Visibility Problems in Crest Vertical Curves

M. LIVNEH, J. PRASHKER, and J. UZAN, Technion, Israel Institute of Technology

The length of a crest vertical curve is governed by visibility considerations. The minimum length is based on the stopping sight distance; the maximum length is based on the passing sight distance, and overtaking is allowed throughout its length. The object of the present paper is theoretical determination of the zone of overtaking visibility in a curve designed on a below-maximum basis. The analysis covers 2 cases: (a) overtaking vehicle inside and oncoming vehicle outside the curve and (b) both vehicles outside the curve. The corresponding curve geometries were also considered. The equations obtained were computer-solved for curves with slope difference ranging from 2 to 12 percent, passing sight distances corresponding to the design speed range of 50 to 110 km/hr, and length limits corresponding to the stopping and passing sight distances respectively. Results were rendered in convenient graph form, permitting determination of the type of division line and the length of the no-overtaking zone to be marked on a 2-way 2-lane highway in the vicinity of the curve. The length of the no-overtaking zone increases with the overall length of the curve, up to the maximum (unrestricted overtaking). The conclusion is that, in order to reduce the no-overtaking zone in below-maximum cases, it should preferably be as short as possible within the requirement limits of overtaking visibility and driving convenience.

•CONVENTIONAL DESIGN of a crest vertical curve (hereinafter referred to simply as curve) in a 2-way 2-lane highway involves 2 extreme cases: minimal, with overtaking prohibited altogether, and maximal, with overtaking allowed throughout its length, based respectively on the sight distance required for stopping and passing (overtaking) at a given speed. Even in a minimal curve a point exists beyond which visibility is insufficient for overtaking. The design is frequently based on a criterion radius that generally satisfies this minimal requirement, and overtaking is allowed along part of the curve in such cases. A solid division line throughout the curve and beyond it would be unrealistic and make every driver a potential offender, as is bound to happen wherever overtaking is prohibited along obviously safe stretches. Removal of the overtaking restriction is indicated on the highway by a broken line parallel to the solid line, and advance notice of the reduced sight distance is indicated by a special closely spaced broken line (warning line) equal in length to the passing sight distance and originating at the point beyond which there is no overtaking visibility. A vehicle that starts to overtake short of this point will be able to complete the move; after this point, the warning line is equivalent to the solid line.

The object of this paper is the theoretical determination of the different zones from the viewpoint of overtaking safety. This was done graphically or by means of practical field tests in situ.

Paper sponsored by Committee on Operational Effects of Geometrics.

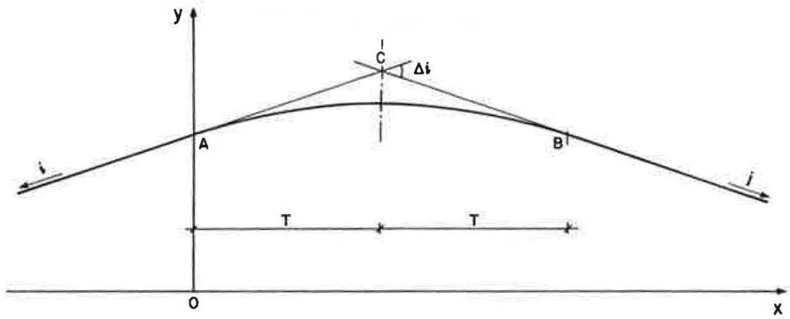

Figure 1. Parabolic crest vertical curve.

DETERMINATION OF VISIBILITY ZONES

The design is generally based on a parabolic curve (Fig. 1) with the following equation:

$$y = -\frac{\Delta i}{4T}x^2 + ix + H_A \tag{1a}$$

$$\Delta i = i - j \tag{1b}$$

where

y = elevation of a point on the curve;
x = distance of the point from the origin of the coordinate system;
Δi = algebraic difference of the tangential slopes i and j;
$2T$ = length of the curve in plan; and
H_A = elevation of point A, the initial point of the curve.

In the range of longitudinal slopes used in road design, it can be shown that the radius of curvature is constant at close approximation:

$$1/R = \frac{y''}{\left(1 + y'^2\right)^{3/2}} \tag{2}$$

Because the slope y' is relatively small, y'^2 in the denominator is negligible with respect to unity, and we have

$$R \cong 1/y'' = \frac{2T}{\Delta i} \tag{3}$$

An additional assumption is that the true length and projection of a sloping line are equal. In these circumstances, the results would not be affected by slight rotation of the system. Accordingly, the absolute values of slopes i and j are immaterial in the following analysis; the only significant parameters are h, level of driver's eyes above the ground, and the difference Δi. The sight distances in both directions are symmetric with respect to the intersection point C of the 2 tangents.

In determining the driver's sight distance with respect to an oncoming vehicle, the 4 cases given in Table 1 should be distinguished. According to the German Code (1), h equals the height of the oncoming car, so that cases 1a and 2b are identical.

Case 1a

Case 1a, in which the overtaking vehicle is inside and the oncoming vehicle is outside the curve, is shown in Figure 2. The parabola equation is used to obtain the elevation of point E′, driver's eye level in the overtaking car, which is

$$H_{E}' = H_A + ix - \frac{\Delta i}{4T} x^2 + h \tag{4}$$

The slope of the line of sight, y′, is

$$y' = -\frac{\Delta i}{2T}(x + m) + i \tag{5a}$$

and

$$m = \sqrt{\frac{4T}{\Delta i} h} \tag{5b}$$

m equals at least the stopping sight distance in the minimal case. The elevation of point D′, level of roof of oncoming vehicle, obtained from the slope of the sight line from Eq. 5a is given by

$$H_D' = H_E' + S_p y' = H_A + ix - \frac{\Delta i}{4T} x^2 + h + S_p \left[i - \frac{\Delta i}{2T}(x + m) \right] \tag{6}$$

where S_p is the passing sight distance. The same elevation obtained from the parabola equation is

$$H_D' = H_A + Ti + (S_p + x - T) j + h \tag{7}$$

TABLE 1

CASES USED IN DETERMINING SIGHT DISTANCE

Case	Vehicle	Position in Curve	Case	Vehicle	Position in Curve
1a	Overtaking Oncoming	Inside Outside	2a	Overtaking Oncoming	Outside Outside
1b[a]	Overtaking Oncoming	Inside Inside	2b	Overtaking Oncoming	Outside Inside

[a]For a curve not based on passing sight distance, case 1b is irrelevant.

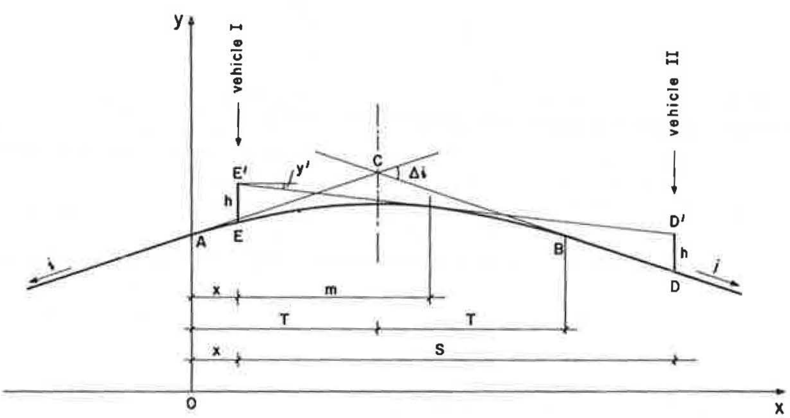

Figure 2. Sight line—one vehicle inside and one vehicle outside curve.

Equating both expressions, we have

$$\frac{\Delta i}{4T}x^2 + \left(\frac{\Delta i}{2T}S_p - \Delta i\right)x + \left[\left(T - S_p\right)\Delta i + \sqrt{\frac{\Delta ih}{T}}\,S_p\right] = 0 \tag{8}$$

For a given curve with known Δi and 2T and for S_p corresponding to a given design speed, Eq. 8 yields x, the coordinate of the point beyond which there is sufficient visibility for overtaking. Of the 2 roots of the quadratic equation, only the one satisfying the following 3 conditions is relevant:

$$x \geq 0 \tag{9a}$$
$$(x + m) \leq 2T \tag{9b}$$
$$(x + S_p) \geq 2T \tag{9c}$$

Case 2a

Case 2a, in which both vehicles are outside the curve, is shown in Figure 3. This situation exists only for $S_p > 2T$. As the diagram shows,

$$S_p = v + m + n + w = v + T + w \tag{10}$$

where

v = the distance from the vehicle situated outside the curve to the intersection point F of the line of sight of the vehicle with the grade line of the beginning of the curve (Fig. 3), and

w = the corresponding distance when the vehicle is on the other side of the curve.

The slope of the sight line is

$$y' = -\frac{\Delta i}{2T}x + i \tag{11}$$

The angles between tangent and sight line are $(i - y')$ and $(-j + y')$ respectively, and we have

$$v = \frac{h}{i - y'} = \frac{h}{x/R} \tag{12}$$

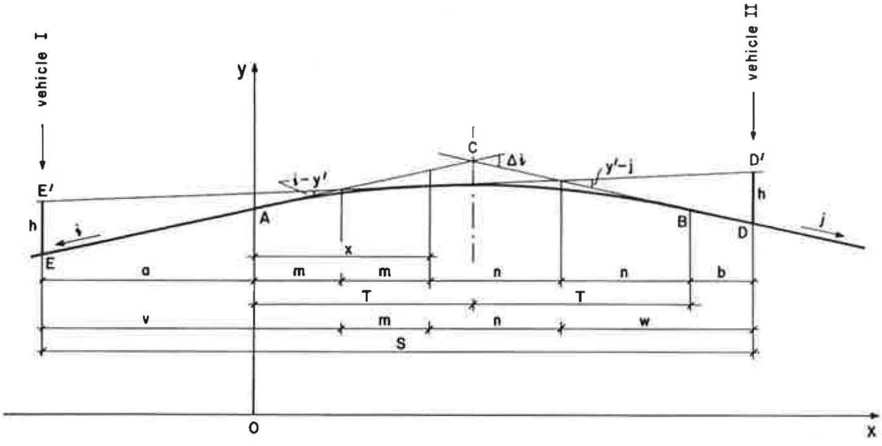

Figure 3. Sight line—both vehicles outside curve.

$$w = \frac{h}{-j + y'} = \frac{h}{-x/R + \Delta i} \tag{13}$$

Where R is the curve radius. Substituting these in Eq. 10, we have

$$(2S_p - R\Delta i)x^2 - R\Delta i(2S_p - R\Delta i)x + 2R^2h\Delta i = 0 \tag{14}$$

x_1 and x_2, the roots of Eq. 14, are symmetrical with respect to the vertical through C, namely,

$$x_1 = 2T - x_2 \tag{15}$$

The positions of the vehicles in the coordinate system are

$$a = v - m = \frac{Rh}{x} - \frac{x}{2} \tag{16}$$

for the overtaking vehicle, measured from the initial point of the curve, and

$$b = S_p - a - 2T \tag{17}$$

for the oncoming vehicle, measured from the end point of the curve.

Identification of Relevant Case

Case 1a, one vehicle inside and one vehicle outside the curve, is represented by Eq. 8. For this case, one of the solutions must obey conditions in Eqs. 9a, 9b, and 9c. Case 2a, both vehicles outside the curve, is represented by Eq. 14. For this case, both a and b, obtained from Eqs. 16 and 17, must be positive. Exclusion of one case indicates validity of the other.

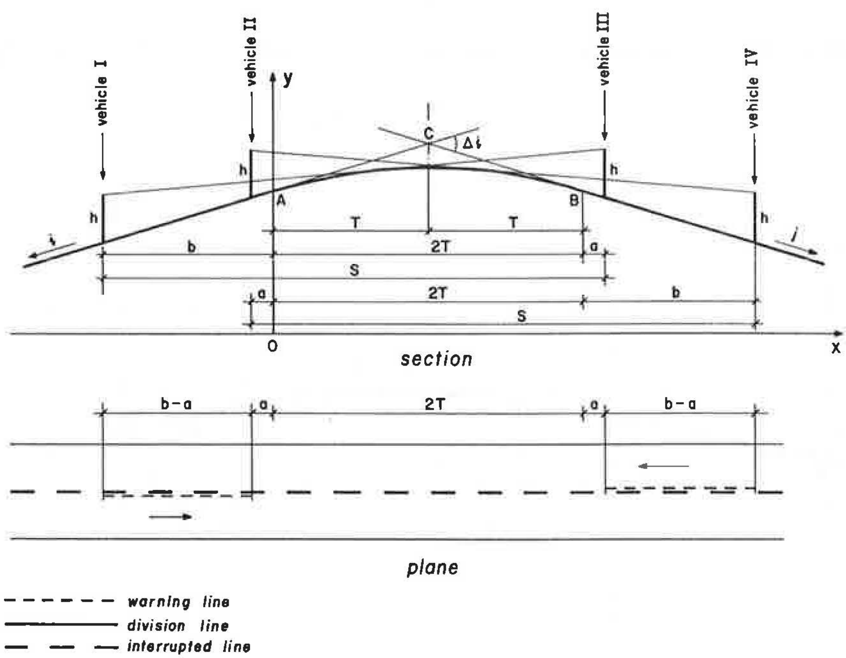

Figure 4. Determination of no-overtaking zone—both vehicles outside curve.

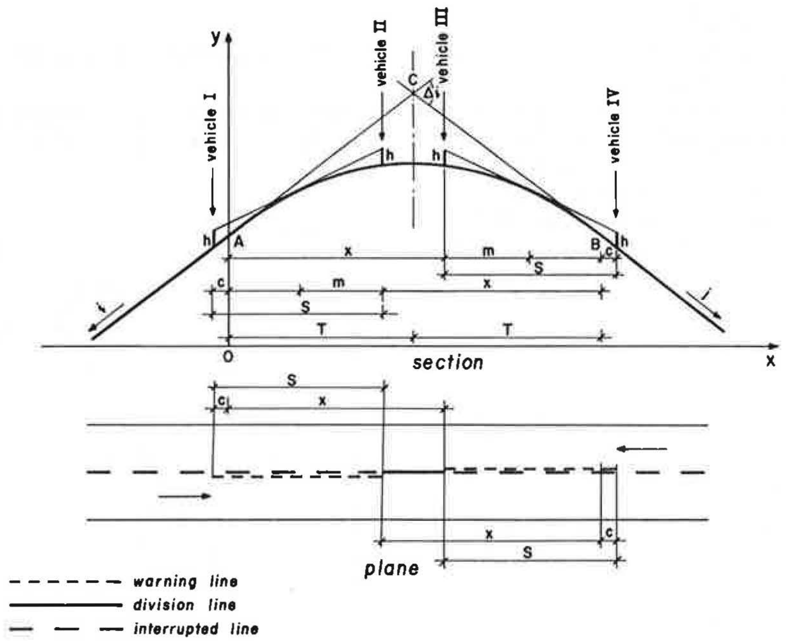

Figure 5. Determination of no-overtaking zone—one vehicle inside and one vehicle outside curve.

Marking of Division Line

Figure 4 shows the lines for case 2a, both vehicles outside the curve. For reasons of symmetry, a and b determine the length of the no-overtaking zone on both sides of the curve. This case is confined to a small angle Δi.

Figure 5 shows the lines for case 1a, one vehicle outside and one vehicle inside the curve. The distance x is obtained from Eq. 8, and the initial point of the division line is given by

$$c = S_p + x - 2T \tag{18}$$

where c is the position of the oncoming vehicle, measured from the end point of the curve.

TABLE 2

STOPPING AND PASSING SIGHT DISTANCE ACCORDING TO DESIGN SPEED

Design Speed (km/hr)	Passing Sight Distance (m)	Stopping Sight Distance (m)
50	340	60
65	460	85
80	550	110
95	640	145
105	700	170
110	760	185
130	820	230

COMPUTATION DATA

Equations 8 and 14 were solved on an Elliott 503 computer for the following i values: 0.02, 0.04, 0.06, 0.08, 0.10, and 0.12. Sight distance or S-values age given in Table 2 (2).

The curve length corresponding to each Δi and S was determined for h = 1.20 m, the elevation of an object on the road being taken as zero. 2T values ranged from $2T_s$ for stopping to $2T_p$ for passing and are shown at 100-m intervals in Figures 6 through 11. For given Δi, 2T, and S_p, the corresponding x is found subject to the relevant conditions. Results of Eq. 14 were not plotted because of the low frequency of the cases involved.

Numerical Example 1

Given $\Delta i = 0.08$, $2T = 1{,}600$ m, $R = 20{,}000$ m, design speed v = 95 km/hr (corresponding to $S_S = 145$ m, $S_P = 640$ m).

The length of the curve based on stopping sight distance is $2T_S = 700$ m; and the length based on permitting overtaking throughout the curve is $2T_P = 3{,}413$ m. As $S_P < 2T$, the relevant case is that of Eq. 8.

The data shown in Figure 9 yield x = 1,320 m, c = 1,320 + 640 - 1,600 = 360 m, and the resulting marking scheme is shown

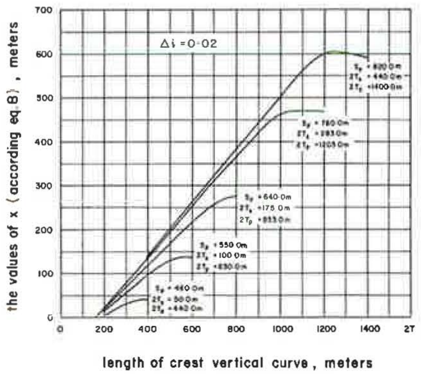

Figure 6. Determination of x values for S_P distances, various lengths of vertical curve 2T, and for $\Delta i = 0.02$.

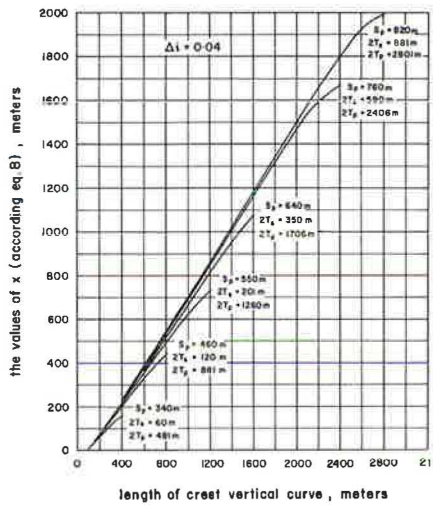

Figure 7. Determination of x values for various S_P distances, various lengths of vertical curve 2T, and for $\Delta i = 0.04$.

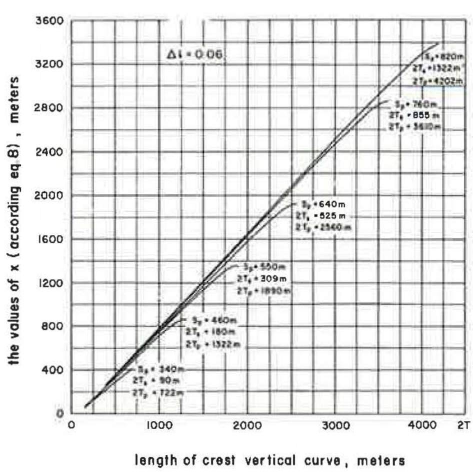

Figure 8. Determination of x values for various S_P distances, various lengths of vertical curve 2T, and for $\Delta i = 0.06$.

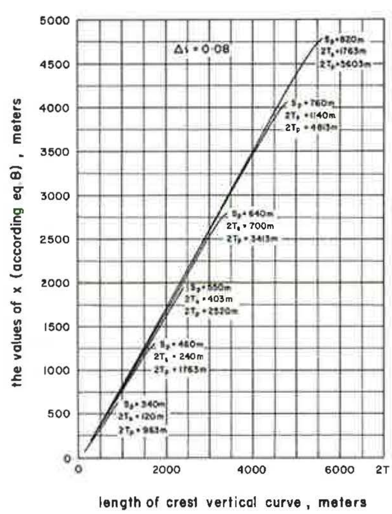

Figure 9. Determination of x values for various S_P distances, various lengths of vertical curve 2T, and for $\Delta i = 0.08$.

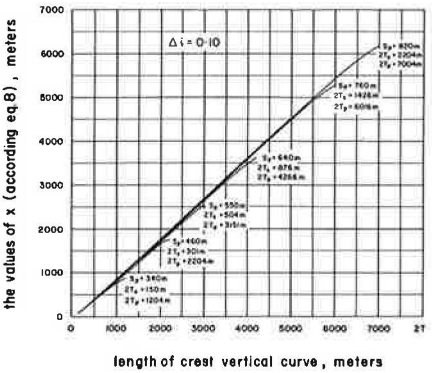

Figure 10. Determination of x values for various S_p distances, various lengths of vertical curve 2T, and for $\Delta i = 0.10$.

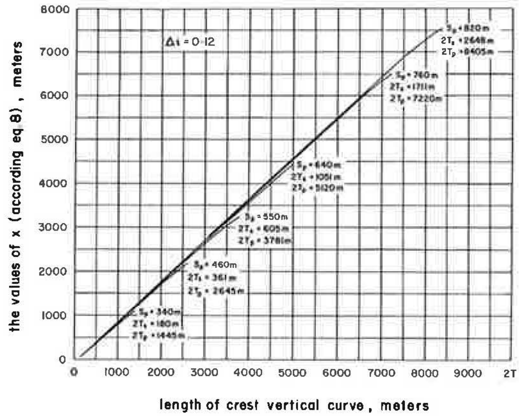

Figure 11. Determination of x values for various S_p distances, various lengths of vertical curve 2T, and for $\Delta i = 0.12$.

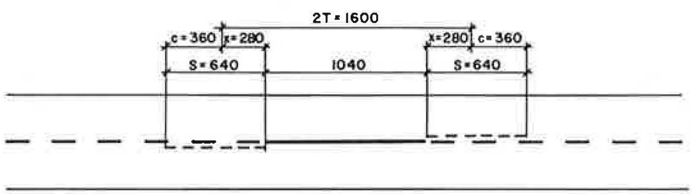

Figure 12. Example of proposed marking.

in Figure 12. For the same data and R = 25,000 m, we have 2T = 2,000 m and, accordingly, x = 1,700 m, c = 1,700 + 640 - 2,000 = 340 m. The corresponding length of the division line is 1,400 m, compared with 1,040 m for R = 20,000 m. The radius permitting unrestricted overtaking under these conditions is 42,667 m.

Numerical Example 2

Given $\Delta i = 0.04$, 2T = 800 m, R = 20,000 m, design speed v = 80 km/hr (corresponding to $S_s = 110$ m, $S_p = 550$ m).

The length of the curve based on stopping sight distance is $2T_s = 202$ m; and the length based on permitting unrestricted overtaking throughout the curve is $2T_p = 1,260$ m. As $S_p < 2T$, the relevant case is again that of Eq. 8.

The data shown in Figure 7 yield x = 1,320 m, c = 356 + 550 - 600 = 306 m. For the same data and a smaller radius R = 10,000 m, we have 2T = 400 m. Now $S_p > 2T$, and Eq. 14 is relevant; its roots are $x_1 = 38$ m and $x_2 = 362$ m. For x_1, Eqs. 16 and 17 yield a = 297 m and b = -187 m, a negative value, whereas it is required to be positive. Hence, the case of both vehicles outside the curve does not apply here, and the division line is marked as for the case with one vehicle outside and the other inside the curve.

SUMMARY AND CONCLUSIONS

This analysis permits determination of the zone of overtaking visibility for a crest vertical curve in a 2-way 2-lane highway. The proposed marking (Figs. 4 and 5) comprises a warning line, equal in length to the passing sight distance, indicating nearness of a zone of reduced overtaking visibility. In this zone the driver is allowed to complete an overtaking move begun earlier but not allowed to attempt a new one once he has passed the initial point of the warning line in the right lane. Analysis of the results shows that the length of the no-overtaking zone increases with that of the curve, up to the maximum where overtaking is unrestricted.

The sudden vanishing of the division line in the diagrams is explained as follows: As $2T$ tends to $2T_p$, x tends to $2T - S_p$. In other words, the solution tends to the case of the oncoming vehicle near the end point of the curve and the other within the curve at distance x. At $2T = 2T_s$, $x = 2T - S_p$; in other words, one vehicle is at the end point of the curve and the other at a distance S_p from it. This corresponds to the case of both vehicles inside the curve with overtaking visibility throughout its length. Both x and $2T - x$ increase as $2T$ increases; $c = S_p - (2T - x)$ decreases accordingly; but as the rate of increase of x is steeper than that of c, the length of the division line increases with 2T.

The conclusion is that, in order to reduce the no-overtaking zone where the design cannot be based on the passing sight distance, the curve should be as short as possible but still comply with the requirements of stopping sight distance and driving convenience. The curve length complying with the latter aspect is obtainable, for example, from

$$\frac{v^2}{R} = 1 \text{ ft/sec}^2 \quad (19)$$

REFERENCES

1. Ueberarbeiteter Entwurf der Richtlininen fuer die Anlagen von Landstrassen (RAL). II Teil, Linienfuhrung (RAL-L), 1963.
2. A Policy on Geometric Design of Rural Highways. American Association of State Highway Officials, 1965.