Analysis of Concrete Slabs on Ground Subjected to Warping and Moving Loads

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A theory has been developed whereby stresses and deflections could be calculated for a series of rectangular slabs lying on a viscoelastic foundation and subjected to a moving load. The stresses and deflections are caused by the weight of the slab, the moving concentrated load, and the linear temperature (or moisture) variations that cause sufficient warping so that the slab is only partially supported by its foundation.

The support conditions were simulated by a Kelvin viscoelastic model, and zones (which depended on the value of subgrade reaction) were set up so that the solutions to the governing differential equations could be reduced to a set of simultaneous algebraic equations. The equations were solved with the aid of an IBM 7090 digital computer using a FORTRAN source program.

It was found that when partial support caused by warping exists, the tensile stress in the slab can increase with increasing velocity of load. Moreover, the maximum deflection (downward) need not occur when the velocity of the load is equal to zero. The reduction in subgrade support over a narrow region (8 ft or less) leads to deflections and stresses that are higher than those calculated using the initial value of subgrade reaction. This is particularly evident when the load is over the region of reduced subgrade reaction.

•IMPROVEMENTS in the performance of concrete pavements for highways and airports have been for many years the concern of civil engineers interested in these problems. This concern has been due not only to the ever-increasing cost of construction and maintenance but also to the preponderance of cracks that exist in pavements.

The development of these cracks depends on several factors, such as type of subgrade, deterioration of the concrete, temperature effects, and load (52), and, although they may or may not represent a failed condition, they do indicate deficiencies in the analysis, design, or construction of highway pavements. Cracks not only detract from the general appearance of a highway but also often lead to driving discomfort and pavement deterioration, especially when load transfer and subgrade support are lost.

Regarding the construction of concrete pavements, advancement in the development of the principles of soil mechanics has limited the use of poor subgrades and has virtually eliminated the use of soils susceptible to frost action and pumping. In addition, specifications that are geared to promote the use of sound concrete and the practice of good construction methods have become widespread (1, 47).

Refinements in construction procedures have been accompanied by development of methods of monitoring the factors influencing concrete pavements (15, 16), and strains as well as deflections of pavements can be determined for both static and dynamic loadings (11, 18). Temperature differences between the upper and lower faces of the slab can

Paper sponsored by Committee on Theory of Pavement Design and presented at the 48th Annual Meeting.

be measured with the use of thermocouples (51), and variations in moisture content can be established from the dielectric properties of concrete (3).

Today, the problem of material weakness has been reduced, if not solved, and significant progress has been made in the ability to build pavements and to measure their important properties. However, little has been done to upgrade the methods of analysis. Current procedures for designing and evaluating pavements (52) are still based on static loads and, except for introducing equivalent static loadings (2), they do not account for the dynamic response of the pavement to moving loads. Moreover, the pavement is usually assumed to be fully supported at all times even though this is often not the case (11, 21, 22).

The present trend in transportation is toward heavier loads and higher speeds and consequently there is an increasing need to be able to predict performance. This can be done only when more is known about the input and causal factors of traffic loads, curling, and loss of subgrade support. It is to help satisfy this need that this research was undertaken.

BACKGROUND

In the early stages of development, pavement design and road construction consisted of a collection of rule-of-thumb procedures based on empirical knowledge. As early as 1901, test roads were being used to determine the best type of pavement for the prevailing traffic (38). Between 1920 and 1940, much work was done on the classification of soils (5) and highway engineers were able to correlate pavement performance with subgrade types.

During World War II engineers were faced with the problem of designing pavements for greater wheel loads than previously thought necessary. Because they could not afford the long period of time that was required to develop design procedures based on past experiences, they sought a more rational basis of design. This search ultimately led to procedures that form the core of pavement design today.

Static Load Solutions

In 1884, Hertz (13) first published a solution to the problem of elastic plate on a Winkler-type foundation (50). However, it was not until the advent of Westergaard's solution in 1926 that a highway pavement was treated in this manner and the problem was approached from a purely theoretical point of view. Today, Westergaard's work still forms the basis of the analytical bent in pavement design.

Westergaard (48) solved the problem arising when a slab is fully supported on a Winkler foundation and subjected to static loads applied at the interior, free edges, and corners of the slab. Later, Kelley (23), Spangler (43), Pickett (34), and Westergaard (49) himself extended these original solutions to account for linear temperature variations. In 1957, Freudenthal and Lorsch (10) used the three fundamental models (Maxwell, Kelvin, and Standard Solid) to study the problem of an infinite beam on a viscoelastic foundation. Then in 1958, Hoskin and Lee (19) solved the problem of an infinite plate on a viscoelastic foundation.

All of these solutions neglect the shearing forces generated at the pavement-base interface. Several mechanisms have been offered to account for this effect. Filonenko-Borodich (9) considered a set of springs held together by a membrane, whereas Schiel (42) took a fluid that exhibited surface tension as his soil model. Pasternak (32) and Kerr (25), on the other hand, considered a beam of unit depth resting on a bed of interrelated springs as a foundation, and Pister and Williams (33) used the shear interactions suggested by Reissner (40). Perhaps the most realistic approach offered to account for shear in an elastic base is that given by Klubin (26), who expressed the pavement reaction by an infinite series of Tschebyscheff polynomials, which also accounts for the two elastic constants (modulus of elasticity and Poisson's ratio).

It should be noted that Klubin's solution is in fact an elastic solution and the others, which impose Winkler assumptions, are not. In spite of this, because the Winkler foundation affords a much simpler analysis and generally gives good agreement with field data, it will no doubt remain popular.

All of these analyses are based on the assumption that the slab maintains contact with its support at all points and at all times. Experimental and field studies (16, 21, 22) have shown this assumption to be seriously in error, and a few investigators have accounted for the effects of only partial support. In 1959, Leonards and Harr (28) solved the problem relating to a partially supported slab on a Winkler foundation subjected to linear temperature and/or moisture variations. Later, Reddy, Leonards, and Harr (39) extended this analysis by introducing nonlinear temperature variations as well as a viscoelastic foundation. In all of these analytical procedures, only symmetrical, statically applied loads were considered.

Dynamic Load Solutions

For many years, the problem of determining the stresses and deflections in a vibrating plate has been of interest primarily to mathematicians. Raleigh (37) and Lamb (27), using the classical beam theory developed by Euler (7) and Bernoulli (4), studied several problems dealing with the vibration of bars, membranes, and plates. Later, Ritz (41) elaborated on this work and made significant contributions toward the study of vibrating rectangular plates.

Recently, engineers have felt the need to account for the dynamic response of pavements, and several solutions have evolved. Pioneering these solutions was the work of Timoshenko (45), Hovey (20), and Ludwig (30) in their studies of the dynamics of rails subjected to moving loads. In 1943, Dorr (6), using Fourier integrals, extended the idea to a beam, but in all of these solutions the foundation was represented by a Winkler model that exhibited no viscous effects.

In 1953, Kenney (24) added the effects of linear damping and, by means of the method of undetermined coefficients, was able to examine the relationship between deflections and critical velocity. In a more recent work, Thompson (44) elaborated on Kenney's solution and showed that the solutions for the deflections fall into three distinct regimes. Specifically, Thompson showed that these regimes were defined by the value of the discriminant of the fourth-order characteristic equation. If the value of the discriminant as defined by

$$\Delta = 16\,[4\,(1 - \epsilon^2)\,\theta^8 - (8 - 36\epsilon^2 + 27\epsilon^4)\,\theta^4 + 4] \qquad (1)$$

where ϵ = the damping ratio, and θ = the velocity ratio, is greater than zero, the characteristic equation has no real roots. If Δ is equal to zero, there is one real, double root; and if Δ is less than zero, there are two real, unequal roots.

The solutions presented by Thompson demonstrate that at static conditions the deflection curve is symmetrical (with maximum deflection occurring under the load); but as velocity increases, the point of maximum deflection falls farther and farther behind the load. Upward deflections occur behind the moving load when Δ is greater than zero; however, when Δ is less than zero, there are no upward deflections (i.e., the deflected surface does not intersect the axis of zero deflection, but simply approaches it at some great distance behind the load).

Several investigators have also considered the road loading system as an interaction between two major components that are interdependent, namely vehicles and roads. Fabian, Clark, and Hutchinson (8) examined the elements of each component and developed some basic mathematical models. Analysis of their vehicle subsystem showed that the magnitude of the dynamic load is a function of vehicle dynamic properties and apparent road profile and may, in fact, be significantly greater than the static load. Quinn and De Vries (35) used an experimental procedure to determine the highway and vehicle characteristics and showed that, if these quantities are known, the reaction of a vehicle on a highway can be predicted.

In general, most of the analytical studies reported in the literature on moving loads on pavements (52) do not account for the interaction between vehicle and road. For the most part these reports show that, for fully supported pavements, the lower the velocity the greater the deflections and stresses in the slab. Measurements of displacement (15, 17, 18) generally support this finding; however, some data (11) do exist to suggest

the possibility that displacements may increase at velocities greater than 30 to 40 mph. Also to be noted is the fact that tests on tire forces on pavements (36) indicate that the "average" force produced on the pavement by a moving load increases with velocity.

Obviously, there are several interdependent factors, such as the vehicle's suspension system, tire pressures, and road profile, that should be included in the analysis of pavements. However, at the present time, the interaction between vehicle and pavement is not well understood. In spite of this, further insight into the highway problem can be gained by examining some of the simplifying assumptions that appear in current analyses of highway pavements. Thus, it is the object of this paper to obtain by analytical means the stresses and deflections in a partially supported concrete slab when subjected to a moving load. Such partial support may be caused by temperature and moisture gradients or by the weakening and partial or complete loss of subgrade.

FORMULATION OF PROBLEMS

Considered first is the problem of a slab lying on a foundation while subjected to moving loads as well as to temperature (and/or moisture) gradients that would cause upward curling. (Upward curling is understood to exist when temperature gradients cause the ends of the slab to rise vertically.) Second, an investigation will be made of weaknesses in the subgrade caused by conditions where (a) water has infiltrated under the pavement from the side of the roadway, or (b) a joint has opened or a crack has occurred in a warped pavement to such an extent as to enable it to regain complete contact with the foundation. When such an opening exists, water may infiltrate through the surface of the pavement. Under both conditions, the infiltration may weaken the subgrade and may eventually lead to pumping. The following assumptions are made for both problems in order to render them tractable:

1. A highway pavement can be represented by an array of rectangular plates.
2. The usual assumptions of plate theory hold; that is, the plate is homogeneous, is isotropic, and obeys Hooke's law; deflections are small in comparison to thickness; plane cross sections normal to the middle plane in the unstressed condition remain normal to this surface after bending; and the effects of rotatory inertia and shear deformation can be neglected.
3. The highway base material acts like a set of linear viscoelastic elements. The inertia of the material is neglected.
4. External forces acting on the plate are those caused by a constant-velocity line load and gravity.
5. The plates are subjected to changes in temperature (and/or moisture) that vary linearly with depth. The variation in temperature is constant on all planes parallel to the upper and lower plate surfaces and is independent of time.

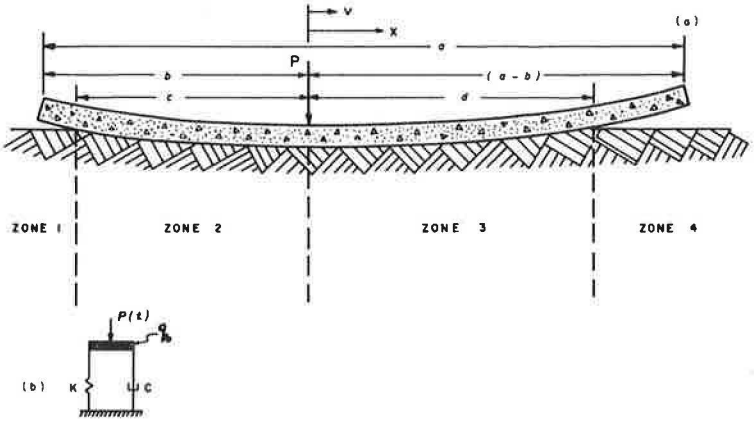

Figure 1. Section of warped slab.

PROBLEM OF PARTIAL SUPPORT CAUSED BY WARPING

In addition to the foregoing assumptions, further assumptions regarding the interaction between slabs are required. For the case studied here, it is assumed that no bending moment exists between slabs (this will be discussed later). However, some load transfer can be accounted for by specifying a shearing force between slabs equivalent to that existing in an infinite slab.

In general, the governing differential equation describing the pavement section illustrated in Figure 1 can be expressed as follows (31):

$$D\left(\frac{\partial^4 w}{\partial x_1^4} + 2\frac{\partial^4 w}{\partial x_1^2 \partial y_1^2} + \frac{\partial^4 w}{\partial y_1^4}\right) + \rho H \frac{\partial^2 w}{\partial t^2} = q(x_1, y_1, t) - p(x_1, y_1, t) \tag{2}$$

where

D = flexural rigidity of the slab,
w = midplane deflection of the slab (positive downward),
x_1, y_1 = fixed coordinates,
ρ = density of the slab,
H = slab thickness,
q = surface loading,
p = foundation reaction, and
t = time.

Using assumption 3, the foundation reaction, p, may be expressed (Fig. 1b)

$$p(x_1, y_1, t) = C\frac{\partial w}{\partial t} + Kw$$

where C is the damping coefficient and K is the modulus of subgrade reaction. Applying the loading conditions implied by assumption 4, and assuming that the deflection does not vary in the y_1 direction, Eq. 2, for a constant cross section of pavement, becomes

$$D\frac{\partial^4 w}{\partial x_1^4} + \rho H\frac{\partial^2 w}{\partial t^2} + C\frac{\partial w}{\partial t} + Kw = q_0 + P(x_1, t) \tag{3}$$

in which q_0 = unit weight of slab and P = the moving line load.

Employing the transformation, $x = (x_1 - vt)$, the equation reduces to a function of only the one variable, x. If in addition the moving load, P, is introduced with the boundary conditions, the following differential equations are obtained:

For zones 1 and 4,
$$D\frac{d^4 w}{dx^4} + \rho H v^2 \frac{d^2 w}{dx^2} = q_0 \tag{4a}$$

For zones 2 and 3,
$$D\frac{d^4 w}{dx^4} + \rho H v^2 \frac{d^2 w}{dx^2} - Cv\frac{dw}{dx} + Kw = q_0 \tag{4b}$$

The boundary conditions for the indicated zones may be summarized as follows:

$$M_1(-b) = 0 \tag{5a}$$

$$V_1(-b) = \overline{V}(-b) \tag{5b}$$

$$w_1(-c) = 0 \tag{5c}$$

$$w_2(-c) = 0 \tag{5d}$$

$$w_1'(-c) = w_2'(-c) \tag{5e}$$

$$w_1''(-c) = w_2''(-c) \tag{5f}$$

$$w_1'''(-c) = w_2'''(-c) \tag{5g}$$

$$w_2(0) = w_3(0) \tag{5h}$$

$$w_2'(0) = w_3'(0) \tag{5i}$$

$$w_2''(0) = w_3''(0) \tag{5j}$$

$$w_3'''(0) - w_2'''(0) = P/D \tag{5k}$$

$$w_3(d) = 0 \tag{5l}$$

$$w_4(d) = 0 \tag{5m}$$

$$w_3'(d) = w_4'(d) \tag{5n}$$

$$w_3''(d) = w_4''(d) \tag{5o}$$

$$w_3'''(d) = w_4'''(d) \tag{5p}$$

$$M_4(a-b) = 0 \tag{5q}$$

$$V_4(a-b) = \bar{V}(a-b) \tag{5r}$$

Solution of Problem

To obtain the solution for the case where the warped slab is as representd by Figure 1, the value of Δ (Eq. 1) is determined and Eqs. 4a and 4b are solved to obtain expressions for the deflections and stresses in the slab (29). The constants in the resulting set of nonlinear equations are then evaluated using the conditions listed in Eq. 5. In solving the set of nonlinear equations, an initial estimate of the solution was made using the fully supported slab. Then a method of functional iteration equivalent to an N-dimensional Newton's method was used with an IBM 7090 computer. With the constants determined, the stresses and displacements were then obtained. Using this same procedure, the other deflection patterns resulting from the moving load were analyzed (Fig. 2).

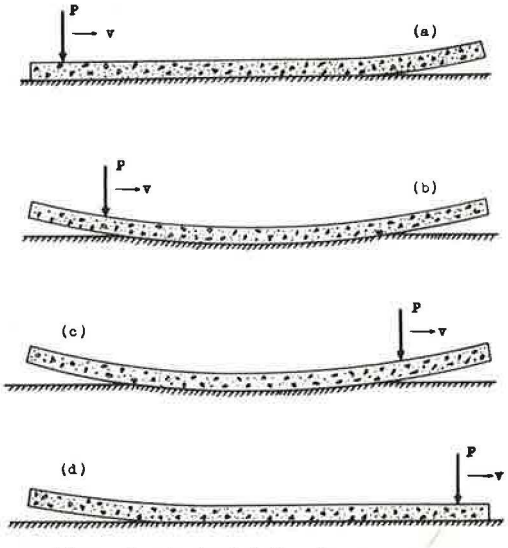

Figure 2. Typical deflection patterns.

Results

Because of the large number of variables involved, it is impractical to present the results for all cases considered. Instead, numerical results are given for a few typical cases to illustrate general trends and for purposes of comparison with results previously obtained by others.

Using the left edge of the slab as origin, the moving load was considered at intervals of 4 ft and stresses and deflections were determined for combinations of the following data:

$\mu = 0.15$
$E = 4 \times 10^6$ psi
$a = 480$ in.
$H = 8,\ 10,\ 12$ in.
$K = 100,\ 200$ pci
$C_r = 1.5,\ 2.0$
$v = 0,\ 20,\ 40,\ 60,\ 80$ mph
$P = 125$ lb/in. (the value of P was determined using an axle load of 18,000 lb over a pavement 12 ft wide)
$q_0 = 150.9$ pcf
$\Delta T = 20,\ 30,\ 40$ F
$\beta = 6 \times 10^{-6}$ in. per in. per deg F

Typical curves for deflections and stresses are shown in Figure 3; Figures 4 and 5 show how maximum deflection and maximum stress vary with velocity when the difference between slab surfaces is 20 and 40 F. A record of the movement experienced by the ends of the slab is shown in Figures 6 and 7 as a function of the load position.

Discussion of Results

In developing the theory for this part of the analysis, it was assumed that a pavement could be represented by an array of rectangular slabs. Furthermore, it was assumed

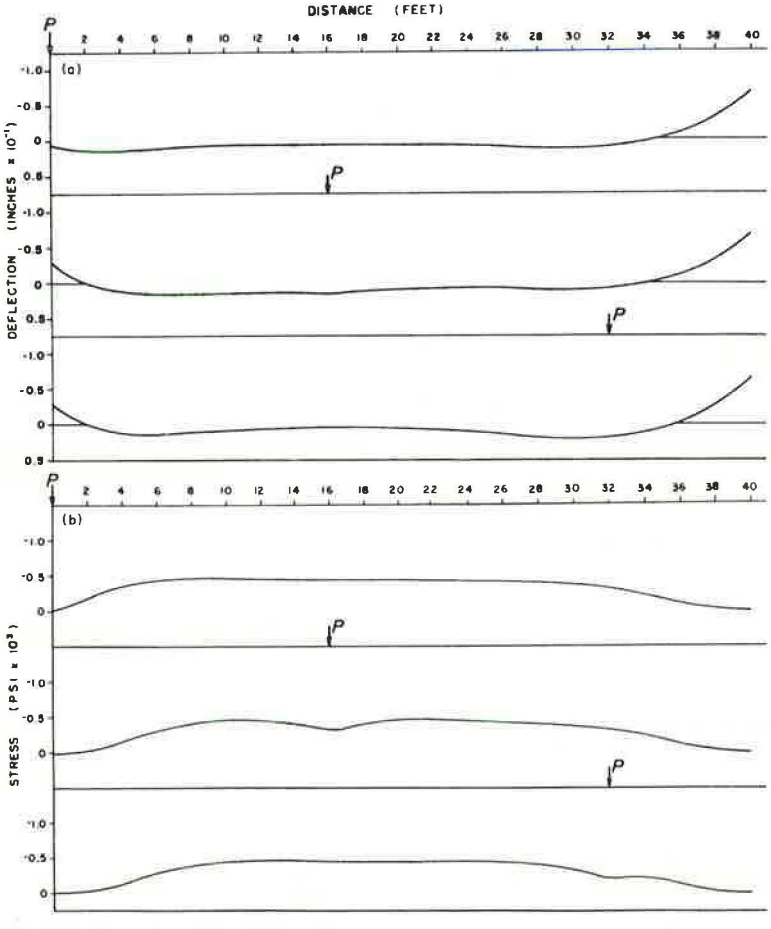

Figure 3. (a) Deflection vs position of moving load; (b) Tensile stress vs position of moving load; $K_0 = 100$ pci, $H = 8$ in., $C_r = 1.5$, $\Delta T = 30$ F, $v = 40$ mph.

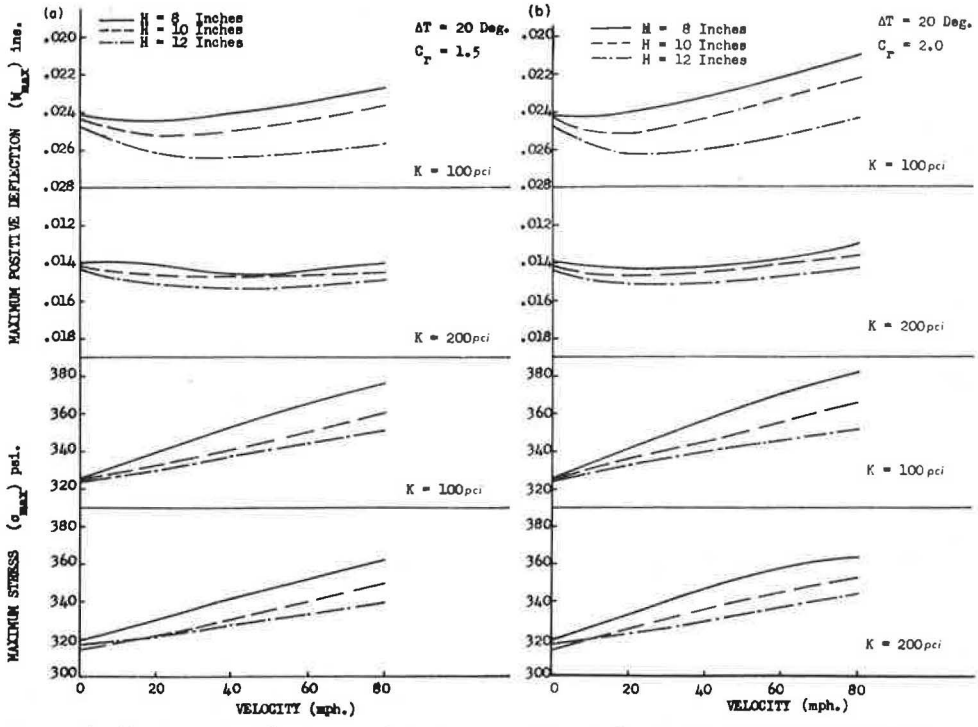

Figure 4. Maximum tensile stress and maximum positive deflection vs velocity of moving load: (a) $C_r = 1.5$, $\Delta T = 20$ F; (b) $C_r = 2.0$, $\Delta T = 20$ F.

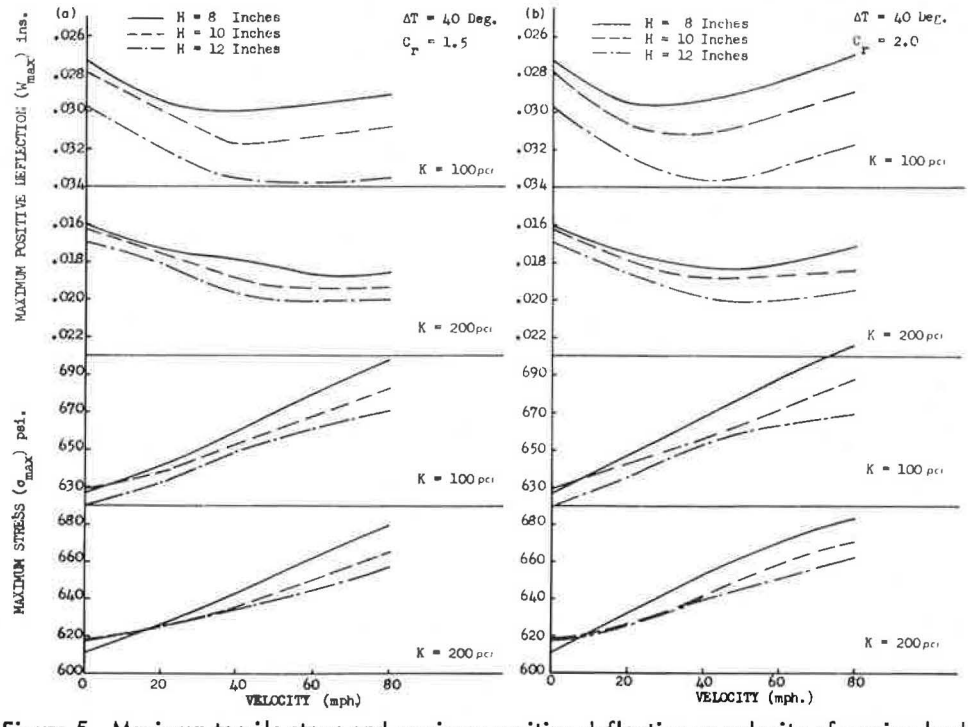

Figure 5. Maximum tensile stress and maximum positive deflection vs velocity of moving load: (a) $C_r = 1.5$, $\Delta T = 40$ F; (b) $C_r = 2.0$, $\Delta T = 40$ F.

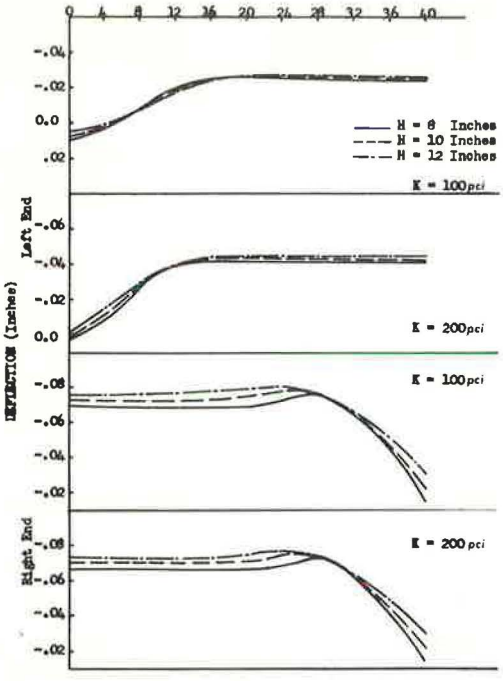

Figure 6. Deflection at ends of slab vs position of moving load (C_r = 1.5, ΔT = 30 F, v = 40 mph).

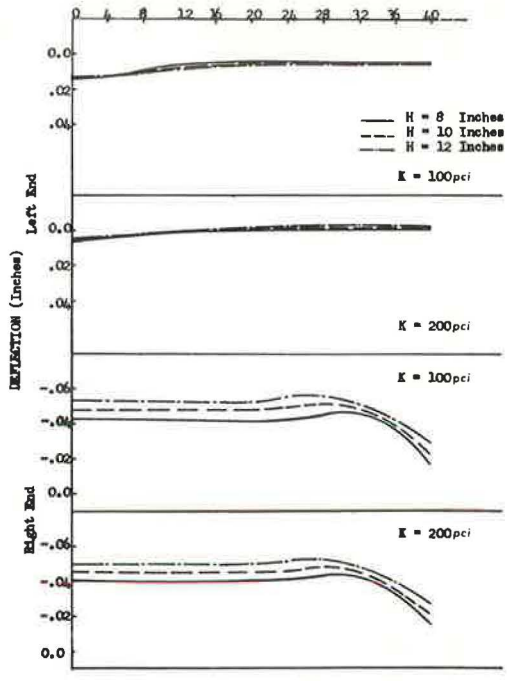

Figure 7. Deflection at ends of slab vs position of moving load (C_r = 1.5, ΔT = 30 F, v = 80 mph).

that the bending movement between slabs could be neglected while a shearing force equivalent to that in an infinite slab could be used to provide shear transfer. This seemed justifiable because of the relatively short depth of dowel embedment, and because the general nature of expansion and contraction joints is such as to permit the transfer of only very limited bending. For most highway work, dowel bars are only approximately 2 ft long, whereas the length of a slab averages about 40 ft; moreover, they are generally smooth and lubricated at one end to maintain freedom of horizontal movement between slabs. Under repetitive loading, these joints become looser and conceivably act more like a hinge, thus offering little or no moment transfer. On the other hand, substantial shear transfer could be experienced if the joint opening is small and the deflections are large enough to cause the development of a bearing pressure between dowel and concrete.

In any case, it should be noted that the magnitude of the moment and shear is really only significant in the near vicinity of the load itself. As a result, the solution is seen to be influenced by the values used for moment and shear transfer only when the load approaches the edges of the slab.

Figures 3, 4, and 5 demonstrate that for the case studied here, the damping ratio, C_r, does not greatly influence the values obtained for deflection and stress at low to moderate velocities; however, the higher value of C_r does result in a wider and less deflected trough (i.e., the depression under the load is wider but of smaller amplitude). In general, these figures tend to indicate that the pattern of the deflection and stress curves is mainly determined by temperature difference between slab surfaces and by the position of the moving load.

In the case considered here, temperature differences cause the ends of the slab to curl upward and become unsupported, while points midway between the midpoint and the ends of the slab experience an increase in positive deflection. For the positions of load shown in Figure 3, there is an increase in positive deflection and a decrease in tensile stress in the near vicinity of the load. However, as Figures 6 and 7 indicate, the radius of influence (wavelength of deflected surface) is not only a function of load

position but also of velocity. At low velocities, the radius of influence is small, but as velocity increases, this radius increases significantly behind the moving load. This characteristic, which was also observed by Thompson (44), plays an important role in the explanation of Figures 4 and 5.

For the case of an overdamped pavement ($C_r > 1.0$), it was initially anticipated that both the maximum deflection and stress would decrease with increasing velocity. However, Figures 4 and 5 indicate that, if curling is in evidence, this may not be the case. As was stated earlier, the maximum positive deflection in an unloaded slab subjected to moisture and/or temperature gradients that cause upward curling at the ends occurs somewhere near the one-quarter and three-quarter points of the slab. When a moving load is introduced, a wave train is set up and the deflected trough that lags the load becomes wider as velocity increases. In other words, the influence of the deflected trough behind the moving load increases with increasing velocity.

Therefore, if a slab that is initially curled upward at the ends is subjected to a moving load, there will be a tendency for the portion of the slab behind the load to become flatter and more fully supported as velocity increases (Figs. 6 and 7). As this flattening occurs, the maximum positive (downward) deflection behind the load increases until the velocity reaches a value that produces a reasonably flat, fully supported slab. Then a decrease in maximum positive deflection is experienced with further increases in velocity. Thus, in a slab that is curled upward at the ends, the maximum positive deflection does not occur at zero velocity, as is the case for fully supported slabs not subjected to moisture and/or temperature gradients, but at some velocity greater than zero.

As far as stresses are concerned, a line of reasoning similar to that used previously to explain the deflection pattern may be employed to account for the increase in maximum stress with increasing velocity shown in Figures 4 and 5. Increased velocities result in a greater tendency to flatten the slab and this in turn causes an increase in the tensile stress at the top of the slab.

As may be expected, the pattern as well as the magnitude of deflections and stresses obtained depends on the stiffness of the slab, the firmness of the subgrade material, and the temperature difference between slab surfaces. For the range of velocity studied, maximum positive deflection decreased as the value of subgrade reaction increased, and increases were experienced as pavement thickness and temperature gradients became greater. However, as Figures 4 and 5 show, the greater the resistance to flattening, the higher the velocity at which the curled surface becomes flat enough to result in a decrease in maximum positive deflection. In the case of stresses, the increase in temperature gradient from 20 to 40 F produced almost a 50 percent increase in stress, whereas the variation in pavement thickness and subgrade reaction did not appear to have much influence. The influence of the higher value of C_r on deflection and stress is also small. However, the fact that it produces a wider and less deflected trough is quite evident.

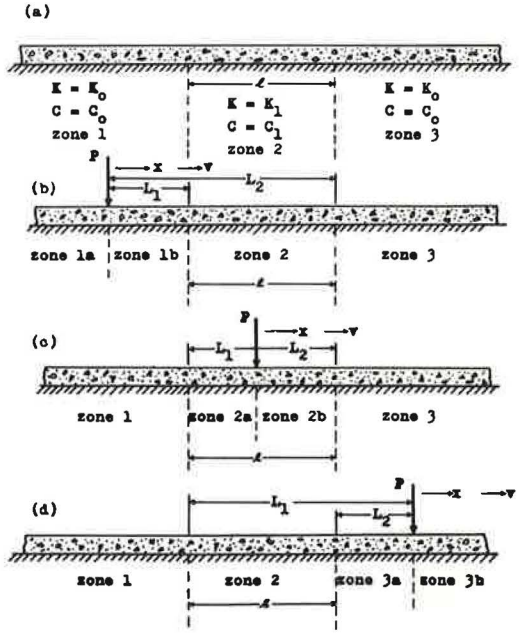

Figure 8. Infinite slab over region of reduced subgrade reaction.

EFFECT OF REDUCTION IN SUBGRADE SUPPORT

In this part of the paper, the main objective is to determine what effect a

reduction of the subgrade reaction would have on the stresses and deflections in a pavement subject to imposed vehicular loadings. For this case there is little loss in generality by assuming a particular value of shear and moment transfer between slabs. The procedure to be followed can be adapted to any degree of transfer. Therefore, for the present purpose where there is primary concern only for the region evidencing subgrade reduction, it is expedient to consider primarily the case of an infinite slab. As a special case, the condition that 50 percent shear and no moment transfer exist between two semi-infinite slabs is also examined.

The pavement section for the case of the infinite slab is shown in Figure 8. Here, the subgrade in its original form is represented by zones 1 and 3, whereas the area over which the reduction of subgrade reaction occurs is represented by zone 2. The differential equation describing the surface of the pavement is given by Eq. 4b.

To introduce the equivalence of a moving load, another zone must be added to the pavement. For example, in Figure 8 zone 1 is seen to be divided into two zones, 1a and 1b, and the force P is accounted for in the boundary conditions.

Solution of Problem

For the case when the moving load is over zone 1 and approaching zone 2, Eq. 4b is applied to each of zones 1a, 1b, 2 and 3, and solutions are obtained using the appropriate subgrade properties. The boundary conditions used in evaluating the constants are as follows:

$$w_{1a}(-\infty) = q_0/K_0 \tag{6a}$$

$$w'_{1a}(-\infty) = 0 \tag{6b}$$

$$w_{1a}(0) = w_{1b}(0) \tag{6c}$$

$$w'_{1a}(0) = w'_{1b}(0) \tag{6d}$$

$$w''_{1a}(0) = w''_{1b}(0) \tag{6e}$$

$$w''_{1b}{}'(0) - w''_{1a}{}'(0) = P/D \tag{6f}$$

$$w_{1b}(0) = w_2(L_1) \tag{6g}$$

$$w'_{1b}(L_1) = w'_2(L_1) \tag{6h}$$

$$w''_{1b}(L_1) = w''_2(L_1) \tag{6i}$$

$$w'''_{1b}(L_1) = w'''_2(L_1) \tag{6j}$$

$$w_2(L_2) = w_3(L_2) \tag{6k}$$

$$w'_2(L_2) = w'_3(L_2) \tag{6l}$$

$$w''_2(L_2) = w''_3(L_2) \tag{6m}$$

$$w'''_2(L_2) = w'''_3(L_2) \tag{6n}$$

$$w_3(\infty) = q_0/K_0 \tag{6o}$$

$$w'_3(\infty) = 0 \tag{6p}$$

After evaluating the constants in the resulting set of linear equations (29) using Crout's method of reduction with an IBM 7090 computer, the deflections and stresses were determined. The complete solution to the problem was obtained by applying this same procedure with suitable modifications to the schemes shown in Figure 8.

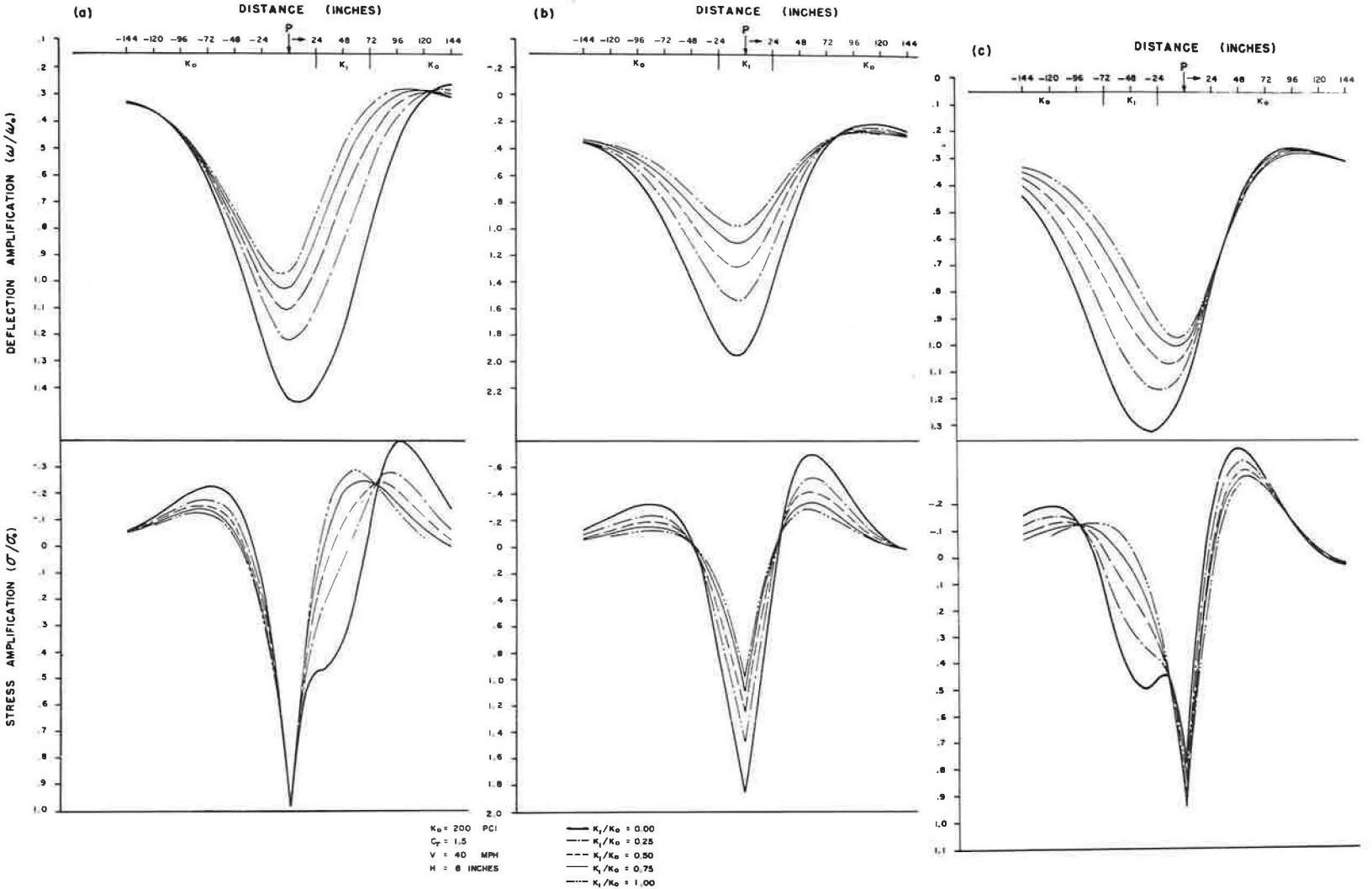

Figure 9. Deflection and stress amplification for load: (a) 2 ft behind region of reduced subgrade reaction; (b) at midpoint of region of reduced subgrade reaction; (c) 2 ft ahead of region of reduced subgrade reaction.

TABLE 1

DEFLECTION AND STRESS UNDER A STATIC LOAD (125 lb/in.)

H, in.	K_0 = 100 pci		K_0 = 200 pci	
	w_0, in.	σ_0, psi	w_0, in.	σ_0, in.
8	0.019144	150.607	0.010722	126.645
10	0.019017	113.948	0.010481	95.819
12	0.019449	90.726	0.010573	76.291

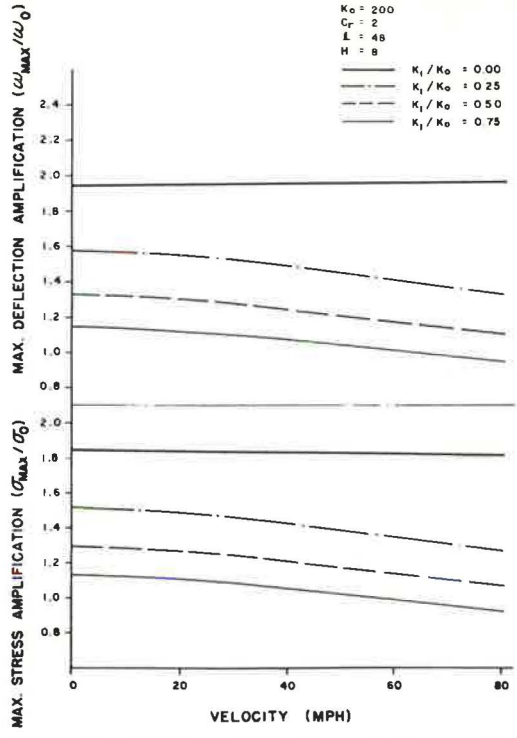

Figure 10. Maximum deflection and stress amplification vs velocity (K_0 = 200 pci, C_r = 2.0).

Results

To investigate the effect of a reduction in subgrade reaction, numerical results were obtained for combinations of the following data:

μ = 0.15
E = 4 × 10^6 psi
ℓ = 24, 48, 96 in.
H = 8, 10, 12 in.
K_0 = 100, 200 pci
K_1 = 0, 25, 50, 75, 100, 150, 200 pci
C_r = 1.5, 2.0
v = 0, 20, 40, 60, 80 mph
P = 125 lb/in.
q_0 = 150.9 pcf

Some typical curves (in nondimensional form) for displacements and stresses for three positions of the load are shown in Figure 9. Values for the deflection and stress under a static load, P, are given in Table 1. These values may be used in Figure 9, with appropriate amplification ratios, to obtain the magnitudes of deflection and stress at a point.

Figure 10 shows the variation of maximum stress and deflection with velocity, whereas Figures 11 and 12 show the influence of the parameters ℓ and H. Curves similar to those in Figure 9 are shown in Figure 13 for the special case where there is only 50 percent load transfer and no moment transfer across a discontinuity in the pavement surface.

Discussion of Results

As shown in Figure 9, there is an increase in positive deflection amplification as the value of K_1/K_0 decreases. For the load positions shown, the increase is greatest when the load is over the zone of reduced subgrade reaction and least when the load is past this zone. As may be expected, when the load is moving over a homogeneous subgrade material, the point of maximum positive deflection lags the position of the load. However, if there is a zone of reduced subgrade reaction and the load is approaching this zone (as shown in Fig. 9a), the maximum deflection can lead the position of the load at low values of K_1/K_0. For the cases when the load is over the weakened region or has passed it, maximum positive deflection lags the load. This characteristic tends to increase as the load moves out of the weakened area.

With regard to stresses, a reduction in the value of K does not appear to influence the maximum stress very much unless the load is within the area of reduced K. However, as Figure 9 shows, it can affect significantly the shape of the curve at very low values of K_1/K_0.

For the three positions of load considered, the maximum stress, which always occurred under the moving load, reached its greatest value when the load was over the

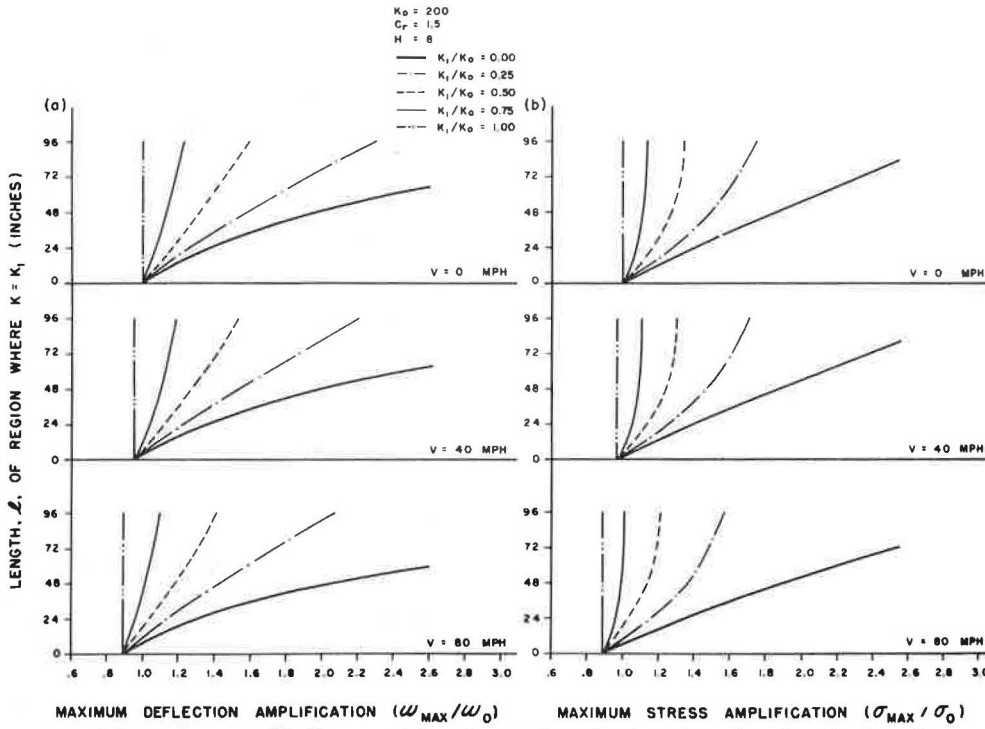

Figure 11. (a) Maximum deflection amplification vs length of region of reduced subgrade reaction; (b) Maximum stress amplification vs length of region of reduced subgrade reaction; K_0 = 200 pci, C_r = 1.5.

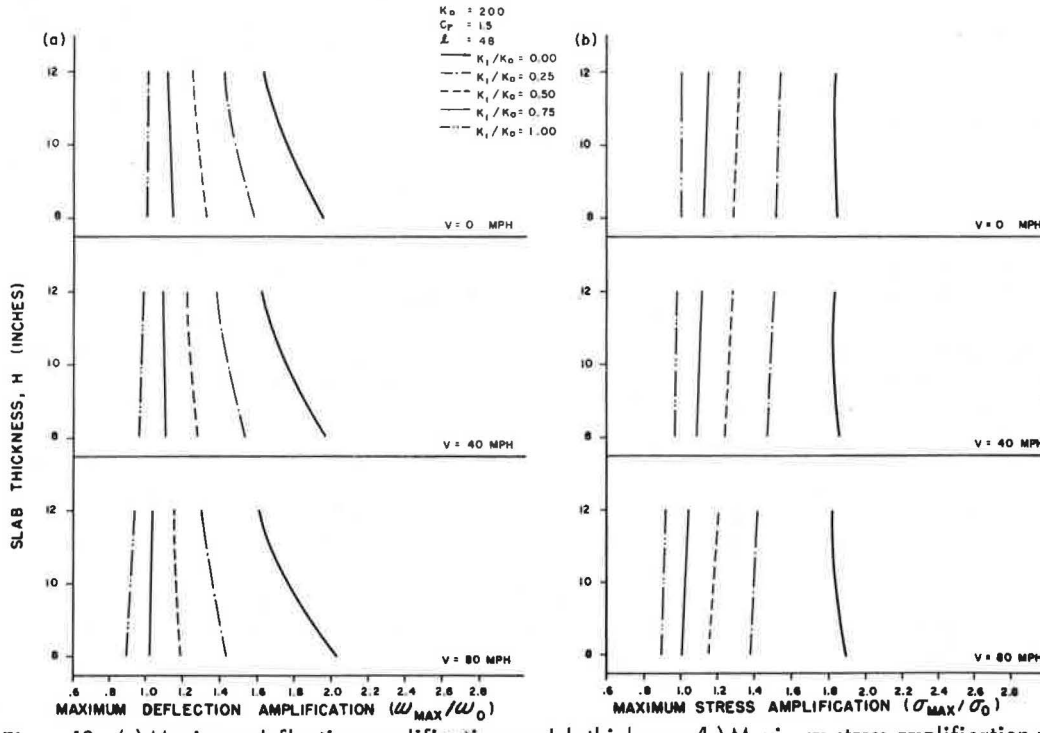

Figure 12. (a) Maximum deflection amplification vs slab thickness; (b) Maximum stress amplification vs slab thickness; K_0 = 200 pci, C_r = 1.5.

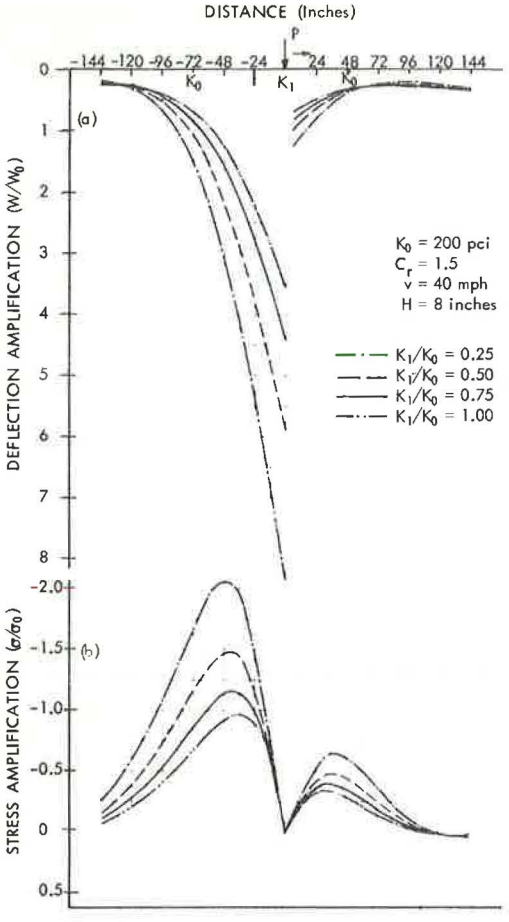

Figure 13. Deflection and stress amplification for load at midpoint of region of reduced subgrade reaction (no moment transfer across midpoint of region).

weakened zone; here, stress increased as K_1/K_0 decreased. In the case when the load is approaching the weakened zone, stress is again seen to increase with decreasing K_1/K_0. But after the load has passed this zone, the trend is reversed and the lower the value of K_1/K_0, the lower the stress under the load. This means that if a slab is lying on a subgrade exhibiting a soft spot, the stress experienced in the slab is greater when the load is moving toward the center of this weakened zone than when the load is moving away. Consequently, if failure caused by overstressing does occur, the signs of distress should appear somewhere between the center and the "back" edge of the soft zone.

Figure 10 shows that, except for very low values of K_1/K_0, deflection and stress amplification decrease with increasing velocity; however, the decrease appears to be more pronounced for the smaller value of initial subgrade reaction, K_0. Here also the higher value of C_r yields the lower value of maximum deflection and stress.

The influence of the length of the weakened region is shown in Figures 11a and 11b. In these figures, both the maximum stress and deflection are seen to increase, for all values of K_1/K_0 less than 1, as the region of reduced subgrade reaction becomes larger. The stiffer the subgrade material is initially, the greater the increase in deflection and stress. However, in the case of stress, this is only discernible at low values of K_1/K_0.

In general, the influence of the thickness of the pavement on stress and deflection amplification was only significant at low ratios of K_1/K_0. As shown in Figures 12a and 12b, deflection amplification decreases as thickness increases when the value of K_1/K_0 is approximately equal to 0.7 or less, whereas stress amplification remains relatively insensitive to variation in thickness. Again, these plots show that for constant K_1/K_0 ratios less than 1, the initially stiffer subgrade leads to the greater increase in stress and deflection. This suggests that, if the possibility of development of soft spots in the subgrade material exists, it may be better to avoid the use of stiff subgrades.

In Figure 13, there is a clear indication of what may happen when, in addition to the reduction in subgrade reaction, 50 percent of load transfer is lost. Deflections are substantially increased near the point of discontinuity, especially when the load is over the weakened region. For example, for $K_1/K_0 = 0.5$ in Figure 13, the deflection amplification for a point under the moving load is 5.85, compared to 1.26 in Figure 9b. Another important aspect to note is the large relative movement that occurs between the edges of the slab (i.e., edges situated at the midpoint of the weakened region). This movement not only causes bumpy driving, but may also lead to further pavement distress.

As for stresses, Figure 13 shows that for the special case considered the maximum stress need not occur under the moving load. In fact, the maximum stress experienced in this case is not only behind the load, but is also correspondingly higher than that

obtained in Figure 9b. For the other two positions of the load, the use of no-moment-transfer and 50 percent shear transfer did not greatly influence the values previously obtained in Figure 9 for maximum stress as long as the ratio of K_1/K_0 was high (approximately = 0.8). However, when the ratio of K_1/K_0 was lowered to 0.25, significant reduction in stress was experienced. Obviously, at high values of K_1/K_0, the load in this case has to be fairly close to the weakened region before conditions existing at the midpoint of the region become important.

SUMMARY AND CONCLUSIONS

In the first part of this study it is shown that, when a pavement is subjected to upward curling, it is possible to experience an increase in maximum positive deflection as velocity is increased, and then a decrease in maximum positive deflection with further increases of velocity. The velocity at which the decrease in maximum positive deflection sets in seems to depend on several factors, including the thickness of the slab, the temperature difference between its surfaces, the stiffness of the foundation, and the degree of damping. In general, it appears that higher velocities are needed to effect a decrease in maximum positive deflection as slab thickness, stiffness of foundation, and temperature increase, and as the degree of damping decreases.

In the case of stresses, maximum stress increased with increasing velocity. The thinner the pavement and the lower the value of subgrade reaction and damping coefficient, the higher the stress. However, there was not a great difference in the values of stress obtained. What seemed to matter most was the difference in temperature between the surfaces of the slab. Increases in the temperature difference resulted in large increases in stress and maximum positive deflection for all values of velocity studied. This observation tends to indicate that temperature difference between slab surfaces is the overriding factor governing the magnitude of stress and deflection that may be obtained, whereas factors such as velocity, thickness of pavement, modulus of subgrade reaction, and degree of damping act more or less to exaggerate or minimize the pavement distress caused by temperature and/or moisture gradients.

In the second part of the study, where the influence of a reduction in subgrade reaction was studied, it was clearly shown that both maximum deflection and maximum stress increased as the region became weaker. The increase experienced was quite pronounced when the load was within the weakened region and moving toward its center. Thus, if a pavement is designed with a particular value of subgrade reaction and a weakened zone develops because of the softening of the ground (as may be the case during spring thaw), the stresses produced in the slab in the near vicinity of the weakened zone will very likely be higher than anticipated.

If there is a significant reduction of subgrade support (even to within 75 percent of the original value of K_0), maximum deflection and stress should still follow the well-known trend and decrease with increasing velocity. However, when there is complete loss of subgrade support, both the maximum stress and deflection can increase with increasing velocity. For the hypothetical case where pavements of equal thickness are built on different subgrade material, it appears that for a constant percentage loss of subgrade support, the pavement built on the stiffer subgrade material should experience a greater increase in stress and deflection. In the case where there is also a loss of load transfer, deflections as well as stresses may be substantially increased and large relative movement may be experienced near the points of discontinuity in the surface of the slab.

In conclusion, we find that on the basis of the assumptions stated herein, this study indicates that (a) contrary to common opinion, an increase in velocity can produce an increase in deflection and stress; (b) of all the variables considered herein (temperature differences, velocities, thickness of pavement, moduli of subgrade reaction, and degrees of damping), the temperature difference between slab surfaces was shown to be the overriding factor governing the magnitude of maximum stress and deflection; (c) there is reason to believe that any meaningful interpretation of the performance of pavements as determined by measured strains and/or deflections of the slab must give due regard to the effects of warping; and (d) stresses and deflections within a slab can be sensitive to localized reductions in subgrade support.

ACKNOWLEDGMENT

This study was made possible by the financial assistance of the Joint Highway Research Project between the Indiana State Highway Commission and Purdue University.

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