# A Three-Dimensional Approach to Highway Alignment Design 

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The growing concern with the natural appearance of our roads makes it desirable, if not necessary, to adopt a design method in which both function and form are equally considered.

Although the determination of the location corridor and the alignment itself depends on many variables in the engineering and the economic field, equal emphasis should be placed on psychological and emotional values such as scenery and aesthetics. The optimum solution of the location and alignment problem in regard to psychological and emotional values can be obtained by means of a central perspective of three-dimensional design technique.

The author describes some methods whereby the electronic computer is used to produce various types of spatial motion pictures for the evaluation of scenic routes within the location corridor and roadway movies for the final test of the internal and external alignment. Additional improvement is obtained by the stereoscopic approach which provides an ideal condition for depth perception.

It is concluded that the central perspective method, in connection with the electronic computer, can be successfully used to assist the highway engineer in evaluating routes within the location corridor and testing and revising the three-dimensional alignment design before construction commences.
-ALIGNMENT design is still done by treating the horizontal and vertical layout separately. Even with the application of advanced engineering standards, the constructed road is not satisfactory because of the disharmony in the internal alignment itself and its relation with what is external to it. The common defects of such an alignment are well known and can be summarized into two main groups: (a) distorted vista such as roller coaster, broken back and break in the natural skyline, and (b) sudden change of direction.

Figures 1 and 2 illustrate a few of these undesirable conditions.


Figure 1. Distorted vista.

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Figure 2. Sudden change of direction.

Only recently have we begun to realize that the highway needs much more than the consideration of functional requirements. A well-designed road should contribute not only to the safety of road users but it should also present an aesthetically satisfying and pleasing picture. Whether the road is seen from the internal view or from the outside, it should fit the landscape harmoniously. Thus, we need to incorporate the human factor in design work by visualizing road location and alignment design in three dimensions.

## OBJECTIVE AND SCOPE

The prime objective, therefore, is to describe briefly some methods of a threedimensional approach in the dual problem of selecting a route, within the location corridor, and designing the alignment. The scope covers the rural highways.

## LOCATION CORRIDOR AND ALIGNMENT

The determination of the location corridor depends on many factors while the location of the alignment itself, within the corridor, is influenced by road and rail networks, right-of-way, stream crossings, drainage, utilities, etc.

The essential factors in determining the location corridor can be grouped into: (a) psychological values, (b) public interest values, and (c) functional values. Figure 3 shows these main values with their correlated subvalues.

Because of the many variables, there is no infallible answer to the location problem. Conflicting points often force compromises.

The alignment itself consists of a series of straight lines connected by curves and spirals. It commences with long tangents and short curves, applied especially in the early years of railroad construction. Unfortunately, this technique is still in use.

Improvements can be made by shortening the tangents and increasing the length of the curves. It has been found that the minimum length of a circular curve should not be less than 1500 to 2000 ft . Finally, when longer spirals are applied, we arrive at a continuous curvilinear horizontal alignment.

The vertical alignment is just as important as the horizontal one (Fig. 4). Long grades have the same monotonous effect as straight horizontal lines. The gradual


Figure 3. Relation between location corridor determinants.


Figure 4. Development of horizontal and vertical alignment.
change between tangent grades is effected by a vertical curve which can be either circular, simple parabolic or cubic parabolic. The simple parabola generally is preferable because this curve has a constant rate of change and the vertical offsets from the tangent vary with the square of the horizontal distance from the curve end. The most used mathematical relation is

$$
\frac{\text { Length of curve ( } \mathrm{ft} \text { ) }}{\text { Algebraic difference between tangent grades }}=\mathrm{K}
$$



Figure 5. Superimposing of horizontal and vertical layout.
as a measure of curvature; it expresses the horizontal distance, L, infeet required to effect 1 percent change in grade.

The ideal form of alignment should have approximately the same scale for horizontal and vertical curves and coincide roughly with the beginning and end of both layouts. For comparative investigations between both types of curves, the parabolic curve can be transformed into circular curves; whereby for small angles $\mathrm{R}=\mathrm{K} 100$. Figure 5 shows various superimposing positions. In the ideal form, the tangent


Figure 6. Direction of view.


Figure 7. Difference between vertical and horizontal view.


Figure 8. Sketch of central perspective.
points on crest and sag curves are always located at the superelevated section while the reverse points of the horizontal alignment lie within the descending and ascending grades.

## TOOLS FOR OPTIMIZING THE CORRIDOR AND THE ALIGNMENT PROBLEM

The success of optimizing the corridor and alignment problem depends on applying engineering and economic principles, and also consideration of landscape. Since, in the final analysis, we will be judged by the appearance of our highways, special emphasis must be laid on psychological values which include scenery and aesthetics.

In order to arrive at an acceptable solution in regard to psychological considerations, a device is necessary to perceive a three-dimensional view. Although aerial photogrammetry provides a three-dimensional perception, it is seen only in the vertical direction and not in the horizontal view as the driver observes the roadway. A more realistic view perhaps can be obtained through the central perspective which satisfies this desired view in the horizontal direction (Fig. 6).

To exhibit the difference between vertical and horizontal view sight, a short movie of a route corridor model has been produced. From the same selected line, we see first the landscape from the vertical view of an airplane, followed by the same site but in the horizontal direction (Fig. 7).

## MATHEMATICAL BASIS OF PERSPECTIVE

The perspective deals with the projection of a spatial object on a projection plane seen from a central point or eye point. The picture itself is projected on the plane by all the view rays between the central point and the object. Figure 8 shows the relation of main rays and points whereby the main axis or view axis is the prime reference line for the geometric build-up. The following mathematical relation can be obtained from the principle of similarity:

$$
\begin{equation*}
\frac{\Delta Z^{\prime}}{\Delta Z}=\frac{d}{D} \tag{1}
\end{equation*}
$$

and with $\mathrm{d}=1$, we derive the fundamental formula for the central perspective

$$
\begin{equation*}
\Delta Z^{\prime}=\frac{1}{D} \Delta Z \tag{2}
\end{equation*}
$$

If we project spatial object points perpendicular to the vertical view axis and notate the effects with $\Delta X_{i}$, Eq. 2 can then be transformed into two-dimensional perspective coordinates

$$
\begin{align*}
\Delta \mathrm{Z}_{\mathrm{i}}^{\prime} & =\frac{1}{\mathrm{D}} \Delta \mathrm{Z}_{\mathrm{i}} \\
\Delta \mathrm{X}_{\mathrm{i}}^{\prime} & =\frac{1}{\mathrm{D}} \Delta \mathrm{X}_{\mathrm{i}} \tag{3}
\end{align*}
$$



Figure 9. Perspective projection of spatial objects.

Each object point is determined by the coordinates $\Delta Z_{i}, \Delta X_{i}$ and the distance $D_{i}$ (Fig. 9 ). The perspective illustration of any object can then be plotted by coordinates of a number of connected points.

A further development of Eq. 3 leads then to the ordinate $\Delta X_{i}$ and the abscissa $\Delta Y_{i}$. With $D_{i}=\Delta Y_{i}$ we derive first

$$
\begin{align*}
& \Delta \mathrm{Z}_{\mathrm{i}}^{\prime}=\frac{1}{\Delta \mathrm{Y}_{\mathrm{i}}} \Delta \mathrm{Z}_{\mathrm{i}} \\
& \Delta \mathrm{X}_{\mathrm{i}}^{\prime}=\frac{1}{\Delta \mathrm{Y}_{\mathrm{i}}} \Delta \mathrm{X}_{\mathrm{i}} \tag{4}
\end{align*}
$$

and by the simple mathematical relation

$$
\begin{aligned}
\overline{\mathrm{E}_{\mathrm{O}}-\mathrm{I}_{\mathrm{O}}} & =\sqrt{\left(\mathrm{Y}_{\mathrm{I}}-\mathrm{Y}_{\mathrm{E}}\right)^{2}+\left(\mathrm{X}_{\mathrm{I}}-\mathrm{X}_{\mathrm{E}}\right)^{2}} \\
\operatorname{cotan} \alpha_{\mathrm{I}} & =\frac{\mathrm{Y}_{\mathrm{I}}-\mathrm{Y}_{\mathrm{E}}}{\mathrm{X}_{\mathrm{I}}-\mathrm{X}_{\mathrm{E}}} \\
\operatorname{cotan} \alpha_{\mathrm{N}} & =\frac{\mathrm{Y}_{\mathrm{N}}-\mathrm{Y}_{\mathrm{E}}}{\mathrm{X}_{\mathrm{N}}-\mathrm{X}_{\mathrm{E}}} \\
\alpha & =\alpha_{\mathrm{I}}-\alpha_{\mathrm{N}}
\end{aligned}
$$

Finally, with $\alpha$ and the distance $\overline{\mathrm{E}_{\mathrm{O}}-\mathrm{I}_{\mathrm{O}}}$ we derive

$$
\begin{align*}
& \Delta \mathrm{X}_{\mathrm{i}}=\sin \alpha \overline{\left(\mathrm{E}_{\mathrm{O}}-\mathrm{IO}_{\mathrm{O}}\right)} \\
& \Delta \mathrm{Y}_{\mathrm{i}}=\cos \alpha \overline{\left(\mathrm{E}_{\mathrm{O}}-\mathrm{IO}_{\mathrm{O}}\right)} \\
& \Delta \mathrm{Z}_{\mathrm{i}}=\mathrm{Z}_{\mathrm{E}}-\mathrm{Z}_{\mathrm{I}} \tag{5}
\end{align*}
$$

Eq. 4 for perspective coordinates can then be completed by substituting $\Delta X_{i}, \Delta Y_{i}$ and $\Delta Z_{i}$ from Eq. 5:

$$
\begin{align*}
& \Delta \mathrm{Z}_{\mathrm{i}}^{\prime}=\frac{1}{\cos \alpha \overline{\left(\mathrm{I}_{\mathrm{O}}-\mathrm{IIO}_{\mathrm{O}}\right)}}\left(\mathrm{Z}_{\mathrm{E}}-\mathrm{Z}_{\mathrm{I}}\right) \\
& \Delta \mathrm{X}_{\mathrm{i}}^{\prime}=\frac{1}{\cos \alpha \overline{\left(\mathrm{E}_{\mathrm{O}}-\mathrm{IO}_{\mathrm{O}}\right)}} \sin \alpha \overline{\left(\mathrm{E}_{\mathrm{O}}-\mathrm{IO}_{\mathrm{O}}\right)} \tag{6}
\end{align*}
$$

## PROGRESSIVE STAGING OF THE DUAL PROBLEM

The central perspective can be applied in a progressive staging process in the dual problem of the location corridor and alignment design. Three phases are proposed from the very early stage during the process of planning:

1. Preliminary spatial motion pictures with the eye point approximately 100 ftabove the ground, to make the first rough selections of lines with the best scenic qualities.
2. Refined spatial motion picture with the eye point approximately 25 ft above the landscape to evaluate the selected lines.
3. Roadway movie to make the final test with incorporated horizontal and vertical alignment.

All three types of motion pictures can be produced by either the cross-section methodor by the grid method (or even by a combination of the two). In the first case the perspective pictures are derived from cross-section of the ground; in the latter one, from grid points.

Perspective movies have been applied very recently in Europe but are restricted to the roadway only. In this paper, a much broader approach and application is made to use the electronic computer for a progressive staging to optimize the problem. The input data consist of the digital model and the geometric data of the horizontal and vertical alignment.

## COMPUTER PROGRAM

Perspective movies can be obtained by feeding three-dimensional coordinates of terrain points into the electronic computer and transforming them into perspective drawings at desired intervals which are finally photographed. Instead of a time-consuming plotting device, an instruction decoder can be used in connection with a cathode ray tube to trace the picture on the face of the tube.

The computer program itself can be introduced by a general block diagram which explains the procedure of various steps. Since the motion pictures are based on either the cross-section method or the grid method, two different programs are necessary. Figures 10 and 11 show the flow chart for these two methods which are self-explanatory.

NOTE ONLY TO BE APPLIED FOR ROAD MOVIE
: I-J FUTURE IMPROVEMENT
Figure 10. Block diagram for cross-section method.


Figure 12. Staging of the dual problem.

In the first two stages, the perspective projection of the surface and significant topographical features of the route corridor are illustrated. These two features interfere with one another and influence the quality of the perspective view drawings. This disadvantage can be avoided by a separated process which results in two different view drawings. The perspective picture of the surface is then screened and superimposed on the one showing the topographical features.

A similar procedure can be applied when the grid method and the cross-section method are combined. Figure 12 shows both methods and their combinations and a selection for the staging into three phases.

## STEREOSCOPIC PROJECTION

An additional improvement can be obtained by stereoscopic projection of a pair of perspective drawings of the same object, seen from two slightly different positions. When a person looks simultaneously at these two drawings, viewing one with each eye, or using a stereoscope, he can see in three dimensions. This depth perception, already known in photogrammetry and other fields, can be successfully used for perspective drawings. Figure 13 shows this stereoscopic projection of a spatial object onto two picture planes with eye point left and right.

The calculation of the coordinates of these two projections can be done in one unit. Equation 6, which refers only to one spatial object, is then extended accordingly:

$$
\begin{align*}
& \Delta Z_{i(L)}^{\prime}=\Delta Z_{i(R)}^{\prime}=\frac{1}{\cos \alpha \overline{\left.\left(\mathrm{E}_{(\mathrm{L}}\right) \mathrm{O}-\mathrm{I}_{\mathrm{O}}\right)}}\left(\mathrm{Z}_{\mathrm{E}(\mathrm{~L})}-\mathrm{Z}_{\mathrm{I}}\right)=\frac{1}{\cos \alpha \overline{\left(\mathrm{E}_{\left.(\mathrm{R}) \mathrm{O}-\mathrm{I}_{\mathrm{O}}\right)}\right.}\left(\mathrm{Z}_{\mathrm{E}}(\mathrm{R})-\mathrm{Z}_{\mathrm{I}}\right)} \\
& \Delta \mathrm{X}_{\mathrm{i}(\mathrm{~L})}^{\prime}=\frac{1}{\cos \alpha \overline{(\mathrm{E}(\mathrm{~L}) \mathrm{O}-\mathrm{IO})}} \sin \alpha \overline{(\mathrm{E}(\mathrm{~L}) \mathrm{O}-\mathrm{IO})}  \tag{6}\\
& \Delta \mathrm{X}_{\mathrm{i}(\mathrm{R})}^{\prime}=\frac{1}{\cos \alpha \overline{\left(\mathrm{E}_{(\mathrm{R}) \mathrm{O}-\mathrm{IO})}\right.} \sin \alpha \overline{\left(\mathrm{E}(\mathrm{R}) \mathrm{O}-\mathrm{I}_{\mathrm{O}}\right)}} \quad \text { (stereoscopic) }
\end{align*}
$$



Figure 13. Stereoscopic projection of a spatial object.


Figure 14. Spatial movie based on grid method.


Figure 15. Roadway movie based on cross-section method.

The block diagram is similar to the previous one, with the exception that the transformation of three-dimensional coordinates are separated in perspective coordinates of the left and right perspective plane.

## EXAMPLES OF MOTION PICTURES

The central perspective can be applied for various types of drawings: (a) single illustrations, (b) stereoscopic illustrations, (c) motion pictures of single perspective drawings, and (d) motion pictures of stereoscopic illustrations.

Several examples of perspective movies have been developed by the planning branch in cooperation with the electronic computing branch of the Ontario Department of Highways. The previous program has been carried out with the IBM 7044 computer and the EAI 3500 plotter; the production of the film was done by photographing the perspective plottings. However, the program is in the process of being improved and simplified and steps are already under way to perform the work with the much faster IBM 360 and an attached cathode ray tube.

For reasons of comparison, these typical examples are presented by two projectors with syncronized filmstrips. The first pair of pictures (Fig. 14) illustrates a short section of a spatial movie with the eye point 100 ft above the ground, based on the grid method. A second section of film (Fig. 15) deals with the roadway itself where the


Figure 16. Tunnel movie with varying road width.
cross-section method was applied. Both sections exhibit on the left side the first approach and on the right side an improvement. Both types of motion pictures can be further developed to stereoscopic projections.

Motion pictures can also be made for very special projects. To investigate the effect on the driver in relation to various tunnel cross-sections and to opposing traffic, a tunnel movie was produced for a 24 ft and 30 ft road width, incorporating traffic from the opposite direction (Fig. 16). For perceptional reasons, the presentation was made by two projectors so that the two proposals could be observed by a group of people at the same time. In addition, each movie was copied twice and spliced together so that the observer perceived each scheme simultaneously.

## FURTHER RESEARCH

This new approach in highway engineering opens up further application in the environmental and operational field. However, more experimentation and research is desired. In the further study we have undertaken, the following points are of special interest:

1. Road movies and single drawings of special locations, of complex interchanges and ramps, including merging and diverging operations;
2. Road movies to check the highway user's perception of signs and signals in regard to various locations;
3. Scale relation between horizontal and vertical alignment;
4. Relation between alignment, landscaping and rhythm of the roadway; and
5. Spatial motion pictures to test the external view of the road.

## CONCLUSIONS

The application of the central perspective is able to assist the highway engineer in optimizing the dual problem of the location corridor and alignment design. With this new tool, function and form can be equally considered by evaluating various routes in the corridor through progressive staging of two sets of spatial motion pictures.

The final testing of the selected route can be carried out by a road movie, seenfrom the driver's eye, along a designed roadway whereby the vertical alignment is combined with a three-dimensional layout.

Any fault of the design, the appearance and disharmony with the landscape can be detected not only by one person but by a group of people. Since it is not difficult to define the location of any section which needs to be improved, the adjustable section can be replaced by a new design and the whole movie presented again. Inthis way, an optimum solution can be obtained.

This technique can be used for both internal and external alignment and further improved by the stereoscopic approach to provide depth perception.

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