A Utilitarian Theory of Travel Mode Choice

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Intensified study of public transit service has placed demands on the transportation planner for improved choice of travel mode modeling techniques. An underlying theory of modal choice is needed to correct inadequacies in our present approach. A theory is proposed that is built on the suppositions that individual choice of mode is utilitarian, that common measures of individual trip utility are subject to chance errors describable by the normal distribution error function, and that deviations result from predictable influences. A difference in disutility measure is set forth for comparison of travel utility. The measure combines time, convenience, and dollar cost into a common unit of equivalent time. The probability of free choice of a given mode is described mathematically as a function of the possible disutility savings. The formulation is postulated to be the normal probability density function, predicting 50 percent probability at zero disutility difference. Submodal-split study results pertaining to both transit and highway route choice are examined and found to support the free modal choice mathematical description. Deviations to be expected in applying the theory to choice of prime mode are examined. It is assumed that long-term captivity to transit or auto can be expressed as a constant probability, and the resultant constrained formulations are illustrated. Effects of excessive trip length and desirable operational refinements are discussed. It is concluded that the proposed theory is readily applicable to modal-choice forecasting and multimode analysis and may lead also to broader applications.

•THE CURRENT INTEREST in providing significant public transit service as part of the total transportation system shows no likelihood of diminishing in this era of concern with urban needs. Demands on transportation planners for improved evaluation of the interplay between private auto and public transit usage can thus be expected to continue and grow.

Predictive models for forecasting choice of travel mode have undergone extensive improvement in the past decade. Nevertheless, travel analysis still suffers from lack of a generally accepted underlying theory of modal choice. Present operational models are mostly individually tailored, empirical formulas or hand-drawn experience curves.

THE ADVANTAGES OF A THEORY

A satisfactory theoretical explanation of observed modal-choice behavior would provide benefits in forecasting travel and in understanding user evaluation of transportation system attributes. A theory is needed to produce advantages such as the following.

1. A modal-choice technique based on a satisfactory theory would allow use of a pretested model requiring only local calibration using standardized procedures. This would put less strain on available survey sample sizes and save time and talent now expended on developing individually tailored models for each application.

2. A proven theory would provide a sound basis for extrapolation beyond those sets of time, convenience, and cost alternatives presently observed in the environment of conventional transportation systems. In contrast, the validity of an empirical modal-

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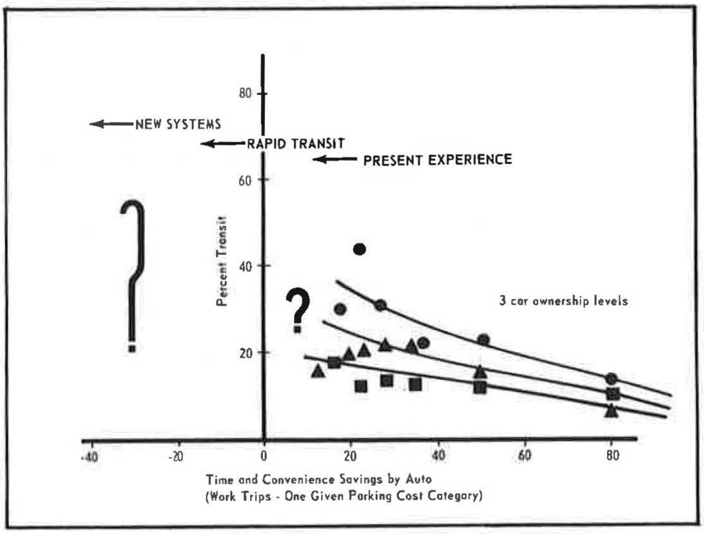

Figure 1. The empirical curve extrapolation problem.

split model under conditions not found in the existing city is dependent in large measure on the experience and judgment of the analyst and on his skill in extrapolating the predictive relationships. Figure 1 shows the difficulties encountered with such present-day models.

3. A model structured on theory would provide a reference point for measurement of travel preferences and thus would broaden the understanding of what travel characteristics are considered important by transportation consumers and how they are weighted. This in turn would provide more accurate input into cost-benefit analyses and would allow identification of those transportation system attributes that deserve priority attention in development and design.

4. A modal-split model with a theoretical framework can be more readily understood, defended, and subjected to critical examination. An empirical model must be judged primarily on the basis of consistency of results. A model based on theory can be evaluated both on this basis and by examining the inherent logic or experience with the postulates expressing user behavior and preference from which the model has been derived. The technical strengths of the model can be identified and used to their best advantage; the weaknesses of the model can be isolated and made the subject of further research.

THE BASIC PROPOSITION

The choice of mode theory presented here for consideration is comprised of primary suppositions and corollary statements as follows.

Supposition 1—Individual choice of mode is utilitarian. The trip-maker's concern is to minimize the sum total of personal disutility involved in the travel action. Corollary: If an individual's unique perception of the sum disutility of using each of the alternative travel modes could be measured for a given trip, his choice of mode could be absolutely predicted.

Supposition 2—Description of individual utility perceptions with standard network analysis techniques is affected by a multitude of chance errors. These include discrepancies caused by the variations in behavior related to differing individual value systems. Corollary: Individual choice of mode cannot be forecast; but as a substitute, the probability of an individual choice can be predicted using the error function associated with the normal distribution.

Supposition 3—Deviation from the normal distribution can be explained by such predictable influences as captive riding and resistance to long trip length.

The corollary of the second supposition is simply a restatement of the central limit theorem of probability mathematics, provided one agrees with the related supposition that the errors involved are errors of chance. The following is a partial list of discrepancies and errors that may occur in measuring individual perception of travel utility.

1. Errors inherent in averaging, specifically the use of one set of travel parameters to represent conditions facing all individuals in a given trip interchange population;
2. Discrepencies caused by variations among individuals in the perception of the utility parameters, in other words, variations caused by individual responses based on imperfect information;
3. Discrepancies caused by behavioral variations among individuals in the evaluation of utility;
4. Random network measurement errors in determining and using the mean travel parameters of alternative modes available to the trip interchange population; and
5. Network biases and errors in specification of the utility measure.

It seems quite reasonable to classify the first four categories as comprising errors of chance, satisfying the proposition in this regard. The fifth type is comprised of consistent, nonrandom errors and must be eliminated insofar as possible in any successful application of the theory.

A COMMON UTILITY MEASURE

For testing and applying the postulated theory, it must be possible to have a realistic and common measure of trip utility. If trip-generation rates are held constant, benefits accruing from reaching the trip destination are irrelevant; and modal-choice evaluation becomes simply a function of the relative opportunity to reduce travel time, inconvenience, and cost. An appropriate and apparently satisfactory measure is "difference in trip disutility," already used by D. A. Quarmby (1) and others in travel forecasting.

In this common measure, all trip costs, including but not necessarily limited to travel time, inconvenience, and money, are converted to equivalent values. They are summed for each traffic interchange and mode under consideration. The comparison of alternative modes is then made for each interchange on the basis of the algebraic difference in their respective disutilities.

In the theory being presented here, the probability of an inferior mode being used is described as a function of misclassification of individual utility perceptions for individuals within a population. This requires knowing the difference between the two measured values for the alternative modes; thus ratios cannot be considered properly for use. Methods for developing the equivalence values used in constructing the disutility difference measure are secondary to the choice of mode theory itself and are discussed in reference to applications.

FREE CHOICE MATHEMATICAL DESCRIPTION

Using the "difference in disutility" measure, the mathematical relationships implicit in the first and second suppositions of the postulated theory can be derived. The logic is outlined using a hypothetical comparison of travel mode B with alternate mode A as shown in Figure 2. Free choice of mode is assumed.

In accordance with the theory, individually perceived disutility differences will occur at variance with the value as measured by the traffic analyst. This is represented in the upper part of Figure 2 by a normal error distribution drawn around each of five measured disutility differences. Now, in line with the supposition of utilitarian choice, a trip-maker will choose mode B if the difference is positive in favor of B. Some individuals, because of the variance that has been described, will perceive the disutility saving to be positive even though it has been measured otherwise. These individuals are misclassified.

The misclassified individuals are a function of the area under the normal curve where the disutility difference is of opposite sign from the measured value. It follows that the probability of a trip-maker choosing the mode that has been measured as being inferior is equal to the probability of misclassification. Following this line of reasoning, the probability of using mode B can be plotted for different disutility differences (bottom of Fig. 2). The curve obtained is mathematically described by the normal probability density function.

The most pertinent elements of the utilitarian-theory mathematical description as it pertains to the free choice of mode can be summarized as follows.

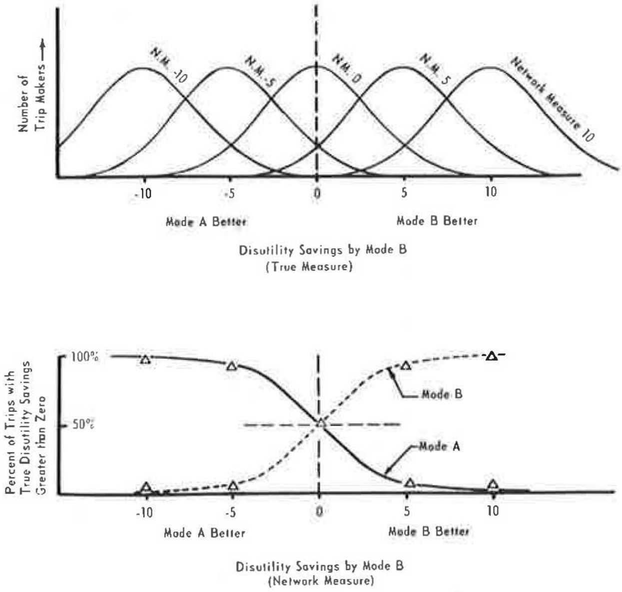

Figure 2. Free choice of mode mathematical relationships.

1. The probability of free choice of a given travel mode is a function of the disutility savings obtained through use of that mode as compared to the alternate.
2. The probability is described by the normal probability density function.
3. The resultant predictive curve has its point of inflection at 50-percent probability and zero measured disutility savings.

When travel captive to a particular mode is considered, the mathematical description must be modified. The effect of captive travel will be discussed subsequently as will the effect of excessive trip length.

EXAMPLES FROM SUBMODAL SPLIT

Submodal-split traffic analyses provide a specialized source of data appropriate for testing the first two suppositions of the proposed theory before proceeding to describe the likely form that deviations from the normal distribution may take when captivity and resistance to long trip length are involved. Submodal split applies to the special case where the trip-maker has already been assigned to the transit or auto mode. The remaining question to be answered is whether he will choose bus or rail routing if he is a transit rider, or freeway or arterial routing if he is an auto user. Obviously, no transit rider is captive to rail or bus if both services are available to the public. Neither is any auto driver captive to using either freeways or arterial streets. All decisions are free choice and the normal probability density function should be found to hold without deviation.

The first example is provided by the derivation of a rapid transit versus surface transit diversion curve as discussed in a paper by the author and Thomas B. Deen (2). The data used were from travel surveys covering the Chicago Transit Authority's "Skokie Swift" rapid transit operation and paralleling bus routes.

In the case studied there was no fare differential between transit submodes. Therefore, difference in disutility was expressed using only time and convenience measures, summed and designated as "equivalent time." Convenience was quantified in terms of "excess time" and was comprised of the walking, waiting, and transfer time involved in any door-to-door trip. Excess time was weighted by a factor to

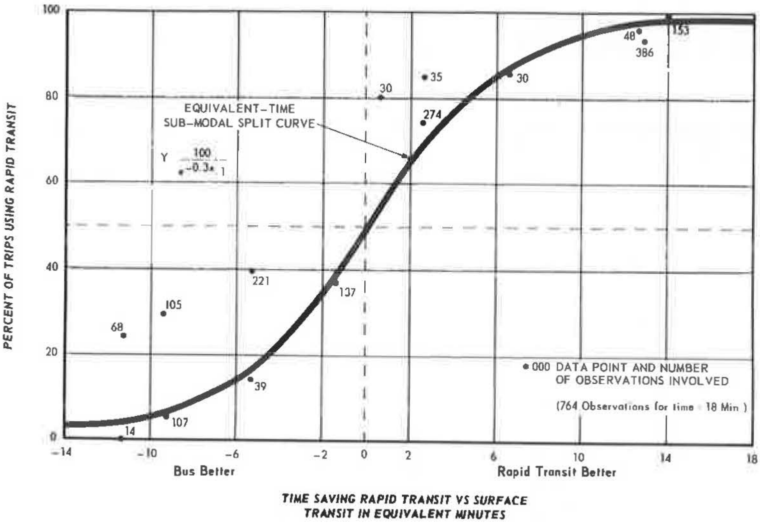

Figure 3. Transit submodal-split relationship. (From Richard H. Pratt and Thomas B. Deen. Estimation of Sub-Modal Split Within the Transit Mode. Highway Research Record 205, 1967, pp. 20-30.)

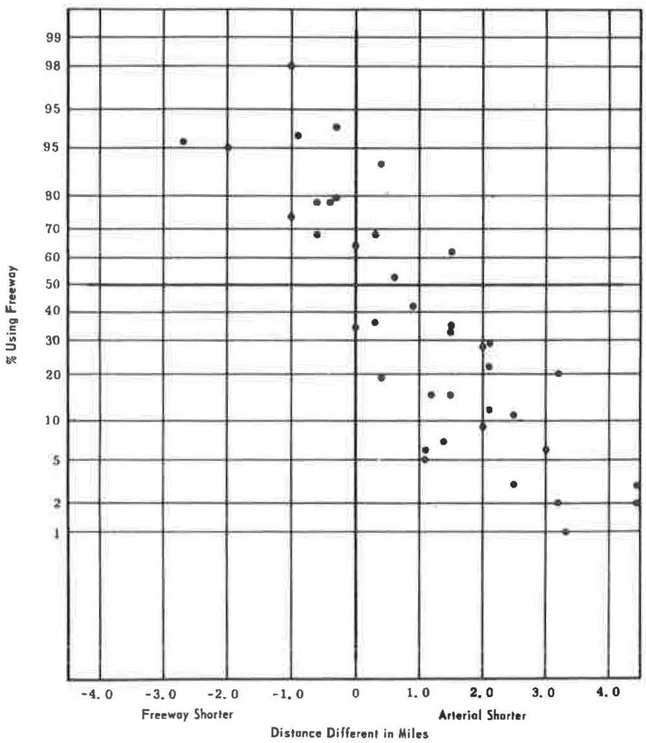

Figure 4. Freeway usage related to distance difference. (From Howard W. Bevis. Estimating a Road User Cost Function From Diversion Curve Data. Highway Research Recrod 100, 1965, pp. 47-54.)

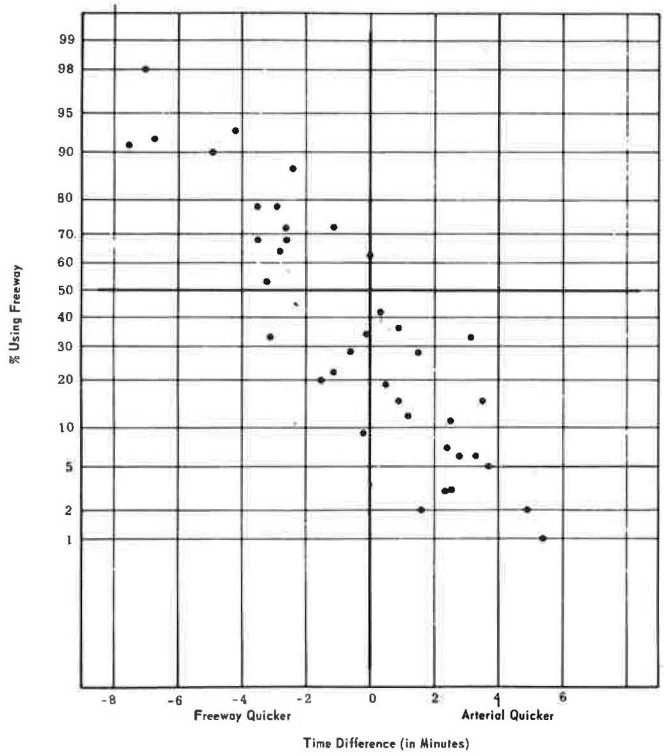

Figure 5. Freeway usage related to time difference. (See Fig. 4 for source.)

render it psychologically equivalent to running time. Several factors were tested and the value 2.5 was chosen.

The percentage of transit trips observed using rapid transit was plotted against the corresponding equivalent time saving (Fig. 3). The R^2 of the S-shaped curve that fitted, comparing predicted and actual percentage of submodal split on an interchange basis and weighting by the number of observations, was 0.886. The formulation, as illustrated, was a logistics curve with the point of inflection at 50 percent and zero difference. For purposes of this discussion, it can be considered an acceptable approximation of the normal distribution function. Thus, support is provided for the proposed theory.

A second submodal-split test of the utilitarian theory is provided by the work of Howard W. Bevis (3) on road-user cost functions. Bevis theorized that auto driver choice of route could be described through use of the normal probability density function if one compared routes in terms of generalized cost. This is an equally acceptable way of expressing difference in disutility, the common unit of measure simply being cents instead of minutes.

Bevis analyzed freeway diversion curve data for freeways in Washington, Dallas, Houston, and San Diego. Figures 4 and 5 show the percentage of freeway trips plotted on probability paper against distance difference and time difference respectively. The linearity of the plots in both instances provides support for the assumption of normality.

Combining the distance and time components into a generalized user cost, Bevis formulated equations constrained to reflect 50-percent diversion at zero cost difference. Correlation coefficients in excess of 0.80 were obtained for each of the data sets tested. Bevis concluded that the data fit the constrained normal cumulative distribution function very well.

APPLICATION TO PRIMARY CHOICE OF MODE

Examination of the utilitarian theory of modal choice using submodal-split data gives every indication that the normal cumulative distribution holds without any significant deviation when only free choice is involved. Using an assumed variance, the generalized predictive curve on standard linear coordinates is shown in Figure 6-A.

Turning to the prime choice of mode, auto versus transit, it is reasonable to expect certain specifiable deviations. In particular, captivity to either mode will restrict the portion of the trip-making population having free choice and should correspondingly affect the shape of the predictive curve.

If it can be assumed that a given range of average incomes defines some constant probability of transit captivity, then the free choice riding can be segregated out and investigated for applicability to the basic theory. Figure 6-B shows a hypothetical low-income population having a 20-percent constant probability of transit captivity. The remaining 80 percent of the population is assigned to free choice probability space and is allocated using the normal cumulative distribution. The theory will be tested using this type of probability space allocation.

It could be argued in opposition to the constant probability assumption that, even for a given income group, auto ownership should decrease as transit service improves, with transit captivity varying accordingly. This argument, which is valid in the usual sense of transit captivity, can be met through use of a more rigorous definition. Transit riders will be considered captive only if long-term unavailability of auto transportation is involved. The auto unavailability must be primarily independent of the quality of transit service available for the trip under consideration.

A transit rider will be considered captive only if the following criteria are met.

1. He does not have an auto available for the trip.
2. He cannot afford to buy and operate an auto.
3. He cannot find a ride or afford a taxi.

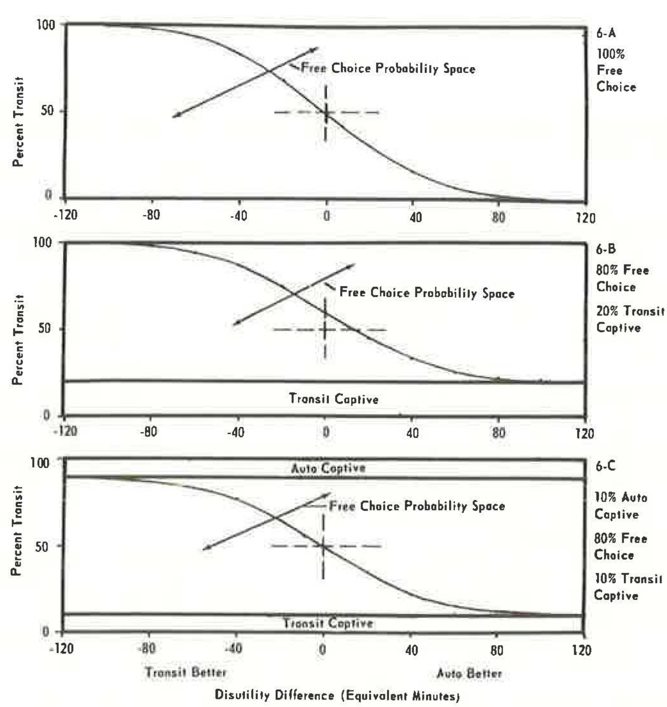

Figure 6. Captive and free choice probability space allocation.

With these criteria, a transit rider who can afford a car, even if he has none, will always be classified as a choice rider.

Auto captivity, too, must be considered. An auto user will be defined as captive if either of the following conditions are met:

1. He requires an auto at his destination (such as for work); or
2. He requires an auto on one leg of his trip (such as to make an intermediate stop on the way home).

It is important to note that the auto captivity definition, like its transit counterpart, is designed to be independent of the quality of transit service for the trip under consideration.

Figure 6-C shows a hypothetical moderate-income population with 10 percent constant probabilities of both transit captivity and auto captivity. Again, the remaining noncaptive population is assigned to a free choice probability space and is allocated as before.

The normal cumulative distribution as shown in Figure 6-A is a straight line on normal probability paper (Fig. 7). Also shown in Figure 7 are the hypothetical curves of Figures 6-B and 6-C replotted. They exhibit a curvature on normal probability paper that is introduced by the assumptions of constant captivity. The hypothetical plots of Figure 7 are compared with observed data in the next section.

EXAMPLES FROM PRIMARY MODAL SPLIT

A few prime modal-split analyses have been prepared with trip interchange comparisons between auto and transit service expressed in a manner approximating the difference in disutility measure. There is enough information, however, to allow tentative evaluation of the proposed theory. All data presented here are for work purpose trips.

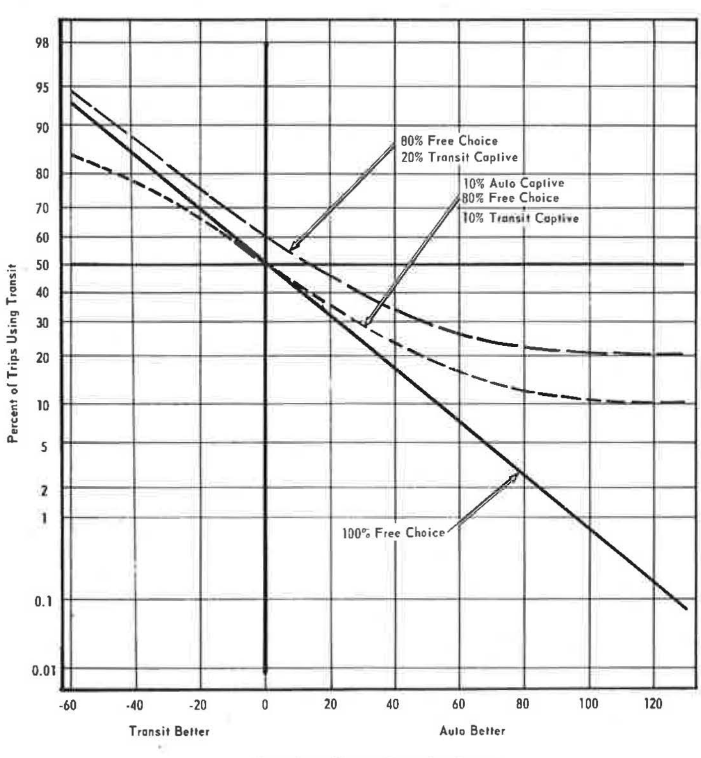

Figure 7. Captivity effects plotted on probability paper.

The first example is from the Twin Cities area of Minneapolis-St. Paul and uses input developed for modal-split model calibration by Alan M. Voorhees and Associates (4, 5). The trip data base was a 5 percent home interview sample taken in 1958.

Trip disutility measures were constructed from the following travel impedance components.

Auto	Transit
Driving time	Running time
Parking time	Walking time
Walking time	Waiting time
Auto operating cost	Transferring time
Parking charge	Transit fare

Time measures were estimated using highway and transit networks. Parking charges were split between the going and return auto trips. Auto-related dollar costs were not divided by auto occupancy on the assumption that cost savings obtained by carrying passengers are counterbalanced by the time and inconvenience involved in passenger pick-up and delivery.

Time and dollar quantities were converted to a common disutility measure using a series of equivalence factors. These factors were for expediency based on experience and prior studies by others. Driving and running time was used as the basic unit. Excess times, including auto terminal time, were factored by 2.5 as an expression of inconvenience. Dollar costs were converted to time equivalents by valuing time at 25 percent of the wage rate implied by the applicable zone-of-origin average-income estimate.

The results of comparing the percent of work trips using transit with the difference in disutility between auto and transit service are reproduced in Figure 8 as plotted on probability paper. Three income strata have been separately analyzed using as the break points origin-zone average family incomes of $6,000 and $8,100 per year.

The predictive curves superimposed on the plots have been hand fitted in conformance with the proposed theory. They are based on the same variance throughout and on the following constant probabilities of captivity.

	Transit (%)	Auto (%)
Lower income	7	7
Medium income	3	7
High income	1	25

The transit captivity probabilities were derived from inspection of the data points at high disutility difference values favoring auto usage. The percent using transit appears to approach a minimum value rather than zero. This minimum percentage has been taken to represent the transit captivity as defined.

Lack of data points at the other end of the curve prevented similar determination of auto captivity percentages. As a substitute, theoretical requirements of the proposed theory were employed. The theory, if the trip disutility measures used are accepted as valid, requires that at zero disutility difference the modal-split curve must predict 50 percent usage of each mode by free choice trip-makers. On this basis, auto captivity probabilities were chosen to force the curves through the intersection of coordinates thus specified. It can be seen that the results are auto captivity percentages that fall in logical relationship among the income groups and conform in general with available information on auto captivity.

Goodness-of-fit computations have not been carried out with the data in the format shown in Figure 8. Except for minor noise at the high-income level, however, very close conformance is evident.

It is interesting to note that although the Twin Cities data stratified by income provide considerable support for the proposed theory, the same data stratified by auto ownership produce noticeably skewed data plots. It can be concluded that whereas

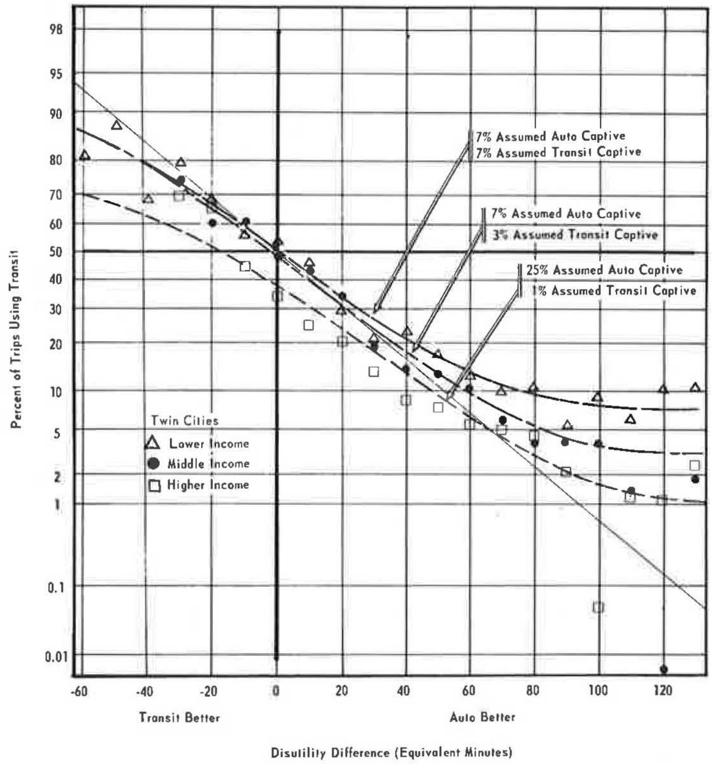

Figure 8. Twin Cities modal-choice relationships.

income provides a relatively unbiased basis for stratification, auto ownership levels do not. Income cannot be directly influenced by choice of mode; car ownership logically can be.

The second example using primary modal-split data is derived from 1955 Washington, D. C., home interview survey, work-trip information prepared by J. Royce Ginn (6). Ginn reprocessed the original Traffic Research Corporation modal-split model inputs (7), classifying trip interchanges by income rank and difference in equivalent time. The equivalent time measure incorporated the same travel cost components as were used for difference in disutility in the more recent Twin Cities studies except that auto-related costs were divided by auto occupancy. Most of the trip interchanges involved downtown-destined trips.

Ginn experimented with various equivalent time measures using a wide range of time equivalents. The data sets selected for presentation here used equivalents as follows: 2.00 equivalent minutes per excess time minute, 1.50 equivalent minutes per penny cost (lower incomes), and 0.75 equivalent minute per penny cost (upper incomes). These equivalents are in the same order of magnitude as the Twin Cities factors.

Figure 9 shows the Washington data in identical form as the Twin Cities data of Figure 8. The hand-fitted curves are based on the following constant probabilities of captivity:

	Transit (%)	Auto (%)
Lower income	18	3
Higher income	6	10

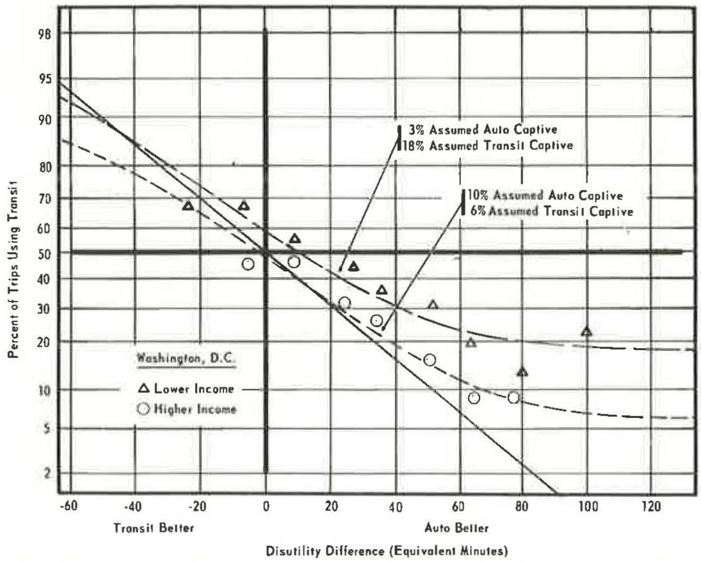

Figure 9. Washington modal-choice relationships.

The analysis of the utilitarian theory of mode choice has been carried one step further by adjusting both the Twin Cities and the Washington data to remove the assumed captive trips from consideration. This was done using the following computation.

$$\text{Free choice percent transit} = \frac{100\ [(\text{overall percent transit}) - (\text{percent transit captives})]}{100 - [(\text{percent transit captives}) + (\text{percent auto captives})]}$$

The results of analyzing the free choice trips alone are shown in Figure 10 as plotted on normal probability paper. The vertical axis represents the percentage of noncaptive trip-makers who choose transit. All data points from both cities and all income groups are included. Occasional negative values result from the observed transit usage being below the estimated transit captivity percentage.

Use of the normal probability coordinate system on the vertical axis amplifies the dispersion of data points at the larger positive and negative disutility difference values. To assist in evaluating the results, corresponding curves at ±5 percentage points have been provided in addition to the principal hand-fitted predictive curve.

Figure 10 was prepared by plotting the more extensive Twin Cities data first, thus establishing the curve. The Washington data were then added on a trial basis prior to setting the final captivity percentages. These percentages were hand adjusted until the best fit was obtained. It was in this manner that the Washington captivity percentages were established.

The exercise serves to show how the theory might be applied to limited data and to a model calibrated to fit an expression predetermined elsewhere in a more comprehensive study. It is not known at this point, of course, if more information on the 1955 Washington travel patterns would substantiate the captivity percentages chosen. The higher transit captivity and lower auto captivity postulated for Washington is consistent, at least, with the comparative urban characteristics of the two cities.

If it were known with certainty that the Washington captivity probability percentages are correct, then the evidence would clearly indicate that not only is the normal probability density function applicable to both cities but that the variance is the same. This would, in turn, mean that the noncaptive populations of the two cities make the choice between transit and auto according to a common, and therefore presumably universal,

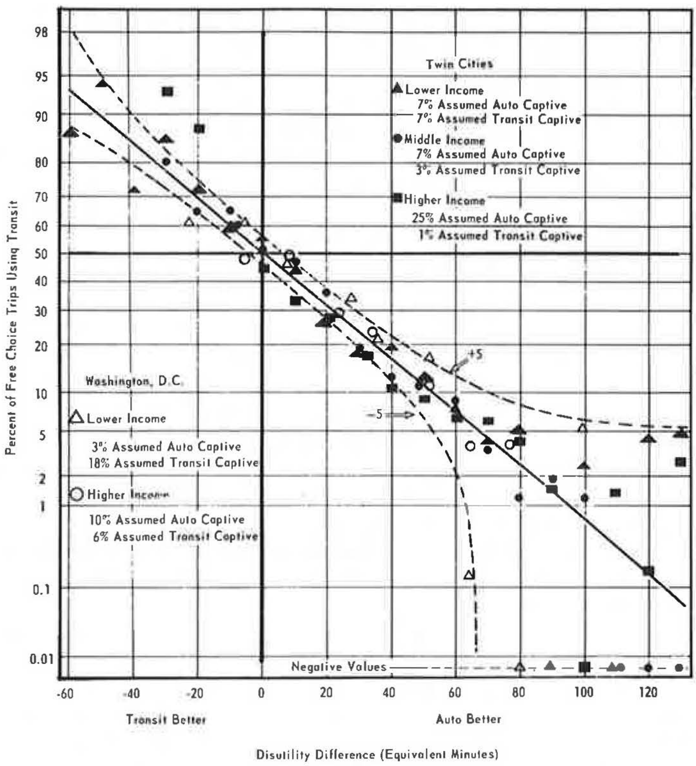

Figure 10. Twin Cities and Washington free choice of mode.

relationship. Final judgment, however, should be withheld until comprehensive data are available from another urban area. It should be noted that the response to disutility difference clearly differs in variance at different levels of decision. This can be seen by comparing the submodal-split results with the prime modal-split examples.

Use of the single curve for free choice of mode at all economic levels does not imply that income has no influence. Income is a determinant of the relative dollar value of time used in constructing the disutility values. The extent of the resultant effect of income on modal choice depends in part on the characteristics of the transit system. Given a situation where transit is consistently cheaper but slower than auto, rising incomes will cause a marked decline even in free choice patronage as forecast by the curve. This, of course, parallels the general experience of surface transit operations over the past two decades of increasing affluence.

EXCESSIVE TRIP LENGTH EFFECTS

As was alluded to previously, the captivity effect is joined by a second significant deviation from the normal distribution. This additional effect takes the form of lack in observed as compared to estimated transit riding at high disutility difference values unfavorable to transit. In the Twin Cities case, very little transit riding is observed at difference in disutility values over 130 equivalent minutes unfavorable to transit regardless of the postulated constant probabilities of transit captivity. Comparable effects are common in the results of other modal-split modeling work.

A logical explanation would appear to be that the deficiency in observed transit trips is a reflection of the overall metropolitan area trip generation and distribution characteristics. Trip-length analyses covering both auto and transit travel indicate that there is a travel time threshold, not sharp but rather of gradual form, above which trips in urban areas normally do not occur in quantity.

Implicit in a very high disutility saving by one mode is the fact that the less favored mode must be quite undesirable and presumably take a very long time. Thus, at high disutility savings by auto, the level of transit riding can be expected to drop below the estimated value simply because the trips that should be allocated to transit are in reality either not being made or are being diverted to an alternate, less desirable, destination. Through analysis of the difference between estimated and observed transit usage in the critical range, the postulated theory indeed might lead to a quantitative measure of trips not now generated but that would be produced at improved transit service levels.

THEORY DEVELOPMENT AND APPLICATIONS

The tests of the postulated theory conducted to date give every indication that the suggested relationships are indeed valid. The results are generally compatible with work along a similar vein by Thomas E. Lisco (8) and Peter R. Stopher (9).

Desirable further testing would include an application where observations could be made of trip interchanges with transit service superior to auto in door-to-door travel time. The Twin Cities and Washington data have not included examples of this condition. A successful test would provide significant corroboration of the postulated theory.

The possible universality of the normal distribution function variance in application to free choice of mode at specified decision levels should be further explored. The theory also needs to be evaluated in reference to choice of transit mode of access and to nonwork choice of prime mode. One nonwork application has been performed as part of the Alan M. Voorhees and Associates project in the Twin Cities (4).

Operational refinements should include improved techniques for determining the equivalence values used in constructing the disutility difference measures, perhaps using a regression technique such as described by Bevis (3) in connection with road-user cost function studies. The method for describing captivity probabilities also needs attention. It should be feasible to estimate captivity probabilities on the basis of trip-maker and metropolitan area characteristics. It should also be possible to predict deficiencies in the number of long transit trips by reference to the implications of the trip distribution model being used.

The postulated theory has immediate application for estimating modal split and route choice within the standard travel forecasting sequence. It should also be applicable to simultaneous multimode assignment upon development of an appropriate multipath assignment technique.

A further interesting possibility is that a comparable theoretical description of travel activity may be possible on a broader scale. If the modal choice and route choice facets of urban travel can be described by utilitarian behavior, disutility measures, and probability theory, perhaps trip generation and distribution can be handled similarly. This could be a fruitful area for detailed research and evaluation.

CONCLUSION

It is concluded that choice of travel mode can be treated as an economic response to the transportation system characteristics. As such, it can be predicted using quantitative disutility comparisons and probability mathematics.

The proposed structure of the descriptive equation set forth gives a logically possible explanation of modal choice and appears to produce good results even with the thin data inherent at the trip interchange level of analysis.

Although further testing and refinement would be highly desirable, the results and the credible framework of the postulated utilitarian theory would indicate that it is readily applicable at the present time to forecasting modal-split and submodal route choice. It is further thought that the basic concepts should be investigated for usefulness in describing other aspects of urban travel.

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