the analysis. It should be noted that a FEM numerical scheme exists for three-dimensional solids (7) but, at the present stage of development, even to perform a relatively simple analysis would overtax the capacity of the computer or require an excessive amount of time, thus obviating this as a feasible approach to the problem. Further advances in computer technology and programming techniques will remove these difficulties. Nonlinear material properties may be considered by performing an analysis wherein the stress-strain law is taken as piece-wise linear and employing a constitutive law in which the physical

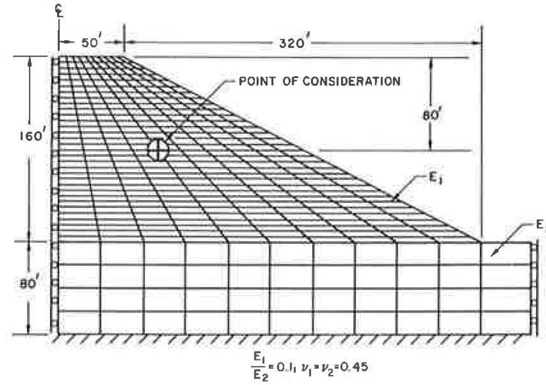


Figure 1. Standard embankment.

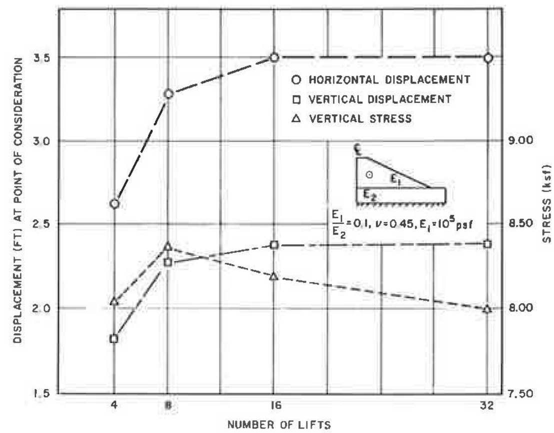
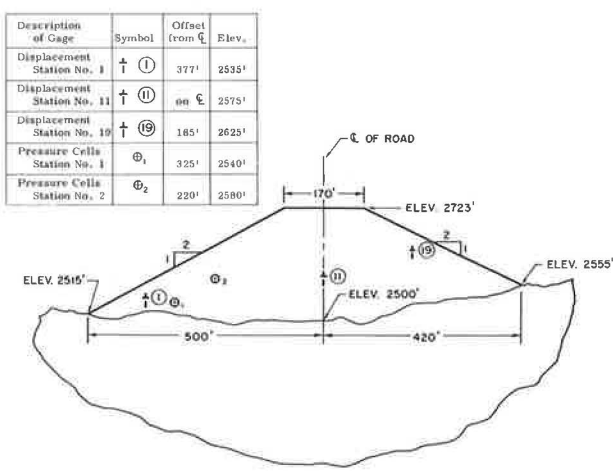


Figure 2. Convergence with respect to number of lifts.



Description of Gage	Symbol	Offset from ℄	Elev.
Displacement Station No. 1	† ①	377'	2535'
Displacement Station No. 11	† ⑪	on ℄	2575'
Displacement Station No. 19	† ⑲	185'	2625'
Pressure Cells Station No. 1	$\oplus_1$	325'	2540'
Pressure Cells Station No. 2	$\oplus_2$	220'	2580'

Figure 3a. Cross section of embankment at Osito Canyon.

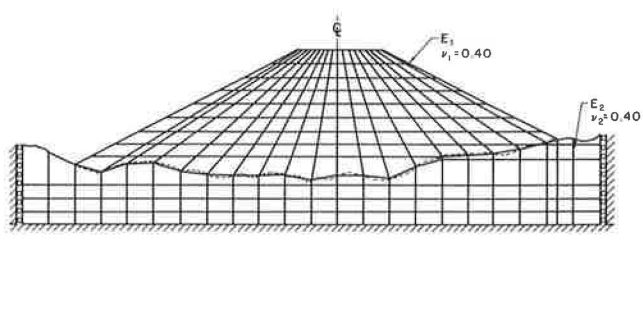


Figure 3b. Finite element idealization of embankment at Osito Canyon.

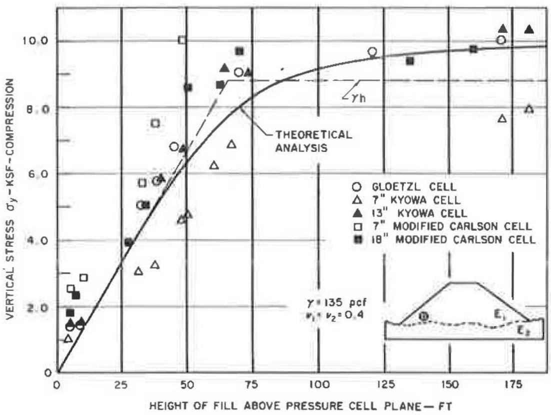


Figure 4. Comparison of analytical and field results at pressure cells No. 1.

nonlinearities are analytically described (3, 4). Inasmuch as the material properties in the present case have not been precisely ascertained, incorporation of material nonlinearity did not seem to be warranted. This does not imply that such effects are unimportant. Clearly the term which soil mechanicians call modulus of elasticity is dependent upon the confining pressure.

RESULTS AND COMPARISON

The particular embankment under consideration is located in Osito Canyon on the new section of US 99, approximately 10 miles north of Castaic, California. Its general vicinity may be described as being in the mountains immediately north of Los Angeles. The cross-sectional configuration together with location of the instrumentation are shown in Figure 3a. Instrumentation shown in this figure was a random selection of the total instrumentation in the field study. A finite element idealization is shown in Figure 3b.

The incremental construction process was simulated with 9 lifts, each approximately 25 ft in thickness. FEM analyses for various ratios of E_1/E_2 (fill to foundation moduli) were performed and the results are given in Figures 4 through 11. In Figures 4 and 5 are shown the vertical stress at 2 points as a function of height of fill above the pressure cells. The horizontal and vertical displacements at 3 stations are plotted in Figures 6 through 11 as a function of height of fill above the instrumentation. The ratios E_1/E_2 serve as the parametric variable for the family of curves in these figures.

As indicated earlier, the Materials and Research Department (MRD) initiated this program to develop reliable measurement devices. Different types of pressure cells were used, viz., Gloetzl, Kyowa 7 in., Kyowa 13 in., Modified Carlson 7 in., and Modified Carlson 18 in. Displacement measuring devices consisted of settlement platforms and horizontal movement indicators. The details of the instrumentation were not of immediate interest to the authors, who were more concerned with the assessment of the mathematical model. Therefore, readers interested in the instrumentation should contact the MRD.

From the geological exploration, the MRD indicated that the modulus of elasticity E_1 was of the order of 600 ksf with Poisson's ratio approximately 0.40. It was also suggested that the foundation material was such that the appropriate value of E_1/E_2 is 0.1. Therefore, in comparing field data, attention should be directed to that curve with $E_1/E_2 = 0.1$.

The field data for the vertical stress shown in Figures 4 and 5 fall generally along the analytical results. The curve for the theoretical results was insensitive to the ratio of E_1/E_2, indicating a relative independence of the vertical stress upon this moduli ratio. The stresses for $E_1/E_2 = 0.1$ were approximately the same

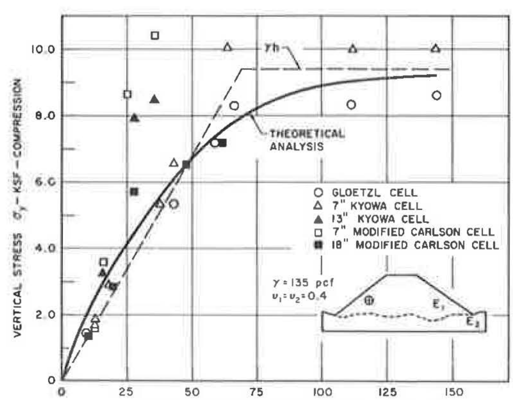


Figure 5. Comparison of analytical and field results at pressure cells No. 2.

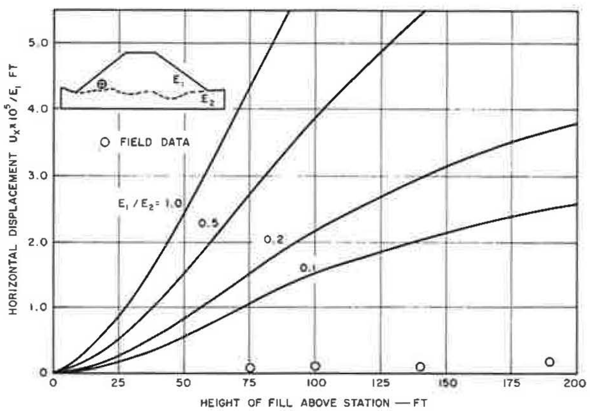


Figure 6. Analytical and field displacements at station No. 1.

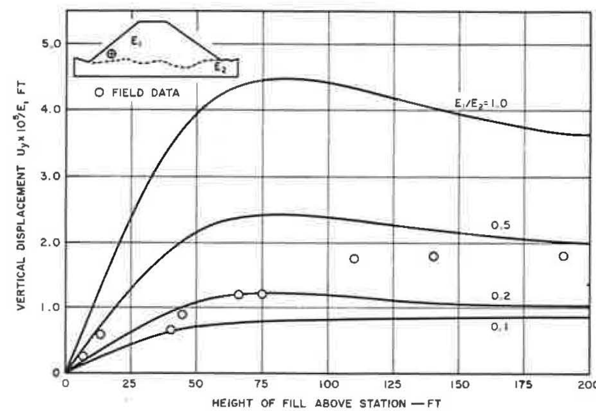


Figure 7. Analytical and field displacements at station No. 1.

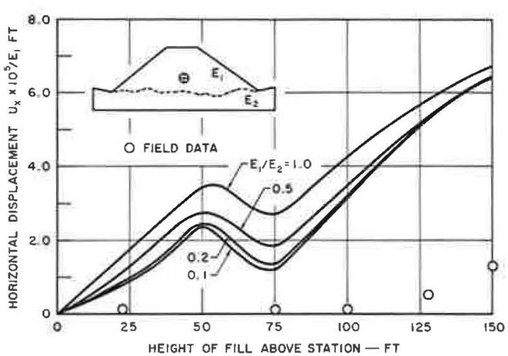


Figure 8. Analytical and field displacements at station No. 11.

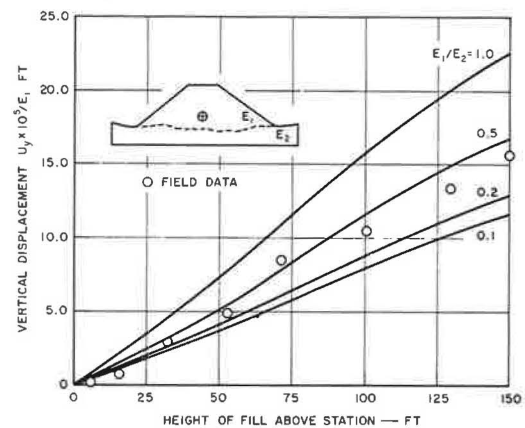


Figure 9. Analytical and field displacements at station No. 11.

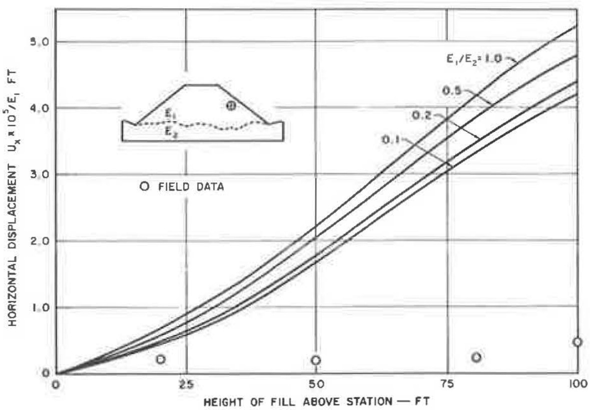


Figure 10. Analytical and field displacements at station No. 19.

as those for $E_1/E_2 = 1.0$. This appears logical because the vertical stress should primarily be a function of the specific weight γ of the fill material and height of fill. A dotted line representing γh, which is equal to the product of the specific weight and actual height of fill directly above the pressure cell, was also drawn. This line represents the approximate engineering estimate of the vertical stress. Both field data as well as the two analytical estimates are within reasonable proximity of each other.

The horizontal and vertical displacements are shown in Figures 5 through 11 for various ratios of E_1/E_2. The ratio of interest for comparison is approximately $E_1/E_2 = 0.1$. Field data for the vertical displacements fell within a reasonable range of the theoretical results. Field data for the horizontal displacements were consistently low when compared with theoretical results.

CONCLUSIONS

From an examination of the analytical results and comparison with the field data it is quite clear that the mathematical model can at least lead to qualitative predictions. Figures 4 and 5 indicate that the capability to predict the vertical stresses by this approach is at least as accurate as the current methods for measuring the field results. In addition, Figures 7, 9, and 11 show that with proper values of the foundation and embankment moduli, satisfactory estimates for the vertical displacements are obtained. Better correlation might be attained by accounting for material stiffening with increased confining pressure.

On the other hand, Figures 6, 8, and 10 clearly indicate that the mathematical estimates for lateral deflections are in error by an order of magnitude. (In a recent communication, the MRD has indicated that the validity of the field measurements for the lateral deflections is doubted.) From this it might be expected that theoretical predictions of the lateral stresses are in error as well. Unfortunately MRD did not make any measurements of the horizontal stresses in the embankments so this point cannot be resolved.

The weakest feature of the mathematical model is the material characterization. Replacement of the linear elastic stress-strain relation by a more accurate nonlinear constitutive law should lead to significant improvements. Whether this will overcome the apparent order of magnitude error in the horizontal displacements remains to be seen. However, this improvement can only be made

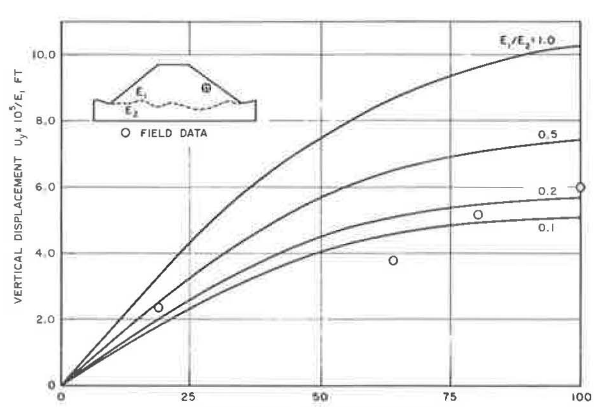


Figure 11. Analytical and field displacements at station No. 19.

after suitable testing of the actual field material. In situ properties of the fill and foundation may be estimated by sonic measurements. This would be particularly useful for determining the ratio of foundation and fill moduli.

Another possible improvement concerns the role of the foundation. This displacement field, and to some extent the stress field, depend on the amount of foundation included in the finite element idealization. This feature has not been considered in this study.

Based on these results the following recommendations are made for future analytical and field studies:

1. Estimates for stress and displacement fields should be obtained prior to placement of instrumentation.
2. It is hoped that lateral pressures will be measured as well as vertical stresses. In this way, a better assessment of the model can be made.
3. Since vertical stresses are dependent primarily on density and height of fill, vertical pressure cells might be used to calibrate cells measuring horizontal stresses.
4. It is necessary to determine accurately the material properties of the earth fill when instrumenting an embankment. Only then can an accurate analysis be performed and accuracy of the predictions be assessed.

The research program of the MRD in this area is still in progress. The results in this paper are based on the preliminary findings from this continuing research. As other embankments are instrumented and more and better information becomes available it will be necessary to review the results presented here.

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The opinions, findings, and conclusions expressed in this publication are those of the authors and not necessarily those of the Bureau of Public Roads or of the California Division of Highways.

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Discussion

WILLIAM G. WEBER, JR., President, W. G. Weber and Associates, Sacramento, California—The authors are to be complimented on an excellent presentation of stresses and strains in an earth mass. As the authors have indicated, there is considerable disagreement between the theory and measured horizontal strains. Therefore, it is desirable to comment briefly on the type of installation that was used under this commentor's direction.

A schematic of the first of these installations is shown in Figure 12. This installation was made on the San Luis Reservoir relocation. A trench, 2 to 3 feet deep and 2 feet wide, was dug across the embankment. Two-inch plastic tubing with slip joints every 20 to 40 feet was placed on the bottom of this trench. Vertical platforms were placed at the desired spacing. Plastic-coated stainless steel ¼-in. aircraft cable was then connected to each platform and run through the plastic tubing to the edge of the embankment. The platform on centerline had a cable extending in both directions through the fill. The cables came through a gage box set in concrete at the edge of the fill and were held taut by a 50-lb weight system. The gage box was also referenced to a system of hubs outside the fill influence so that its movement could be independently observed. Also, water level-type settlement platforms were installed to record the vertical movement.

Major damage occurred to the weight system on this first set of installations because of construction operations, therefore little usable data were obtained during construction. On the Ridge Route installations, a metal housing was used to protect the gage box (Fig. 13), and a spring loading system was used to keep the cables taut. Complete data were obtained with these units during construction.

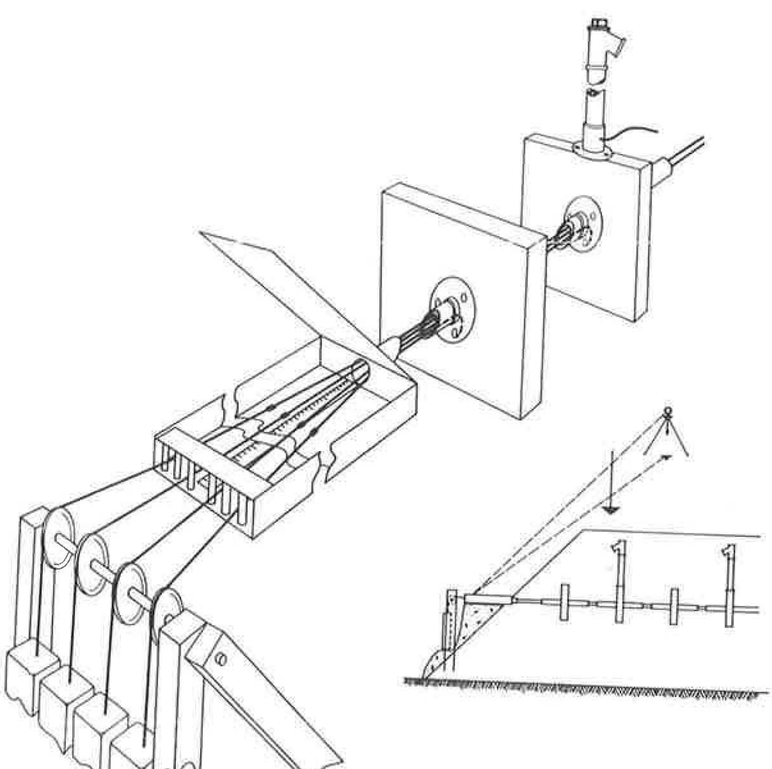

Figure 12. Horizontal and vertical movement indicator.

Figure 13. Side of embankment on Ridge Route showing the instrument shelters.

In comparing the movement of the gage box as indicated by surveys and the cables connected to the centerline platform (cable through the fill), a serious discrepancy was noted. For example, at one elevation the surveys indicated that the two gage boxes on opposite sides of the fill had moved apart 0. 32 feet, while the cables indicated they moved apart 0. 09 feet. At the present time it is not known which movement is correct; however, there are indications that the cables are tending to move as a unit at the gage box and all tend to have about the same indicated movement.

As a result of this discrepancy in the horizontal movement, the installation made in northern California during the summer of 1967 included electrical potentiometers between adjacent platforms. To date, the electrical potentiometers have indicated movements several times in excess of the movement indicated by the cables. Thus, the field data from the horizontal movement platforms are of questionable accuracy. Thus, the field data in Figures 6, 8, and 10 are probably in error. The field data from the vertical movements and vertical stresses are considered reasonably reliable.