Ramp Capacity and Service Volume As Related to Freeway Control

KENNETH A. BREWER, JOHANN H. BUHR, DONALD R. DREW, and CARROLL J. MESSER, Texas Transportation Institute, Texas A&M University

The problem of establishing a policy for freeway control can be structured into an ordered set of control problems within a hierarchy of levels of control. At each level associated with the hierarchy of the problem structure, an interrelation exists among freeway-capacity, freeway-demand, ramp-capacity, ramp-demand, outside-freeway-lane-flow, and ramp-merging characteristics and merging-control-system operation. The extent to which an optimum control policy can be established at each level of control is directly related to the extent to which the pertinent traffic and control system variables can be interrelated.

In this report, ramp capacity and service volume are defined in terms of entrance-ramp geometry and gap-acceptance and merging-controller-operational characteristics. Subsequently, an optimal control policy is established for an isolated entrance ramp and the integrated total freeway control policy modeled.

•DURING the last decade freeway control has evolved from a research experiment to an operational reality. Freeway control systems have been installed and are operational in Chicago, Detroit, and Houston (6, 7, 8). Others are being initiated in Seattle and Dallas. The various forms of control have included freeway main-lane control (9), ramp metering (7), and ramp merging control (8).

In order to optimally control a freeway system, the capacity and flow characteristics of the freeway and the ramps must be known and understood. The capacity of freeway sections has been studied (1), and its effect on freeway control philosophy is well documented (10). The capacity of an uncontrolled ramp is not as well defined nor as precisely quantified, even though much work (1, 2, 3, 4, 5) has been done to relate the outside freeway lane flow to ramp service volumes. Optimizing freeway control system operation requires that the control of an individual ramp be understood in the context of overall freeway system control. Therefore, the capacity and service volume relationships for controlled merging conditions must be determined in terms of variables that functionally define freeway control. These variables include speed and flow on the outside freeway lane, total freeway flow, freeway capacity, ramp capacity, ramp demand, efficiency of the merge, and extent of queuing on the ramp. Furthermore, these variables can be grouped according to those that can be controlled (e.g., ramp flow), those evaluated within a control system (e.g., ramp geometry and vehicular characteristics), and those used to effect the desired control action (e.g., controller settings).

This paper is directed toward determining the relationships among these variable groups, their influence on controlled ramp capacity, and their effect on freeway operation.

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CONTROLLED RAMP SERVICE VOLUME MODEL

General Merging Control System Configuration and Operation

An entrance ramp with a "stopped vehicle gap acceptance" mode of control searches for a gap, t, in the outside freeway lane such that $t \geq T_s$, where T_s is the service gap setting on the controller. A selected gap is projected in time to a decision point so that the vehicle waiting at the ramp signal may be released into the selected gap (8, 13).

When a queue of vehicles is always waiting at the ramp signal, the merging control system output is primarily a function of the efficiency of the system components and freeway traffic conditions. In the event that a ramp driver rejects the selected gap, he will suffer some delay while searching for a gap in the traffic stream that is acceptable to him. If during such time the driver stops in the merging area, a presence detector will hold the merging controller in the inoperative state until the merging area is clear. Any additional vehicles that are released at the ramp signal and that are behind the driver who stopped in the merging area (due to the travel time from the ramp signal to the merge area) will also miss their assigned gaps. When a vehicle stops in the merging area, the controller will be held in the inoperative state until all vehicles between the signal and the merge area have departed. Then, the normal gap selection-gap projection process will continue.

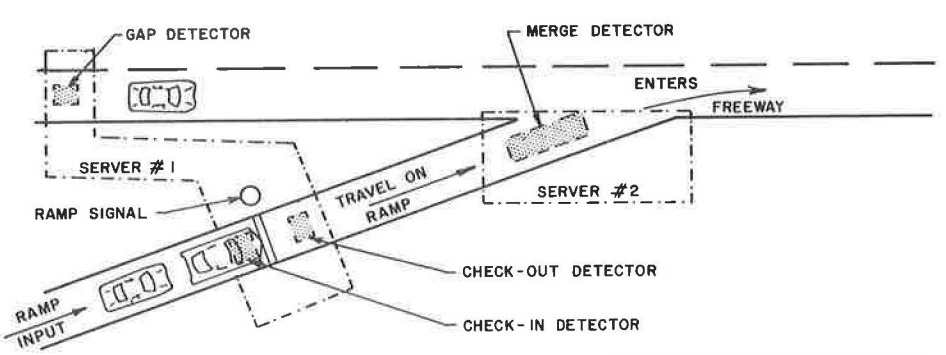
As each vehicle arrives at the head of the queue, it will stop over a "check-in" detector in front of the signal. From the time the ramp signal displays a green indication until a "check-out" detector just beyond the signal is actuated, the controller will dwell in the inoperative state. This "server dwell time" also corresponds to the time required for the next vehicle in the queue to move up over the check-in detector.

A controlled ramp as just described can be considered similar to a two-server-in-series queuing facility. Figure 1 shows a typical controlled ramp facility with each server function physically identified. In order to maximize the output per unit time of such a ramp, the average service time in the total service facility must be minimized.

"Controlled ramp capacity" will be a function of several variables, including (a) availability of gaps in outside freeway lane, (b) size of service gap setting, (c) geometry of ramp, and (d) driver-vehicle characteristics. Control of the ramp is basically a problem of detecting and projecting freeway gaps for a ramp vehicle to utilize. Functional relation of this problem to ramp capacity will be in terms of these variables.

Development of Control System Total Service Time

If time headways in the outside freeway lane are distributed according to an Erlang distribution with a probability density function, f(t), then



NOTE: GAP ACCEPTANCE MERGING CONTROL COMPUTER FUNCTIONS AS SERVER #1. A FEEDBACK LOOP FROM THE MERGE DETECTOR TO THE CONTROLLER INTERCONNECTS SERVER #1 AND SERVER #2.

Figure 1. Ramp control system functional components.

$$f(t) = \frac{\left(aq_f\right)^a t^{a-1} e^{-aq_f t}}{(a-1)!} \tag{1}$$

where t is time headway, q_f is average outside lane flow rate, and a is an Erlang constant.

The expected delay to a ramp vehicle searching for a gap, t, in a stream of gaps has been developed mathematically when the driver has a constant threshold gap (17). For a controlled merge, an equivalent search is conducted by the control computer. Therefore, the average waiting time, T_c, for a vehicle at the ramp signal while the merging controller is searching for a gap (16) is expressed by

$$T_c = \frac{e^{aq_f T_s} - \sum_{i=0}^{a} \frac{\left(aq_f T_s\right)^i}{i!}}{q_f \sum_{i=0}^{a-1} \frac{\left(aq_f T_s\right)^i}{i!}} \tag{2}$$

where T_s is the gap set on the merging controller. Then, the average service time in server No. 1 (Fig. 1), S_1, is given by

$$S_1 = R + T_c \tag{3}$$

where R is the average server dwell time between the release of a ramp vehicle at the signal and the start of the next gap search.

In order to consider the service time in server No. 2, the ramp driver's probability of accepting the assigned gap must be evaluated. Research into the gap-acceptance phenomenon has developed the concept of a gap-acceptance probability, P(t), such that a ramp driver has a probability P(t) of accepting a gap, t, in the merge area (17, 18, 19, 20). The gap-acceptance function is

$$P(t) = \int_0^t g(t)\,dt \tag{4}$$

where g(t) defines the functional change in the probability of accepting a gap as gap sizes change.

Two distributions that have been found to be appropriate for g(t) are the long-normal distribution (17) and the Erlang distribution (21). Since an Erlang distribution is a reasonable description of gap acceptance (Fig. 2), the gap-acceptance function used in this development will be a cumulative Erlang form. Then P(t) becomes

$$P(t) = 1 - e^{-k\mu t} \sum_{i=0}^{k-1} \frac{(k\mu t)^i}{i!} \tag{5}$$

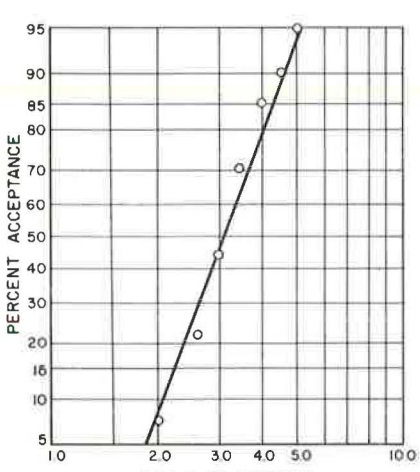


Figure 2. Dumble gap acceptance.

where k is an Erlang constant and μ is the inverse of the mean accepted gap for moving merges. It follows then that the probability of accepting a gap, t, given that $t \geq T_s$, which will be denoted by P(a), is expressed by

$$P(a) = \frac{\int_{T_s}^{\infty} P(t)\, f(t)\, dt}{\int_{T_s}^{\infty} f(t)\, dt} \tag{6}$$

Substituting Eqs. 1 and 5 into Eq. 6 gives

$$P(a) = 1 - \frac{\int_{T_s}^{\infty} \left[e^{-k\mu t} \sum_{i=0}^{k-1} \frac{(k\mu t)^i}{i!} \right] \frac{\left(aq_f\right)^a}{(a-1)!} t^{a-1} e^{-aq_f t}\, dt}{e^{-aq_f T_s} \sum_{i=0}^{a-1} \frac{\left(aq_f T_s\right)^i}{i!}} \tag{7}$$

The integral in the numerator of the fractional portion of Eq. 7 can be expanded into a series of integrals:

$$\frac{\left(aq_f\right)^a}{(a-1)!} \frac{(k\mu)^n}{n!} \int_{T_s}^{\infty} e^{-\left(k\mu + aq_f\right) t}\, t^{a+n-1}\, dt, \quad (n = 0, 1, 2, \ldots, k-1)$$

By letting $b = k\mu + aq_f$, this expression can be manipulated into a series of Erlang integrals with the general term being

$$\frac{\left(aq_f\right)^a (a-1+i)!\, (k\mu)^i}{b^{a+i} (a-1)!\, i!} \int_{T_s}^{\infty} \frac{e^{-bt}\, b^{a+1}\, t^{a-1+i}}{(a-1+i)!}\, dt, \quad (i = 0, 1, 2, \ldots, k-1)$$

Integration of each integral results in a series of cumulative Poisson series that has a general term of the form

$$\frac{\left(aq_f\right)^a}{b^{a+i}} \frac{(a-1+i)!}{(a-1)!} \frac{(k\mu)^i}{i!} \left[e^{-bT_s} \sum_{j=0}^{a-1+j} \frac{\left(bT_s\right)^j}{j!} \right], \quad (i = 0, 1, 2, \ldots, k-1)$$

Substituting this expression into Eq. 6 gives

$$P(a) = 1 - \frac{\dfrac{\left(aq_f\right)^a e^{-bT_s}}{b^a} \displaystyle\sum_{j=0}^{k-1} \left[\frac{(k\mu)^j}{b^j} \frac{(a+j-1)!}{(a-1)!\,j!} \sum_{i=0}^{a+j-1} \frac{\left(bT_s\right)^i}{i!} \right]}{e^{-aq_f T_s} \displaystyle\sum_{i=0}^{a-1} \frac{\left(aq_f T_s\right)^i}{i!}} \tag{8}$$

Let p(n) be the probability of n consecutive drivers accepting an assigned gap before a driver rejects his gap. Then, enumerating the possible combinations gives

$$p(0) = 1 - P(a)$$

$$p(1) = P(a) \cdot [1 - P(a)]$$

$$p(2) = [P(a)]^2 \cdot [1 - P(a)]$$

$$\vdots$$

$$p(n) = [P(a)]^n \cdot [1 - P(a)]$$

The expected number of consecutive drivers accepting their assigned gap before one driver rejects his gap is

$$E(n) = \sum_{n=0}^{\infty} n \cdot p(n) = \frac{P(a)}{1 - P(a)} \tag{9}$$

The expected number of vehicles, E(q), on the ramp when a driver rejects his assigned gap is related to the average travel time on the ramp and the service time at server No. 1 by

$$E(q) = 1 + \frac{T_r}{S_1} \tag{10}$$

where T_r = average travel time on the ramp.

The average delay suffered in the merging area by drivers who reject or miss their assigned gap can be estimated by assuming each driver looks for a gap larger than a critical gap, T_m. The average delay for this merging problem (17) is

$$T_d = \frac{e^{aq_fT_m} - \sum_{i=0}^{a} \frac{\left(aq_fT_m\right)^i}{i!}}{q_f \sum_{i=0}^{a-1} \frac{\left(aq_fT_m\right)^i}{i!}} \tag{11}$$

which is the same form as Eq. 2. The average total delay, E(d), to all vehicles on the ramp when the first driver of a group rejects his gap is

$$E(d) = T_d[1 + 2 + 3 + \ldots + E(q)]$$

$$= \frac{E(q)\,[E(q) + 1]}{2}\,T_d \tag{12}$$

The average service time in server No. 2 (Fig. 1), S_2, is the average total delay suffered by vehicles delayed when a ramp driver rejects his assigned gap divided by the sum of the average number of vehicles delayed and the average number of vehicles between such delayed groups. The expression for S_2 is

$$S_2 = \frac{E(d)}{E(q) + E(n)} \tag{13}$$

The expected total service time in the merging control system is the sum of the average service times in each server. The ramp service volume under merging control, q_r, is the inverse of the expected total service time so that

$$q_r = \frac{1}{S_1 + S_2} \tag{14}$$

Effects of Ramp Geometry and Ramp Operation

Institution of merging control on a freeway entrance ramp should reduce the importance and influence of geometric characteristics since a driver approaches a merging area that is free of ramp congestion. There are, however, basic considerations of ramp geometry and operational characteristics that will influence the controlled merge operation. Previous research has been reported dealing with the effect of ramp vehicle speed relative to freeway speed, the effect of angle of convergence of the ramp with the freeway, and the effect of acceleration lane length on uncontrolled gap acceptance (5, 17, 22). Much of the restrictive influence of these variables has been removed by the merging controller as it selects a gap for the driver. However, relative speed at the ramp nose, sight distance as the driver approaches the merging area, angel of convergence, acceleration lane length, and ramp grade do appear to influence controlled merging. In the absence of any quantifiable evidence of the effect of geometrics on the operation of a controlled ramp, and in view of past findings for uncontrolled ramps (17), the type of ramp operation anticipated under controlled conditions may best be classified on the basis of expected relative speed, angle of convergence, and acceleration lane length (Table 1).

TABLE 1

TYPE OF RAMP OPERATION

Relative Speed = Average Freeway Speed - Average Ramp Speed at Nose									Angle of Convergence (deg)
< 5 mph			5 mph-20 mph			> 20 mph			
Acceleration Lane Length (ft)			Acceleration Lane Length (ft)			Acceleration Lane Length (ft)			
0-300	300-600	> 600	0-300	300-600	> 600	0-300	300-600	> 600	
I	H	H	I	H	H	L	I	H	0-6
I	H	H	L	I	H	L	L	I	6-12
L	I	I	L	L	I	L	L	I	>12

H = high type, I = intermediate type, L = low type.

Gap-Acceptance Parameters

The classification of high-, intermediate-, and low-type operations can be assumed to correspond to gap-acceptance characteristics for moving vehicle merges described by Erlang distributions (Eq. 5) with the following Erlang constant, k, and mean accepted gap, $1/\mu$:

1. High type: $k = 2$, $1/\mu = 2.4$ sec;
2. Intermediate type: $k = 6$, $1/\mu = 3.0$ sec; and
3. Low type: $k = 10$, $1/\mu = 4.0$ sec.

Examples of each are shown in Figure 3.

Gap acceptance regressions for fast- and slow-moving vehicles provided a basis for selecting values of T_m, the critical gap for slow and stopped merges, associated with each classification of ramp operation (17). For the relative speed categories used in Table 1, the corresponding representative critical gaps for drivers who had rejected a gap were estimated to be

1. High Type: $T_m = 3.0$ sec;
2. Intermediate type: $T_m = 3.5$ sec; and
3. Low type: $T_m = 4.0$ sec.

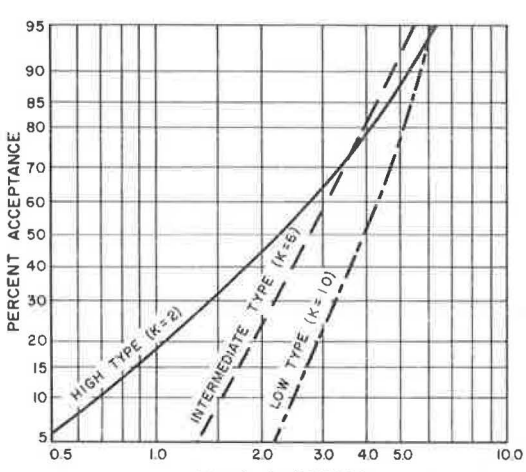


Figure 3. Gap acceptance characteristic for operation types.

Gap Distribution Parameters

In addition to the gap acceptance parameters, the outside lane freeway flow enters into the expression for ramp service volume. So that a specific Erlang distribution may be selected to describe a particular flow level, the Erlang constant, a, must be evaluated. Drew et al (5) found an empirical relation between

TABLE 2

ERLANG a AND OUTSIDE FREEWAY LANE FLOW

Erlang a:	2	3	3	4	5	6
Flow (vph):	800	1000	1200	1400	1600	1800

TABLE 3

SERVER DWELL TIME IN CONTROLLED MERGING AT DUMBLE RAMP

Sample (no.)	No. of Vehicles	Mean (sec)	Variance (sec^2)
1	35	2.73	3.39
2	74	2.72	1.02
3	65	2.83	3.99
4	25	2.25	0.26

the freeway lane flow rate, q_f, and the Erlang constant, a, from which the values in Table 2 were used to evaluate Eq. 14.

Server Dwell Time

A range of likely values from 1.5 to 4.0 sec for server dwell time could be established from previous research (23, 24). But measurements of server dwell time as given in Table 3 indicate that R = 2.6 sec is a good estimate for high- and intermediate-type operation. A value of 3.0 sec for R would appear to be a reasonable estimate for low-type ramps since the initial ramp speeds would likely be lower than for high- and intermediate-type ramps.

Ramp Travel Time

Ramp travel time in the development of Eq. 14 is the time to travel from the check-out detector to the merge area. Travel times, as reported in a companion report (15), can be adjusted for the server dwell time resulting in the values for each type of operation (Table 4). The same travel times are valid for ramp lengths different from those indicated because the ramp grade, sight distance, and ramp traffic composition obviously alter the value of travel time, but these variables are all reflected in the relative speed parameter.

Service Volume Control Charts

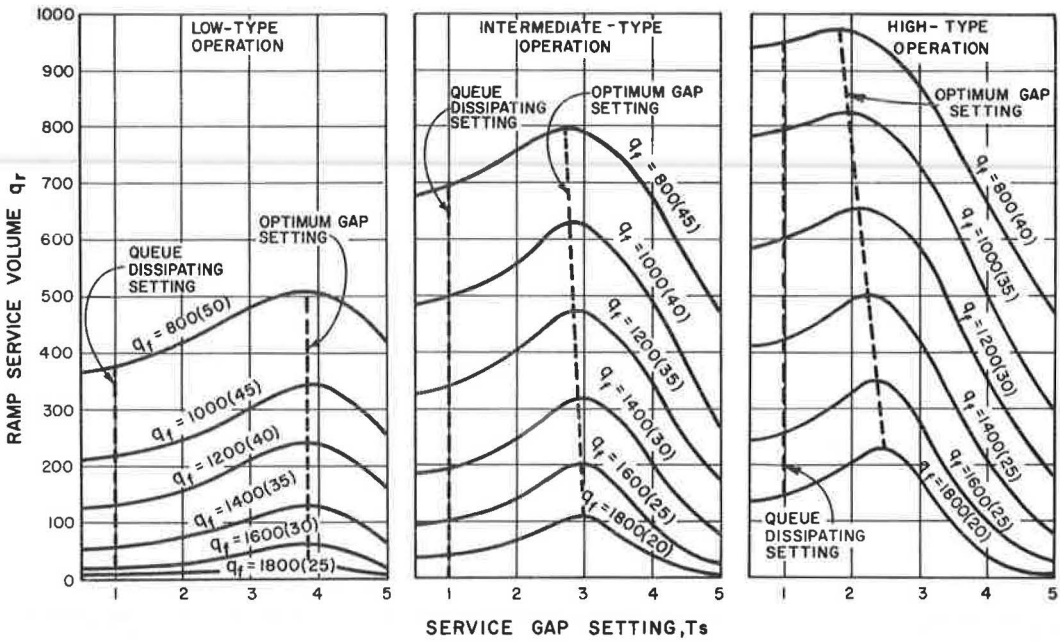
The family of curves in Figure 4 shows the ramp service volumes that can be attained with a certain service gap setting when the outside freeway lane flow is at a given level, as predicted by Eq. 14 under the assumptions stated. Each type of operation has an optimum controller setting (service gap) for a given level of freeway flow. As the controller setting is decreased from the optimum, vehicles are released more rapidly, but more drivers reject their assigned gaps, thus decreasing the service volumes. As the controller setting is increased above the optimum, the service time at the signal becomes excessively long, again decreasing the ramp service volumes. The optimum setting varies quite slowly as the freeway flow level varies.

When the congestion created by the queue of vehicles waiting at the ramp signal begins to reduce the capacity of frontage roads or arterial street intersections, it may be necessary to temporarily store queued vehicles on the ramp between the ramp signal and the merge area. Even under such conditions, it is still desirable to release the vehicles at the signal for an assigned gap. Thus, a queue-dissipating setting is indicated that reduces the overall efficiency of the merging control system operation, but, since the service gap is small, many gaps in the traffic stream larger than it are identified, thereby increasing the rate of release of vehicles at the ramp signal.

TABLE 4

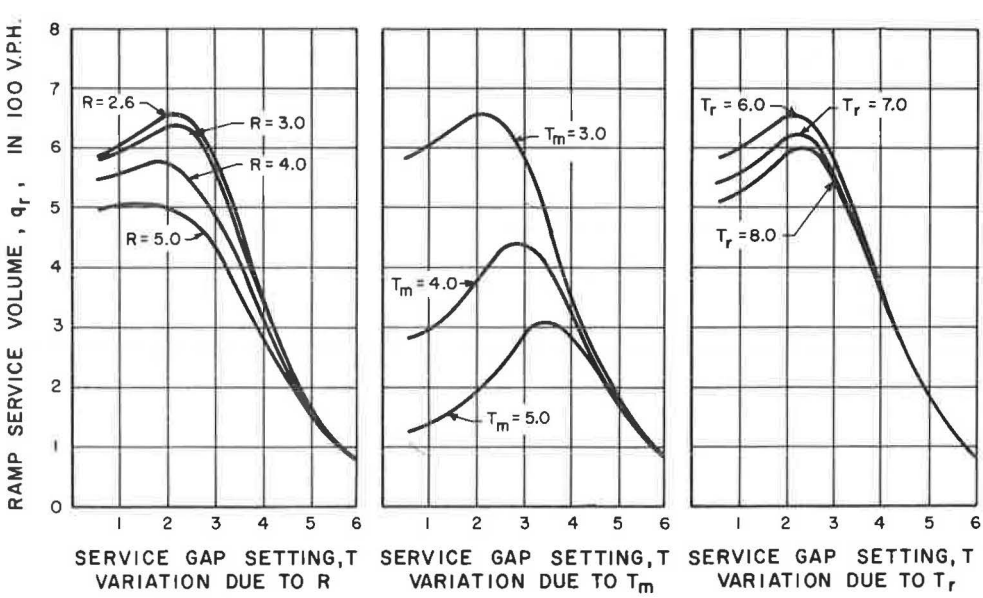
RAMP TRAVEL TIME CHARACTERISTICS

Type Operation	Travel Time (sec)	Distance From Signal to Merging Area for Level Ramps (ft)
High	6.0	150
Intermediate	7.0	200
Low	8.0	250



NOTE: q_f = 1200(40) IS THE OUTSIDE LANE UPSTREAM FREEWAY DEMAND WITH THE SPEED OF THE TRAFFIC STREAM IN PARENTHESES

Figure 4. Service volume control curves.



NOTE: INTERMEDIATE TYPE RAMP AT A FREEWAY FLOW OF 1200 V.P.H. (OUTSIDE LANE)

Figure 5. Parameter sensitivity.

Parameter Sensitivity

In order to indicate how sensitive the ramp service volume is relative to variations in the server dwell time, the critical gap for stopped vehicles, and the ramp travel time, control curves were developed holding all variables constant except R, T_m, and T_r. The results (Fig. 5) indicate that ramp service volume is relatively insensitive to server dwell time, R, and to ramp travel time, T_r. Variations in the critical accepted gap for slow and stopped merge vehicles, T_m, do produce wide ranges in the maximum ramp service volume (Fig. 5). However, the value of T_m for any specific ramp is a constant that is not related to merging control. When necessary, the curves of Figure 4 can be adjusted for the measured T_m characteristic of a particular location (17).

Ramp Capacity as a Function of Service Volume

The capacity of a controlled ramp is the maximum expected service volume for a given level of flow on the outside freeway lane. For a specific ramp, the geometry and the gap-acceptance parameters will be constant. Thus, ramp capacity will only be a function of the outside freeway lane flow and the service gap setting. Ramp capacity, q_{rc}, under merging control may be defined as

$$q_{rc} = q_r \text{ at } \frac{\partial q_r}{\partial T_s} = 0, \text{ at } q_f = \text{constant} \tag{15}$$

for a specific freeway flow level, q_f. Since evaluating the derivative in Eq. 15 is quite tedious, the approximate ramp capacity as a function of outside freeway lane flow can be taken from Figure 4. The ramp service volume for a given freeway flow level at the optimum service gap setting is the ramp capacity for that flow level. Figure 6 shows the variation of ramp capacity as freeway flow varies for each type of ramp operation.

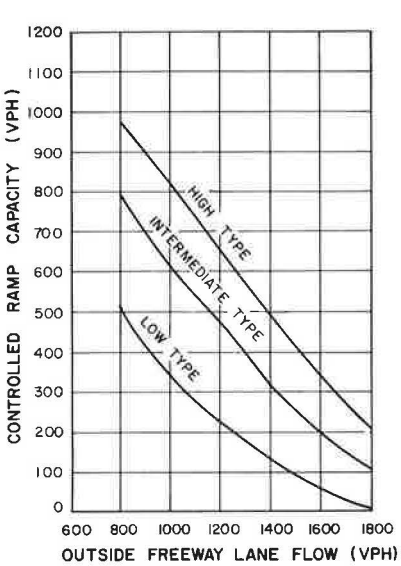


Figure 6. Ramp capacity under merging control.

VERIFICATION OF SERVICE VOLUME RELATION

Study Site

The Dumble Street entrance ramp to the northbound Gulf Freeway in Houston was selected for the data collection site to verify the mathematical model for ramp service volume. Figure 7 shows the site plan and relative location of the detection equipment. This particular site was selected for the following reasons:

1. A gap acceptance merging controller was available;
2. The ramp grade and the freeway grade in the vicinity of the ramp were nearly level;
3. Unrestricted sight distance was present at the location; and
4. Freeway flow in the vicinity of the ramp was usually free-flowing.

Data Collection and Data Analysis

Detection and control-function hardware components of the merging control system

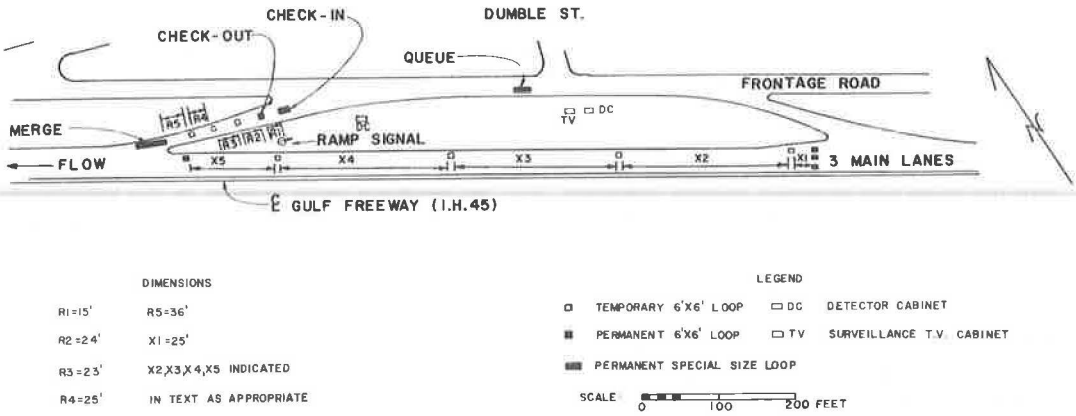


Figure 7. Layout of site.

were monitored during the morning peak-period control. The permanent control system inputs were supplemented with observers and temporary loop detectors placed on both the ramp and the outside freeway lane. All of the data were recorded on a 20-pen graphic recorder at the field site. The graphic record was run through a digital X-Y coordinate analyzer to produce punched card data records for analysis by digital computer.

Parameter Measurement Results

Ramp travel times from the check-out detector to the merge detector were measured, and the results are given in Table 5. The results for the four data sets are quite consistent. The Dumble Ramp is 135 ft long from the check-out detector to the merge detector, and is on a level grade with an acceleration lane of 350 ft and an angle of convergence of 12 deg. The ramp-freeway speed characteristics are given in Table 6. On the basis of the relative speed and the given geometric characteristics, Table 1 indicates an anticipated "intermediate-type" controlled merge operation.

Figure 8 shows the observed and predicted service volume rates for two service gap settings tested at the Dumble Street ramp (T_S = 2.5 sec and T_S = 3.0 sec). Each curve in Figure 8 represents the expected service volume for this intermediate-type ramp as the merging volume (combined freeway flow and ramp flow) varies. The majority of the observations are close to the expected curves, which tends to substantiate the model.

LEVELS OF CONTROL AND RAMP CAPACITY

Control of a freeway can be decomposed into a set of component subproblems with each subproblem responding to a particular level of control within a hierarchial structure of control functions (12). Within each level of control, ramp and freeway capacity assume varying degrees of influence on the establishment of an optimum control policy.

TABLE 5

RAMP TRAVEL TIME

Data Set	Number of Vehicles	Mean Time
1	35	5.6
2	74	5.5
3	65	5.6
4	25	5.5

First Level

The lowest level of control deals only with entrance ramps and considers each ramp to be independent of the total freeway or surface street operation. Merging control at this level seeks to put ramp vehicles into gaps in the outside lane of the freeway. Whenever a gap in the traffic stream arrives that is greater than or

TABLE 6

RAMP-FREEWAY SPEED IN MERGE AREA

Data Set	Average Speed at Ramp Nose (mph)	Average Freeway Speed in Merge Area (mph)	Average Outside Freeway Lane Flow (vph)	Relative Speed (mph)
1	20.0	28.4	1180	8.4
2	19.2	37.4	1210	18.2
3	18.0	38.0	1220	20.0
4	19.0	39.0	1185	21.0

equal to a service gap setting on the controller, a vehicle is released at the ramp signal (Fig. 1). If the ramp driver accepts his assigned gap, the controller continues to operate. If the ramp driver rejects his assigned gap, thereby blocking the merging area, the controller dwells until the merging area clears. Since no consideration is made of freeway capacity, the optimal control policy at this level is to put as many ramp vehicles into gaps as is possible. This means the ramp should be controlled to operate at its "controlled ramp capacity" (at the optimum setting of Fig. 4).

Second Level Control

Within the second level of the problem structure, merging control is expanded to include not just the ramp, but also variations in the speed and volume of the outside freeway lane flow and control alternatives for excessive queuing at the ramp signal. Operation of the merging controller is now functionally related to ramp service volume rather than capacity. The controller seeks to optimize the ramp service volume as the outside lane freeway operation (speed and volume) varies and the ramp signal queue fluctuates. Thus the controller setting, T_S, will move from one optimum to another as the outside freeway lane flow rate varies (Fig. 4), and as the ramp signal queue becomes excessively long the controller setting will drop to the queue-dissipating setting (Fig. 4).

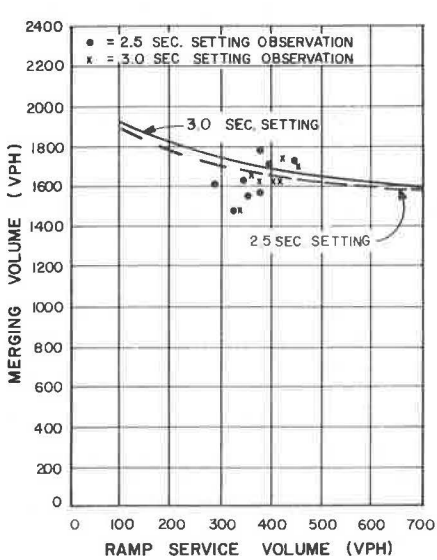


Figure 8. Observed and predicted service volume.

Third Level Control

An entrance ramp cannot often be controlled to operate at its maximum capacity without considering its effect on freeway operation in the vicinity of the ramp. Similarly, merging control will be more effective when it is responsive to capacity reductions on the freeway and to other deviations from established values of control model parameters. In the third level of merging control, controller operation is a function of ramp service volume as related to freeway capacity (Fig. 6).

Fourth Level Control

When the control of a freeway spans several ramps so that control of individual ramps becomes a problem of interrelating ramp and freeway operation over a freeway system, another level of control evolves. An overall system control concept establishes control criteria for each

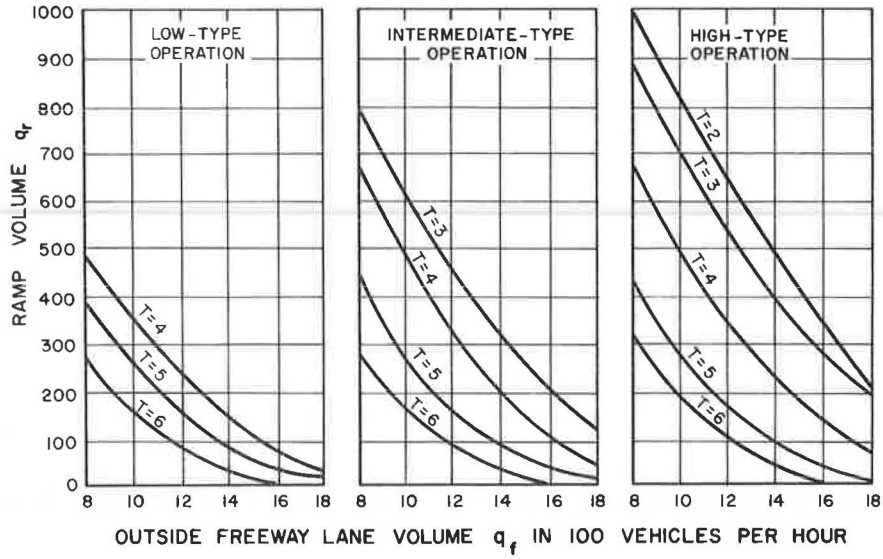


Figure 9. Curves illustrating the theory of reduced capacity operation.

ramp based on each ramp's controlled capacity, each ramp's service volume, freeway capacity constraints, freeway demand, and each ramp's demand. A linear programming model has been developed for long-time measurement periods (10). A mathematical programming model is being developed for short measurement periods (25).

APPLICATION TO FREEWAY CONTROL

Initiation of Control on an Isolated Entrance Ramp

An initial freeway control installation may consist of only a few merging controllers just as an initial traffic signal network installation may have signals at only a few intersections. Individual entrance ramps with merging operation problems may be identified and placed under control much as isolated problem intersections in a street network would be signalized. An isolated entrance ramp could be controlled with either first-level control or first- and second-level control.

If a ramp were to be controlled by a first level of control sophistication, a single service gap setting would be needed to achieve the most nearly optimum operation for expected outside freeway flow conditions during control periods. From Figure 4, the controller service gap setting would be about 2.4, 2.8, or 3.6 sec, depending on whether the ramp was considered a high-, intermediate-, or low-type ramp. This control policy would dictate operating the ramp at or near capacity.

When both first and second levels of control are installed at an isolated entrance ramp, the merging controller becomes responsive to variations in outside lane freeway flow and to the extent of queuing at the ramp signal. Figure 4 shows how the merging controller would vary its operation to achieve such responsiveness. As changes are detected in the speed and flow on the outside freeway lane, the controller service gap setting moves along the optimum setting line to maintain capacity operation (providing that demand exceeds controlled capacity) until the ramp signal queue becomes excessively long. The merging controller service gap setting is then adjusted to the queue-dissipating setting, which is shown to be 1.0 sec in Figure 4. Figure 4 shows that the ramp service volume will decrease but the congestion will be moved from the ramp signal to the ramp merging area. Such a trade-off would be appropriate when a ramp signal queue backs up into a frontage-road intersection or reduces frontage-road capacity at a critical location. Control function and hardware component interrelations to accomplish this operation are described in a companion report (12).

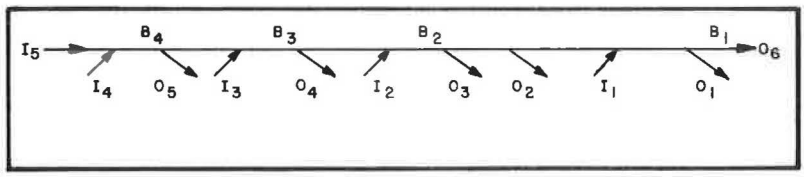


Figure 10. Schematic of freeway system control.

Integrated Multilevel Freeway Control

With a freeway control system capable of exercising various levels of control, each ramp will be controlled as previously described for an isolated ramp under the first and second levels of control (12), but each ramp will also be constrained to broader system-control optimization at higher levels of control.

If a traffic incident, a permanent geometric constriction, or an ambient condition reduces the capacity of the freeway downstream from the controlled ramp, it may be necessary to operate the merging controller so that the ramp service volume is less than optimum. This means that the third level of control will establish a service gap setting larger than the optimum shown in Figure 4. The theory of reduced capacity operation can be illustrated with Figure 9. The basic steps of controller operation are

1. Measure volume at downstream block (bottleneck capacity, Q);
2. Measure upstream freeway demand, q_f;
3. Enter abscissa at q_f and ordinate at $(Q - q_f)$; and
4. T = controller gap setting.

For example, if the ramp is an intermediate-type ramp and the bottleneck capacity of the outside lane is 1500 vph, then, with an outside freeway lane flow of 1200 vph, the allowable ramp volume is 300 vph. This locates a point below the curve for T = 4 so that the service gap setting must be about 3.5 sec or larger.

For a freeway control system to utilize the fourth level of control and achieve system optimization, the control policy must incorporate both the macroscopic (10) and the microscopic (11) approaches inherent in the various lower levels of control into a total system control policy. Messer has proposed a control model that has this capability (25).

The approach basically uses linear programming techniques to determine the optimum input (entrance) ramp flow rates on a system basis. Once the allowable input flow rates and the active input constraints are known, the appropriate service gap settings can be determined. The resulting service gap settings are then used as control inputs to the merging controllers for the particular ramps. This yields the desired flow rate (service volume) and a solution to the merging control problem. To illustrate the approach further, a freeway control example will be evaluated.

Consider the freeway schematic shown in Figure 10 as being the entire freeway system under control. Merging control is in effect at all input ramps with a central computer monitoring and controlling the total system operation.

The linear programming model used to determine the freeway inputs is

$$\text{Maximize} \sum_{j=1}^{5} X_j$$

$$\text{subject to} \sum_{j=1}^{5} A_{jk} X_j \leq B_k \qquad k = 1, \ldots, 4$$

and $$b_m P_a \sum_{j=a+1}^{5} A_{ja} X_j + X_a \leq C_m \qquad a = 1, \ldots, 4$$

and $$X_j \leq D_j \qquad j = 1, \ldots, 4$$

$$X_5 = D_5$$

and $$X_j \geq 0 \qquad j = 1, \ldots, 5$$

X_j is the input volume to the freeway system; A_{jk} is the decimal fraction of vehicles entering at input j that pass through section k; B_k is the bottleneck capacity (in this example) of freeway section k; A_{ja} is the decimal fraction of vehicles entering at input j that pass by ramp a. The percent of the total freeway volume upstream of input a, which is in the lane adjacent to entrance ramp a (25 percent was used in all cases in this example (5)) is P_a; D_j is the present demand at input j. To determine the b_m and C_m values for a particular ramp input a, it is necessary to first classify ramp a according to type—high, intermediate, low, (m). Then the b_m and C_m values are the slope and the y intercept, respectively, of the linear estimate of the ramp flow vs freeway flow capacity curves in Figure 6.

The required inputs to the model are given in Table 7, and Table 8 gives the optimum flow rates and the appropriate service gap settings for the merging controllers. These service gap settings will always be the optimum setting for the particular freeway flow unless the active constraint for that input ramp is the "bottleneck" constraint. If the "bottleneck" constraint is the active constraint, then the "throttled" gap setting as determined from the appropriate operating curve in Figure 4 is used to compute the required service gap.

TABLE 7

REQUIRED INPUT TO L-P

CLASSIFICATION OF INPUT RAMPS

INPUT RAMP	1	2	3	4
TYPE	INTERMED.	LOW	INTERMED.	INTERMED.

RAMP FREEWAY LANE 1 CAPACITY EQUATIONS

HIGH	$q_r + .770\, q_f$	≤ 1590
INTERMEDIATE	$q_r + .695\, q_f$	≤ 1320
LOW	$q_r + .494\, q_f$	≤ 840

PRESENT DEMAND

INPUT	1	2	3	4	5
DEMAND	300	80	500	400	5400

INPUT		1		2		3		4		5	
FRACTION DESTINED TO X	x	k	a	k	a	k	a	k	a	k	a
	1	1		.86	.76	.52	.60	.48	.59	.79	.82
	2			1.		.93	.93	.91	.91	.90	.90
	3					1.		1.	1.	.94	.94
	4							1.		1.	1.

TABLE 8

RESULTS OF L-P MODEL

INPUT RAMP	1	2	3	4
OPTIMUM FLOW (VPH)	380	61	233	380
BOTTLENECK ACTIVE ?	NO	YES	YES	NO
SERVICE GAP	2.86	5.47	3.92	2.89

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