Change and Equilibrium in the Urban System

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•A SYSTEMS VIEW of urban problems should be much more than a catalog of interactions or a platform from which to launch highly specific proposals for action or research. Systems considerations as such are of little more than trivial interest if they do not provide major insights into problems through some general theoretic concepts. These theoretic concepts should have the property of sustaining new deductive conclusions.

I propose to discuss two or three concepts having to do with urban modeling and, more particularly, having to do with the relationships between various urban policies and the goals of urban development in the context of system equilibrium. The principal topics that are discussed have to do with problems of form (or morphology), problems of change, and problems of measurement.

Before taking up these topics, let me comment briefly on my view of the city as a system. We know, naturally, that nearly every system is a subsystem or an element in some larger system, and frequently the degree of interaction with the external environment is so strong that the independent study of it is fruitless. Sometimes we have to distinguish different aspects of this central problem. For example, man is a self-contained biological system in many respects and can be studied as such. But it is almost useless to study a man as a social system, even though he is a major element in any such system. Similarly, it may be argued that cities are far from independent of the national and world economies, that their import-export relationships are powerful and even dominating, and that, therefore, the economic life of the city is too open usefully to be considered as a system. I would agree that this is the case regarding culture, technology, economic function, and national politics. On the other hand, as a labor market area, a pattern of settlement, a dense concentration of land development, and a site for daily social interaction, the metropolitan area functions as a coherent and identifiable system. From this point of view, the other considerations become a part of the long-term development that impinges on, but is to a considerable extent independent of, the metropolis as a system. This paper should indicate that I regard this second aspect of urban affairs as deserving systems study.

FORM OF METROPOLITAN AREAS

In discussing the form of metropolitan areas, I refer to the patterned distribution in three-dimensional Euclidean space in a metropolitan area of artifacts, people, and their attributes. Form is not necessarily plainly visible because, for example, the relative distribution of occupational groups or religious groups might be a significant element of form but would not be immediately obvious to the observer. Form also includes in a sense flows and interactions, because these are attributes both of people who occupy an urban area and of their artifacts.

Urban form in its most general sense is an important object of policy manipulation; it implicitly controls many of the aspects of the quality of life that people appreciate (positively or negatively). The cost, location, and quality of housing, the amounts of private and public open space, the length of the journey to work, the social environment, pollution or its absence, and public safety are all aspects of urban form that affect people's lives and that are more or less subject to public control. It is therefore

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important to know, among other things, how form is determined, so that the cost-effectiveness of public policies can be improved. This needed knowledge implies some understanding of the urban system.

There are essentially two complementary ways of looking at the genesis of urban form, static and dynamic. It is tempting and indeed useful to note the similarities among a wide variety of cities and to speculate that these similar forms represent the conclusion of an equilibrating process, an end state toward which, under present circumstances, many large urban conglomerations converge. If this were correct, the problem of emerging tendencies in urban form could be studied as a problem in general equilibrium, and this is a view to which I tend to subscribe.

This view is frequently counterposed to the picture of the metropolis as an evolving organism, in which the processes of change are more important than the states that exist at any particular time. In addition, it is suggested that the dynamic moving forces that motivate this process of change are so strong, so persistent, and themselves so changeable that the system never can achieve equilibrium. I agree that this view is true in its most literal sense, but I am inclined to believe that, in spite of external shocks and stimuli, any particular exemplar of the urban system is always tending toward equilibrium. With a proper definition of that equilibrium and with a proper understanding of lags, we can use the equilibrating tendencies to explain much of the dynamic picture.

These two views of urban equilibrium play complementary roles in the evaluation of policy. In the theory of general equilibrium in economics, equilibrium is frequently identified with optimality, and probably we should examine the extent to which this is true of spatial equilibrium. The opposite proposition also has considerable merit—that a study of the dynamic properties of systems operating over time will illuminate their anticipated behavior under a variety of policy assumptions. The static view neglects the path by which some desired equilibrium might be achieved, whereas the dynamic view tends to neglect ultimate objectives and to focus principally on the immediate implications of policies. The dynamic view also turns out to be a very clumsy way of testing paths of arriving at desirable configurations.

A certain note of caution must be struck regarding the optimality implications of competitive spatial equilibria. The Hotelling problem regarding the location of two hot dog vendors on a beach is the simplest possible example of a general and pervasive spatial problem. Competition leads the two vendors to locate side by side at the center of the beach, even though social welfare would dictate their being located one-fourth of the way from each end, where they would equally divide the market. This example indicates that globally optimal solutions are not necessarily reached by "natural growth processes" as replicated in models, especially when there are indivisible units and spatial monopolies.

SPATIAL DISTRIBUTION MODELS

There are two very broad classes of urban models of spatial distribution. One class provides an equilibrium description of urban distributions in the static sense without arriving at this conclusion by way of an examination of the equilibrating process. Three examples of this type of model might be mentioned with varying degrees of explicitness in their definition of equilibrium. First, Lowry (1) defines an equilibrium distribution of population and service activities for Pittsburgh based on the transportation systems, travel patterns, and location of export or "basic" industry. The equilibrium implications of this model are difficult to determine, but they reflect some stability in travel patterns. A second group of equilibrium models belongs to a class of gravity models used in the location of retail trade. These models if applied to a uniform distribution of purchasing power and a uniform class of commodities will, like central place theory, arrive at a distribution of equal-sized market areas. It can also be shown that this type of model tends, in a somewhat indirect way, to minimize total travel time for shopping subject to certain constraints and to a stochastic distribution of shopping trip lengths. The equilibrium that exists is quite explicitly between spatially located supply and demand, and if an equilibrium were disturbed, it is implied that some centers would be more prosperous than others. Finally, we may mention the Herbert-Stevens model (5) of residential location, which is based on the Alonso theory of the land market and which uses linear programming to achieve a Pareto optimum that is also a behavioral, competitive, market-clearing solution and therefore a form of equilibrium.

The second case of static equilibrium has a certain number of interesting properties. It connects ideas about the statistical behavior of users of a transportation system with ideas about the equilibrium of land use and location. At the same time, it may seem to produce an optimal situation from the point of view of the users. But it turns out that the equilibrium and the optimum are not exactly the same thing. This finding suggests that spatial behavioral models with equilibrium-seeking properties may not possess all of the same optimality properties that nonspatial economic models have.

The model in view here is one of the location of retail trade that has been extensively discussed in earlier literature by Berry, Garrison, Huff, Carroll, and others. Recently somewhat similar ideas have been applied without any equilibrium properties by Lowry (1), and the equilibrium model was developed simultaneously by Lakshmanan and Hansen (2) and by me. Because the Lakshmanan and Hansen model is simpler and more directly related to the problem that I wish to discuss, I will use that rather than my own model. All of the models mentioned produce results that are very similar to the results of central place theory. Each marketplace is surrounded by an area of market dominance, and the areas of market dominance exhaust a plane. Unlike central place theory, however, these models, which are based on gravity models of trip interaction, admit of overlapping trade areas, and if the trade centers are of unequal size, the boundaries between their areas of dominance are neither straight nor equidistant between the centers.

The model developed by Lakshmanan and Hansen assumes that we are dealing only with a uniform type of subregional shopping centers with floor areas in the vicinity of 500,000 sq ft. Repeated applications of the model yield estimates of the number of tripmakers who will be attracted by each center as it competes with other centers. (Hypothetical center locations are an input to the model.) If the purchasing power of a center exceeds some predetermined average (say, $55 per sq ft), then the center is expanded on the next iteration. Centers that become too small are dropped out, and centers that become too large may be split in two if hypothetical sites are available. The outcome of this process is a form of equilibrium in which nearly all centers have an equal level of sales per square foot of floor area.

Lakshmanan and Hansen found, as a by-product of their procedures, that the pattern of centers produced by this process also appeared to involve the minimum total miles of travel for the users of shopping centers. If this observation were absolutely correct, it would provide a useful consequence of the equilibrium aspects of the model. However, it may readily be seen that the equilibrium postulated in the model is primarily a producer's equilibrium. Sales at less than $55 per sq ft are uneconomic and cause some firms to go out of business, whereas sales at over $55 per sq ft on the average are excessively profitable and cause new firms to enter any particular center, thereby expanding its floor area and attractive power. There is a large element of consumers' or users' preferences involved in these equilibria in that, owing to the convenience aspects of shopping as reflected in the gravity model of trip-making, it is impossible for all shopping to become concentrated in one center, and the distribution of centers becomes fairly even. This evenness produces the apparent optimality from the point of view of the user, but it must be stressed that there is no guarantee of such user optimality built into the model.

A simple way of viewing the paradoxical nature of this model may now be presented. The following three assertions have been made:

1. Purchasers or consumers tend to behave as in a gravity model for any particular class of trips, e. g., food shopping.
2. Producers achieve a spatial equilibrium by adjusting the size of their activity to serve just precisely the level of activity that it will attract.
3. This arrangement represents a minimum travel cost scheme from the point of view of the consumer.

It is not difficult to show that only in very restricted circumstances can all three of these assumptions be true. Consider the layout of market areas along a radial axis with a declining density gradient. It is apparent that if two centers are of equal size they will not have equal radii of service under the second assumption that the size of the center adjusts to the available market. If, on the other hand, they have equal radii of dominance, they will have unequal total markets and therefore be of unequal size. We must note, however, that under the gravity model formulation of trip distribution a market area boundary, defined as a line of equal probability, will be equidistant from two centers only if these centers are equal in size, inasmuch as the interaction probabilities are generally proportional to the size of centers at equal distances. Finally, to a good approximation, it is evident that equal radii of market areas are necessary and sufficient for consumer travel times to be at a minimum. If, as a consequence of unequal sizes of centers, market boundaries are shifted toward one or another center, a substantial proportion of consumers will make trips longer than those to the nearest center. This contradicts the hypothesis of consumer optimality.

With this line of reasoning, we are usually constrained to give up at least one of our three original hypotheses. There is, however, one condition that seems to permit us in part to escape this trilemma. We can assume that centers are of equal size, but unequally spaced. At the same time, the lines of equal influence are perpendicular bisectors of lines joining the centers. Thus centers will have radii of influence that are shorter on the "up-hill" side of the density gradient and will not be located in the center of their service areas. This seems to be the usual pattern of shopping center location, and Lakshmanan and Hansen seem to have been fortunate in their selection of potential sites, making it possible to arrive at a configuration that would approximately satisfy this set of conditions.

This solution, however, still contains a residual paradox. In the postulated configuration, the shopping centers are not necessarily at the centroids of their service areas. Within any one area, if a center could relocate and retain its customer allegiance, it could reduce total travel cost. But some customers would be disadvantaged in their choices of centers, and their consequent shift of allegiance would result in a change in center sizes. The equal size condition could no longer be maintained.

The model therefore permits all three assumptions to hold only on an isotropic, equal-density configuration. It thus seems that this example raises serious questions about the rationality and reality of the gravity model of trip-making, or of this family of retail trade models, or of the assumptions of optimality implicit in the equilibrium model. This line of inquiry is thus a powerful means for exploring certain aspects of models.

DYNAMIC MODELS

A broad class of dynamic models that have equilibrium and final state implications is becoming very popular in metropolitan planning circles; it goes under the general name of urban or regional growth models. The general form of such models is a system of differential or difference equations, not necessarily linear and sometimes quite large.

A recent publication by Forrester (3) makes quite clear the structure of a system of simultaneous differential equations applied to urban phenomena. These systems of equations have the properties of embodying many feedback loops, of possibly providing contra-intuitive results, and of producing projections that for any particular phenomenon are not necessarily monotonically increasing or decreasing. All of these features have some considerable attraction in that they correspond with our intuitive views of the real world. Nevertheless, Forrester's presentation has a number of difficulties, most particularly in the nature of the assumptions regarding interregional change and the lack of detail regarding intra-urban distributions. Forrester also suggests that his ideas in their application to cities are novel, although this is clearly not the case.

At least three major modeling efforts have been made in which an interacting set of models, used recursively in steps of 5 years or less, provides a much larger and richer mix of feedbacks than appear in the Forrester system. The argument is not essentially changed by the fact that these models are all based on difference equations rather than differential equations and that their results are cruder, but computationally more convenient. Models of this type include the EMPIRIC model for Boston (which had a short-lived companion in the differential equation formulation, POLIMETRIC), the Penn-Jersey Transportation Study model package, and the Time-Oriented Metropolitan Model developed by Crecine (4) for the Pittsburgh community renewal project and later further expanded. We might also include the Dyckman-Robinson model for the San Francisco community renewal project.

There is thus no shortage of relevant dynamic models, but very little attention has been paid to their properties. I now propose to explore some of these by illustrating a number of points. First, I will look at the connection between equilibrium and dynamic models, and I will suggest that these ideas immediately provide another powerful means of examining both policy issues and the construction of the models themselves. Second, I will develop in brief a particularly simple model of urban location patterns and examine the properties of its equilibrium solutions in slightly more detail. Finally, I will look at a group of statistical problems that arise in connection with these ideas and, indeed, in connection with a great deal of urban research.

Relationship Between Equilibrium and Dynamic Models

The role of feedback and dynamic performance of systems in relation to homeostasis and equilibrium is complex, subtle, and not understood in sufficient detail. For example, the models that I will discuss have linear feedback loops. If these loops were nonlinear or discontinuous, it is probable that in many cases the equilibrium tendencies of any particular system so described would depend on the initial state of the system as well as on its structural characteristics. Such dependency is common in biological systems and must exist in some social situations—indeed, quite commonly at least in any situation that has to do with matters of life and death. I will, however, investigate only linear and generally continuous systems.

Positive and negative feedback are of course distinctively different in their influence on dynamic systems. Positive feedback implies positive and self-reinforcing experiences and, consequently, leads to growth and to extended exploitation of the environment. Negative feedback, on the other hand, leads to decline or to equilibrium-seeking performance. It is important to realize that positive feedback and exponential growth cannot continue to operate indefinitely. Systems possessing this characteristic ordinarily encounter one of two modes of change that limit the growth. The ordinary or "liberal" solution results from a shift in relationships either internal to the organism or between the organism and the environment such that positive feedback is converted into negative feedback. Typically this happens when expansion is limited by the increasing cost of resources, or when the agglomeration economies begin to be offset by the diseconomies that result from congestion or pollution. The "radical" or less automatic solution arises when basic changes resulting from the growth of the system create conflicts or problems that necessitate new laws and new institutions. In the first case, the growing system reacts to changed circumstances. In the second case, either the system or the larger system in which it is embedded adapt by change of form. In society these changes of form are changes in institutions, laws, and social relations.

Models of Urban Location Patterns

As a consequence of this distinction between positive and negative feedback, we can logically and practically ask two questions about models of urban systems: Do they generate any unlimited tendencies and do these tendencies in fact correspond to those that can be observed in the real world? Unlimited growth, decline, concentration, or dispersion would in general seem to be contrary to our intuitive view of urban arrangements, but, were they realistic, they could in any case be expected to create various types of severe institutional stress. Systems that behave in this way have no equilibrium or homeostatic tendencies except when they have reached boundary conditions such as the concentration of national population in a single or very large city, ultrahigh urban densities or uniform densities, or giant corporate monopolies. If an exploration of a model leads to the conclusion that it does not imply any normal equilibrium, it will be a matter of considerable delicacy to decide whether the abnormality lies in the construction of the model or in the true behavior of the system.

It seems much more likely that for well-constructed models a set of equilibrium solutions will be available for most inputs of policies and environmental conditions. Such an equilibrium is, for example, displayed by the long-term solutions of the Forrester models of urban dynamics (3). More generally, various types of equilibrium probably exist corresponding to no change (or a steady-state turnover of individuals, households, or firms), or to variously defined conditions of equiproportional growth. As I have suggested, in the case of linear models these equilibria are probably independent of starting conditions and rates of initial growth, but in the event that there are long lags such as may be identified with respect to the redevelopment and redeployment of the urban capital investment, the ultimate equilibrium might take a long while to achieve in any realistic growth situation.

Evolution and Equilibrium

When we explore the possible equilibrium positions of urban systems implied by dynamic models, we must take account of the many aspects of the relationship between equilibrium and dynamic performance. Not only do actual physical investments tend to persist over long periods of time, but agglomeration economies, once established, may long outlast their original impetus. Thus, for example, urban financial centers are typically located near the original port centers of major cities, even though these may no longer be in the central business district. Given this resistance to change, it is my view that cities probably tend toward their equilibrium position. This, however, may be constantly changing as a result of external impulses, and thus the homeostatic mechanism is aiming at a moving target. Tendencies that affect the rate of movement of the target are most particularly the rates of growth of metropolitan regions, the rates of change of economic function, the rates of increase or decrease of personal income, and the rates of change of technology—particularly in building, transportation, and communication.

It is attractive to consider that the manifest form in which cities are cast is a joint product of the evolutionary tendencies and their underlying equilibrium tendencies. Such a view might at the same time accommodate an explanation both of the convergent similarities of cities and of certain specific and evident differences. It is also attractive to compare this process, if it exists, with processes of biological morphogenesis and evolution. These comparisons are perhaps more dangerous than helpful, especially as long as our knowledge of both biological and metropolitan morphogenesis is so qualitative and so inadequately explored.

Aside from long-term speculations in the philosophy of science, the relationships that we have sketched between equilibrium and dynamics suggest that at any particular point in time the equilibrium that could be achieved for given environmental conditions, existing sunk capital, and policy determinations might tend to represent some sort of optimal arrangement. Actual anticipated development that takes into account short-run decisions that will result in capital investment and therefore foreclose some aspects of the long-term equilibrium would then be by definition less than optimal. In making use of this hypothesis we must constantly bear in mind the qualifications developed earlier regarding the possible mismatch between equilibrium and optimality. We must also recognize that avoiding currently attractive decisions that in the long run are less than optimal will usually impose costs on either government, investors, or users. Given all these qualifications, the equilibrium condition for dynamic systems may be extremely useful for the exploration of ideal future states and the policies that are related to their attainment.

EMPIRIC MODEL

To give this statement some realistic content, I should like to discuss briefly a modified version of the EMPIRIC model, originally developed by Donald Hill and his associates (6) for application in the Boston region. This model, as mentioned, is a multiple-equation, multiple-variable difference equation model that will be considered here in a modified form for simplicity of discussion. The dependent variable in the EMPIRIC model is a large set of area-specific and locator-specific rates of change—actually deviations from regional rates of change. The right-hand variable in these equations falls into four classes. First, the changes in all other locator quantities in a given area are assumed to affect the rates of change on the left. For example, if during a given period the volume of manufacturing in an area increases greatly, the rate of increase of residential location will be depressed. This formulation is necessary for a difference equation formulation, especially one with a time interval as long as 10 years, but because it is not relevant to a differential equation formulation, we omit it from further discussion. The second class of variables defines the density of each locator variable in each area. In most cases it is anticipated that high densities discourage additional location. In the original EMPIRIC model these densities appear in a concealed form in relation to zoning policy variables, but we will consider them explicitly. The third class of variables has to do with accessibility, a constructed variable that, in this case, is calculated by weighting the locator volumes in all other areas by a declining function of time-distance from the area under consideration. Although the distance functions are nonlinear, the weighting process is linear and the locators in various areas enter into the calculation in a linear way. Ordinarily, except possibly for conflicting land uses, the signs of the coefficients of accessibility are positive, thus differing from density. The fourth and final group of variables has to do with neighborhood qualities. These in principle may be both variables that are exogenous to the planning process—such as those having to do with slope, elevation, microclimate, and the like—and control variables such as water and sewer service and many other planned neighborhood characteristics.

Given this general description of the model, if we have N areas and M locators, we have MN equations for MN locators. Owing to the construction of the accessibility variables, all of the variables appear in all of the equations, or everything influences everything else, and there are MN feedback relationships involving all the variables. There are of course many less than $(MN)^2$ basic parameters in the model, because of the manner in which the accessibilities are calculated. Here the network conditions, which are themselves policy variables, generate a large number of coefficients. In general, each equation will contain some positive and some negative coefficients so that, for a properly selected vector of locator groups of length MN (with all elements positive), it may be possible to force all rates of change to zero. There are in fact two different cases under which this might occur. If the left-hand side of this MN equation is set to zero, we have on the right-hand side a set of terms involving the locator groups and a set of terms involving neighborhood conditions. The dual problems are as follows:

1. Given a certain set of neighborhood conditions, what would be the equilibrium distribution of locators?
2. Given a desired distribution of locators, what would be the necessary configuration of neighborhood conditions?

In both of these cases, certain mathematical difficulties arise.

Equilibrium Distribution of Locators

In the first case, there is almost certain to be a unique solution. Not only is the number of unknowns equal to the number of equations, but the combined neighborhood conditions provide a nonzero vector of constants. It seems unlikely on somewhat cursory examination that the (very large) matrix of coefficients applying to the locators would be singular. Such singularity could arise, however, in a case where the behavior of a locator is exactly similar to the behavior of any other locator (that is, where its coefficients are proportional to another locator), or indeed if any locator's coefficients can be defined as a linear combination of any other locators. From a certain point of view this might be taken as reason for reducing the number of locators to be considered. From a different point of view, however, it makes considerable sense to say, for example, that banking is indeed 50 percent retail trade and 50 percent business services and to analyze its behavior accordingly. This may be so even if it does not make very much sense to use the algebraically equivalent statement that business services are equivalent to banking doubled less retail trade.

These problems, though somewhat novel, are a perfectly legitimate field of inquiry in locational models, and they suggest that other methods for dealing with this type of problem need to be explored. It is obvious, for instance, that for iterated solutions such as have actually been used for the EMPIRIC model and by Forrester the singularity of a coefficient's matrix may not create the same type of difficulty.

Another problem that may arise in this first case is that certain constraints on the solution have not so far been built into our formulation of the problem. The first of these constraints is that none of the values of the locator groups shall be negative. The second is that the total volume of each locator must exactly match some predetermined or input value of population or business that has to be accommodated. The second problem can be converted into the first by eliminating M equations and M variables—for instance, replacing the N th subarea variable for each locator by the predetermined total less the sum over all other areas. This calculated quantity itself cannot be negative. The existence of negative locator values in such a solution would indicate that the system described has no equilibrium, because if the negative values are replaced by zeros, various rates of change will be nonzero.

Necessary Configuration of Neighborhood Conditions

The second case in this dual problem has to do with the circumstance when we have projected a possible pattern of equilibrium of locators and wish to know what public policies could bring this equilibrium about. We will not discuss the subcase of the influence of transportation networks via accessibility on the equilibrium, because this leads into the solution of a problem that involves not only nonlinear functions, but also the combination of links into least-cost paths in a network. Given a network configuration, however, it seems practical to ask what levels of other government services are necessary to ensure a certain pattern of development, short of direct controls. Where such controls are everywhere binding, the notion of equilibrium is no longer applicable. The mixed case where some controls are binding and others are not is most vexatious, not only as applied to solution methods, but also with respect to the observation of "natural" locational tendencies.

The first thing to be observed is that there are apt to be many more locator variables (and hence equations) than policy variables influencing location. This will always be the case if there are more types of locators than there are policy variables. Thus, the equation system for determining what policies are necessary may be overdetermined. There are two general ways out of this dilemma. One of these is to make a least-squares fit of policies to the desired configuration. In this case, the RMS error could be interpreted as a measure of the lack of realism in the policy. The second means of dealing with the problem is to reduce the number of equations by combining locators. This could be done along the lines discussed earlier or by any other reasonable procedure based on past locational behavior.

No matter which method is followed in solving this dual problem, the same difficulty regarding potential negative (or, more generally, unrealistic) policy values will probably be observed. In this case, the implicit advantages of an iterative scheme are not available, and it is necessary to conclude that the desired configuration cannot be achieved by way of influencing the behavior of the locators, but only by outright regulation. It seems likely that such regulation of locators implies some departure not only from equilibrium, but also from optimality. In other words, the imposition of a preconceived pattern of location may satisfy certain planners' goals, but does not necessarily best serve the interests of the locators.

STATISTICAL IMPLICATIONS OF EQUILIBRIUM MODELS

My last major point in this discussion has to do with certain statistical implications of equilibrium models. In dealing with spatial location and perhaps even more with dynamic spatial processes, we often find that statistical problems are gravely complicated by aspects of multi-collinearity. Statisticians sometimes argue that this problem should be avoided by reducing the number of variables, because it is "quite evident" that some of these variables must measure the same thing. This approach suggests the desirability of step-wise regression methods among others, but in my view is not entirely satisfactory. It seems to me that the preceding discussion leads directly to some conclusions that are at variance with this interpretation of multi-collinearity.

First, to clarify the situation somewhat, we must refer to the earlier discussion of the uniqueness and linear independence of locational behavior. If, in fact, some locational behaviors can be represented as linear combinations of other locational behaviors, and if the system is approaching equilibrium, then the corresponding consequent locational patterns may be linear combinations of other locational patterns. Because the locational patterns enter uniformly into the variables of density and accessibility that make up many of the independent variables of this model, these variables will in turn be linearly dependent and the correlation problem is in principle not soluble. There may be some difficulty in identifying this case separately from the more difficult and more important case that follows. A simple way to deal with it, however, would be to component-analyze the locational patterns of all the locators over all areas. The locators themselves could then be replaced in the model by a set of component scores that would ordinarily be less than the number of locators. We have used this type of analysis to reduce the dimensionality of measures of accessibility, without sacrificing any of the information provided by taking a rich and detailed view of this set of variables.

The second and more serious difficulty arises out of equilibrium considerations. Even assuming that each locator entering into the model is truly independent, a correlation analysis might still break down. Consider the circumstances that arise when an urban system has either reached equilibrium or has approached it and is "tracking" equilibrium in a relatively uniform way. In this case, for each locator the rates of growth for areas on the left side of the difference equations are zero or uniform. In a regression analysis to determine the coefficients of these equations, the vector of correlations between the dependent and independent variables is zero. In this case, the equation for the coefficients yields only a trivial solution if the matrix of correlation coefficients for the independent variables is nonsingular. We may justifiably generalize this situation slightly by saying that the closer an urban system approaches equilibrium, the more likely it is that any analysis of the rates of change will create a singular correlation matrix.

We may put the same problem in more intuitively attractive terms. The model we have outlined depends on the interaction of factors that attract and repel the locators, e. g., accessibility and density. For any particular size of city and location within it, there is some appropriate balance between these, indicating that their weighted algebraic sum is zero. (This discussion, of course, assumes a linear model.) If this condition is satisfied everywhere, the system is in equilibrium and has no impetus to change, yet this condition of a zero weighted sum of two or more vectors is precisely the condition for linear dependence in a set of variables. In practice in correlation analysis, we observe this phenomenon in the form of a very small determinant of the correlation matrix, followed by high and "unreliable" B values. Alternatively, if we correlate the dependent variable with component scores for the independent variables, we find that components with very small eigenvalues play a very large role in the analysis.

It is quite evident in this situation that throwing out variables is not the appropriate solution, although I hasten to add that the exact selection of appropriate methods is not altogether clear to me. However, it is clear that throwing out one of two highly correlated variables may be a disaster if in fact some phenomenon of locational change is closely related, for example, to their difference. Because density and accessibility (as illustrative variables) obviously measure quite different phenomena, the original assumption of overlapping concepts and variables is no longer applicable. In other words, equilibrium provides an alternative explanation for collinearity.

From the preceding discussion it is quite evident that the stronger the equilibrating forces and the more responsive the system is to them, the less confidence statistical measures would give to the coefficients describing growth relationships. For example, if I observed an SMSA in which accessibility and density were very highly correlated, I would take this as a confirming instance of my basic view of urban dynamics; yet if I used these variables in a correlation analysis of change in the same urban area, statistics would tell me that the influence of these variables is measured in a highly unreliable way. I must say that I cannot accede to this view, and I think that the problem of sorting things out is up to the statisticians. It is important because it is closely related to the predictive power of models.

SUMMARY

In this brief case I have tried to develop in an illustrative way a cluster of ideas about how the relationships between dynamics, equilibrium, and optimality could be used to explore more fully our understanding of models and of urban phenomena. I think that the ideas presented are perhaps somewhat naive and oversimplified, but I am confident that further exploration in greater depth would be more rewarding. I trust that these explorations will be widely undertaken.

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