

# Plan Design Model for Urban Area Use Allocations

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This paper reports the development of an urban area use model. The model's normative solution is to be utilized in the allocation of uses to locations so that net community return is maximized. A heuristic procedure for finding a good solution has been developed and programmed for the computer. The computer program was tested in an application to simplified data on the Lafayette, Indiana, plan area. The objective of urban area use allocations is to maximize the gross utility return from the use of area and from economies of scale, less the utility costs of transportation, of adaptation of locations to uses, and of incompatibility between proximate uses. In order to provide a computationally practical means for obtaining a near optimal solution to the model norm, we devised an iterative solution procedure, using an initial feasible solution. From the existing solution, the change in net return, as a result of a unit change in the allocation of each use to each location, is evaluated, and a number of the more beneficial interchanges are made to obtain a better solution. The maximum permissible changes in each use per location are then decreased by one-half of their current values. If the new limits are greater than preset minima and if further significant increases in return may be expected, the cycle is repeated.

•LAND USE ALLOCATION MODELS attempt to distribute the urban area activities to the available land in such a way as to achieve certain objectives. A worthy objective might be the maximization of benefit to all of the citizens of the community. The problem then would be one of identification and quantification.

The objective of this work was to develop a model of the urban system and to identify its component parts. In order to test the validity of the model, a computer program was developed that required reasonable quantifications of the system elements. Further refinements will result as each component is studied in depth.

The basis for the model is that each urban area will require a somewhat unique set of activities and uses and that the way these activities are assigned to the land will affect the net benefits received. We further assumed that the benefits, both positive and negative, are quantifiable and can be expressed in monetary value. The model will tend toward optimization, given some validity to assumptions made.

## MODEL FORM

There are many feasible configurations of activities within any area. The design problem is to determine the best allocation of given activities in an urban area, or perhaps even the allocation of part or all of available activities, in order to maximize return per person. The latter implies the determination of an optimum size, and thus is a more general problem than allocating given activities to maximize total return. This

work was restricted to the specific problem of allocating given activities, but this does not preclude an a priori assumption regarding optimum population size to determine the given activities. The solution to the problem will be constrained to some feasible region by available areal and other resources.

The problem was one of determining from a bounded set of many feasible allocations of area uses the one that maximizes return. Thus, any one of many mathematical programming techniques might be suitable. A heuristic method, which utilizes linear programming principles to obtain a good allocation plan, was used.

An urban structure has been shown to be a group of activities linked by communications. The latter can be divided into 2 classes, the desired flows and the undesirable by-products of both the activities and the flows. An urban form that accommodates this structure can be represented by a group of locations, a network of channels linking the location centroids, and barriers between each pair of locations. It is implied that each location should be delineated so that it is homogeneous with respect to local and general topography, flora (natural cover) and fauna, foundation materials, water conditions, and access to the centroid and, thence, to the channel network. If these characteristics can be assumed to be similar within the location, then average values for each characteristic for each location can be used in the model.

Barriers to interlocation by-product transmission are assumed to exist between all pairs of locations, but for proximate locations their effectiveness tends to approach zero. The flow channels available for the movement of the generated flows include, but are not necessarily restricted to, channels in the plan area. In most cases, the flow channels would consist of major and minor arterials and collector streets. Where there are large numbers of locations involved, it may be necessary to simplify the computations on long-distance flow costs. This can be achieved by aggregating locations with similar major thoroughfare access to form districts. Flows then may be treated in 2 groups, those whose destinations lie within the district containing the origin location and those whose destinations lie in another district.

The functional relationships between these model factors can now be expressed more precisely under the following categories:

1. Required activities and uses,
2. Locations and use allocations,
3. Flows, and
4. By-product transmission.

#### Required Activities and Uses

The number of units of each activity that will take place within the area is a function of the characteristics of the local population, local natural and cultural resources, and the extramural interactions with the surrounding region and with urban realms. In a general model, the amounts of each activity could be variables whose levels are determined by the solution of the model. But, in the model developed here, the amounts of each activity are fixed as model inputs. These amounts can be estimated or predicted from population and economic surveys.

The given data for a model must include not only the number of units of each activity but also the number of units of an activity that will be accommodated by each use. Further, the uses and activities must satisfy the following:

$$\sum_{K \in S_T} \sum_{I=1}^N Y_{TK} \cdot X_{IK} = A_T \text{ for every } T \quad (1)$$

where

$\epsilon$  = "an element of the set of,"

$S_T$  = set of uses that can accommodate activity T,

$N$  = number of locations in the plan area,

$Y_{TK}$  = number of units of activity T per acre of use K,

$X_{IK}$  = number of acres of use K allocated to location I, and  
 $A_T$  = number of units of activity T.

### Locations and Use Allocations

The plan area must be represented as a group of locations, each having a finite area and homogeneous characteristics. The area of each location is fixed. Because all of that area must be assigned to one or more uses (one of which may be the vacant use), the use allocations must satisfy the following:

$$\sum_{K=1}^M X_{IK} = P_I \text{ for every } I \quad (2)$$

where

$X_{IK}$  = number of acres of use K allocated to location I,  
 $M$  = number of uses, and  
 $P_I$  = size in acres of location I.

The use allocations must be non-negative and thus satisfy the following:

$$X_{IK} \geq 0 \text{ for every pair of } I \text{ and } K \quad (3)$$

### Flows

Flow costs, as used here, are caused primarily by person trips. Accordingly, to simplify the model, we can express other significant flows in terms of person trips. The total of all types of flows then could be represented as equivalent person trips. To simulate flow costs more precisely, however, we might weight some types of trips according to their peaking or time density characteristics.

Total flows are input to the model as the number of equivalent person trips generated per acre of origin use. Each use will have associated trip generation rates, one for each trip purpose as defined by the destination use. The model has assumed that these rates are fixed. Further, it has been assumed that trip attractions are balanced by trip productions and are distributed uniformly over all acceptable destination use allocations. That is, trip ends must satisfy the following conditions:

$$\begin{aligned} O_{IKL} &= Z_{KL} \cdot X_{IK} \text{ for every combination of } I, K, \text{ and } L \quad (4) \\ &= \text{equivalent person trip origins of type } KL \text{ (that} \\ &\quad \text{is, originating at use } K \text{ and destined to use } L) \\ &\quad \text{generated by location } I \end{aligned}$$

where

$Z_{KL}$  = equivalent person trips of type KL generated per acre of use K and  
 $X_{IK}$  = number of acres of use K allocated to location I,

and

$$\begin{aligned} D_{JKL} &= \sum_{I=1}^N O_{IKL} \cdot \frac{X_{JL}}{\sum_{H=1}^N X_{HL}} \text{ for every combination of } J, K, \text{ and } L \quad (5) \\ &= \text{equivalent person trip destinations of type } KL \text{ attracted} \\ &\quad \text{to location } J \end{aligned}$$

where

$X_{JL}$  = number of acres of use L allocated to location J,  
 $X_{HL}$  = number of acres of use L allocated to location H, and  
 $N$  = number of locations.

The flow route choices are made such that the total flow cost of the entire community is minimized. Thus the flow routes are fixed for a given allocation of uses and a given transportation system.

### By-Product Transmission

Incompatibility of use allocations occurs only if the harmful by-products are transmitted. Between all pairs of uses there are spatial or physical barriers or both to such transmission. We assumed for model purposes that these barriers modify the standard (unrestricted) by-product transmission rate between 2 adjacent locations of average size, as follows:

1. The transmission values are proportional to the ratio of the given interface length to a standard interface length determined for each model application.
2. The transmission values are reduced by some factor if there is an intervening use.
3. The transmission values are reduced by some factor if there is a physical barrier to transmission.

In the general case the transmission rate will not be the same for each by-product. For example, earth embankments may be effective as noise barriers but not as odor barriers. However, until further research has been conducted into incompatibility, we deemed it advisable to assume that all by-products had a single transmission rate.

### MODEL NORM

The goal of the normative land use model is to obtain a feasible solution that will maximize public as well as private utility returns. All the factors of the model norm can be represented as a net utility return in terms of location return less adaptability costs, a net utility cost in terms of flows, an incompatibility cost caused by proximity, and a negative incompatibility cost resulting from economies of scale.

### Net Location Returns

An effort was made to recognize that variations in the physical determinants of lands lend them to one or several uses rather than others. Furthermore, if the model would optimize with proper weighting given to existing land uses, then adaptation costs must be included. Based on the literature reviews on adaptability of uses to locations and on sensitivity of costs and returns to density and organization, we hypothesized that a net location return (location return less adaptability costs) could be expressed as follows:

$$R_{IK} = K_{1K} + K_{2K} \cdot Cd^{-1} + K_{3K} \cdot Ad^{-1} + f_{4K}(T_I) + f_{5K}(F_I) + f_{6K}(D_I) + f_{7K}(N_I) + f_{8K}(S_I) \quad (6)$$

where

- K = the Kth use, K = 1, . . . , M;
- I = the Ith location, I = 1, . . . , N;
- $R_{IK}$  = net location return for use K in location I;
- $K_{JK}$  = constant for use K, J = 1, 2, 3;
- $f_{JK}()$  = nonlinear function of the bracketed variable, J = 4, . . . , 8;
- Cd = channel density;
- Ad = areal density;
- $T_I$  = topography rating of location I;
- $F_I$  = foundation rating of location I;
- $D_I$  = drainage rating of location I;
- $N_I$  = natural resources (excluding man-made) of location I; and
- $S_I$  = cultural resources of location I.

The first 3 terms on the right side of the equation are of no concern if the amount of each use is fixed because they are constant (not a function of  $I$ ) for a given use, irrespective of location.

Each component of the adaptability costs (the remaining 5 terms) is relatively constant throughout a location, except for the cultural or man-made facilities. When there are existing structures or other facilities on part of a location, there will be a very large saving in adaptation costs if the existing use is reallocated back to that part of the location and thus to the existing structures. Consequently, 2 different net location returns will be calculated for each use in each location. The net return value that includes a zero cultural feature cost is applicable only when a use is reallocated to a currently existing use. It will be designated as  $R_{IK}'$ . In the other net location return,  $R_{IK}''$ , there will be a nonzero cultural feature cost that should cover the cost of demolition and relocation of the existing activity as well as the cost of construction of new facilities. In practice, however, it might be more expedient to add the demolition and relocation costs as returns in  $R_{IK}'$  and neglect them in  $R_{IK}''$ . If this were not done, a separate  $R_{IK}''$  would have to be calculated for each existing use. This expediency results in constant terms being added to the objective function, but the relative values of the returns are correct, and thus the allocations are not affected.

The 2 net location returns are necessary input data. They are independent of the allocation of uses.

### Channel Flow Costs

The total cost of flow on a channel link may be composed of the following:

1. Channel capital cost,
2. Channel operation and maintenance costs,
3. Vehicular capital costs,
4. Vehicular operation and maintenance costs, and
5. Time costs.

Several of these components are at least partially fixed, that is, independent of flow volumes. Furthermore, they are also independent of the area use allocations. Thus the channel costs will be split into fixed costs, which need not be considered directly in the allocation process, and flow costs, which are assumed to be related linearly to flows and thus must be considered in the allocation process.

The unit flow costs, which are inputs to the model, are user costs for flows from district centroid to district centroid or location centroid to location centroid. These are determined by summing the flow costs of the channel links that comprise the least-cost route. We assumed also that the channel flow costs are independent of the flow volumes. This implies that there are no capacity limits on any route, unless the flow costs are set sufficiently high to limit volumes.

### Inter-Use Incompatibility

The review of operating and performance standards noted that positive incompatibility costs will result from differential operating standards. We assumed that the positive incompatibility cost is (a) proportional to the difference in operating standards of the incompatible uses, (b) proportional to the length of the interface where the incompatibility occurs, (c) reduced by barriers such as major transport routes or topography, and (d) reduced by spatial separation of the incompatible uses.

The utility cost of incompatibility at a given distance might be determined from what the user will pay to increase the distance of separation from, or to reduce the operating level of, the offensive use. By summing the cost distance curve over a typical location and dividing by area, we obtain a linear unit cost figure.

An example of a possible procedure is shown in Figure 1, where hypothetical values are used. The figure is based on the assumption that the incompatibility cost of use L to use K in adjacent locations may be determined by the use of average values for successive bands of 200 feet in width. As an illustration, the area of use L within the bands around use K was set at 65, 80, and 95 acres, for 0 to 200, 200 to 400, and 400 to 600

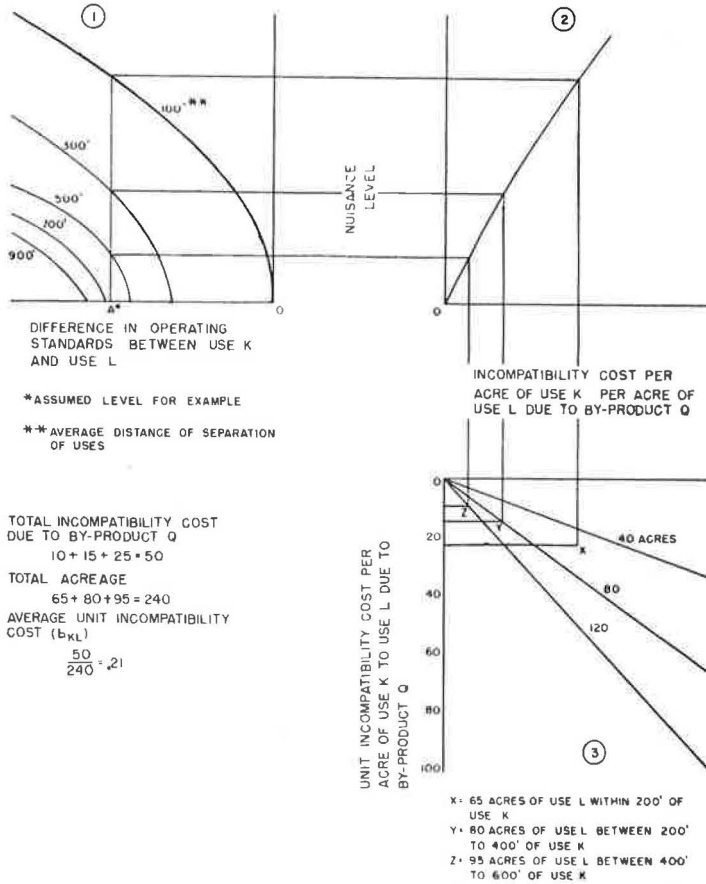


Figure 1. Determination of the cost of incompatibility between one acre of use K and of use L caused by by-product Q.

feet respectively. The difference in operating standards of use K and use L was designated as A. By entering graph 1 at A, we can determine from 1 and 2 respectively the differential nuisance level and the unit utility cost for each band at its median separation distance from use K. Then by utilizing the area of each band, we can determine from 3 the total incompatibility cost for each band. The aggregate of these band costs would then be divided by the total area to derive a unit cost for the 2 locations. This procedure may be applied to 2 average locations for any by-product under the assumptions that all of one location is occupied by use K and all of the other location by use L.

If we assume that there are P significant by-products, incompatibility costs may be expressed as follows:

$$I_{IK} = \sum_{J=1}^N \sum_{L=1}^M \sum_{Q=1}^P f_{KLQ} (O_{KQ} - O_{LQ}) G_{IJQ} \quad (7)$$

where

$I_{IK}$  = incompatibility cost of use K in location I to all other uses,

- $f_{KLQ}$  = functional relationship of cost of differential transmission of by-product Q between uses K and L,  
 $O_{KQ}$  = operating standard of use K for the Qth by-product,  
 $O_{LQ}$  = operating standard of use L for the Qth by-product,  
 $G_{IJQ}$  = relative incompatibility transmission rate from location I to location J for the Qth by-product,  
 $N$  = number of locations,  
 $M$  = number of uses, and  
 $P$  = number of by-products.

Little is now known about the quantification of by-product incompatibility costs. Accordingly, we assumed that an average cost and transmission figure for all transmitted by-products between any pair of uses would be utilized. That is,

$$B_{KL} = \sum_{Q=1}^P f_{KLQ}(O_{KQ} - O_{LQ}) \quad (8)$$

- the average utility lost because of the incompatibility effect of one acre of use L on one acre of use K when allocated to the same location or to adjacent locations I and J whose  $G_{IJ} = 1$

and

$$G_{IJ} = \sum_{Q=1}^P G_{IJQ} / P \quad (9)$$

- = the average relative incompatibility transmission rate from location I to location J for P uses

The incompatibility costs have been based on homogeneous uses in each location. For a location having a mixture of uses, these assumed costs will be in error for 2 reasons. The true incompatibility costs will be higher than those assumed in the model because smaller units have a greater interfacial contact per unit area and also a higher utility cost because of a lower average distance of separation of the incompatible uses. But on the other hand, the designer will reduce the incompatibility by arrangement of the uses within the location, and this may tend to reduce the preceding errors.

### Economies of Scale

There is a relatively high return from the allocation of some uses in large blocks. It is in reality a reduction in activity costs, but it may be treated as a return in the objective function because this maintains the proper relative values for allocation purposes. These scale economies of a use are a cohesive force and thus are equivalent to negative incompatibility of the use with itself. We assumed that the economy of scale returns or savings are related linearly to the homogeneous use allocation size. Thus the incompatibility can be treated simultaneously with economies of scale. The economies of scale or negative incompatibilities then form the diagonal of the inter-use incompatibility matrix.

### The Objective Function

The objective function or norm may now be stated as follows: Maximize

$$\begin{aligned}
 & \sum_{I=1}^N \sum_{K=1}^M R_{IK} \cdot X_{IK} - \sum_{I=1}^N \sum_{J=1}^N U_{IJ} \cdot F_{IJ} \\
 & - \sum_{I=1}^N \sum_{J=1}^N \sum_{K=1}^M \sum_{L=1}^M X_{IK} \cdot X_{JL} \cdot B_{KL} \cdot G_{IJ} - F_{cc}
 \end{aligned} \quad (10)$$

subject to the constraints in Eqs. 1 through 5,

where

- $R_{IK}$  = net location return from allocating one acre of use K to location I (this can equal either  $R_{IK}'$  or  $R_{IK}''$  as discussed previously);
- $X_{IK}$  = number of acres of use K allocated to location I;
- $U_{IJ}$  = unit flow cost from location I to location J (excluding fixed channel costs);
- $F_{IJ}$  = number of equivalent person trips from location I to location J;
- $X_{JL}$  = number of acres of use L allocated to location J;
- $F_{cc}$  = fixed (for a given transportation plan) component of channel costs;
- $B_{KL}$  = average utility lost because of the incompatibility effect of one acre of use L on one acre of use K when allocated to the same location or to adjacent locations I and J whose  $G_{IJ} = 1$  (Eq. 8);
- $G_{IJ}$  = average relative incompatibility transmission rate from location I to location J (Eq. 9);
- $N$  = number of locations;
- $M$  = number of uses;

and all trips must be distributed at the least total cost and in such a manner that trip origins and destinations in each location satisfy constraints in Eqs. 4 and 5.

## A PROPOSED ASCENT SOLUTION PROCEDURE

### Additional Assumptions

In addition to those made in the formulation of the model and the norm, the following assumptions were made to devise a practical solution procedure:

1. Given an existing solution, that a satisfactory area use move evaluator may be formed from an approximation of the change in the objective function that would result from a unit increase in that allocation in the given location (thus there will be one evaluator for each use in each location);
2. That these evaluators may be calculated in 3 separate parts, corresponding to the first 3 terms of the objective function;
3. That the fourth term of the objective function, fixed channel costs, may be neglected in the formation of the evaluator (this will result in an attempt to optimize the use allocations for the given transportation plan);
4. That the change in the interactions of the area uses that have been moved simultaneously (without updating the evaluators) may be neglected in the formation of the evaluators, provided that the magnitudes of the allocation moves are limited;
5. That the flow cost contribution to a move evaluator (excluding fixed channel costs), although dependent on the magnitude of the move, can be approximated by one of two unit costs, again if it is assumed that the magnitude of the allocation move is limited; and
6. That a better solution will be obtained by decreasing each use allocation and re-allocating these uses with the objective of maximizing the sum of the products of the reallocated uses and their respective move evaluators.

### The General Procedure

The solution procedure that has been developed to solve the problem requires an initial feasible solution (step 1 of the algorithm given in a succeeding section). Then movements of the allocated uses, which would result in an increase in the objective function, are identified and some of the more beneficial moves are made. This forms a new initial feasible solution for repeating the entire process for the second of several iterations. The iterative procedure is utilized because the interaction effects of simultaneous moves are neglected in forming the evaluators of potential moves. As a result, the evaluators must be updated periodically. The primary decision variables in the procedure are the use allocations. But, the channels or transportation system may



be modified on a judgment basis either between iterations or in some cases during iterations.

We assumed that the sum of the changes in the objective function terms, when some use  $K$  is superimposed on the existing feasible plan in location  $I$ , is a satisfactory evaluator of the value of additional allocations of use  $K$  to location  $I$ . In the algorithm these changes in the first 3 terms of the objective function are evaluated separately for every use in every location. Because the fourth term is not changed in land use evaluation, it may be neglected.

The unit change in the first term of the objective function is known exactly because it is equal to the appropriate net location return. However, the second term, flow costs, is not related linearly to the change in allocation. Thus 2 unit flow cost estimators, which reflect an average change in flow costs for different amounts of change in the given use allocation, were developed and used in such a manner that the flow costs are never underestimated. This should result in a conservative evaluator of a potential move. With each successive iteration the amount of the overestimation is reduced, permitting the identification of the less desirable moves. These unit flow costs are calculated in steps 3 and 4 of the algorithm. Interaction effects caused by simultaneous changes in several uses are neglected. The change in the third term of the objective function, incompatibility costs, is calculated in step 2 of the algorithm for a unit increase in the given use. Thus, it also neglects interaction caused by simultaneous changes of several uses.

Once all of the evaluator components have been calculated, they may be combined. However, there are 2 net location return components ( $R_{IK}'$  and  $R_{IK}''$  as discussed previously) and 2 flow cost components. The choice of the appropriate values is dependent on the level of the current allocations. For each use in each location, there are 4 possible levels of the evaluator.

In order to permit reallocation of the uses, with the objective of increasing return, the problem is converted to a capacitated transportation problem. In step 5 of the algorithm, the origins (unallocated activities) and destinations (uncommitted areas) are created by decreasing each use allocation in each location. The evaluators are then formed in step 6 for the new decreased allocation level. The evaluator then is equivalent to the flow cost of the transportation algorithm, but the problem is one of maximization rather than minimization. In step 7 a capacity limit on each additional allocation is imposed. This capacity is set equal to the lesser of (a) the maximum amount that the use can be increased without resulting in an infeasible solution, or (b) the maximum amount that the use can be increased without invalidating the evaluator, that is, up to the level at which either the net location return or the assumed unit flow cost changes.

Given these capacitated transportation formulations, the initial levels of the dual variables are calculated and as much of the unallocated uses as possible are reallocated (step 8). Because the evaluators are not constant over the entire range of possible allocations, an optimal solution of the simplified problem may not be determined. In step 9, the evaluators, capacities, and dual variables are updated simultaneously and as many additional reallocations as possible are made. This step is repeated until all uses have been assigned.

#### Flow Cost Components for the Evaluators

Two unit flow costs, one for allocations at levels less than those of the previous allocation,  $X_{IK(a)}$ , and one for all other allocations, are developed (Fig. 2). In view of the basic premise that only one move at a time is considered in the formation of the unit costs, any increase in the allocation of use  $K$  to location  $I$  results in the attraction of trips at increasing cost. Thus, as the allocation of use  $K$  increases, the average flow cost for trips from use  $K$  in location  $I$  tends to increase and never decreases (Fig. 2). However, for low levels of the allocation, the unit flow cost contribution to the objective function may be negative because some trips are cancelled (this is discussed more fully later). We assumed that, if the current allocation of use  $K$  to location  $I$ ,  $X_{IK}$ , is less than that in the previous plan,  $X_{IK(a)}$ , then the flow cost will be assumed to be equal to

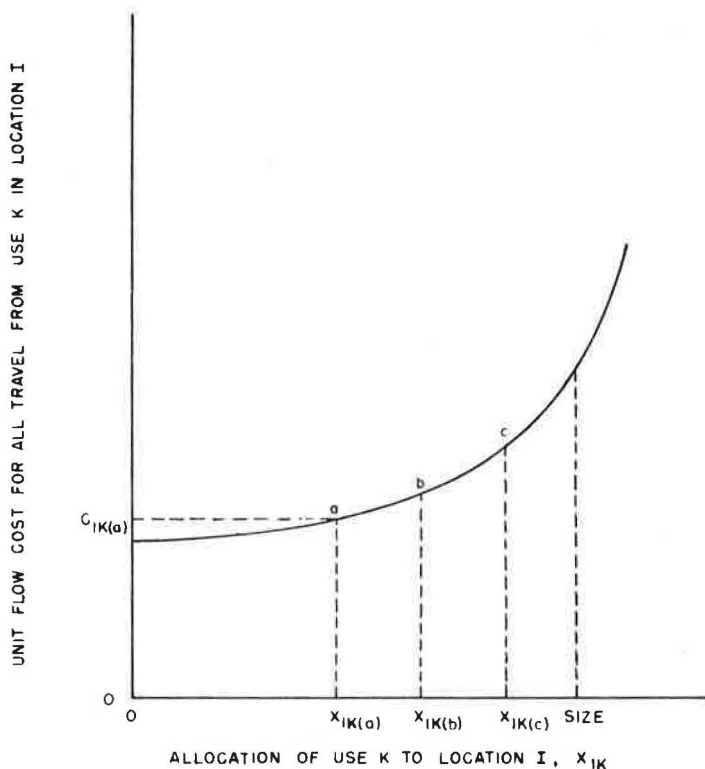


Figure 2. Flow costs for use K in location I for a given allocation plan.

$C_{IK}(a)$ . Therefore, if  $X_{IK}$  is less than  $X_{IK}(a)$  the cost tends to be overestimated. In all other cases, the unit flow cost will be assumed to be equal to the average unit flow cost for an increased allocation of use K to location I, where this increase is the maximum permissible,  $X_{IK}(c) - X_{IK}(a) = \Delta X_{IK}$ . The maximum permissible increase is the lesser of the interchange limit (to be outlined in the next section) or the location size,  $P_I$  less  $X_{IK}(a)$ . This assumed unit flow cost,  $C_{IK}'$ , is equal to the area under the curve ABC divided by  $\Delta X_{IK}$ . If, however, the new allocation level is less than  $X_{IK}(c)$ , such as  $X_{IK}(b)$ , the  $C_{IK}'$  is an overestimate of the theoretical cost, which would be the area under the curve AB divided by the net change in allocation over the previous solution. Thus, no flow costs are underestimated and some are overestimated.

As the interchange limit is decreased on successive iterations, the maximum permissible allocation change will decrease, and thus the conservative bias of  $C_{IK}'$  tends to decrease also. This permits the identification of new moves, even if in lesser quantities and with lower net benefit.

In some cases,  $C_{IK}(a)$  may be greater than  $C_{IK}'$ .  $C_{IK}(a)$  is calculated from the existing shadow prices. (The shadow price of an origin is the change in the total optimal cost of all flows caused by a unit increase in the production at an origin. Similarly, the shadow price of a destination is the change caused by a unit increase in the trip attractions of a destination.) But, the shadow prices consider only flow costs and do not account for the savings resulting from the elimination of some interlocation trips. When  $X_{IK}$  is augmented, each  $X_{JL}$  is assumed to be augmented in proportion to its current level, so that all trips may be allocated. This means, however, that some trips from location J to location I may now satisfy their destination use desire in location J resulting in a saving, and similarly some of the new trips originating in location I can

now satisfy their destination in location I. These savings are not accounted for by the shadow prices because they consider only additional trips from pure origins to pure destinations, while in the preceding some of the locations became both origins and destinations. On the other hand, these savings are accounted for in  $C_{IK}'$  because it considers the total change in the objective function, which sums the cost of new flows less the savings caused by flows that are eliminated. If  $C_{IK(a)}$  is greater than  $C_{IK}'$ , then  $C_{IK(a)}$  is assumed to be equal to  $C_{IK}'$ . This will still result in an overestimation of the flow cost in many of these cases and thus the positive bias of the assumed unit flow costs is maintained.

### Interchange Limits

We assumed that the average change in the objective function caused by a temporary additional amount of a given use in a given location is an evaluator of the worth of increasing that use in that location. Provided that only one move is made and provided that its magnitude is equal to that of the temporary addition, then the evaluator is exact. However, if more than one move is made without recalculating the evaluator, the second and subsequent moves are based on incorrect evaluators. The error in the evaluator is caused by neglect of the interaction term between the changed uses. Also, because the unit flow cost for a given use in a given location is dependent on the magnitude of that use allocation, the magnitude of the proposed move must be equal to that assumed in the formation of the unit flow cost for the move evaluator to be exact.

Both of these errors may be neglected, provided that only small moves are made before the evaluators are recalculated. Thus maximum interchange limits have been established. The magnitude of the limits will vary considerably according to the type of use.

In step 10 of the solution algorithm (the conclusion of one iteration), the interchange limits are arbitrarily decreased by one-half of their current values. Because the larger of the assumed unit flow costs (one of the move evaluator components) is calculated for a maximum permissible increase in the allocation, the decrease in the limits results in a decrease in the assumed unit flow cost for the next iteration. The new flow cost leads to a less conservative move evaluator, and thus it permits the identification of less beneficial moves on the next iteration. The choice of the fraction (one-half) for the decrease in the interchange limit per iteration was completely arbitrary.

Minimum interchange limits have been provided as a device for terminating the algorithm. However, termination based on the rate of convergence toward an upper bound on the objective function certainly would be preferable if a reasonable estimating procedure for this bound could be devised.

### Evaluator Synthesis

The formation of the unit move evaluators, the second phase in the solution procedure, is shown in Figure 3. The appropriate location return is selected in step A. In step B, the unit incompatibility cost is subtracted from this return. Finally, in step C the algorithm selects one of the two unit flow costs, depending on the relationship of  $X_{IK}$  to  $X_{IK(a)}$  as outlined earlier, and subtracts this cost from the output of step B. The result,  $E_{IK}$ , is an evaluator of the return, net of cost, for allocating one acre of use K to location I. Because the assumed flow costs have a positive bias, the evaluator tends to be conservative.

### The Solution Algorithm

The general solution algorithm proceeds according to the following steps. All terms are defined at the conclusion of the algorithm.

1. Find a feasible allocation of the required uses to the available locations (a good starting solution, such as an existing plan, will speed convergence toward the optimum).
2. Calculate the unit incompatibility cost,  $In_{IK}$ , resulting from both negative and positive incompatibility, for an infinitesimal increase in  $X_{IK}$ . Repeat for every I and K. Set  $K = 0$ .

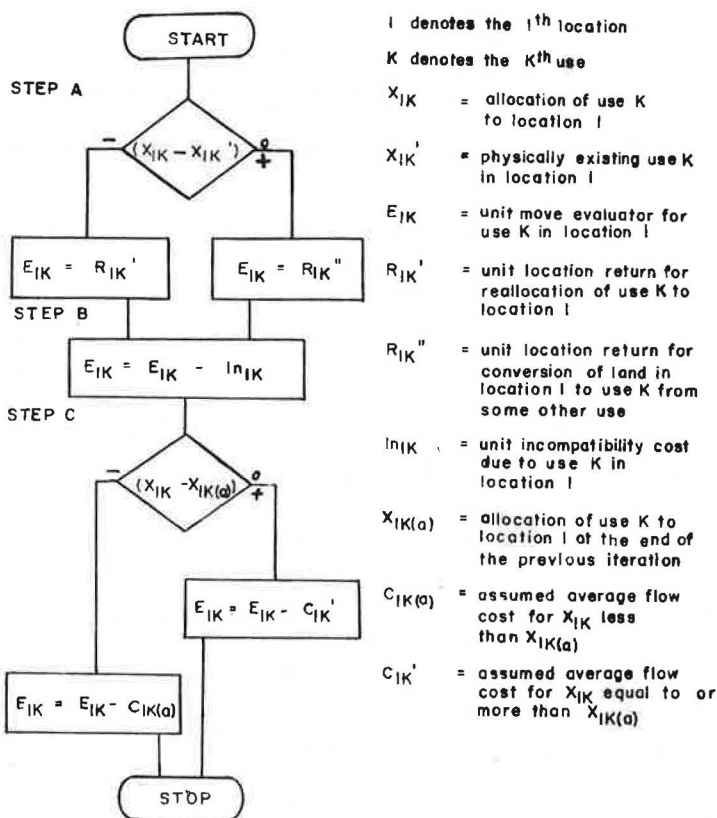


Figure 3. Move evaluator formation.

3. Calculate the interdistrict flow costs and the interdistrict components of the assumed unit flow costs as follows:

3.1. Set  $K = K + 1$ . If  $K$  is greater than the number of uses, go to step 4. Otherwise, set the flow evaluators  $C_{DK}$  and  $C_{DK(a)} = 0$  for every  $D$ . Set  $L = 1$ .

3.2. Using the appropriate input inter-use and intra-use flows per activity unit and the current allocations, calculate origins and destinations for flow type  $KL$  (a flow type is designated by the origin and destination uses respectively). Allocate these flows at least cost by means of a transportation algorithm, recording the district shadow prices, the individual total cost for each flow type, and the total cost for all flows allocated.

3.3. If a decrease in the allocation of use  $K$  to district  $D$ ,  $X_{DK}$ , is feasible, utilize the shadow prices to calculate the cost savings for the reduction in interdistrict flow type  $KL$  that would result from an infinitesimal decrease in  $X_{DK}$ , that is, the products of the shadow prices for the destinations and their respective fractions of the destination totals summed over all possible destinations, plus the shadow price of the origin. Add twice this saving to  $C_{DK(a)}$ . Repeat for every  $D$ .

3.4. Temporarily increase  $X_{DK}$  by  $Rm_K$ , the maximum interchange limit for use  $K$ . Assume that the destination use  $L$  is augmented at each district  $E$  in proportion to the current  $X_{EL}$ , such that the total number of new destinations is equal to the number of new origins created by augmenting  $X_{DK}$ . Calculate the new flows of type  $KL$  and allocate them at least cost. Record the change in total cost for flow type  $KL$  divided by  $Rm_K$ . Add twice this figure to  $C_{DK}^'$ . Decrease  $X_{DK}$  by  $Rm_K$  in order to revert to  $X_{DK(a)}$ . Repeat for every district,  $D$ .

3.5. Set  $L = L + 1$ . If  $L$  is less than or equal to the number of uses, return to step 3.2. Otherwise, return to step 3.1.

4. Calculate the interlocation flow costs and the interlocation components of the assumed unit flow costs (within the origin district only) as follows:

4.1. Set  $D = 0$ .

4.2. Set  $D = D + 1$ . If  $D$  is greater than the number of districts, go to step 5. Set  $K = 0$ .

4.3. Set  $K = K + 1$ . If  $K$  is greater than the number of uses, go to step 4.2. Otherwise, set  $C_{IK}' = C_{DK}'$  and  $C_{IK}(a) = C_{DK}(a)$ , where  $D$  denotes the district containing location  $I$ . Repeat for every  $I \in S_D$ . Set  $L = 1$ .

4.4. Using the appropriate input inter-use and intra-use flows per activity unit and the current allocations, calculate origins and destinations for flow type  $KL$ . Allocate these flows at least cost by means of a transportation algorithm, recording the location shadow prices, the total cost for flow type  $KL$ , and the total cost for all flows allocated.

4.5. If a decrease in  $X_{IK}$  is feasible, calculate the intralocation cost savings for the reduction in flow type  $KL$  that would result from an infinitesimal decrease in  $X_{IK}$ , that is, the products of the shadow prices for the destinations and their respective fractions of the destination totals, summed over all locations within the district. Add twice this saving to  $C_{IK}(a)$ . Repeat for every  $I \in S_D$ .

4.6. Temporarily increase  $X_{IK}$  by the maximum permissible amount,  $\Delta X_{IK}$ . Assume that the destination use  $L$  is augmented at each location  $J$  in proportion to the current  $X_{JL}$ , such that the total number of new destinations over all destinations in all districts is equal to the number of new origins created by augmenting  $X_{IK}$ . Calculate the new intradistrict flows of type  $KL$  and allocate them at least cost. Record the change in total cost for flow type  $KL$  divided by  $\Delta X_{IK}$ . Add twice this amount to  $C_{IK}'$ . Decrease  $X_{IK}$  by  $\Delta X_{IK}$  in order to revert to  $X_{IK}(a)$ . Repeat for every  $I \in S_D$ .

4.7. Set  $L = L + 1$ . If  $L$  is less than or equal to the number of uses, return to step 4.4. Otherwise return to step 4.3.

5. Reduce every  $X_{IK}$  [ $X_{IK} = X_{IK}(a)$  prior to this reduction] by either  $0.5 \cdot Rm_K$  or  $X_{IK}(a)$ , whichever is the lesser. Record the amounts of each use left unallocated and of each location left uncommitted.

6. Calculate the move evaluator  $E_{IK}$  for every  $I$  and every  $K$  (Fig. 3).

7. For every  $I$  and  $K$ , calculate  $Cap_{IK}$ , the maximum amount that  $X_{IK}$  may be increased without invalidating  $E_{IK}$  as calculated in step 6.  $Cap_{IK}$  is the least of (a) the location size ( $size_I$ ) minus  $X_{IK}$ ; (b)  $X_{IK}'$  minus  $X_{IK}$ , for  $X_{IK}$  less than  $X_{IK}'$  only ( $X_{IK}$  denoted the physically existing level of use  $K$  in location  $I$ ); (c)  $X_{IK}(a)$  minus  $X_{IK}$ , for  $X_{IK}$  less than  $X_{IK}(a)$  only; or (d)  $X_{IK}(a)$  plus  $Rm_K$  minus  $X_{IK}$ .

8. Allocate as much of the unallocated uses as possible by means of a capacitated transportation algorithm at the initial shadow price (dual variable) levels.

9. Reevaluate all  $E_{IK}$  and  $Cap_{IK}$  and then revise the shadow prices. Allocate as much of the unallocated uses as possible. Repeat this step until all uses have been allocated.

10. Reduce  $Rm_K$  by one-half of its current value for every  $K$ . If the new values are more than the minima set as criteria and if further improvements seem possible, return to step 2.

The terms of the algorithm are defined as follows:

$In_{IK}$  = incompatibility cost produced by allocating one acre of use  $K$  to location  $I$

$$= \sum_{J=1}^N \sum_{L=1}^M (B_{KL} + B_{LK}) \cdot X_{JL} \cdot G_{IJ}$$

$B_{KL}$  = average utility lost because of the incompatibility effect of one acre of use  $L$  on one acre of use  $K$  when they are allocated to the same or adjacent locations;

$X_{JL}$  = allocation of use  $L$  to location  $J$ ;

$G_{IJ}$  = average relative incompatibility transmission rate from location  $I$  to location  $J$ ;

$N$  = number of locations;

$M$  = number of uses;  
 $\epsilon$  = "an element of the set of";  
 $C_{DK}'$  = average cost of interdistrict flows resulting from the increase in allocation of use  $K$  from  $X_{DK(a)}$  to  $X_{DK(a)} + Rm_K$

$$= 2 \cdot \left( \sum_{L=1}^M \sum_{E=1}^P F_{DEKL}' \cdot C_{DE} - \sum_{L=1}^M \sum_{E=1}^P F_{DEKL} \cdot C_{DE} \right) / Rm_K;$$

$P$  = number of districts;  
 $X_{DK}$  = allocation of use  $K$  to district  $D$ ;  
 $F_{DEKL}$  = net flow of type  $KL$  from district  $D$  to district  $E$  for  $X_{DK} = X_{DK(a)}$

$$= T_{KL} \cdot \frac{(X_{DK} \cdot X_{EL})}{\sum_{F=1}^P X_{FL}} - F_{EDKL}, \text{ for } F_{EDKL} < \text{the first term,}$$

$F_{DEKL}'$  = 0 in all other cases;  
 $F_{DEKL}'$  = net flow of type  $KL$  from district  $D$  to district  $E$  for  $X_{DK} = X_{DK(a)} + Rm_K$

$T_{KL}$  = number of trips from one acre of use  $K$  destined to some use  $L$ ;  
 $C_{DE}$  = minimum cost of flow for one trip from district  $D$  to district  $E$ ;  
 $C_{DK(a)}$  = average saving in interdistrict flow costs caused by an infinitesimal decrease in  $X_{DK}$

$$= 2 \cdot \left( \sum_{L=1}^M Pr_D \cdot T_{KL} - \sum_{L=1}^M \sum_{E=1}^P Pr_E \cdot T_{KL} \cdot X_{EL} \right) / \sum_{F=1}^P X_{FL};$$

$Pr_D$  = shadow price of district  $D$ ;  
 $X_{EL}$  = allocation of use  $L$  to district  $E$ ;  
 $C_{IK}'$  = average cost of flows resulting from an increase in  $X_{IK}$ , where location  $I$  is in district  $D$

$$= 2 \cdot \left( \sum_{L=1}^M \sum_{J \in S_D} F_{IJKL}' \cdot C_{IJ} - \sum_{L=1}^M \sum_{J \in S_D} F_{IJKL} \cdot C_{IJ} \right) / \Delta X_{IK} + C_{DK}';$$

$F_{IJKL}$  = net flow of type  $KL$  from location  $I$  to location  $J$  for  $J \in S_D$  and  $X_{IK} = X_{IK(a)}$

$F_{IJKL}'$  = net flow of type  $KL$  from location  $I$  to location  $J$  for  $J \in S_D$  and  $X_{IK} = X_{IK(a)} + \Delta X_{IK}$

$S_D$  = set of locations comprising district  $D$ ;  
 $C_{IJ}$  = minimum cost of flow for one trip from location  $I$  to location  $J$ ;  
 $\Delta X_{IK}$  = maximum permissible increase of allocation of use  $K$  to location  $I$ ;  
 $C_{IK(a)}$  = average saving in flow costs caused by an infinitesimal decrease in  $X_{IK}$  where location  $I$  is in district  $D$

$$= C_{DK(a)} + 2 \cdot \left( \sum_{L=1}^M \sum_{J \in S_D} Pr_J \cdot T_{KL} \cdot X_{JL} \right) / \sum_{H=1}^N X_{HL};$$

$Pr_J$  = shadow price of location  $J$ ;  
 $R_{IK}'$  = location return for conversion of land in location  $I$  to use  $K$  from some other use; and  
 $R_{IK}''$  = location return for reallocation of land in location  $I$  to the existing use  $K$

## Computer Programs

Two series of computer programs were developed to carry out the calculations. The first series was used to test the algorithm on a hypothetical case of 7 uses allocated to 9 locations. Based on the excessive time consumed, adjustments were made in the second series, which computed flow costs in 2 parts. One component of the flow cost was for interdistrict flows and the other was for intradistrict flows.

All the programs except INCOMP, which carried out step 2, made extensive use of variants of the out-of-kilter network algorithm. This algorithm was chosen because it offers ease of programming and because it rapidly solves network and transportation problems that are minor modifications to the problems attacked here.

### SUMMARY

The work that culminated in the formulation of this model was initiated to provide a normative means of area use allocation resulting in maximum net community return. We decided that the factors to be incorporated in the objective function were transportation, incompatibility, and adaptability costs. We recognized, however, that a great deal of research will be required to quantify these costs accurately and precisely, and that some assumptions regarding the inputs were required.

The primary assumption was that all costs can be represented satisfactorily by piecewise linear approximations. This requires that the data supplied to the model be chosen for the expected range of the solution variables. Because land use allocation is not sensitive to small changes in the input data, this should not be a serious limitation.

A second major assumption in the solution procedure was that the transportation costs are fixed. This is a serious drawback, because it requires that each transportation plan be studied separately. Even so, if the initial iterations in the solution procedure indicate that improvements, such as upgrading arterials to freeway status, may reduce overall costs, the new plan may be evaluated by changing a few data plus a minimal amount of computation. Because the interdistrict and intradistrict flow costs are computed separately, changes in the major thoroughfare plans and addition of congestion tolls to reduce unrealistically high volumes might be made without repeating the entire iteration.

The third major assumption required for the solution was that the rate of generation of trips, and thus the major cost, was independent of the solution. This may not be true, but by proper calibration of the trip rates the resultant errors will be minimized.

Finally, the solution procedure considers only a deterministic static case. If the static case is a distant horizon year, the resultant solution may not be satisfactory for the aggregate of the intervening years. A potential modification of the model to provide a good solution over a long term would be to construct the evaluators as the sum of several components. Each component should reflect the average value of a given allocation over a given period.

The consideration of the deterministic case has led to the deletion of all intra-use flows. In the transportation algorithm utilized, all intra-use flow destinations and origins are allocated to the same location because this results in least cost. The true selection of a trip destination is not always to the least cost location even though the probability of such a selection is high. Thus errors are introduced. Certain types of trips, such as the interresidential social-recreation trips, may be best deleted from the model because the destinations of these trips deviate considerably from the economist's rational choice and are not easily predicted. Thus, the intraresidential flows were deleted from the Lafayette input data. Some inter-use flows may be deleted advantageously if their volumes are so low that their effects on the solution are insignificant. These deletions could reduce the computations considerably.

Although external-internal trips were not considered in the Lafayette model, this may not be advisable in practice because such trips will affect some use allocations. Dummy external zones, whose trip loading points are placed at the intersection of major arterials or highways near the plan area boundaries, may be used to consider their effect.

One of the drawbacks to the solution algorithm proposed here is that successive applications of the transportation algorithm to various trip types preclude capacitation of transport routes. We have assumed that such route capacities can be increased as required. However, it is hoped that the multicommodity transportation algorithms will be improved to the point where they may be substituted with desired capacity constraints, for the present algorithm. If the present algorithm is used, volumes could be limited only by calibrating the flow costs to reflect congestion.

Because of the macroscopic nature of the model, the data it requires need not be more detailed than conventional land use model data. However, there is very little known on area use economies of scale (negative incompatibility), positive incompatibility caused by by-product transmission, and the dollar-utility transform. All of these areas currently are considered on a subjective basis in planning. The provision of this model permits their quantitative consideration and, it is hoped, may help stimulate research in each of the areas. Also, the area of goals formulation and evaluation requires further research.

Incompatibility costs were considered only between adjacent locations in the Lafayette application. This can be extended to any number of locations for such by-products as smoke, which may cover a considerable area on the lee side of prevailing winds. The additional computation time is trivial, and additional computer storage requirements would likely not be a problem.

Although the model was developed for use on one urbanized area, the same methodology could be applied, for example, to the design of a region or neighborhood. Major modifications would be required in the input data. For instance, in neighborhood design the walking trip would be a significant factor.

In the Lafayette application, economies of scale were neglected. This was partly because of a lack of knowledge of the appropriate data, but a more important reason was that the use categorization used for the test did not appear to be sufficiently definitive to realistically apply average scale economy factors. Although this certainly casts some doubt on the true value of the results, the test served its purpose, which was merely to test whether the procedures are practical.