The Mathematical Form of Travel Time Factors

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The travel time factor represents the effect of travel time on trip interchanges. In its use in gravity model studies, the travel time factor is an empirical hand-down curve that is considered constant over time. This paper questions constancy over time by attempting to express travel time curves in parametric form and relating the parameters to variables within the study area that are known to change over time. Driving time curves for three trip purposes were selected from ten widely scattered transportation study areas of different sizes. By curve moment techniques, the travel time factors were closely modeled by Pearson Type I and Pearson Type III curves. The travel time curves of home-based work and non-home-based trips were best modeled by Pearson I curves, whereas the factors for shorter length shopping trips were better modeled by Pearson III curves. Using factor analysis and correlation analysis for the selection of independent variables, regression equations of acceptable statistical significance were derived relating the parameters of the Pearson models to variables calculated on a citywide basis. The study found significant relations between curve parameters and such ephemeral variables as numbers of trips made, trip-making rates, car ownership, and ratios of trips made for various purposes.

•THE CURRENT use of the gravity model in the trip distribution phase of transportation planning has evoked interest in the form of the effect of distance or travel time on travel. Traditionally, research was into the effect of separation function suggested by calibrations of the form with respect to available data. This work sought to further that research, still in an empirical manner, by an examination of the travel time factors as used in various transportation studies. After observations of the form of existing travel time curves, conclusions concerning the general form of such curves could be drawn. From these conclusions, suitable parametric curves were derived and fitted to the travel time curves.

Having achieved suitable parametric fits, the parameters derived were themselves subjected to examination to determine whether relationships existed between these parameters and characteristics of the study area and its population. It appeared reasonable that if relationships between curve parameters and areawide variables could be found, then changes in parameters (and changes in the travel time curves themselves) could be predicted from changes in the areawide variables. Another area of application would arise in the generation of a travel time curve for a study at the start of the calibration phase. An initial curve could be obtained from the parameters obtained from the regression analysis on areawide variables. Current methods of estimating the travel time factor curve for the first calibration cycle generally evolve on a guess based on past experience in other study areas. An improvement of this technique would result

Paper sponsored by Committee on Origin and Destination and presented at the 48th Annual Meeting.

from a modeling approach. Such an approach would decrease the number of cycles needed to calibrate the gravity model within required limits of accuracy, with an ensuing decrease in computer and technical time involved in the calibration process.

GENERAL DISCUSSION OF THE GRAVITY MODEL

The use of the theory of gravity to describe the interaction of human populations dates back to the early nineteenth century.

Voorhees (1) utilized a gravity formulation in his "General Theory of Traffic Movement." His work used a constant exponent for the influence of distance, but Voorhees also indicated that travel time rather than physical spatial separation was a realistic measure of impedance to travel. This is reflected in current practice which uses the following form of gravity model:

$$T_{ij} = \frac{P_i F_{ij} \cdot A_j}{\sum_{\text{all } k} F_{ij} A_k}$$

where

T_{ij} = the trip interchange from i to j,

P_i = the number of trips produced at zone i destined to all zones,

A_j = the number of trips attracted to zone j from all zones, and

F_{ij} = the friction factor derived from the travel time curve for a travel time equaling that time from i to j.

For clarity of presentation the model is shown without the often necessary social-economic adjustment factors. In early work in the Washington, D. C., area, Hansen (2) used travel time as a measure of spatial separation, and found that a constant exponent was not usable. The negative exponent appeared to increase with increasing separation. This was highly apparent in the case of the work trip. The need for a variable exponent in the use of a gravity model has been found necessary in many city studies carried out since Hansen's work. The San Mateo study, for example, used travel time exponents that varied with time, ranging from 0 to 1.2. More common practice is the use of ordinates from an empirical hand-drawn "friction factor curve," equivalent to using an exponent of time that varies over the whole time range. Tanner (3) investigated the question of a constant exponential of spatial separation from a mathematical approach. It was found that it was not mathematically possible for the distance exponent to remain constant with the distance. This work indicated that short trips required, under assumptions of uniform population density, an exponent between 2 and 3, which was theoretically impossible in the case of long trips, and which led to a ridiculously high vehicle-mileage of travel in an area. Rather than a constant exponent where

$$f(d) = d^n$$

Tanner suggested a more general form of curve, the gamma function,

$$f(d) = e^{-\lambda d} d^n$$

where

$f(d)$ = the functional form of the effect of distance as it would appear in a gravity model formulation,

λ = some nonnegative constant,

d = the measure of spatial separation,

n = some constant, and

e = the base of natural logarithms.

Such a form would permit a sufficiently rapid decay of the function to prevent errors in long trip computations.

Voorhees (4) has suggested that the general gamma density function can be utilized as a parametric substitute for hand-fitted travel time factors.

CONSTANCY OF TRAVEL TIME FACTORS

An examination of range of travel time factors of various cities was made by Whitmore (5). This work found that the travel time factor could be represented by general polynomials, and that regional friction factors are similar from region to region, but vary considerably from city to city. Whitmore indicated that the best fit to travel time factors could be found with a polynomial of the form

$$f(t) = A_0 + A_1 t + A_2 \cdot \frac{1}{t-3}$$

where A_0, A_1, and A_2 are constants, and t is the travel time including terminal time. An examination of the form of function indicates that as t tends to infinity, the function itself becomes infinite. Although it gives an apparently adequate fit over certain ranges, it cannot be held as a completely rational form of the travel time factor. Possible forms should certainly have characteristics similar to that of the function suggested in Tanner's work, in which the value of the function decreases at an increasing rate with time.

It is felt that a great deal of information on the behavior of the travel time factor is being lost by current use of hand-fitted travel time factors. This problem has been recognized for some time by the Bureau of Public Roads which states:

> It is important to keep the "line of best fit" smooth and as straight as possible for the following reasons:
>
> a. Smooth curves can be approximately defined in a mathematical expression; possibly, one that is not complex.
> b. If these curves can be approximated by a mathematical expression, meaningful comparisons can be made between these expressions for different urban areas with various population and density characteristics.
> c. These comparisons would eventually help quantify, with a mathematical function, the effect of spatial separation between zones on trip interchange.

If nonparametric curves continue to be used in the gravity models of transportation studies, little advance can be made with respect to the assumption of constancy. Such an assumption is not likely to be greatly in error in a slowly developing community. In an area of considerable development, and significant social change, the assumption may well be unjustified. It is precisely within areas of radical change that the transportation modeling process is of greatest value. The use of parametric travel time curves is recommended so that information relating the form of the curve to the character of study area can be retrieved by statistical relationships. Statistical modeling is in widespread use throughout the remainder of the transportation planning process, and it is felt that it can well be extended into the modeling of the travel time factor itself. Any model used to describe the form of the travel time factor must be sufficiently flexible to fit the various shapes of the factors.

The Pearson system of curves, a highly flexible system derivable from a basic differential equation appears to have adequate variety of form to permit its adaptation for use as a travel time factor curve. Pearson curves have as many as three shape parameters and one shift parameter. All curves are derivable from the basic formula

$$f(x) = \frac{x-a}{b_1 + b_2 x + b_3 x^2}$$

where a, b_1, b_2, b_3 are constants.

PURPOSE OF RESEARCH

It was decided that this research would serve three main purposes:

1. To determine how satisfactorily this curve system could serve as travel time factors.
2. To establish relationships between the parameters of fitted curves and the characteristics of the study area and its population; and to derive suitable regression equations.
3. To indicate how well parameters derived from regression relationships would serve for initial estimates of the travel time factor.

Further advantages from the use of parametric curves occur from the fact that the calibration cycle itself can be completely computerized, thus eliminating the present methods of hand-fitting empirical curves at each stage of calibration of the gravity model. The modeling method indicated in this work is directly applicable to the fitting of a parametric curve to points, rather than to the hand-fitting method recommended by the Bureau of Public Roads.

Curve fitting was carried out by the method of moment, found to give excellent results with Pearson I and Pearson III distributions (6).

The Pearson system of curves may be derived from the basic differential equation

$$\frac{dy}{dt} = \frac{y\,(t + a)}{b_0 + b_1 t + b_2 t^2}$$

where

$y = F(t)$, the functional form,
t = the time separation, and
$a = b_0,\ b_1,\ b_2$ = constants.

This is a very general form of curve, which by proper choice of constants can insure that $y = 0$ as t tends to infinity, that $dy/dt = 0$, and that y can have a local maximum at low values of t. The equation therefore conforms with observed properties of the travel time factor.

Depending on the values of the constants in the basic differential equation, a variety of curve types are obtainable. If the roots of the denominator are real and of different sign, then the differential equation gives the Pearson I curve of the form

$$F(t) = \frac{C_1}{A^{m_1 + m_2 + 1}} \cdot B(m_1,\ m_2) \cdot (t - c)^{m_1} \cdot (A - (t - c))^{m_2}$$

where

$m_1,\ m_2,\ A$ = shape parameters;
c = a shift parameter;
C_1 = a constant, affecting only the magnitude of the curve;
t = the time separation; and
$B(m_1,\ m_2)$ = the value of the beta function with parameters $m_1,\ m_2$.

When $b = 0$, the solution to the differential equation is the Pearson III curve:

$$F(t) = C \cdot \frac{p}{A} \cdot \frac{(p+1)^p}{e^{p+1}\,\Gamma(p+1)} \cdot \left(1 + \frac{t - \mu}{A}\right) \cdot e^{-p/A(t-\mu)}$$

where

$p,\ A$ = the shape parameters;
μ = a shift parameter;
C = a constant, affecting only the magnitude of the curve;
t = the time separation; and
$F(t)$ = the functional form.

Depending on the value of the differential equation constants, eleven different types of curves result. These are examples of the Pearson system of curves. For the purposes of this work, it was found that satisfactory fit could be obtained using Pearson I for work trips and non-home-based trips, and Pearson III for shopping trips. It is of interest at this point to compare this approach from the differential equation basis with Tanner's earlier observation that the time factor could not be constant, but was probably of the form

$$f(d) = e^{-\lambda d} d^n$$

Tanner's suggested form of the travel time curve, derived mathematically, is a special case of the Pearson III distribution, which is itself a special case of the solution to the basic differential equation. The work undertaken here was therefore considered an examination of Tanner's generalized hypothesis.

DISCUSSION OF DATA

For this study, extensive use was made of data compiled in Whitmore's work, including additional data. In order that the data should be compatible for the rational application of regression equations to similar data, the studies selected were those using driving time curves with no inclusion of terminal times. Compatible data were available for ten cities for the home-based work trip, nine cities for non-home-based trips, and five cities for shopping trips. Because of the small sample for shopping trips, caution should be used in accepting results found in this limited study. Other trip purposes such as school trips and social-recreational trips could not be investigated because of the variety of ways in which these trips were treated in the available studies.

WORK TRIP TRAVEL TIME CURVES

It was found that the travel time factor for work trips could be modeled satisfactorily by the use of the Pearson I curve, which has three shape parameters and one shift parameter. The summary of the results is given in Table 1. The Pearson I curve was found to model adequately a full range of travel times that included at least 90 percent of all travel for that purpose. The range of times varied from a low value of 2 minutes to a high value of 50 minutes. In all classes the percentage of trips falling outside the upper limit of the model was sufficiently small that it could be ignored without affecting the validity of the model. No attempt was made to fit parameters at very low travel times.

TABLE 1

SUMMARY OF PEARSON I SHAPE PARAMETERS FOR HOME-BASED WORK TRIPS

Location	m_1	m_2	A	c Shift Parameter	Index of Multiple Correlation	F-Ratio of Regression
Cedar Rapids	-0.27	1.48	55.9	0.72	0.997	13,861
Waterbury	-0.72	4.16	74.2	1.71	0.995	2,593
Erie	-0.37	1.89	40.9	1.21	0.999	26,733
New Orleans	-0.73	2.84	70.7	1.93	0.997	6,288
Providence	-0.66	5.40	104.6	2.25	0.992	2,434
Sioux Falls	-0.35	0.48	15.8	0.97	0.979	831
Hartford	-0.63	3.01	60.6	1.10	0.997	5,776
Fort Worth	-0.83	3.89	54.8	2.23	0.993	1,379
Baltimore	-0.65	2.19	57.3	1.87	0.988	1,702
Los Angeles	-0.77	6.87	128.3	11.37	0.991	1,825

It is apparent that a high degree of fit has been obtained by the use of the Pearson I curve. Figure 1 shows examples of the excellent fits obtained using the Pearson I curve.

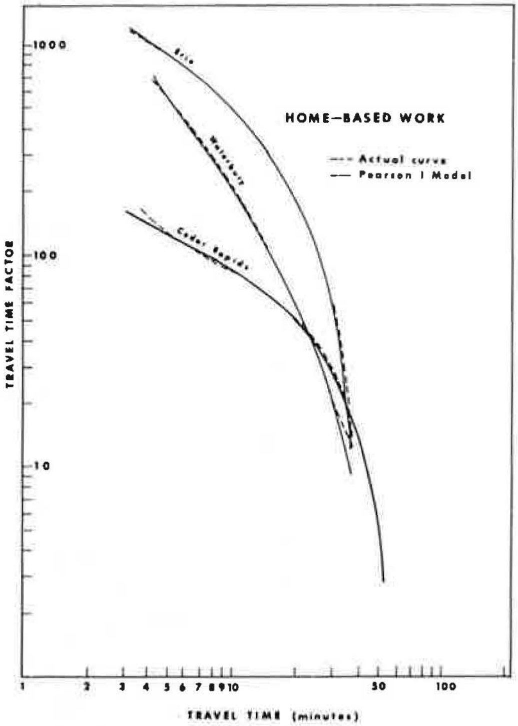

Figure 1. Examples of adequacy of Pearson I fit for home-based work travel time factors.

Relationships Between Curve Parameters and Area Characteristics

The second stage of the research dealt with attempting to find statistical relationships between the parameters of the model for travel time factors, and various citywide variables. Such a relationship would indicate possible predictability of the travel time factor curve under varying conditions, and would shed light on the assumption that travel time factors are constant with time.

Statistically significant trends were found to exist between the shape parameters of the model and citywide variables determined in the O-D studies. The selection of the variables used in the regression was based on correlation analysis, factor analysis, and the suitability of the variable for predictive purposes. Where it was possible variables involving the study area size were avoided. For predictive purposes, such variables would in general be unreliable because the inclusion of large peripheral rural areas could radically affect the value of such variables without a remarkable change on the trip characteristics. Final selection of the regression equation was also selected by minimizing the significance level of both the regression coefficients of the independent variables and the regression equation itself. This procedure was followed for all trip purposes. Table 2 and Figures 2 through 5 summarize the findings of the regression analysis for home-based work trips.

A summary of the statistical findings concerning the home-based work trip travel time factors would indicate that (a) travel time factors curves can be satisfactorily modeled with Pearson Type I distribution curves and that (b) the parameters of the

TABLE 2

SUMMARY OF REGRESSION EQUATIONS FOR CURVE PARAMETERS—WORK TRIPS

Regression Equation	Level of Significance of Regression Coefficients (percent)	Level of Significance of Variables (percent)	Correlation Coefficient of Equation
$m_1 = -0.993 + 0.000933 \times$ home-based work trips per 1,000 population	2	2	0.75
$\ln (m_2) = 3.51 - 1.74 \times \ln$ (total trips per car)	1	1	0.79
$\ln A = -4.995 \times 10^4 \times$ (total home-based work trips)$^{-1}$ + 4.52	0.1	1	0.87
$c = 2.63 - 0.0025 \times$ home-based work trips per 1,000 population	2	2	0.77

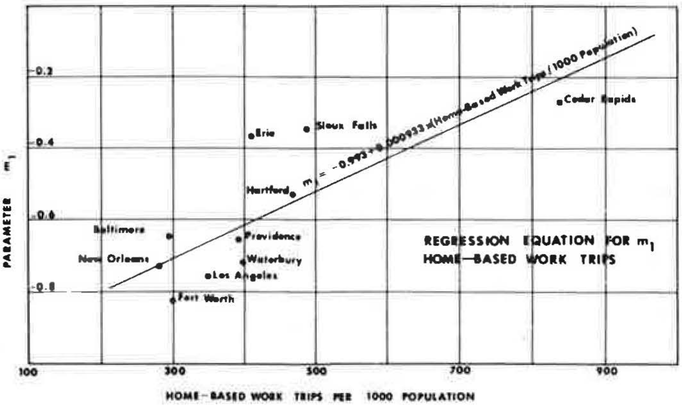

Figure 2. Regression equation for m_1, home-based work trips.

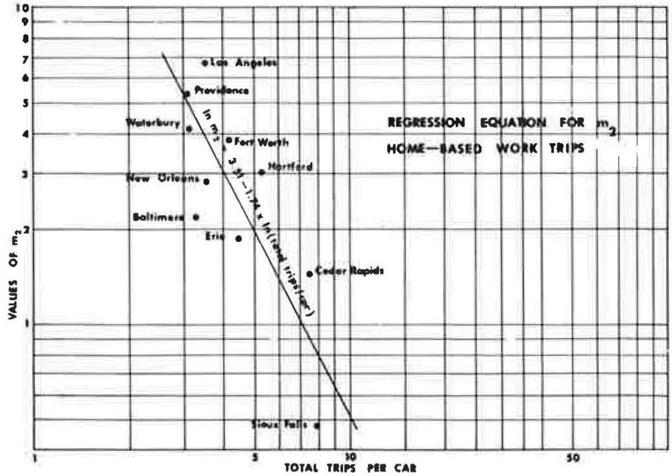

Figure 3. Regression equation for m_2, home-based work trips.

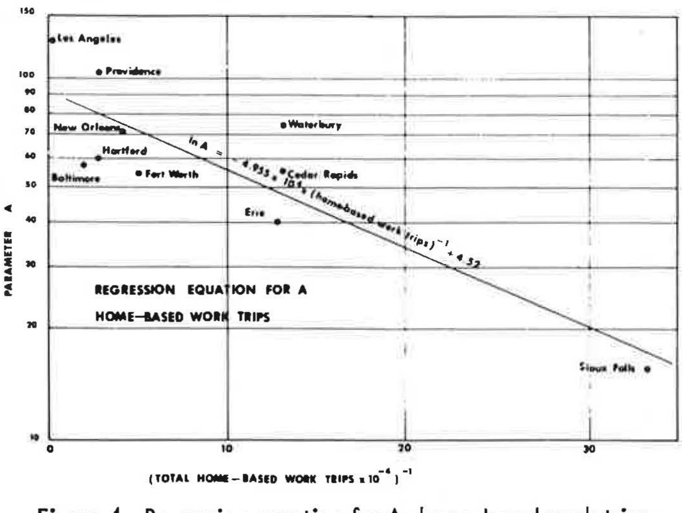

Figure 4. Regression equation for A, home-based work trips.

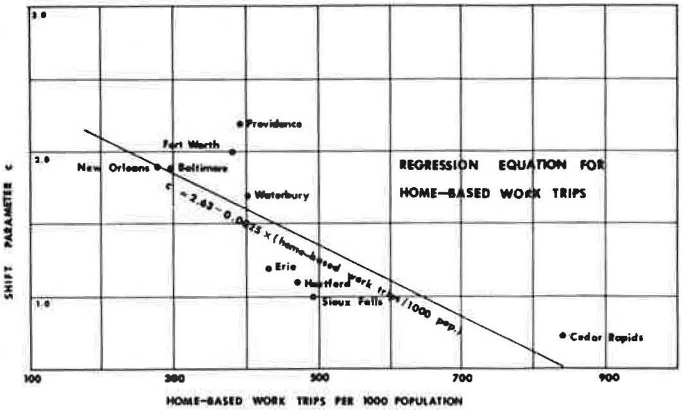

Figure 5. Regression equation for c, home-based work trips.

Pearson I models are found to be statistically related to overall citywide variables. These variables were found to be the number of home-based work trips per thousand population, the total number of home-based work trips, and the number of trips per car. From the statistical relationships found for home-based work travel time factors, it would appear that these factors may not be constant over time as is currently assumed in the calibration of the gravity models for transportation studies. Constancy over time for a particular urban area would indicate an independence of the parameters of the curve from any relationship with city variables, and any change in the trip-making patterns would be assumed to have no effect on the form of the travel time curve.

NON-HOME-BASED TRIP TRAVEL TIME CURVES

In the case of non-home-based trips, homogeneous data were available for nine cities for analysis of the travel time factor curves. It was found that the most satisfactory model for the non-home-based trip curves was the Pearson Type I distribution. Table 3 summarizes the results. This curve was an accurate model over the range of travel times that included at least 90 percent of non-home-based trips. A full range of travel times was therefore considered. In all cases, the percentage of trips falling

TABLE 3

SUMMARY OF PEARSON I SHAPE PARAMETERS FOR NON-HOME-BASED TRIPS

Location	m_1	m_2	A	c Shift Parameter	Index of Multiple Correlation	F-Ratio of Regression
Cedar Rapids	-0.42	1.95	38.7	1.02	0.999	17,097
Waterbury	-0.18	7.02	46.7	0.90	0.997	4,098
Erie	-0.58	2.89	42.6	1.42	0.994	2,135
Providence	-0.61	11.48	78.3	1.20	0.997	2,380
Sioux Falls	-0.54	0.62	16.6	1.10	0.996	3,133
Hartford	-0.91	8.05	68.2	1.07	0.990	791
Fort Worth	-0.68	3.11	48.0	1.68	0.996	2,560
Baltimore	-0.86	9.92	109.0	1.52	0.985	2,380
Los Angeles	-0.64	1.05	57.9	11.37	0.987	2,359

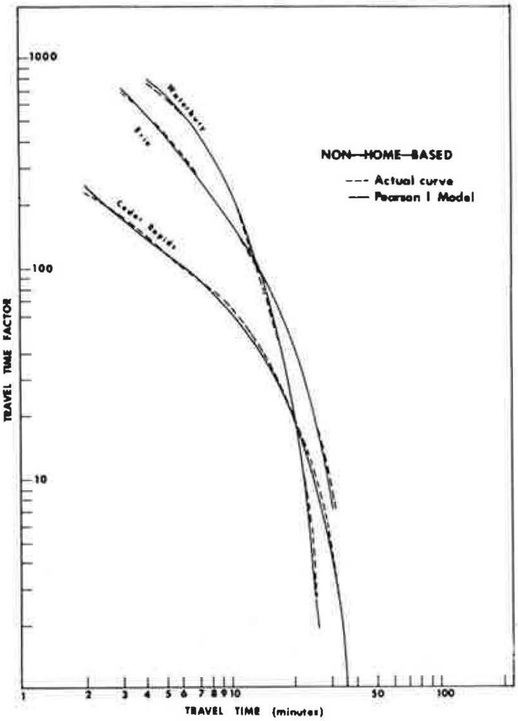

Figure 6. Examples of adequacy of Pearson fit for non-home-based trip travel time curves.

outside the range of applicability of the model was sufficiently small that the model was considered valid.

It is immediately apparent from a comparison of the model values and the actual values that a high degree of fit has been achieved with the use of the Pearson I curve. Examples of the fits obtained are shown in Figure 6.

Relationships Between Curve Parameters and Area Characteristics

The second stage of statistical modeling indicated that significant relationships could be developed between the parameters of the Pearson I models and various citywide variables. The results of the regression analysis are shown in Table 4 and in Figures 7 through 10.

The findings on the travel time factors for the non-home-based trips can be summarized as (a) travel time factor curves can be satisfactorily modeled by the use of Pearson Type I distribution curves; and (b) the parameters of Pearson I models are found to be statistically related to the following overall citywide variables: all-purpose trips per car, non-home-based trips; all trips, total number of trips, non-home-based trips per car, and non-home-based trips; study area.

The dependency of the curve parameters on independent variables would indicate that these parameters may not be constant under conditions where the independent variables noted above are projected to change during the planning period.

Caution must be exercised in the use of the regression equations developed here. The sample size of this study was relatively small, and further research would appear to be necessary to determine whether the findings can be generally applied.

TABLE 4

SUMMARY OF REGRESSION EQUATIONS FOR CURVE PARAMETERS—NON-HOME-BASED TRIPS

Regression Equation	Level of Significance of Regression Coefficients (percent)	Level of Significance of Variables (percent)	Multiple Correlation Coefficient
$m_1 = 0.479 + 0.169 \times$ (total trips per car) $- 1.56 \times \frac{\text{(non-home-based trips)}}{\text{all trips}}$	2	2	0.87
$m_2 = 6.56 + 5.86 \times 10^6 \times$ (total trips) $- 0.207 \times$ (non-home-based trips $\div$ total trips)	2, 5	2, 5	0.89
$\ln A = 6.55 - 0.417 \ln$ (non-home-based trips $\div$ study area in sq mi)	1	2	0.79
$c = 1.51 - 0.71 \times$ (non-home-based trips, trips per car)	7	7	0.71

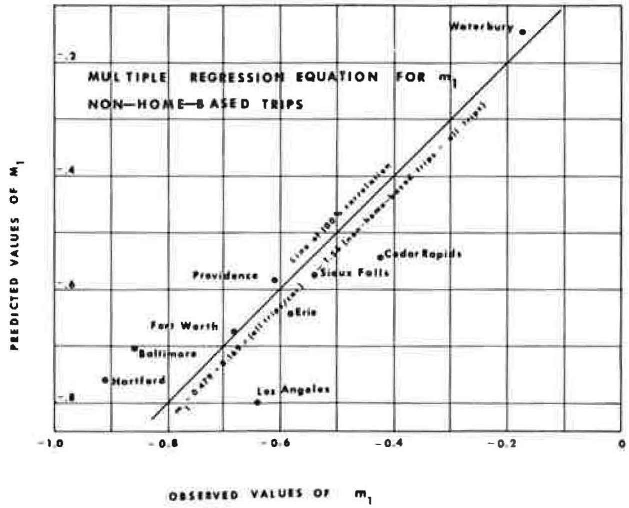

Figure 7. Regression equation for m_1, non-home-based trips.

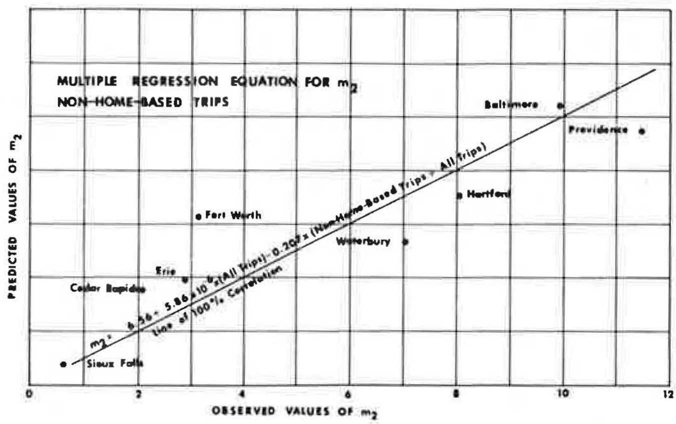

Figure 8. Regression equation for m_2, non-home-based trips.

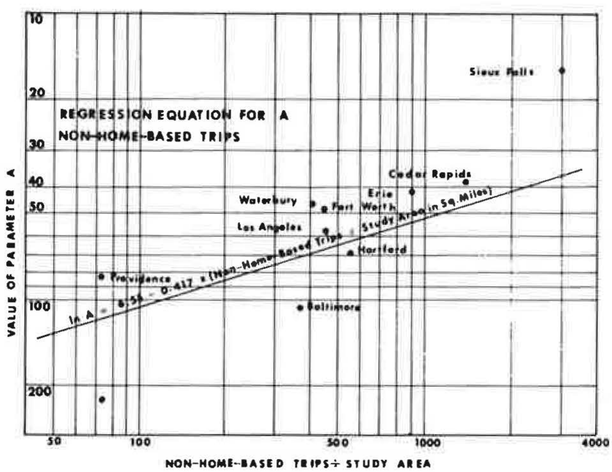

Figure 9. Regression equation for A, non-home-based trips.

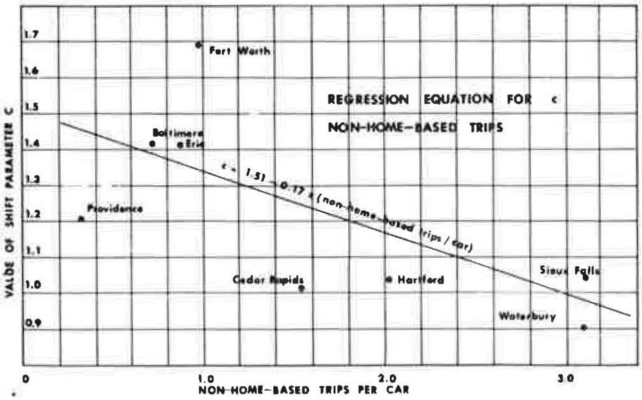

Figure 10. Regression equation for c, non-home-based trips.

SHOPPING TRIP TRAVEL TIME CURVES

The final set of travel time curves analyzed was for shopping trips. Because of the various ways in which shopping trips can be classified for study purposes, this group of travel time factors presented the smallest homogeneous sample of the set. Only five travel time curves were analyzed.

At the time of analysis, an immediate difference became obvious between the shopping trip and the two other types of trips. The median shopping trip length was 7.5 This compared with 12.2 for the work trip. It was found that the best fit to the shopping trip curves was obtained with the Pearson III curve. This distribution curve was found to satisfactorily model at least 90 percent of all trips.

TABLE 5

SUMMARY OF PEARSON III SHAPE PARAMETERS FOR HOME-BASED SHOPPING TRIPS

Location	p	A	Shift Parameter μ	Index of Multiple Correlation	F-Ratio of Regression
Waterbury	-0.35	-1.12	2.85	0.997	2,913
Erie	-0.45	-1.90	3.16	0.991	824
Providence	-0.49	-2.33	3.32	0.996	2,226
Hartford	-0.79	-4.27	1.91	0.997	1,951
Fort Worth	-0.39	-0.97	2.54	0.990	760

TABLE 6

SUMMARY OF REGRESSION EQUATIONS FOR CURVE PARAMETERS—HOME-BASED SHOPPING TRIPS

Regression Equation	Level of Significance of Regression Coefficients (percent)	Level of Significance of Variables (percent)	Correlation Coefficient of Equation
ln (A) = 1.37 + 1.28 × 10^{-3} × total trips per 1000 population	10	10	0.82
ln (p) = 1.11 + 5.10 × 10^{-7} × total trips	5	10	0.87
= 8.06 - 15.79 × (cars per person)	1	1	0.97

Table 5 compares the Pearson III model with the actual travel time factors used in the transportation studies. Excellent fit has been obtained using this form of curve. The F-ratios are high, and the index of multiple correlation is also high (Fig. 11).

The Pearson III curve used for this type of trip is a curve discontinuous at its lower end only. The curve is continuous to infinite travel times. The number of parameters needed to describe it is one less than for the Pearson I curve, which is discontinuous at both ends. The second stage of correlation for the shopping trip curves therefore amounts to the relation of the two shape parameters p and A, and the shift parameter u to the citywide variables and the curve parameters. The results of the regression analysis are shown in Table 6 and Figures 12 through 14.

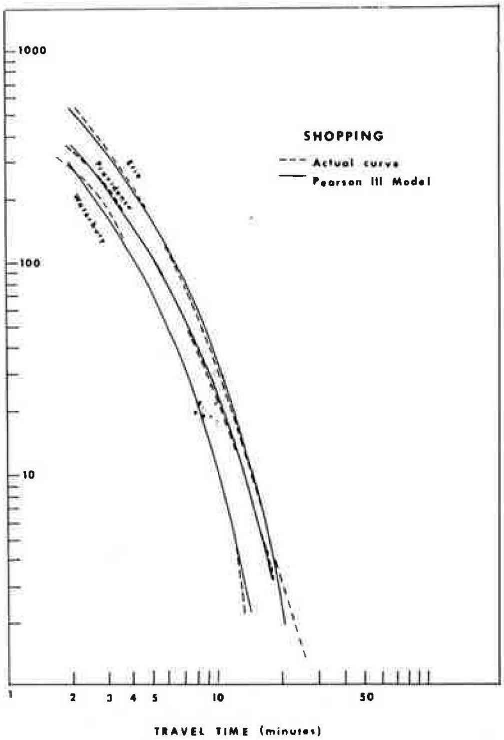

Figure 11. Examples of adequacy of Pearson III fit for shopping trip travel time factors.

Relationship Between Curve Parameters and Area Characteristics

A summary of the statistical findings concerning shopping trip travel time factor curves would indicate that (a) travel time factor curves can be satisfactorily modeled using Pearson III distribution curves, and (b) the parameters of the Pearson III curves that best fit the actual curves were found to be statistically related to the following citywide variables: total trips per thousand population, total trips, and car ownership per person. The regression equations developed indicate a method whereby the change in travel time curve parameter for a given change in travel behavior can be predicted.

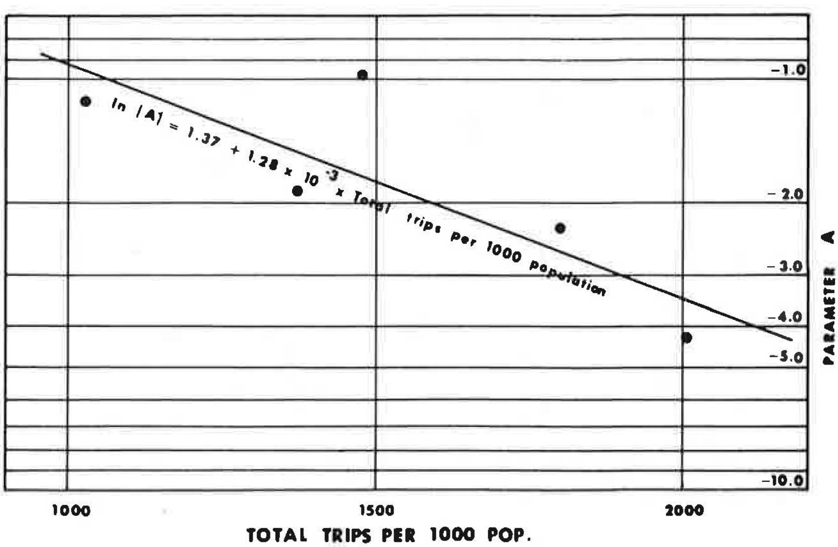

Figure 12. Regression equation for A, shopping trips.

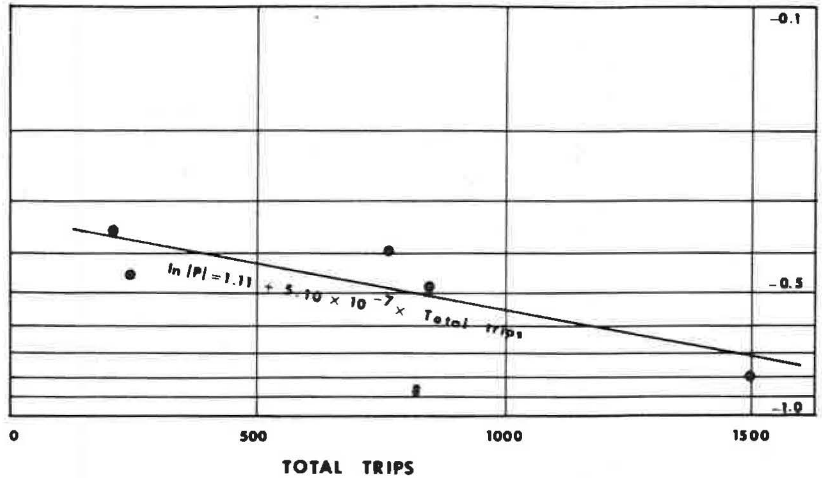

Figure 13. Regression equation for p, shopping trips.

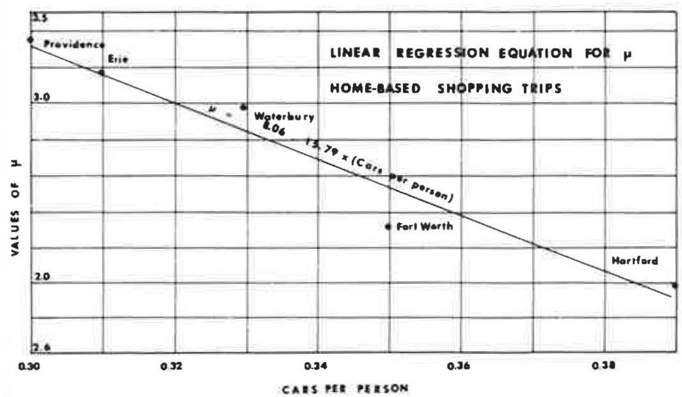

Figure 14. Regression equation for μ, shopping trips.

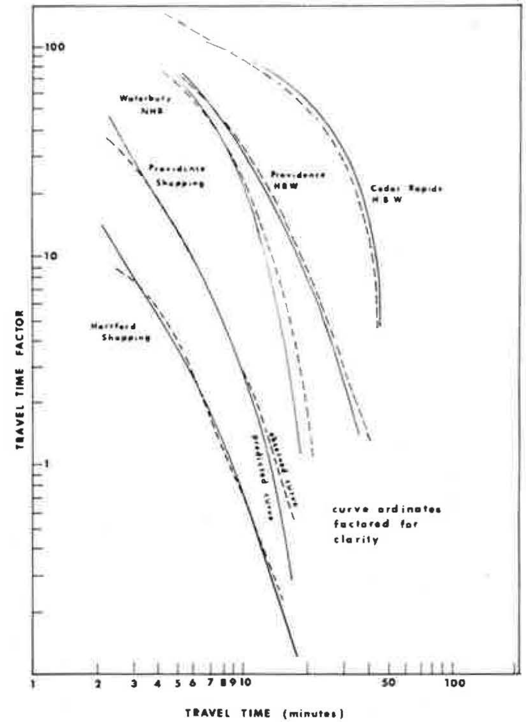

Figure 15. Predictive ability of regression equation parameters.

PREDICTIVE ABILITY OF REGRESSION EQUATION

To determine how well the derived regression equations could be used for predicting travel time curves, an analysis was made comparing curves from regression-derived parameters with actual travel time curves. The results of the analysis for the three trip purposes are shown in Figure 15. For clarity the curves are standardized on convenient ordinates. It is apparent that the regression-derived parameters in some cases gave curves that would have been adequate for final gravity model calibrations, whereas in other cases the curves would have sufficed only for initial estimates of the curves. It would appear that the derived regression equations in Tables 2, 4, and 6 are likely to give good first estimates of travel time curves.

CONCLUSIONS

1. Empirical travel time factor curves for transportation studies can be closely approximated by parametric curves of the Pearson system. The parameters are best estimated by curve moment procedures.
2. Home-based work and non-home-based travel time curves are best modeled by Pearson I curves. Pearson III curves were found to provide a better model for the shopping trip that has a lower mean trip time than home-based work or non-home-based trips. It would appear that the Pearson I distribution provides a better fit to those travel time curves in which the change of time exponent with time is most apparent.
3. Statistical relationships were found between the parameters of the Pearson models and pertinent citywide variables. Among those variables related to model parameters were home-based work trips per thousand population, total trips per car, total home-based trips, ratio of non-home-based trips to all trips, total trips in the study area, non-home-based trips per car, cars per person, and total trips per thousand population.
4. Significant regression equations between study area variables and model parameters can be calculated, indicating that there is statistical probability that these parameters are not constant, but are likely to change as the character of the area itself is modified. Such modification would be reflected in a change in the areawide variables.
5. The regression equations can be used without serious error for a first approximation of the travel time factors. Modification of the initial estimate of travel time curves can be effected in a manner similar to the method suggested in the Bureau of Public Roads Manuals, except that the new curves should be computed by moments rather than by the hand-fitting method currently recommended. This would enable the formation of a data bank of mathematical expressions for travel time curves and study area characteristics so that "meaningful comparisons can be made between these expressions for different urban areas with various population and density characteristics." This curve-fitting technique is easily programmed for high-speed computers, and will speed the present gravity model iterative fitting techniques.
6. The Pearson I and III shape parameters were not found to be highly sensitive. Small errors or small changes in parameters did not give radically different curves.

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Discussion

SALVATORE J. BELLOMO, Alan M. Voorhees and Associates, Inc.—In reviewing the work of Covault and Ashford, certain findings and implications were made regarding use of Pearson I and III statistical relationships for estimating travel time factors for urban transportation planning studies. This discussion will be related to certain of their major conclusions and what I see as shortcomings that would require a limited use of their work, and as applications of their research that will definitely improve our ability to do travel forecasting at a lower cost in our small- and medium-sized urban areas.

It appears that the statistical measures given, such as the index of multiple correlation and F-ratio, are based on a comparison of actual versus estimated travel time curves rather than on a comparison of the actual versus estimated distribution of trips. Research done as part of Factors and Trends in Trip Lengths (4) indicates that any procedure to estimate travel time factors must be sensitive to the first 10 to 15 minutes of travel, the time period in which most trips are made. Figure 15 of the authors' paper shows this comparison of travel time factors and indicates that the Pearson distribution does not fit this critical part of the curve, especially in the first 10 minutes. Because Alan M. Voorhees and Associates, Inc., conducted half the studies mentioned, it well recalls the sensitivity of these travel time factors in this critical area on the synthesis of the trip length distribution.

The applicability of the Pearson distribution formulas based on the areawide characteristics mentioned would be quite dubious in larger metropolitan areas where mode choice and mass public transportation are currently being evaluated. In these studies, person travel time factors are usually developed, and current research on trip lengths has indicated that these travel time factors may be affected by mode as well as route choice. Furthermore, the areawide characteristics, such as car ownership in urban areas with potential mass transit use, would not be applicable to synthesizing travel time factors for people who have no car. Does their travel time curve really depend on areawide characteristics? In conducting research for Factors and Trends in Trip Lengths, it was found that travel time factors within our metropolitan area may be influenced by the spatial arrangement of trip opportunities. This is illustrated in Figure 16 which shows variations in travel time factor curves based on the mean opportunity length and in Figure 17 which indicates the relationship found between the shape parameter of the gamma distribution used to synthesize travel time factors in that research project and the mean opportunity length for several selected zones in the Washington area.

Concerning the sensitivity of the parameters of the Pearson distribution and their effect on the synthesis of the travel time curve, no analysis was conducted by the authors. Varying the areawide characteristics, based on reasonable changes that might occur, should be made to see their effect on travel time factors and on the trip length distribution. In addition, the results of this research should be checked for their applicability over time using historical rather than cross-sectional data. It should be noted that these tests were conducted over time for Washington and Baltimore. No appreciable change in travel time factors was found for these metropolitan areas. Studies are currently being made in Detroit to see if the travel time factors for that metropolitan area change over time. If they do, the effect of these changes on the actual trip length distribution will be determined.

From a practical standpoint, the recommendation to use Pearson distribution parameters and a computer because of cost savings seems unsupported and unwarranted. A recycle of a gravity model costs less than a Pearson III statistical calculation does when one considers staff time, programming, and the cost of turnaround on calibrating gravity models. (To run a gravity model for a 180-zone system costs about $50.) Furthermore, a program is currently available that calculates unsmoothed travel time factors and plots actual and estimated trip length distribution given the O-D trip table and zone to zone travel time or costs as inputs. The computer running time with this program is low.

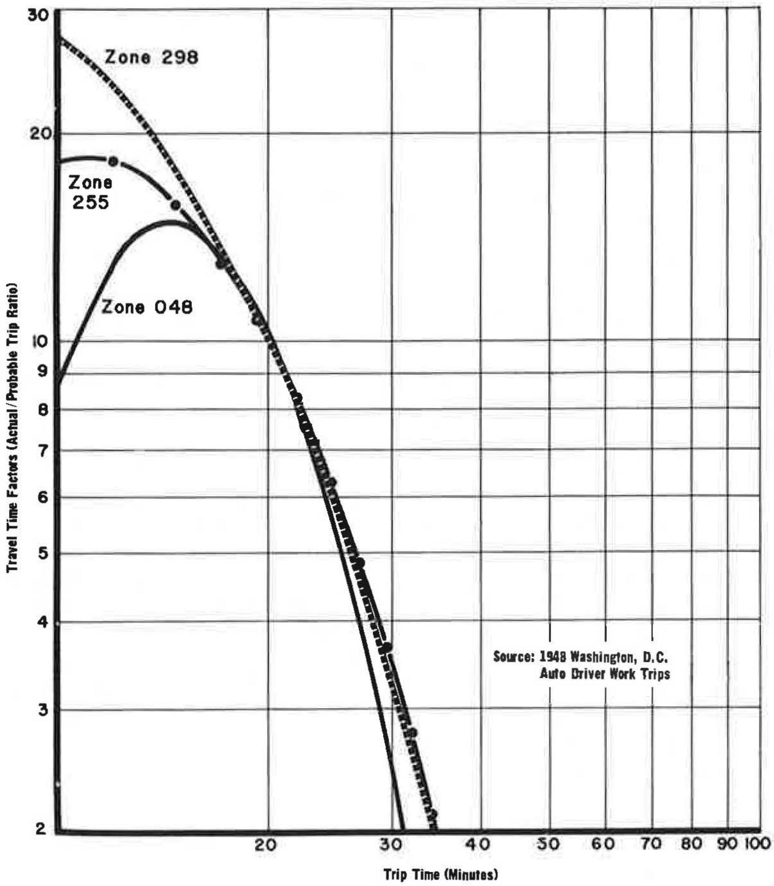

Figure 16. Travel time factors vs trip time for three selected zones in Washington, D.C.

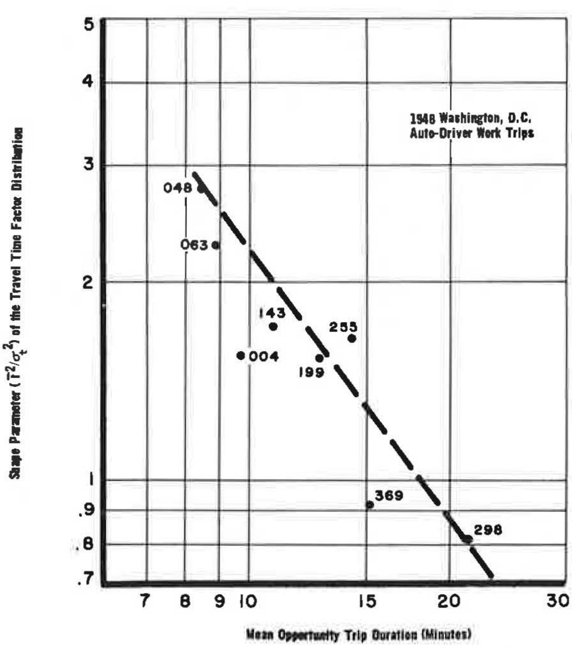

Figure 17. Shape parameter vs mean opportunity trip duration.

The idea of collecting parameters for the Pearson distribution for research purposes is unwarranted, and it would not be applicable to be undertaken by operating transportation agencies. More research and support are required before this work is required by the DOT of on-going studies.

The applicability of these research findings, as I see it, would probably be in our small- and medium-sized cities with small modal split where data collection of costly O-D data could be reduced in lieu of a travel time factor simulation based on areawide characteristics. What could be done is as follows:

1. Covault and Ashford's technique might be used in estimating travel time factors based on areawide characteristics.
2. A small sample O-D survey could be used to establish trip generation rates, and the trip length distribution.
3. A thorough land use and land activity survey could be applied to the trip rates to establish trip productions and attractions.
4. A gravity model run could then be made and the mean and standard deviation of the resulting trip length could be checked against the small sample O-D trip length.

These steps could reduce the cost of data collection and could produce a model more quickly while the transportation and land-use issues are being faced by the community. However, before this is applied, it should be rigorously tested in pilot studies by a transportation study agency working closely with the Department of Transportation. These studies should be conducted for an area of about 50,000 population and for one of about 150,000 population; they should compare alternative ways of developing transportation models in less time and at a lesser cost.

In conclusion, I have attempted to pinpoint what I felt to be areas which needed additional support while at the same time reporting what I thought would be an area of application for their findings in our transportation planning studies. I commend the authors on their work and hope they find this discussion constructive to their research effort.

DONALD E. CLEVELAND, University of Michigan—Ashford and Covault have appropriately directed their efforts toward a problem of some concern to those who use the gravity trip distribution model, the problem of the time stability of the relative attractiveness of destinations at varying distances from the trip origin. Until now little direct evidence has been presented to support or refute the contention that this function is constant over time.

The main thrust of their paper is that (a) there is value in determining a relatively simple mathematical function that can be used to express the relative attractiveness of destinations as a function of travel time; (b) parameters for the appropriate mathematical function can be easily determined; (c) the extent of correlation of these parameters with other study area variables for a cross-sectional sample of data from several cities can be determined, and the existence of a causal relationship inferred; and (d) the time stability of the correlated study area variables is apparent. If they are not stable over time, neither are the travel time factors. If they are stable, so are the travel time factors.

The authors worked with 24 sets of data classified by three trip purposes and developed in several cities. They concluded that the travel time function is not constant. My brief discussion raises certain questions of interest to those who wish to study the details of this procedure and concludes with a statement concerning the validity of the approach adopted by the authors.

1. In the statistical analysis it would be of interest to learn of the variables that did not correlate with the travel time factor parameters.

2. The way in which the authors resolved a number of difficult questions associated with fitting parameters to general Pearson types would be of particular interest to the student of curve fitting. Weighting procedures used would be of particular value.

3. The method of moments is not always an efficient method of obtaining parameters. Some comment on this is requested.

4. In discussing the development of an appropriate mathematical function for travel time factors, it is stated that the value of this function should decrease at an increasing rate with time. It is not clear to this reviewer why this should be the case. In fact, it would appear that this decline with separation would lessen at the larger separations.

5. It is well known that long trips contribute a disproportionate share of vehicle mileage on high-type facilities. What fraction of vehicle miles of travel is covered by the area of satisfactory fit of the Pearson I distribution?

6. It would appear to this reader that travel time factors would be particularly sensitive to spatial variables. If a region were to double in diameter, then it would be expected that there would be some travel between places separated by more than the original diameter of the area. Such a response would appear to require a spatial representation in the areawide variables used in the regressions. Previous studies bear this out.

Finally, it is my opinion that the approach used in this paper, one of inferring a causal relationship as a result of rather weak correlations of cross-sectional data, is weak and that those who believe in the variability of travel time factors will need stronger evidence than that presented here.

NORMAN ASHFORD and DONALD O. COVAULT, Closure—In closing the discussion, the authors wish to comment briefly on some of the important points brought out by Messrs. Bellomo and Cleveland.

The stability of travel time factors over time is an assumption that has been made with little supporting evidence. While the main thought of this paper was an examination of the form of the travel time factor, relationships were found to exist which should cause the planner to seriously question the traditionally accepted time stability.

Sensitivity analysis was carried out with respect to all shape parameters of both Pearson I and Pearson III curves. This analysis indicated that all shape parameters were insensitive, and the curve form was, therefore, applicable.

The method of moments is an efficient method of curve-fitting where the data points to be fitted closely identify with the curve form being used. The close agreement between actual curves and parametric curves lead the authors to believe that this method was justifiable for this work. The ability of the method to closely simulate the observed rapid increase in the logarithmic negative slope for smaller cities would indicate that the form selected was suitable.

The authors agree that some reworking of existing programs would be required to develop parametric curve fits. It is not felt that this is a major drawback which should discourage planners from adopting parametric forms in lieu of hand-drawn curves.