Analytical Estimation of Highway Impedances Within Urban Areas

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This paper proposes an Urban Network Impedance Model (UNIM) to estimate highway impedances within urban areas without employing current network simulation procedures. Point-to-point travel time, distance, and cost are the principal outputs of UNIM. Inputs to the model are the geographic locations of the two points, the service characteristics of the highway network as specified by a function that characterizes the distribution of isotropic speed throughout the metropolitan area, and an assumption concerning an idealized geometric configuration of the highway network. A method for calibrating the speed is presented.

UNIM was tested, using Baltimore, Maryland, as a case study, by comparing the impedance estimates of the model to those provided by conventional network analysis procedures. Test results suggest that the model provides a reliable and efficient procedure to estimate impedances within urban areas. UNIM could be used in the initial calibration of a distribution model, to provide the accessibility inputs to a land use model, or for estimating site accessibility in location studies. Further refinements of the model are possible and would lead to more accurate results and to an even broader range of applications.

•A SIMULATION framework and an analytical formulation are two possible approaches to the estimation of highway impedances. The concept of impedance is used as a substitute for travel time, cost, or distance. The objective of this paper is to present the Urban Network Impedance Model, developed to evaluate analytically highway impedances within urban areas.

Traditional simulation methods for estimating point-to-point impedances involve the representation of the transportation system as a set of links and nodes and the association of some impedances with each element. Spider networks have also been constructed in which all of the transportation facilities between adjacent zones are represented by one set of impedances. Impedances between any two nodes may then be estimated by building minimum impedance trees through the network. System usage and service are derived in the simulation analysis by assigning interzonal traffic movements to the coded network. Thus, such a coded network is used to estimate interzonal impedances and to simulate the operation of the system.

Acquisition and processing of the large amount of data required to adequately represent a network are sources of many problems, e.g., exceeding the available resources or the capacity of the computer. Thus, an alternative technique for estimating usage of a facility has been proposed (1, 2). Furthermore, a simulation strategy is inherently cumbersome, and analytical strategies have been suggested for the analysis of flow and service in idealized situations (3).

The approach of this study follows the efforts of various researchers in this direction. Most of Smeed's models assume that drivers choose a path that minimizes the

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distance between origin and destination (4, 5). Other authors have used a similar approach (6, 7, 8). But, such models ignore the relationship between the service provided by a highway and the volume of traffic using it. Lam has proposed a method that incorporates the volume-capacity relationship into an analytical assignment model (9, 10).

The problem addressed in this discussion is somewhat different from that structured above. Previous work has concentrated on the assignment of the total amount of traffic in an area to an idealized network and to the simultaneous estimation of link impedances. The model presented hereafter is concerned with point-to-point impedances. Such data are required for the accessibility inputs to a number of urban transportation models, for example, those used for both land use forecasting and trip distribution.

UNIM was initially developed to estimate the time, distance, and cost of traveling to a given terminal site from various points within an urban area in the Northeast Corridor. The location of the terminal is an exogenous input to the analysis, because it is unknown prior to the specification of a given design. It is conceivable that access link characteristics can be obtained by coding the internal transportation network of each analysis district and building trees. However, in view of the area covered by the Northeast Corridor, this method would involve coding an extremely large network and would be prohibitively expensive. This suggests that the problem of estimating access impedances should be treated analytically. Such an approach is particularly applicable in estimating access link characteristics, because the problem does not explicitly involve traffic assignment.

PRELIMINARY OBSERVATIONS

In this research, the model to synthesize highway impedances requires three initial information inputs: (a) the geographic locations of the two points between which impedances should be estimated; (b) the service characteristics of the highway network, as specified by a function that characterizes the distribution of speed throughout the urban area; and (c) a definition of the paths between the two points.

Our initial effort to structure a highway impedance model used only the first two inputs, i.e., location and speed distribution. The travel path selected was the one corresponding to a minimum time, given a specified speed distribution over a continuous transport plane. This approach led to a calculus-of-variations problem that can be solved explicitly only by using rather limiting assumptions regarding the speed distributions.

In particular, the problem has been solved in a manner feasible for use in an operational impedance model for the following speed distributions:

$$v = ar$$

and

$$v = ar^2$$

where

- r = the radial distance from a given point to the center point of the speed distribution (usually this center point would be the central business district of the metropolitan area);
- a = a parameter to be calibrated for the metropolitan area; and
- v = an estimate of the isotropic speed (i.e., the speed is independent of the direction of travel) at radial distance r from the center.

The geometry of the path connecting the two points was found to be a spiral for the linear speed distribution and a circle for the parabolic speed distribution. This result provides an interesting basis for evaluating Miller's discussion regarding spiral road networks (11). However, the assumptions regarding the speed distribution to be used in the calculus-of-variations model are unduly restrictive. In particular, the speed at the center of the distribution (i.e., the central business district) is set at zero. As a result, none of the paths may pass through the center.

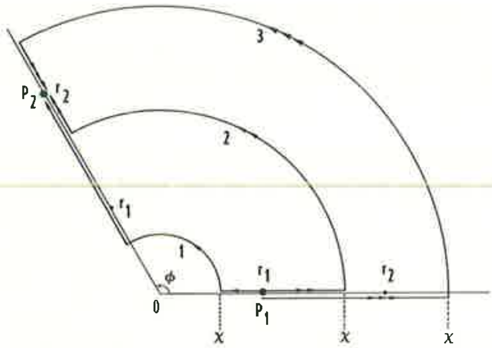

Figure 1. Alternative paths between two points.

In order to avoid the numerical solution of differential equations and to use a more realistic speed distribution, it is necessary to specify explicitly the paths between the two points. In UNIM, it is assumed that movement is restricted to either a radial or a circular path. Further, movement along these directions may occur at any point within the metropolitan area. This formulation allows realistic speed distributions as inputs; it was solved using a logistic curve to define speed.

FORMULATION OF UNIM

Consider a city whose CBD center is designated by O and two points P_1 and P_2 in this city (Fig. 1). Let

$$r_1 = OP_1 \qquad (r_1 \geq 0)$$

$$r_2 = OP_2 \qquad (r_2 \geq 0)$$

$$\phi = (\vec{OP_1}, \vec{OP_2}) \qquad (0 \leq \phi \leq \pi)$$

At a given point, P, the speed, v(r), is assumed to depend only on the radial distance, $r = OP$. The function v(r) is supposed to be defined, continuous, and derivable twice over the range of r. It is hypothesized that the speed increased from a nonzero value, v_0, at the center to a constant value, v_ℓ, at the edge of the city. In mathematical terms, this can be expressed as follows:

$$v(r) \geq v_0 > 0$$

$$\lim_{r \to \infty} v(r) = v_\ell$$

$$\frac{dv(r)}{dr} \geq 0$$

$$\lim_{r \to \infty} \frac{dv(r)}{dr} = 0$$

As mentioned previously, travel can only take place along a radial and/or a circumferential path. Because of the symmetry around O, it can be assumed that $r_2 \geq r_1$. Let x be the radius of the circumferential portion of the path ($x \geq 0$). Between any pair of points, P_1 and P_2, there are three distinct types of travel paths that may be used:

Case 1—(a) inward radial travel from P_1; (b) circumferential travel at a radius x, where $O \leq x \leq r_1 \leq r_2$; and (c) outward radial travel to P_2;

Case 2—(a) outward radial travel from P_1; (b) circumferential travel at a radius x, where $r_1 < x \leq r_2$; and (c) outward radial travel to P_2;

Case 3—(a) outward radial travel from P_1; (b) circumferential travel at a radius x, where $r_1 < r_2 < x$; and (c) inward radial travel to P_2.

Along a circumference of radius x, the speed remains constant and the corresponding travel time is $x\phi/v(r)$. Along a radial path, the speed varies and the travel time is given by the integral $\int dr/v(r)$. The travel times for the three basic cases are respectively

$$\text{Case 1} \quad \tau_1(x) = \int_x^{r_2} \frac{dr}{v(r)} + \frac{x\phi}{v(x)} + \int_x^{r_1} \frac{dr}{v(r)}$$

$$\text{Case 2} \quad \tau_2(x) = \int_x^{r_2} \frac{dr}{v(r)} + \frac{x\phi}{v(x)} + \int_{r_1}^{x} \frac{dr}{v(r)}$$

$$\text{Case 3} \quad \tau_3(x) = \int_{r_2}^{x} \frac{dr}{v(r)} + \frac{x\phi}{v(x)} + \int_{r_1}^{x} \frac{dr}{v(r)}$$

Three possible forms of the speed function have been considered:

Hyperbola $\quad v(r) = \dfrac{r + a}{br + c}$

Exponential $\quad v(r) = a(b + e^{-pr})$

Logistic curve $\quad v(r) = a(b + e^{-pr})^{-1}$

where a, b, c, and p are parameters. The first two functions keep a constant concavity, whereas the last one offers the advantage of a varying concavity. Because of the analytical flexibility to fit empirical data, which is provided by the provision for an inflection point, the logistic curve was selected for the speed function.

The parameters of the logistic curve have the following significance:

$$v_0 = \frac{a}{b + 1}$$

$$v_\ell = \frac{a}{b}$$

i.e.,

$$b = \frac{v_0}{v_\ell - v_0}$$

$$a = \frac{v_0 v_\ell}{v_\ell - v_0}$$

UNIM is based on the assumption that the travel path selected is that which corresponds to the minimum travel time. Three values of x, which result in the minimum travel times via Path 1, Path 2, and Path 3 respectively, are calculated. These minimum travel times, $\tau_1(x)$, $\tau_2(x)$, and $\tau_3(x)$ are evaluated, and the minimum minimorum is selected as the travel time between the two points. This minimum minimorum travel time completely defines the geometry of the travel path, thus allowing the calculation of the travel distance and travel cost along this path between P_1 and P_2.

Substituting for v(r) in the expressions of τ_1, τ_2, and τ_3 and integrating, we obtain

$$\tau_1(x) = \frac{1}{a}\left\{ b\left[(\phi - 2)x + r_2 + r_1\right] - \frac{1}{p}\left[e^{-pr_2} + e^{-pr_1} - (px\phi + 2)e^{-px}\right] \right\}$$

$$\tau_2(x) = \frac{1}{a}\left[b(\phi x + r_2 - r_1) - \frac{1}{p}(e^{-pr_2} - e^{-pr_1} - px\phi e^{-px}) \right]$$

$$\tau_3(x) = \frac{1}{a}\left\{ b\left[(\phi + 2)x - r_2 - r_1\right] - \frac{1}{p}\left[e^{-pr_2} + e^{-pr_1} - (px\phi - 2)e^{-px}\right]\right\}$$

The values of x, which correspond to the minimums of $\tau_1(x)$, $\tau_2(x)$, and $\tau_3(x)$, are found by solving the equations

$$\frac{\partial \tau_i(x)}{\partial x} = 0 \qquad i = 1, 2, 3$$

subject to:

$$\frac{\partial^2 \tau(x)}{\partial^2 x^2} > 0 \qquad i = 1, 2, 3$$

A complete analytical solution of the minimum minimorum problem has been developed. It shows that the value of b is a critical element in the selection of the minimum time path [the ratio v_0/v_ℓ is equal to $b/(b + 1)$]. For certain values of b, the absolute minimum time can be found without comparing the minimum of τ_1, τ_2, τ_3. Figure 2 illustrates the limiting values of b that define the possible paths among which the solution is found. When the pair (v_0, v_ℓ) falls in Regions I or II, the minimum corresponds to that of τ_1; when the pair (v_0, v_ℓ) is in III, the minimum corresponds to the smallest of the minimums of τ_1 and τ_2; finally when (v_0, v_ℓ) is in IV, τ_1, τ_2, and τ_3 have to be considered.

The travel distance corresponding to the minimum minimorum travel time is:

$$\ell(x^*) = r_1 + r_2 + (\phi - 2)x^*$$

where x^* = the radius of the circumferential element of the minimum minimorum travel time path.

The travel cost corresponding to the minimum minimorum travel time is established as follows. It is assumed that the unit cost of travel is a function of the instantaneous travel speed according to the following function:

$$c(v) = a_0 + a_1 v + a_2 v^2 + a_3 v^3$$

where

$c(v)$ = the unit costs of travel at speed v; and

a_0, a_1, a_2, a_3 = parameters to be calibrated.

Because speed, v, is a function of the distance from the center, r, the unit cost of travel can be expressed as a function of r. The travel cost between P_1 and P_2 is obtained by taking the line integral of the unit cost function over the minimum minimorum travel time path:

$$C = \left| \int_{x^*}^{r_1} c(r)dr \right| + \phi x^* c(x^*) + \left| \int_{x^*}^{r_2} c(r)dr \right|$$

where C = the travel cost between P_1 and P_2.

TESTING OF UNIM

In order to test the methodology, it was decided to compare impedance values obtained from the model to impedances estimated from coded networks. Because of the availability of data, Baltimore was chosen as a test city. Before exercising UNIM, it is necessary to calibrate the speed function for the metropolitan area.

A ring-sector zonal structure was superimposed on a map of Baltimore. This investigation utilized the 1962 network coded during the course of the Baltimore Transportation Study. Off-peak speeds for the 1962 network were established through field

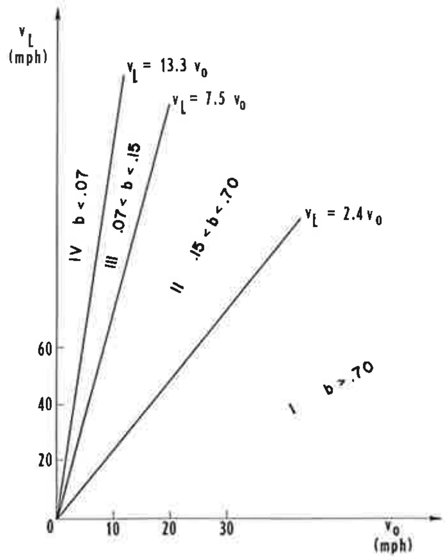

Figure 2. Limiting values of b.

observations made during the summer and fall of 1961. The average speed in each of the 15 rings was estimated as shown in Figure 3. It is evident that the speed in a given sector generally increases with the distance from the center. The average speed in the innermost rings is relatively high, e.g., about 20 mph, but this may have been caused by inaccuracies in the network coding.

The formula $v = a(b + e^{-pr})^{-1}$ can be linearized as follows:

$$\frac{1}{v} = \frac{b}{a} + \frac{1}{a} e^{-pr}$$

where $\frac{a}{b} = v_\ell$, the speed reached in the fringes. Hence:

$$\frac{1}{v} - \frac{1}{v_\ell} = \frac{1}{a} e^{-pr}$$

$$\ln\left(\frac{1}{v} - \frac{1}{v_\ell}\right) = -\ln a - pr$$

This relationship is linear with respect to $\ln\left(\frac{1}{v} - \frac{1}{v_\ell}\right)$ and r. In order to perform a regression analysis to determine a and p, v_ℓ has to be assigned. It should be noted that other techniques are available for calibration.

Three values were assigned to v_ℓ: 40, 45, and 50 mph. The parameters of the speed function were estimated for each of these limiting values using the complete sample of 15 rings, using a sample of 11 for which the first four rings were left out because of the seemingly high values of v at the CBD center, and using a sample of 13

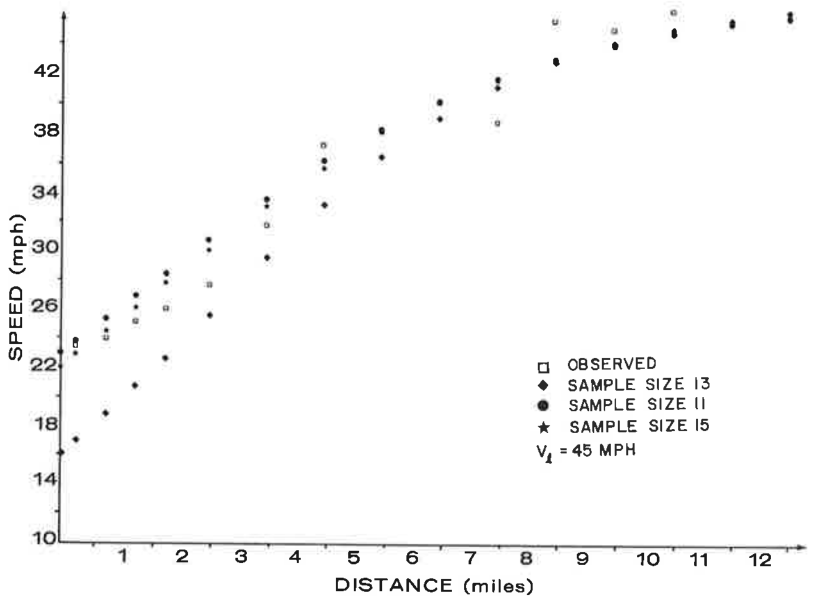

Figure 3. Speed distribution curve.

TABLE 1

RESULTS OF REGRESSIONS

Speed Equation	Sample Size	v_ℓ (mph)[a]	R	ℓn a	p	Equation	v_0 (mph)[b]
1	11	40	0.94	3.00022	0.3289	$v = 20.09\,(0.522 + e^{-0.3289r})^{-1}$	13.2
2	15	40	0.96	3.30896	0.2944	$v = 27.36\,(0.684 + e^{-0.2944r})^{-1}$	16.3
3	11	45	0.86	3.67674	0.2799	$v = 39.52\,(0.878 + e^{-0.2799r})^{-1}$	21.1
4	15	45	0.93	3.59518	0.2887	$v = 36.42\,(0.809 + e^{-0.2887r})^{-1}$	20.1
5	11	50	0.49	4.71429	0.1246	$v = 111.50\,(2.230 + e^{-0.1246r})^{-1}$	34.5
6	15	50	0.76	4.08172	0.1946	$v = 59.26\,(1.185 + e^{-0.1946r})^{-1}$	27.1
7	13	40	0.96	2.73577	0.3582	$v = 15.43\,(0.386 + e^{-0.3582r})^{-1}$	11.1
8	13	45	0.92	3.03539	0.3514	$v = 20.80\,(0.462 + e^{-0.3514r})^{-1}$	14.2

[a]Assumed speed at the edge of the city.

[b]Computed speed at the center of the city.

formulated by adding the following data points to the sample of 11: 10 mph for the first ring and 12.5 mph for the third ring.

The results of the regressions are presented in Table 1. The equations corresponding to a limiting speed of 50 mph were rejected for low correlation coefficients and high speeds at the center. It was also found that arbitrarily adding data points for the first and third rings did not significantly change the coefficients of the equation. The equations corresponding to v_ℓ = 45 mph are graphed in Figure 3.

The model was first applied to estimate impedances from a set of districts within the Baltimore metropolitan area to Friendship Airport. Each of the six equations obtained for the speed distribution was used in this test of UNIM. To this effect, three programs were written in FORTRAN IV for the IBM 360/40 computer. The first program transformed the grid coordinates of 586 Baltimore zone centroids into a system of polar coordinates, the second program computed the time and distance from each centroid to Friendship Airport, and the third program aggregated zonal-level times and distances to the corresponding values for the study districts defined in the Baltimore-Washington Airport Survey. UNIM's estimates of travel time and distance, using Eq. 1 (Table 1) are compared in Figures 4 and 5 to corresponding network values. The calculated travel times are generally within 10 percent of the corresponding network values, whereas travel distances are systematically underestimated.

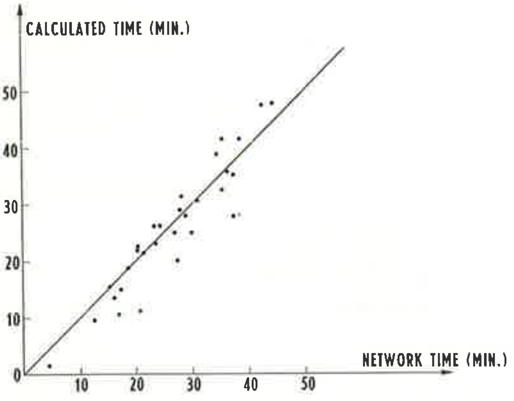

Figure 4. UNIM estimate of travel times vs network.

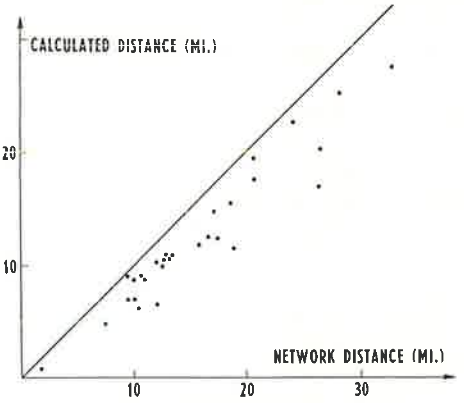

Figure 5. UNIM estimate of travel distances vs network.

TABLE 2

EVALUATION OF UNIM OVER 231 INTERCHANGES

Speed Equation	Time					Distance				
	Mean (min)	Standard Deviation (min)	Correction Factor	R	Standard Error (min)	Mean (mi)	Standard Deviation (mi)	Correction Factor	R	Standard Error (mi)
1	25.1	11.1	0.98	0.99	4.5	11.8	6.5	1.33	0.96	4.0
2	23.2	10.5	1.06	0.99	4.4	11.7	6.3	1.35	0.96	3.9
3	19.8	9.1	1.24	0.98	4.0	11.5	6.0	1.38	0.96	3.8
4	20.0	9.2	1.23	0.98	3.9	11.6	6.1	1.36	0.96	3.9
7	25.6	10.9	0.97	0.99	4.5	12.1	6.9	1.28	0.96	4.0
8	21.8	9.5	1.12	0.99	3.9	11.9	6.7	1.30	0.96	3.9
231 network observations	25.9	11.8				14.7	8.7			

In view of the promising results of this initial test, it was decided to carry out a more comprehensive evaluation. Twenty-two points uniformly distributed over the Baltimore urban area were selected. UNIM was used to obtain the time and distance corresponding to the 231 different possible interchanges. These results were compared to the impedances obtained from the coded 1962 network.

Results of the evaluation are presented in Table 2. The correction factor is obtained by means of regressing the observed network values against the values estimated by UNIM. Because the regression line was forced through the origin, only one correction factor was obtained. Finally, the mean and standard error of estimate of the set of estimated impedances for each speed model were obtained.

Several inferences may be drawn from these results. It would appear that Eq. 1, (Table 1), which uses a limiting speed of 40 mph, could be usefully applied to reproduce off-peak highway impedances in the Baltimore area. The proportion of variance explained and the standard error of estimate are not appreciatively sensitive to variations in the assumed limiting speed, although the same is not true of the correction factor.

The correlation coefficients are all over 0.95, the standard errors of estimate are relatively low, and deviations that occur between an estimated and a network value can be satisfactorily corrected by means of a linear factor. This suggests that UNIM does explain a substantial portion of the variance in interzonal impedances within the test area. As noted in the first test, the model's estimate of travel times is better than its estimate of travel distances. In particular, in the case of Eqs. 1, 2, and 7 (Table 2), the travel time correction factors are very close to one. This is not true, on the other hand, for the travel distance correction factors, which are about 1.3.

CONCLUSIONS

On the basis of these tests, it would appear that UNIM provides an acceptable method for calculating point-to-point highway impedance in urban areas without requiring the preparation and processing of coded networks. Further, there seems to be a close identity between the impedance values estimated by UNIM and network values. The model predicts travel time relatively accurately, although travel distance is systematically underestimated by about 30 percent.

It does not appear overly difficult to acquire the input data necessary to calibrate the speed function used in UNIM. An obvious data source is the network link data (i.e., time, distance, and speed) that are generally acquired during the inventory phase of an urban transportation study. These data could be supplemented by the results of various travel time and delay studies, and/or spot speed studies that may have been conducted within the metropolitan area. Once experience is gained with developing speed functions for a number of urban areas, it will probably be possible to define a generalized speed curve that would vary as a function of the characteristics of the metropolitan area.

UNIM would appear to have application to a number of problems encountered in transportation system analysis. For example, it could be used to provide the accessibility inputs required for land use planning models, to supply the initial impedance estimates to calibrate a trip distribution model, or to estimate the accessibility of various sites in location studies. A computer software package has been developed so that UNIM may be conveniently applied to estimate point-to-point impedances or weighted site accessibilities at a cost level that is substantially lower than current network analysis procedures. The weights used in calculating accessibilities could be any activity distribution for the metropolitan area and may be defined on either a continuous or a discrete basis.

Numerous research projects, both empirical and theoretical, could be defined to further exploit the potential implicit in UNIM. Initially, it would be useful to apply and evaluate the performance of the model in a large number of metropolitan areas and to refine current procedures for acquiring the data needed to calibrate the speed function. It is conceptually easy to extend the model by utilizing three speed functions: one for circular travel, one for inward radial travel, and one for outward radial travel. Finally, it is possible to formulate UNIM on the assumption that a city has a grid rather than a radial-circumferential highway network.

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