An Investment Approach Toward Developing Priorities in Transportation Planning

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This paper discusses the development of priorities in transportation planning as a problem in investment planning or capital budgeting. Determination of a construction program for a regional freeway network is used to illustrate the approach. This analysis is carried out with two mathematical programming formulations; one formulation permits early acquisition of right-of-way, and the other does not. In these formulations, the objective is to order construction of segments of the network so that the net user benefits of these freeway segments are maximized. The maximization of this objective is subject to several constraints. Expenditures on the freeway segments are limited by the budget allocated for that purpose, and the entire network must be completed by a particular date. The results of the analysis permit establishment of project priorities and of a construction program for completing the network in a particular length of time. A significant advantage of the use of mathematical programming techniques in this investigation is the ease with which sensitivity analyses can be accomplished; the impact of alternate budgets and varying lengths of time to construct the facilities can be readily determined.

One of the more critical aspects of the transportation planning process is the determination of construction priorities for the recommended plan. If a transportation plan is to be effectively implemented, a program for construction must be prepared. Specific guidelines on when to start individual projects are needed to ensure completion of the plan by the design year. The development of such a program requires more than subjective rating of individual projects. Even some analytic measure of each individual project's worth will alone not be sufficient. Although these measures are needed for an analysis, they do not take into account how the plan will be financed; therefore, the funding available for new construction must also be examined.

These two elements—the worth of individual projects and the financing available for their construction—are the basic inputs that must be investigated to determine when individual projects should be constructed. But another important factor must be considered; this factor is time, because all but the very simplest transportation plans will require a number of years for completion. The worth of a project usually changes with time when the project is opened to serve traffic. Furthermore, the flow of funds for construction may fluctuate from year to year during the period of time needed to implement the plan.

Thus, the objective for a program of construction should be to take advantage of these changes in the worth of individual projects over time, and to schedule individual projects so as to maximize the total benefit of all projects in the plan. This is subject, of course, to the restrictions imposed by the flow of funds for construction purposes. Consequently, the analysis to evaluate alternate construction programs described in this paper was developed from the following objectives:

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1. Maximization of user return through early construction of heavily used facilities; and 
2. Feasibility of financing—the projects scheduled in any budgeting period cannot exceed the funds available for that period.

The above framework is analogous to an investment planning, or capital budgeting, study (1). This paper adopts investment planning concepts and applies them to a typical problem in regional transportation studies, the determination of a construction program for a regional freeway system. Implicit in this approach is the assumption that the flow of available funds can be predicted with sufficient accuracy to make the resultant program attainable and compatible with the previously stated objectives of construction programming. Also, it must be emphasized that the evaluation of the proposed facilities has already taken place prior to this programming analysis. Improvements to the network have already been justified through the Chicago Area Transportation Study (CATS) evaluation of the entire transportation system. The question here is not whether a facility should be built, but rather when it should be built.

PROBLEM DESCRIPTION

The Final Report of the Chicago Area Transportation Study (2) proposed a transportation plan for the region that has been accepted by the Northeastern Illinois Planning Commission and various operating and government agencies in the Chicago metropolitan area. A regional freeway network, which is now being built, is an integral part of the proposed highway plan. Figure 1 shows the corridors making up this network and the staging of construction recommended in the CATS Final Report. The first stage included all committed freeway facilities in the region as of 1960. This portion of the freeway network is presently near completion, and work is now being started on several portions of the second stage. Figure 2 shows the current status of the network.
Because location studies for most of the remaining facilities in the second stage of the plan and part of the facilities in the third stage are now in progress or about to start, the determination of a preliminary construction program for these segments is of prime importance. The portion of the network for which the construction program is to be developed is shown in Figure 3. These routes are intended to supplement the existing Interstate and primary freeway network in the region and to provide improved access to the suburban areas they serve.

The major reason for these facilities being included in the plan is clear. The suburban Chicago metropolitan area is undergoing rapid development and the increase in the amount of travel generated in these areas will be dramatic. Unfortunately, the costs associated with construction of these supplemental freeways are also expected to rise. Of particular importance is the expected increase in the cost of right-of-way. Land that is presently at a low level of development will be heavily utilized within several years and right-of-way costs may become prohibitive. Therefore, in addition to the problem of scheduling construction of these supplemental freeway facilities, there is timing of the right-of-way purchases to consider. The trade-off between funds going for early acquisition of right-of-way and funds going for actual construction of facilities must be investigated.

There are several reasons why this problem and many similar problems dealing with the scheduling of improvements to transportation facilities should be viewed as investment planning problems. First, a large number of decisions on alternate courses of action must be made at different points in time; in this case, alternate segments of the network may be constructed, or different parcels of right-of-way may be purchased. Second, the absolute and relative worth of these alternate courses of action change over time. The return on each project in the example being considered varies with changing travel demands, the changing cost of constructing the facility, and the increasingly expensive right-of-way. Finally, the flow of funds for investment is reasonably predictable over time and is independent of the selection of a particular course of action. The funds are committed over a period of years for completion of the network.

In this paper, the development of two mathematical programming formulations designed to analyze the problem of scheduling construction of these supplemental freeways is presented. The initial mathematical program does not include the possibility of early acquisition of right-of-way, whereas the second formulation considers this potential course of action. This latter mathematical program balances the benefits resulting from advance acquisition of right-of-way against the benefits resulting from actual construction. Thus, the second formulation is a logical extension of the initial mathematical program.

DEVELOPMENT OF INITIAL MATHEMATICAL PROGRAM

Mathematical programming was selected as the tool for analyzing this problem (3). As a first step, the time between the present and the future completion date of the net-
work is divided into \( n \) equal time intervals. Each time interval corresponds to a time period with a definite budget established for the construction of supplemental freeway facilities. For this problem, five 2-year budgeting periods were used. The next step is to divide the network into \( m \) segments. The network is sectioned so that each individual segment is a major part of the completed network and yet still can be constructed within a single budgeting period. The 11 selected segments of the supplemental freeway network are shown in Figure 4.

Initially, the problem was formulated as a mathematical program without considering early acquisition of right-of-way. This formulation was further idealized by assuming that all benefits created through construction of a freeway segment are accountable; all future benefits can be properly appraised and included in the analysis. Like all mathematical programs, this formulation consists of three parts:

1. The choice variables, i.e., variables corresponding to construction of freeway segments in different budgeting periods;
2. The objective function, i.e., a mathematical expression that computes the benefits associated with a construction program; and
3. The constraints, i.e., relationships among choice variables that limit the construction programs that may be considered.

Mathematically, the objective function may be stated as follows:

\[
\text{maximize } \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=j+1}^{\infty} B_{ik} Y_{ij}
\]

where

\( Y_{ij} = 0 \) if segment \( i \) of the network is not constructed in budgeting period \( j \),
\( Y_{ij} = 1 \) if segment \( i \) of the network is constructed in budgeting period \( j \), and
\( B_{ik} \) = the present worth of the net benefit (user return minus segment costs) in budgeting period \( k \) derived from construction of segment \( i \) in a previous budgeting period.

Variable \( Y_{ij} \) is subject to the following constraints:

1. \( \sum_{i=1}^{m} t_{cij} Y_{ij} \leq C_j \) if \( 1 \leq j \leq n \)
2. \( \sum_{j=1}^{n} Y_{ij} = 1 \) if \( 1 \leq i \leq m \)
3. \( Y_{ij} = 0 \) or 1
where
\[ t_{Cij} \] = the total construction and right-of-way costs of segment \( i \) if built in budgeting period \( j \), and
\[ C_j \] = the budget allowed for completion of the network in budgeting period \( j \).

The first constraint ensures that the sum of all expenditures in a budgeting period is less than the funds budgeted for that period. The second and third constraints together require all freeway segments to be built by the \( n \)th budgeting period.

This formulation assumes that a budget surplus cannot be carried over to the next budgeting period. If this is not the case, the first constraint can be replaced by the following constraint, which allows budget transfers to a later budgeting period:

\[
\sum_{i=1}^{m} t_{Cij} Y_{ij} + S_j = C_j + (I + 1)S_{j-1} \quad \text{if} \quad 1 \leq j \leq n
\]

where
\[ S_j \] = the budget surplus in budgeting period \( j \), and
\[ I \] = the interest rate per budgeting period for a budget surplus.

This mathematical program is an integer linear programming problem, a type of problem that is notoriously intractable. Fortunately, this problem can be solved fairly simply, disregarding the summation of net benefits, \( B_{ij} \), to infinity for the present, because all integer solutions can be generated with exactly \( m \) of \( Y_{ij} \) equal to one (4). Thus, solution of the problem through conventional linear programming techniques can be accomplished by restricting the number of \( Y_{ij} \) in the solution. This restriction can be handled by either limited-basis entry or post-optimal procedures.

**DETERMINATION OF A FREEWAY SEGMENT'S BENEFIT**

The definition and measurement of all benefits and diseconomies resulting from freeway construction have been debated for some time, yet there is common agreement that the evaluation of proposed major highway facilities must be as broadly based as possible. However, in implementing a plan that has been broadly evaluated, a more limited base for making a decision about the priorities of constructing segments or purchasing rights-of-way is sufficient. Thus, in employing only changes in user costs to determine the return on individual freeway segments, the intent is not to provide a warrant for construction of the facility, but rather to assess the relative merits of the alternate freeway segments. Again, the question is not whether a facility should be built, but when it should be built.

In defining user benefits, it is assumed that the future demand for travel in the corridor associated with a particular supplemental freeway segment does not depend on whether the freeway is built. Furthermore, it is also assumed that over time the change in this demand is independent of when the facility is constructed. Within each segment's corridor there exists a certain amount of traffic that would be diverted to a freeway facility in the corridor. If this facility were not built, this traffic would remain in the segment's corridor but would travel over the corridor's other arterial facilities. The user benefit for each freeway segment is then defined as the difference between the cost of operation on a freeway and the cost of operation on an arterial without a freeway in the corridor to handle the divertible traffic.

To calculate the benefits from these freeway segments, existing traffic assignments over the CATS final highway plan were used to provide an estimate of 1985 travel on the proposed supplemental freeway facilities. Each of these estimates was assumed to equal the vehicle-miles of divertible traffic in a freeway segment's corridor. From these estimates, the traffic in each segment's corridor, during the five budgeting periods between 1970 and 1980, was developed by multiplying the 1985 estimate of divertible traffic by traffic increase factors determined for each freeway segment. A factor was defined to be the ratio of daily trip ends in a freeway segment's corridor during a budgeting period to the daily trip ends in the corridor in 1985. For example, the ratio of daily trip ends in a corridor during a budgeting period to the corridor's daily trip
ends in 1985 would be the corridor's traffic factor for the budgeting period. Multiplying this factor by the 1985 estimate of corridor traffic that is divertible to a freeway yields the divertible traffic for the budgeting period. Using this method, an estimate of average corridor traffic that is divertible to the freeway segment was prepared for each budgeting period.

With this traffic volume estimated, the travel speeds in the corridor were then prepared. Speeds on corridor arterials, assuming the freeway did not exist, were first determined. Next, these arterial speeds were estimated assuming that the freeway had been built. Finally, the speed on the corridor's freeway was estimated. Then the costs per vehicle-mile at these speeds, as developed by Haikalis and Joseph (5) but increased 20 percent to reflect the general price trend since the presentation of their paper, were used to calculate the differential cost to users who would be diverted to the freeway facility. This corridor differential cost or user return was determined for each budgeting period and multiplied by the previously determined corridor divertible traffic to obtain the total gross user benefit of the freeway segment in the corridor. Figure 5 is a detailed flow chart of the procedure for determining the total gross user benefit of each freeway segment.

This benefit value is not, however, equivalent to \( B_{ij} \), the net benefit of a freeway segment defined in the objective function. As stated, the foregoing calculation defines the gross user benefits generated by a segment during a particular budgeting period. From this gross benefit, network cost must be subtracted to obtain a net benefit. These other costs include the segment construction and right-of-way costs allocated to a budgeting period and the costs of maintaining the freeway segment during the same budgeting period. More specifically, \( B_{ik} \) equals the user benefit (change in user costs) created by freeway segment \( i \) during budgeting period \( k \), minus the portion of construction and right-of-way costs for segment \( i \) allocated to budgeting period \( k \), minus the costs of maintaining segment \( i \) during budgeting period \( k \).

If early acquisition of right-of-way is not considered, right-of-way for segment \( i \) must be purchased in the same budgeting period when construction of segment \( i \) takes place. Thus, the allocated cost per budgeting period in the objective function is the total cost (construction plus right-of-way) of the segment times the capital recovery factor. This allocation of right-of-way and construction costs to a budgeting period was accomplished with a capital recovery factor at 5 percent annual interest assuming a 50-year life for all freeway segments.

### COST ALLOCATION, INFLATION, AND DISCOUNTING

It should be emphasized that there are differences between the definitions of construction and right-of-way costs in the objective function and the first constraint. These differences arise because of the handling of cost allocation, inflation, and discounting in these two parts of the formulation. As previously outlined, in the objective function only a portion of the total cost of the facility is allocated to each budgeting period over the life of the facility. This is accomplished with the capital recovery factor. However, in the first constraint the variable \( tc_{ij} \) is the total of construction and right-of-way costs associated with the facility if it is built in budgeting period \( j \).
A second major difference occurs in the treatment of inflation in these costs over time. In the objective function, increases in the costs of construction and right-of-way due to inflation are ignored. It is assumed that the inflationary trend in both of these highway costs equals the inflationary trend in prices in general, including the user costs that are used to calculate the gross user benefit (6). The reformulation to include advance right-of-way acquisition does, however, permit increases in the cost of right-of-way above price increases attributable to inflation. In the first constraint, however, the situation is much different. Although construction and right-of-way costs may increase at a certain rate due to inflation, there is no guarantee that the available funds will increase at this rate. Unlike the case in the objective function, there is no reason to suppose that the inflationary trend in the highway costs equals the increase in the funds budgeted for construction and right-of-way expenditures on the supplemental freeway facilities. Thus, increases in construction and right-of-way costs due to inflation must be included in the analysis in the first constraint to ensure that inflated costs do not exceed the amount of available funds.

The final difference is in the relative importance of discounting in the objective function and first constraint. In the first constraint, all costs are measured at the same point in time; all money spent on construction and right-of-way for the freeway segments in budgeting period \( j \) must be less than or equal to the funds available in budgeting period \( j \). Discounting costs and funds available do not affect this constraint because both sides of the equation are multiplied by the same discount factor, and thus discounting can be ignored. But in the objective function, benefits at different points in time are being totaled. In this case, discounting is important because a dollar of future benefit must be distinguished from a dollar of present benefit. In the objective function, the present worth, instead of the absolute dollar value, of all future benefits must be computed and used.

**PERIOD OF TIME OVER WHICH BENEFITS ARE TOTALED**

In the initial formulation of the problem, the summation of \( B_{ik} \) for all values of \( k \) is not limited by an upper bound on \( k \), i.e., limited to any length of time. This unbounded upper limit serves only to represent the concept of a continuous return on a freeway segment through time. In practice, discounting future benefits generally allows a limit to be placed on the length of time over which the benefit of a freeway segment accumulates. Discounting reduces the significance of distant future benefits relative to near-term future benefits. The discounted return on a segment during a year far in the future will not add significantly to the total worth of the segment (7). This allows benefits from the future remaining lives of the segments to be ignored after a certain point in time.

This reasoning is the basis for developing an upper bound on the period of time over which a freeway segment's benefits are totaled. Note that the objective function can be broken into two separate expressions, one for all benefits occurring before the completion date of the network and a separate expression for all benefits accruing after completion of the network. Mathematically, this can be stated as follows:

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=j+1}^{\infty} B_{ik} Y_{ij} = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=j+1}^{n} B_{ik} Y_{ij} + \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=n+1}^{\infty} B_{ik} Y_{ij}
\]

The far right-hand side of this equation (the portion that accumulates benefits accruing after all freeway segments are completed) is nearly constant. Changes in the usable lives of the segments and related changes in their benefits, which are discounted from far in the future, do not greatly affect the total value of the objective function. Benefits from a segment between the completion of the network and before the segment's
usable life is exhausted are relatively independent of when the segment was constructed. The objective function can, therefore, usually be shortened to include only benefits before the network is constructed without changing the problem's solution. This equivalent objective function is

$$\text{maximize } \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=j+1}^{\infty} B_{ik} Y_{ij}$$

## EXTENSION TO EARLY RIGHT-OF-WAY ACQUISITION

In the initial formulation of the problem, early acquisition of right-of-way was not considered. To increase the realism of the analysis, it was decided that this potential course of action could not be ignored. Providing this option meant that the benefits associated with advance right-of-way acquisition had to be accounted for in the objective function. With respect to the costs included in the objective function, the effect of allowing early right-of-way acquisition is fairly obvious. Early acquisition may allow savings in the right-of-way costs allocated to a budgeting period and increase $B_{ik}$. However, these additional benefits accrue only if there is an increase over time in the cost of right-of-way above that explained by general price increases; otherwise, the initial formulation would still be appropriate.

In the initial formulation, the objective function was defined as follows:

$$\text{maximize } \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=j+1}^{\infty} B_{ik} Y_{ij}$$

where $B_{ik}$ is equal to the reduction in corridor user costs in budgeting period $k$ created by construction of segment $i$ in an earlier budgeting period minus the construction, right-of-way, and maintenance costs for segment $i$ allocated to budgeting period $k$.

Use of the same interest rate in the capital recovery factor and present worth calculations makes the present worth of the cost of right-of-way equal to the present worth of the series of allocated right-of-way costs in the objective function. Mathematically this may be stated as follows:

$$\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=j+1}^{\infty} B_{ik} Y_{ij} = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=j+1}^{\infty} B'_{ik} Y_{ij} - \sum_{i=1}^{m} \sum_{j=1}^{n} R_{ij} X_{ij}$$

where

- $B'_{ik} = B_{ik}$ without the allocated right-of-way cost for segment $i$ in budgeting period $k$ included in the calculation, and
- $R_{ij} = \text{the present worth of the right-of-way required for segment } i \text{ if purchased in budgeting period } j$.

In this formulation, the right-of-way is still purchased at the same time that construction takes place.

In order for right-of-way to be purchased in a budgeting period other than when construction takes place, a new variable must be defined. This variable is $X_{ij}$, a variable analogous to $Y_{ij}$ that applies only for the purchase of right-of-way. Placing this variable in the objective function changes the initial formulation to

$$\text{maximize } \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=j+1}^{\infty} B'_{ik} Y_{ij} - \sum_{i=1}^{m} \sum_{j=1}^{n} R_{ij} X_{ij}$$

where

- $X_{ij} = 0$ if right-of-way for segment $i$ is not purchased in budgeting period $j$,
- $X_{ij} = 1$ if right-of-way for segment $i$ is purchased in budgeting period $j$, and
\( \text{R}_{ij} \) = the present worth of the right-of-way required for segment \( i \) if purchased in budgeting period \( j \) (the cost of right-of-way in budgeting period \( j \) includes the uninflated increase in the cost of right-of-way).

In effect, this reformulation adds the objective of minimizing cost through early acquisition of right-of-way that will appreciate rapidly.

As before, the above objective function can be expanded as follows:

\[
\sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=j+1}^{\infty} B'_{ik} Y_{ij} - \sum_{i=1}^{m} \sum_{j=1}^{n} R_{ij} X_{ij} = \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=j+1}^{\infty} B'_{ik} Y_{ij} - \sum_{i=1}^{m} \sum_{j=1}^{n} R_{ij} X_{ij} + \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=n+1}^{\infty} B'_{ik} Y_{ij}
\]

The summation of benefits past the completion date of the network, the term to the far right of the equality, is nearly constant. As was explained earlier, this expression can be dropped from the objective function without affecting the problem's solution. Rewriting the above objective function without this term yields the bounded objective function for the mathematical program that includes early right-of-way acquisition:

\[
\text{maximize } \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=j+1}^{\infty} B'_{ik} Y_{ij} - \sum_{i=1}^{m} \sum_{j=1}^{n} R_{ij} X_{ij}
\]

**REFORMULATION OF THE CONSTRAINTS**

Attention will now be turned toward the constraints on the early right-of-way acquisition objective function. The constraint set prepared for the initial formulation is inadequate for this expanded problem. The first constraint must be revised so that the total cost of a segment, which includes right-of-way, is not always charged against the funds in the budgeting period in which the segment is built. Right-of-way and construction costs can now be charged to different budgeting periods. The second constraint of the original constraint set ensures that each segment is completed within \( n \) budgeting periods, and this constraint is still valid.

Constraints applicable to the advance right-of-way acquisition variables must be added to those constraints dealing with the original choice variables. A constraint that prevents right-of-way from being purchased more than once is needed. More importantly, a constraint to ensure that right-of-way acquisition takes place prior to construction of a segment is required. The formal representation of the reformulated constraint set is as follows:

1. \( \sum_{i=1}^{m} \text{cc}_{ij} Y_{ij} + \text{row}_{ij} X_{ij} \leq C_j \) if \( 1 \leq j \leq n \)
2. \( \sum_{j=1}^{n} Y_{ij} = 1 \) if \( 1 \leq i \leq m \)
3. \( \sum_{j=1}^{n} X_{ij} = 1 \) if \( 1 \leq i \leq m \)
4. \( \sum_{j=1}^{p} X_{ij} \geq Y_{ip} \) if \( 1 \leq i \leq m, \ 1 \leq p \leq n \)
5. \( Y_{ij} = 0 \) or \( 1 \)
6. \( X_{ij} = 0 \) or \( 1 \)

where

\[
\begin{align*}
cc_{ij} &= \text{the construction cost of segment } i \text{ if built in budgeting period } j, \text{ and} \\
row_{ij} &= \text{the right-of-way cost of segment } i \text{ if purchased in budgeting period } j.
\end{align*}
\]

The reformulated problem, like its predecessor, is an integer linear programming problem, a problem that can be solved fairly simply because the integer solutions can be generated with exactly \( m \) of the \( Y_{ij} \) and \( m \) of the \( X_{ij} \) equal to one.

**PRELIMINARY RESULTS**

Approximately 30 runs of the initial mathematical programming formulation have been completed. The reasons for the large number of runs were to test the sensitivity of the solution to alternate budgets and to analyze the effect of allowing the transfer of funds between budgeting periods. Even with this number of runs, the total cost of computer time for the linear programming routine used in the analysis was less than $100. The reformulated mathematical program that includes advance right-of-way acquisition has also been run successfully several times. Although this reformulation created a substantially larger problem, computer costs still remain reasonable; each run of the reformulated problem cost approximately $10.

The results obtained from the initial formulation permitted the establishment of preliminary project priorities and a construction program for completing the 11 segments included in the analysis. An example of the type of program that can be easily developed from this analysis is shown in Figure 6. Budgeting period expenditures for right-of-way and construction are ordered by segment and do not exceed the funds available in each budgeting period. In addition to this information, the results of the reformulation show, by freeway segment, where advance right-of-way acquisition is beneficial.

**CONCLUSIONS**

Perhaps the most important aspect of the use of mathematical programming in this study is that such methods permit sensitivity analyses to be performed quite easily. Effects of alternate budget allocations and segment costs on the problem's solution are readily obtainable. Such information provides a base for cost-effectiveness analysis and similar evaluation procedures, which then feed back information to the budget allocation process.

Even though the present problem considers only a limited number of network segments, the expansion of the mathematical programs in the analysis to large networks is quite practical with the development of extremely efficient and large-scale computer codes for solving mathematical programming problems. Eventually, a similar analysis is planned for the entire network included in the CATS area, the complete Chicago metropolitan area. This intermediate-level planning will provide a link between the long-range transportation study at the regional level and the selection of specific projects to develop the plan on the ground.

**REFERENCES**


