Preformed Elastomeric Bridge Joint Sealers: Thermal Characteristics of Bridge End Movements

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This paper, the third in a series, presents current knowledge on the thermal characteristics of a mass, such as a concrete deck, and on the principles of heat transfer. Using an empirical approach, a method for predicting reasonably accurate concrete temperatures is given. Information is included on the subjects of actual time difference and phase shifts between peaks of solar radiation and air temperatures.

•IN EARLIER PAPERS covering the design and construction of bridge joints sealed by elastomeric material (1, 2), consideration was given to the thermal characteristics of bridge deck end movements. In the meantime, the need for increased knowledge about these thermal characteristics has become apparent, leading to this portion of the subject study.

As a by-product of the research phase dealing with the relationship between deck and air temperatures and joint movements, the knowledge of heat transfer through a concrete slab has been expanded. Using field data made available by the Louisiana Department of Highways, we were able to develop an empirical approach to the problem of calculating concrete temperatures. This work should advance the knowledge of the temperature effects on bridge joint movements.

THEORETICAL BACKGROUND

It has been recognized that the temperature of a mass, in this case a concrete bridge deck, is influenced by a number of rather complex factors, such as solar radiation, ambient air temperature, wind velocity, insolation, re-radiation, evaporation, conductivity, diffusivity, surface conditions, specific heat, and density. By making assumptions of average values of the secondary parameters and a sinusoidal effective daily temperature cycle, a solution is obtained. Figure 1 shows the sinusoidal cycle assumption.

A reasonably accurate solution would necessitate the gathering and evaluation of a substantial amount of secondary parameter data for each specific location. Because such expanded research is not within the scope of this study, further development of this problem will have to be the object of separate analysis. Thus, assumptions made and data used for this particular discussion are mainly informative in nature.

In a paper by Barber (3), an equation was developed for the temperature of a semi-infinite mass in contact with air that can be expressed as follows:

$$T = (T_A + R) + \eta_o (0.5T_R + 3R) \exp\left(-X\sqrt{\frac{\pi}{t_o a}}\right) \sin\left(2\pi \frac{t}{t_o} - X\sqrt{\frac{\pi}{t_o a}} - \epsilon_o\right) \quad (1)$$

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where

$$R = 0.103\frac{b}{\alpha}L;$$

$$\eta_o = \sqrt{\frac{1}{1 + 2\sqrt{\frac{\pi}{H^2 t_o a}} + 2\frac{\pi}{H^2 t_o a}}};$$

$$a = \frac{k}{sw};$$

$$H = \frac{\alpha}{k};$$

$$\alpha = 1.3 + 0.62\, v^{0.75};$$

$$\epsilon_o = \arctan\frac{1}{1 + \sqrt{\frac{H^2 t_o a}{\pi}}}; \text{ and}$$

X = distance as measured from the top of mass (Fig. 2).

It is obvious that for the top surface of this mass X = 0. Thus, the equation for the temperature of the top surface can be written in the following form:

$$T = (T_A + R) + \eta_o\,(0.5T_R + 3R)\sin\left(2\pi\frac{t}{t_o} - \epsilon_o\right) \qquad (2)$$

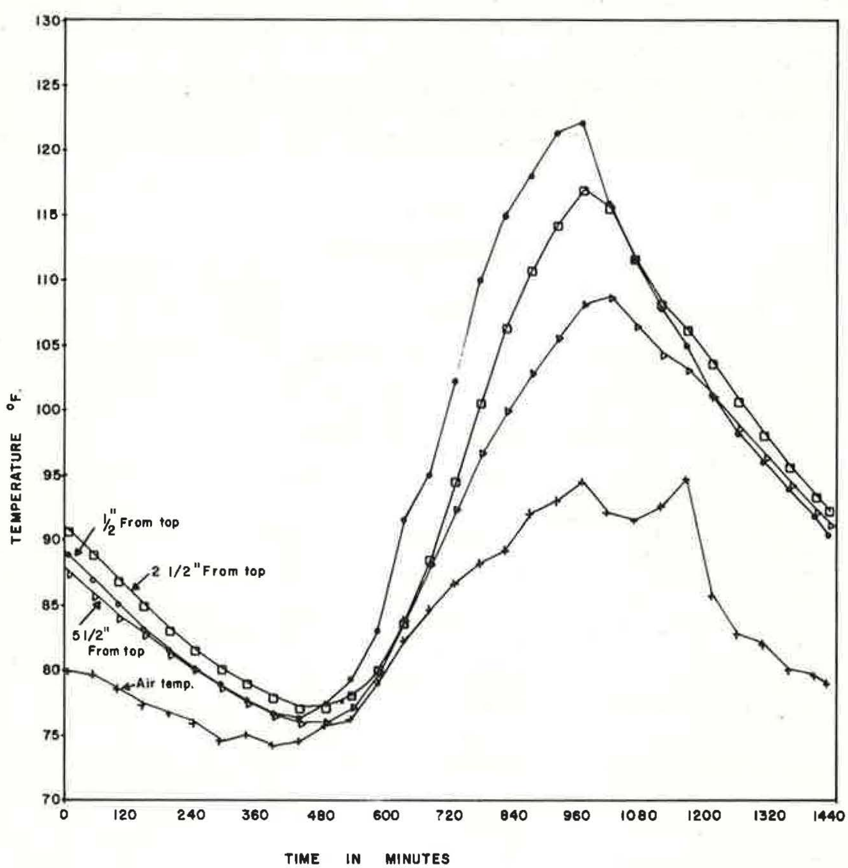

Figure 1. Sinusoidal variation of temperature (Aug. 5, 1967).

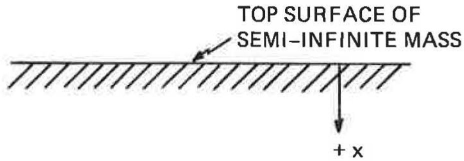

Figure 2. Top surface in contact with air temperature.

For T to become maximum, $\sin\left(2\pi \frac{t}{t_o} - \epsilon_o\right) = 1.0$ and $X = 0$; therefore

$$T_{max} = (T_A + R) + \eta_o (0.5T_R + 3R) \quad (2a)$$

On the basis of previously made assumptions, as given in Table 1, the above equations can be transformed as

$$T = T_A + 0.0135L + 0.687\,(0.5T_R + 0.0405L) \quad (2b)$$

where

L = solar radiation received on a horizontal surface in Langleys ($cal/cm^2/day$);
α = 4.95 for V = 10.7 mph (for New Jersey);
a = 0.0334 for k = 1.00 $Btu/ft^2/hour$, deg F/ft;
s = 0.20 Btu/lb, deg F and w = 150 lb/ft^3;
η_o = 0.687 for H = 4.95 and t_o = 24 hours; and
R = 0.0135L for b = 0.65.

The discussion here pertains to the temperature of a semi-infinite mass—that is, a very thick mass exposed on the top only, such as a pavement slab, with soil below.

Groeber (4, p. 86), defines the harmonic surface temperature oscillations in semi-infinite mass as being of the same period as the temperature of the surrounding air but lagging in time by an amount determined by ϵ_o. It is an analytical expression for semi-infinite mass; in the case of finite dimensions, the thickness and other properties will have to enter into the calculations of the time lag.

In the case of a slab such as a bridge deck exposed on the top and bottom and having a relatively thin mass, the problem becomes different, obviously because of physical limitations exposed to the same influences as previously indicated. By superimposing

TABLE 1

PARAMETER VALUES

Definition	Value	Assumed Average Value	Source
T_A = average daily air temperature (deg F)		Varies	U.S. Weather Bureau
T_R = daily range in air temperature (deg F)		Varies	U.S. Weather Bureau
R = average contribution to effective air temperature (deg F)	Varies	Varies	
L = solar radiation received on a horizontal surface in Langleys ($cal/cm^2/day$ = 3.69 $Btu/ft^2/day$)	Varies	Varies	Heating and Ventilating, July and Jan. 1949, pp. 62 and 72
k = conductivity of concrete ($Btu/ft^2/hour$, deg F/ft)	k = 7 to 16 per in.	$k = \frac{12.0}{12} = 1.0$	Ref. (9), p. 5-14; Ref. (10), p. 178
s = specific heat of concrete (Btu/lb, deg F)	s = 0.19 to 0.27	s = 0.20	Ref. (9), p. 5-4; Ref. (10), p. 1175
w = density of concrete (lb/ft^3)		w = 150	
b = absorptivity of surface to solar radiation	b = 0.65 to 0.80	b = 0.65 (for concrete)	Ref. (10), p. 95
v = wind velocity (mph)	Varies	v = 10.7	Ref. (10), p. 253
a = diffusivity ($ft^2/hour$)	Varies		
α = surface coefficient ($Btu/ft^2/hour$, deg F)	Varies		
t_o = length of period (hours)	t_o = 24		
t = time from beginning of cycle (hours)	Varies		
h = thickness of slab (ft)	Varies		
x = distance as measured from top of mass (ft)	Varies		
y = distance as measured from bottom of slab (ft)	Varies		

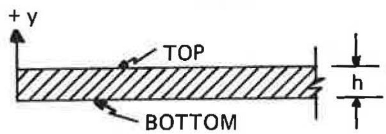

Figure 3. Exposure of bridge deck slab.

the steady-state heat transfer solutions developed by Carslaw and Jaeger (6) for periodic temperature states on an infinite plate, Zuk (5) has evolved the following equation for the time-temperature-depth relation:

$$T = \left(T_A + \frac{y}{h} R\right) + \eta_o \left[(0.5T_R + 3R)\, Y \sin\left(2\pi \frac{t}{t_o} - \phi + \ell n Y\right) + (0.5T_R)\, Y^1 \sin\left(2\pi \frac{t}{t_o}\right) \right] \quad (3)$$

where

$$Y = \left(\frac{\cosh 2my - \cos 2my}{\cosh 2mh - \cos 2mh}\right)^{1/2},$$

$$Y^1 = \left[\frac{\cosh 2m\,(h - y) - \cos 2m\,(h - y)}{\cosh 2mh - \cos 2mh}\right]^{1/2},$$

$$m = \left(\frac{\pi}{t_o a}\right)^{1/2}, \text{ and}$$

y = distance as measured from the bottom of the slab (Fig. 3).

According to Zuk (5), ϕ is to represent the phase angle difference in the sine curves of surface temperatures between the bottom and the top of the slab; for the time being there is no analytical expression for its value, because it must come basically from field observations.

Also, Zuk's opinion was that, although Eq. 1 seems to predict the magnitude of surface temperatures fairly well, it does not adequately take into account the phase shift between peaks of solar radiation and air temperatures, the phenomenon vital to the analytical determination of ϕ. However, his tests indicate the time difference to be of the order of 3 hours. The function of ϕ in Eq. 3 is in no way intended to relate to the ϕ of Carslaw and Jaeger.

Zuk suggests an empirical approach to this problem. In this way the established values will be suitable for use only in specific locations, unless enough data are collected to establish average values of this parameter. It seems, though, that an effort should be made to develop theoretically correct analytical expressions, correlated to the actual field observations. Perhaps it might be possible after such values are secured empirically.

By further developing Eq. 3, the temperature of the top surface of the slab, when $y = h$, is

$$T_{top} = (T_A + R) + \eta_o\,(0.5T_R + 3R) \sin\left(2\pi \frac{t}{t_o} - \phi\right) \quad (3a)$$

Again, for T_{top} to become maximum, $\sin\left(2\pi \frac{t}{t_o} - \phi\right) = 1.0$ and $y = h$; therefore

$$T_{top\,max} = (T_A + R) + \eta_o\,(0.5T_R + 3R) \quad (3b)$$

Equations 2a and 3b are obviously identical. Thus, the temperature of the bottom surface of the slab, when $y = o$, is

$$T_{bot} = T_A + \eta_o\,(0.5T_R) \sin\left(2\pi \frac{t}{t_o}\right) \quad (3c)$$

T_{bot} is maximum when $\sin\left(2\pi \frac{t}{t_o}\right) = 1.0$ and $y = o$; therefore

$$T_{bot\,max} = T_A + \eta_o\,(0.5T_R) \quad (3d)$$

Again utilizing previous assumptions, this equation can be expressed as follows:

$$T_{bot\ max} = T_A + 0.687(0.5T_R) = T_A + 0.3435T_R \qquad (3e)$$

And finally the temperature T of the slab can be determined from Eq. 3 for the length of the period t_o = 24 hours and various values of time t from the beginning of the cycle and the depth of slab y. Again, assumed average values of secondary parameters are given in Table 1.

In a bridge deck, the slab is often rigidly connected to the supporting beams forming a composite section. Within the scope of this report, only the case of steel beams is mentioned because it is not too unreasonable to assume that, considering the high heat conductivity of steel, the temperature of the beams will be the same as that of the ambient air. This was confirmed by Naruoka, Hirai, and Yamaguti (7).

Also significant is the configuration of the temperature distribution curve through a slab (Fig. 4). Being nonlinear, it follows an oscillatory wave of decreasing amplitude, as indicated in the mathematical argumentation of heat transfer analysis by Groeber (4) and confirmed also by the tests outlined by Zuk, and by Naruoka, Hirai, and Yamaguti. Zuk explains, "Generally speaking, in normal bridge structures, a temperature extreme (either hot or cold) at the top of the slab rapidly decays with depth, so that at approximately mid-depth the temperature is virtually the same as at the bottom of the slab" (5). The difference of the top and bottom temperatures can be about 20 F.

Inasmuch as the subject study is bridge end movements reflected in the sealing of joints, not the thermal stresses, the basic problem is to determine the average effective temperature for a bridge deck. Forgetting the monolithic character of a slab of, say, L = 100 ft, with an average coefficient of expansion C = 0.000066 in./ft/deg F, the differential movement between top and bottom (if they were free to move separately) would be $\Delta = c\Delta tL = 0.132$ in. This example is given only to accentuate the problem. The problem is even more pronounced in a composite section where, in the case of steel beams, the difference in coefficients of expansion would also have to be considered.

It is obvious that, because of the monolithic nature of a slab, all that can be expected is a small amount of rotation of the joint sides. For the purposes shown earlier, this differential movement can be neglected and only overall average joint side movements should be considered. In other words, the stabilizing effect of the structural characteristics of a bridge deck will produce, for these purposes, effective average movements. These average effective movements can be expected to result from so-called average effective deck temperatures.

Only continuous data, obtained by field observations for a period of at least 1 year, would give a lead to such a temperature in the spectrum of the temperature variations discussed in preceding paragraphs. Consequently, the preceding synthesis is an attempt to make understandable the complexity of the problems and the difficulty encountered in determining the actual effective temperatures that govern the thermal behavior of the specific bridge elements.

CALCULATION OF MAXIMUM TOP AND BOTTOM TEMPERATURES

A numerical example of a bridge deck slab temperature calculation is given here to illustrate the preceding synopsis.

A bridge in northern New Jersey was arbitrarily selected on July 26, 1966. Maximum daily temperature recorded was 90 F and minimum daily temperature recorded was 68 F. Solar radiation received on a horizontal surface was assumed to be L = 500 Langleys (see average solar radiation charts for month of July in New Jersey area). Thickness of the slab was assumed to be $h = \frac{8}{12}$ ft. From these data T_A = 79 F and T_R = 22 F. Maximum slab top temperature, using Eq. 2b, is

$$T = T_A + 0.0135L + 0.687\ (0.5T_R + 0.0405L)$$

$$= 79.0 + 6.75 + 0.687\ (11.0 + 20.25)$$

$$= 107.2\ F$$

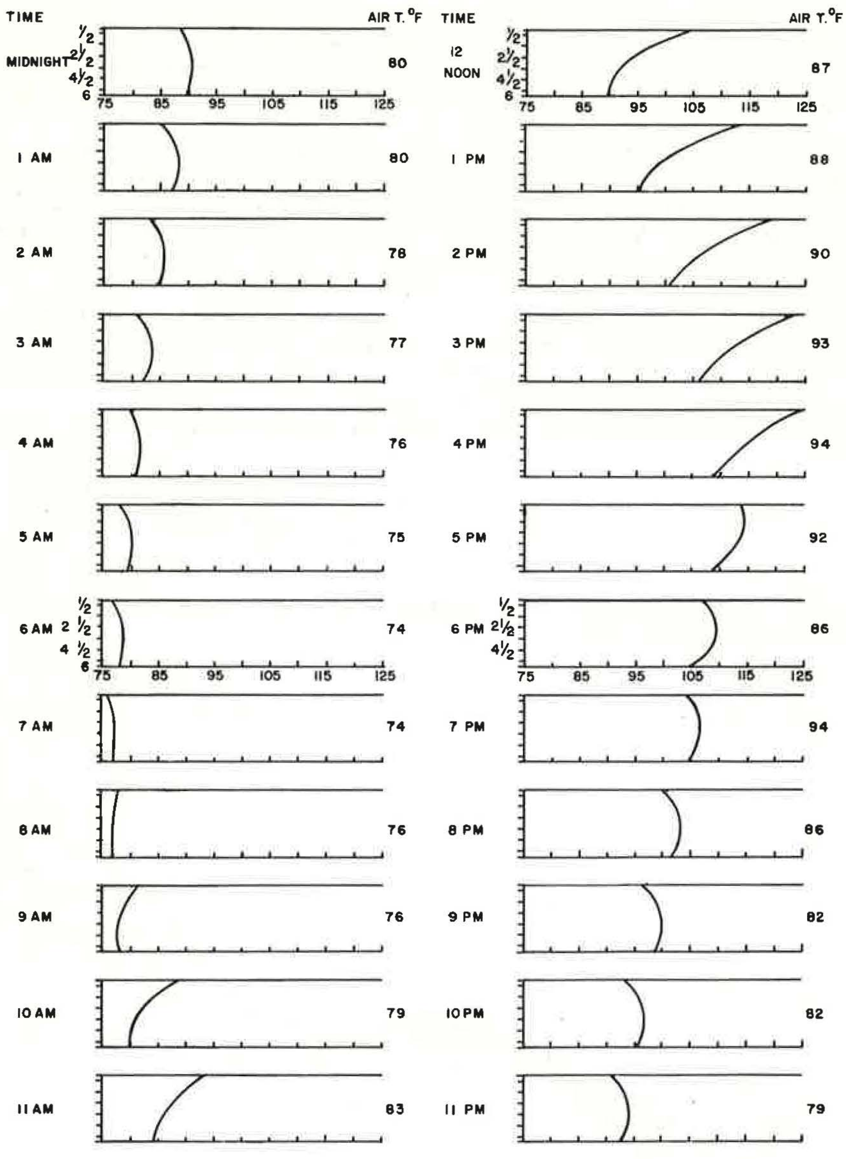

Figure 4. Temperature log at Ruston, Louisiana (Aug. 5, 1967).

Maximum slab bottom temperature, using Eq. 3e, is

$$T = T_A + 0.3435\ T_R$$

$$= 79.0 + 0.3435 \times 22.0$$

$$= 86.55\ F$$

EMPIRICAL APPROACH

The numerical analysis of temperature distribution through a slab is carried out here by an empirical approach as suggested by Zuk. The phase lag, ϕ, is computed from field observations of air and bridge deck concrete temperatures measured at various depths for 24 hourly time periods. These observations were taken from a study conducted by the Louisiana Department of Highways research and development section. The bridge is located where Louisiana 408 crosses over Interstate 20 near Ruston.

The temperature gradient was determined throughout the depth of a 6-in. concrete bridge deck. As stated by Rushing (8), the thermocouples were placed in the concrete at 1-in. intervals starting ½ in. from the top with the last point being ½ in. from the bottom of the slab. Two thermocouples were also used to record air temperatures. Continuous recording was performed for one yearly cycle. In this paper the data for only the two hottest days, August 4 and 5, 1967, are utilized.

Although the temperatures have been measured, the average values of the secondary parameters must be assumed as indicated previously and shown in Tables 1 and 2. Table 2 is a supplement to Table 1 and provides the specific parameter values suitable for use during August 1967 at Ruston, Louisiana.

The variable ϕ is established by rearranging Eq. 3 and solving it for ϕ:

$$\phi = 2\pi \frac{t}{t_o} + \ell n\, Y - \text{arc sin} \left[\frac{T - T_A - \frac{Y}{h} R - 0.5\eta_o T_R Y^1 \sin 2\pi \frac{t}{t_o}}{\eta_o\,(0.5T_R + 3R)Y} \right] \quad (4)$$

As can be seen from Eq. 4, values of ϕ depend on the time, depth of bridge deck, and air and concrete temperatures.

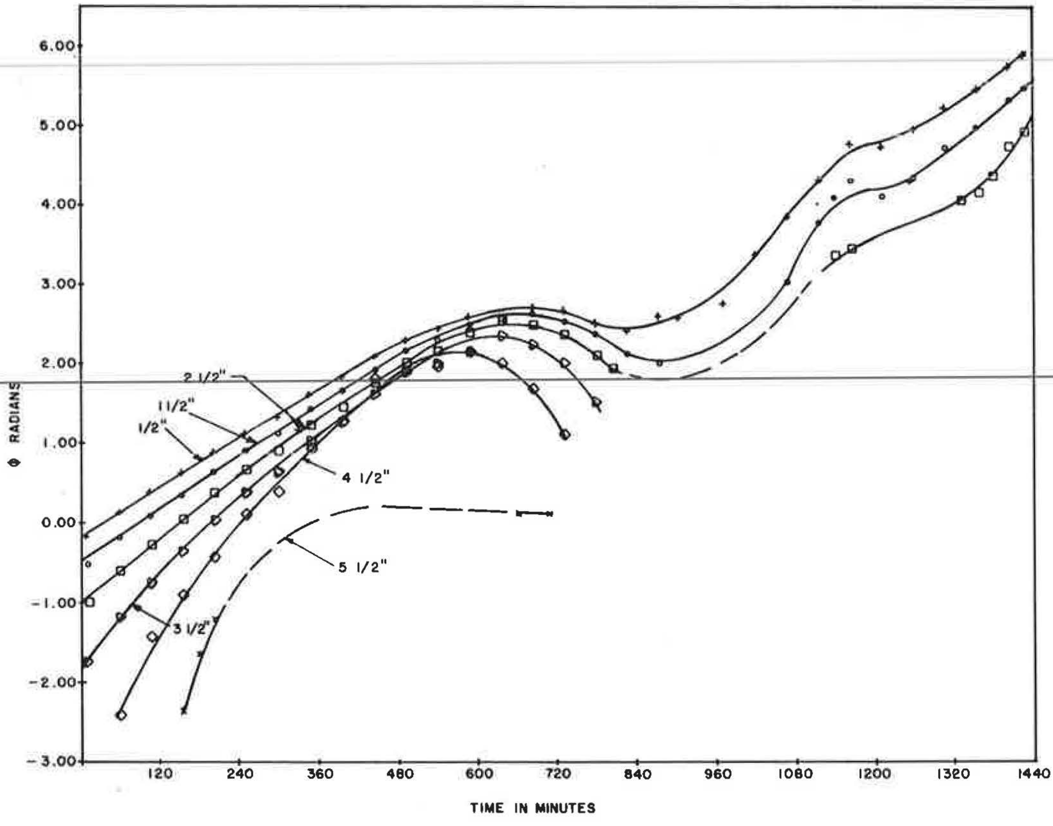

Figure 5. Phase lag between surface and bottom temperatures (Aug. 5, 1967).

TABLE 2

PARAMETER VALUES FOR USE DURING AUGUST 1967 AT RUSTON, LOUISIANA

Definition	Value	Assumed Average Value	Source
T_A = average daily air temperature (deg F)		Varies	Ref. (6)
T_R = daily range in air temperature (deg F)		26.1 for Aug. 5	Ref. (6)
L = solar radiation received on a horizontal surface in Langleys (cal/cm^2/day = 3.69 Btu/ft^2/day)	Varies	531	Ref. (11)
b = absorptivity of surface to solar radiation (dimensionless fraction)	b = 0.6 to 0.9	b = 0.65	Ref. (12)
v = wind velocity (mph)	Varies	V = 10.0 for Louisiana	Ref. (13)

DISCUSSION OF CURVES

For the purpose of the curve discussion of ϕ, as expressed in Eq. 4, the computed values are plotted as follows:

1. ϕ in radians, against time in minutes, for given depths in inches, as shown in Figure 5; and
2. ϕ against depth for given time in hours, as shown in Figure 6.

The computational work was facilitated by a FORTRAN program that was written for use on the IBM 360 computer while plotting was performed with the help of an on-line incremental plotter.

Before proceeding with the discussion of these curves, it should be remembered that ϕ represents the phase angle difference in the sine curves of surface temperatures between the bottom and the top of the slab. As such, it apparently should not vary with depth, but certainly it can vary with time. It is the term $\ln Y$ in the expression

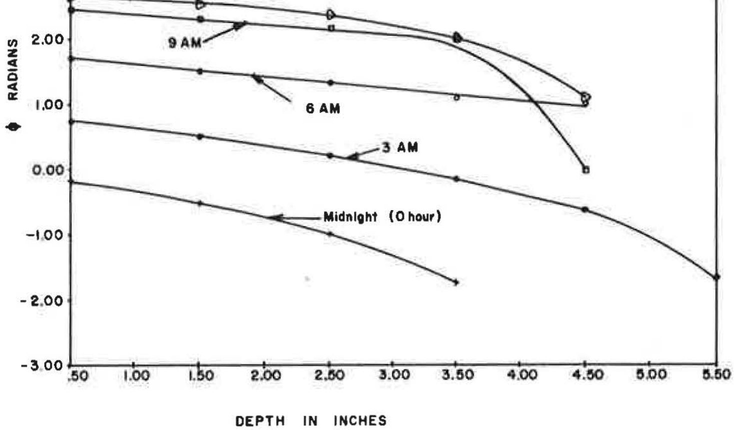

Figure 6. Phase lag variation with increase of depth measured from the top of the slab (Aug. 5, 1967).

$\sin\left(2\pi\frac{t}{t_o} - \phi + \ln Y\right)$ in Eq. 3 that is apparently intended to take care of phase lag variation with depth.

Thus the analysis of these curves reveals that when the variable ϕ is plotted against time the phase lag ϕ seems to decrease with the increase of depth of a slab. This phenomenon indicates the inadequacy of the depth-related factor, possibly of the term $\ln Y$. There should be only one ϕ curve for all depths. The resulting curves are approximately straight lines drooping in the middle, which possibly is caused by the sinusoidal component in Eq. 4. These curves also show that the value of ϕ is increasing with time, thus indicating that ϕ is a time-dependent variable.

In the case of ϕ plotted against depth, the variable ϕ increases with time as already explained. The curves are also approximately straight lines slightly drooping, which again indicates fallibility of depth-related factors. The components Y and Y^1 contain hyperbolic cosines and are depth-related variables. Specifically, as y decreases, so does the term Y, causing the denominator of arc sin also to diminish. As a result, $\ln Y$ becomes smaller as the value of arc sin grows, obviously decreasing the variable ϕ.

The negative values of ϕ indicate a phase lead of the bottom temperature that occurs during the 24-hour time cycle sometime between midnight and morning. Air convection, absence of solar radiation, and nightly heat emission are possible causes of that.

Thus, on August 5, 1967, as can be seen in Figure 5, ϕ becomes positive soon after midnight at the depth of $\frac{1}{2}$ in. from the top, and at $5\frac{1}{2}$ in. down, ϕ is positive several hours after midnight, possibly as late as about 10 a.m. At this time the lag is equal to zero. This obviously occurs because radiation or re-radiation affects the top of a deck sooner than the lower concrete strata.

In going from top to bottom of the slab, the values of Y become relatively small at approximately mid-depth, causing the arc sin to become larger than unity. In such cases, obviously, no ϕ curves can be plotted for such depths. The reason for this phenomenon is the limits of Eq. 3 apparently established by Zuk, as indicated in his correspondence to Kozlov as follows: "...in proposing my temperature equation, I had assumed that the phase lag term would be a constant over a given daily period.... As your computations indicate, it appears to vary with both depth and time of day. Even if it varied only with time my theoretical equation would no longer be valid. This may in part explain the difficulty encountered." Zuk concludes with this remark: "Nevertheless, your paper has brought to light new information which will eventually help resolve the problem of predicting end movements of bridges."

From the preceding analysis, it appears that the validity of Eq. 3 could be measured in degrees, but it is obvious that further studies are needed. Furthermore, the effect that this phenomenon would have on the calculation of the slab temperatures in the region below mid-depth is indicated in the following observations.

If the expression $\left(2\pi\frac{t}{t_o} - \phi + \ln Y\right) = \theta$, then in Eq. 3, the term $\eta_o\left[(0.5T_R + 3R)Y \sin\left(2\pi\frac{t}{t_o} - \phi + \ln Y\right)\right] = \eta_o (0.5T_R + 3R)Y \sin\theta$ ("the term" is of consequence because ϕ cannot be evaluated in this region). For the values in Table 2, T = 26.1 F, $\eta_o = 0.6787$, R = 7.45 F, h = $\frac{6}{12}$ ft, a = 0.0334, and m = 1.98.

The maximum error in calculating the temperature at these depths, if Y will be assumed equal to zero, can be estimated as follows:

y in inches	Y	"The Term" in deg F	Maximum sin θ
$2\frac{1}{2}$	0.4120	9.85 sin θ	Near Unity
$1\frac{1}{2}$	0.2455	5.86 sin θ	
$\frac{1}{2}$	0.0208	0.50 sin θ	

The approximate maximum value of sin θ will occur at the time when variable ϕ is a minimum.

The maximum error, which is in reference to the slab temperatures actually observed, is shown here only in order to illustrate more completely the suspected limitations of Eq. 3, and is actually a would-be error, if Eq. 3 is used outside the limits indicated in this paper.

SIGNIFICANCE OF FINDINGS

The data evaluated are for a period of one day and in one location only, and therefore it is obvious that no generalization is advisable. But it seems to be true that ϕ is a time-related variable and that there is a slight error in a depth-related variable, possibly in the term $\ell n\,Y$.

The cause for these seemingly small deviations of depth-related factors could be the apparent variations from actual conditions, which are mostly in the sinusoidal 24-hour cycle assumption, and also in all other secondary parameters that are assumed to be constant and average values. Nevertheless, it appears that, if the preceding statement is taken into account, Eq. 3 could be made applicable for predicting acceptably accurate temperatures.

The principal idea of this study was to find practical applications and to show ways for predicting temperatures. It was hoped that a nomogram, such as Figure 5, could be developed, giving values of ϕ for any time and thus enabling the engineer to predict reasonably accurate concrete temperatures, their changes, and, therefore, expected bridge deck movements. Perhaps if enough data were collected, for example on a zonal basis, this idea could become a distinct reality. Although it would be only as accurate as all the other parameters, it would nevertheless be much more than what an engineer has to work with at this time.

Thus the information presented in this paper should enlighten an engineer on the subjects of actual time difference and phase shift between air and concrete temperatures and on the effect of solar radiation. The increased comprehension of the problems has enhanced the feasibility of development of an equation for phase angle difference ϕ. The significance of this work is in showing the engineer the complexity of the subject problem.

CONCLUSIONS

In this paper, an attempt is made to summarize the reliable theoretical background of the thermal characteristics of a mass, such as a concrete bridge deck, and the fundamentals of heat transfer principles. Using the empirical approach suggested by Zuk, an attempt to determine the phase angle difference ϕ has been made on a limited basis, which clearly indicates the feasibility of such an approach. Within the limits available, one can draw the conclusion that phase lag ϕ increases with time in a roughly linear manner.

A recommendation, therefore, can now be made to implement Eq. 3 for the conditions established here so that a generalization of the approach shown in this paper could be attempted and a nomogram similar to Figure 5 could be made available.

The correlation between temperature and bridge end movements, in addition to some further information on the subject, will be investigated in future work, but only after the data from the experimental New Jersey sites are available.

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