A Technique to Calibrate Choice Models

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The need for fast, efficient, and inexpensive techniques to calibrate transportation planning choice models led to the development of the methodology discussed in this paper. The calibrated models consist of stratified curves with a minimum of three variables to reflect conditions both at ends of a trip and on the competing transportation systems. The requirements of a stratified curve model are dealt with in considerable detail as well as their advantages to the model-builder and transportation planner. The calibration technique starts with an approximated set of curves and then attempts, by relaxation of one variable at a time, to fit an unknown set of curves in a rational manner. The first adjustment obtains the correct number of trip productions into the system as well as the number of attractions out of the system by traffic zones within certain characteristic strata. The curves regulating trip production and attraction are then adjusted to obtain the correct trip length over the model system variables. The final check on the trips is the origin-destination distribution by spider network assignments or other indirect techniques. This technique was used in Baltimore, Columbus, and Detroit during 1969, and the major results are presented in figures and tables. The model and technique are particularly suitable for computer application to large studies.

•PLANNING for future transportation facilities is dependent on good predictive models. Large sums of money and effort are expended in the calibration of these models. The calibration of the modal-choice model (transit-auto) is often critical to the configuration of the future mass transportation systems. Auto-occupancy models are also choice models that will divide highway trips between auto driver and passenger. This paper explains a simple technique to balance choice models that are constructed using stratified curves. The technique allows a systematic approach that will take a "guessed" set of curves and rapidly manipulate them toward the correct answer, thus saving much time and money. The method is sufficiently systematic to be computerized.

STRATIFIED CURVE MODELS

The stratifying of trips by purpose in the origin-destination (O-D) survey serves to group types of trips to minimize the variance of characteristics within the purpose categories and allows for a more meaningful analysis of travel patterns. (Many cities have successfully used stratified curves for modal-split models. Washington, Buffalo, and Seattle are a few examples.) Stratification may be carried one step further to independent variables to minimize variance within the group as well as to allow for nonlinear model relationships. The production and attraction variables are stratified in a meaningful manner to ensure a relative homogeneity within the strata.

Stratification has the additional advantage that the dependent variable and the trip-end and system variables may be visually examined and plotted to guard against inconsistent occurrences arising from insufficient data.

A choice model should be capable of explanation by the model-builder and understandable to the user. This usually precludes the use of a long complicated relationship

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that duplicates the present but lends no rationale for the possible changes of the variables over time. Complex relationships are at best a risk because the interdependence of many variables may magnify variations. The variables for a choice model should describe

1. The trip-maker or the person making the choice,
2. The trip-maker's origin and destination,
3. The relative merits of the modes of transportation available, and
4. The trip purpose.

The trip-maker's characteristics and his origin are often interdependent, particularly in small and medium-sized cities. The trip-maker and his origin may be adequately described by a single variable such as income or residential density or car ownership.

The attraction end variable has some limitations placed on it simply because of the criteria of splitting travel between transit (public transportation) and the automobile (auto driver and auto passenger) and because the transit is usually aimed at intensive land-use areas such as the central business district (CBD). This implies that the attraction end variable should be selected to reflect intensive land-use areas. Some measures of the intensity of land use are employment density and parking cost.

The most critical variable to be selected is the variable that reflects the competitive position of the modes available for the journey. The competition of the systems may be stated as the ratio or difference (1) of two modal characteristics such as time or money or as some relative measure of the two. The waiting time and the time actually riding can be combined in some equitable manner to measure the apparent efficiencies of both systems (2, 3, 4).

TECHNIQUE

The technique is essentially a process of curve-fitting in a number of dimensions by the successive relaxation of one variable (relaxed parameter) until the estimated relationships approximate the unknown true relationship. A typical modal-choice surface (transit-auto) for a production-attraction pair and for two system variables S1 and S2 is shown in Figure 1. The usual view of the surface is the curve set A-E through D-H on the plane defined by X-S1; the parameter in this instance is the system variable S2. Similar curves may be projected on the plane Y-S2 with S1 and a parameter. The equation for the surface shown in Figure 1 has the form

$$Y = (X_1, X_2, \ldots X_n, S_1, S_2, \ldots S_n)$$

where

Y = percentage of trips taking choice A (area under the surface),
X_i = zonal production or attraction variables, and
S_j = a characteristic of the system or systems that connect the production and attraction zones.

The method assumes certain surface conditions, which for modal-choice models are usually self-evident and may be summarized as follows:

1. No discontinuity will develop on the surface.
2. The dependent variable usually will either monotonically increase or monotonically decrease with respect to independent variables; i.e., very few surface combinations will have valleys (5).

CORRECTION FOR NUMBER OF PREDICTED TRIPS

The starting point for the technique is a set of estimation curves and is a first attempt at approximating the true curve set. The first set of stratified curves may be any assumed approximation to the true unknown curves. If the curves closely approximate the true curves, then the time spent calibrating and refining the model is reduced.

where

Ao_k^{S1} = the area under the observed distribution about the point k along S1, and

Ae_k^{S1} = the area under the estimated distribution about the point k along S1.

The trip-length correction will cause the curves to rotate in such a way as to approach the known true trip-length frequency. The correction may be more than a simple rotation if the curves intersect at more than one point. The corrected curve now has percentage choice A values of

$$\hat{Y}^{(3)}_{\substack{(ij)\\k}} = \left[\hat{Y}^{(1)}_{\substack{(ij)\\k}} + S_{ij}\right] \xi_k \tag{5}$$

This equation is applied to all the curve projections A-E through D-H shown in Figure 1. The entire surface series of curves may be corrected approximately by ξ_k values using a trip-length frequency for the entire population. A refinement may be introduced by stratifying S2 and doing trip-length corrections for each strata of S2. The corrections would be calculated in a similar manner and would be applied to each curve that forms part of the curve set. If detailed trip lengths for stratified S2 values are not available, then the corrections in the S2 dimension would be undertaken in a manner similar to the correction of S1. The corrected percent choice A would be of the form

$$\hat{Y}^{(4)}_{\substack{(ij)\\k,l}} = C_l\, \hat{Y}^{(3)}_{\substack{(ij)\\k}} = C_l\, E_k \left(\hat{Y}^{(1)}_{\substack{(ij)\\k}} + S_{ij}\right) \tag{6}$$

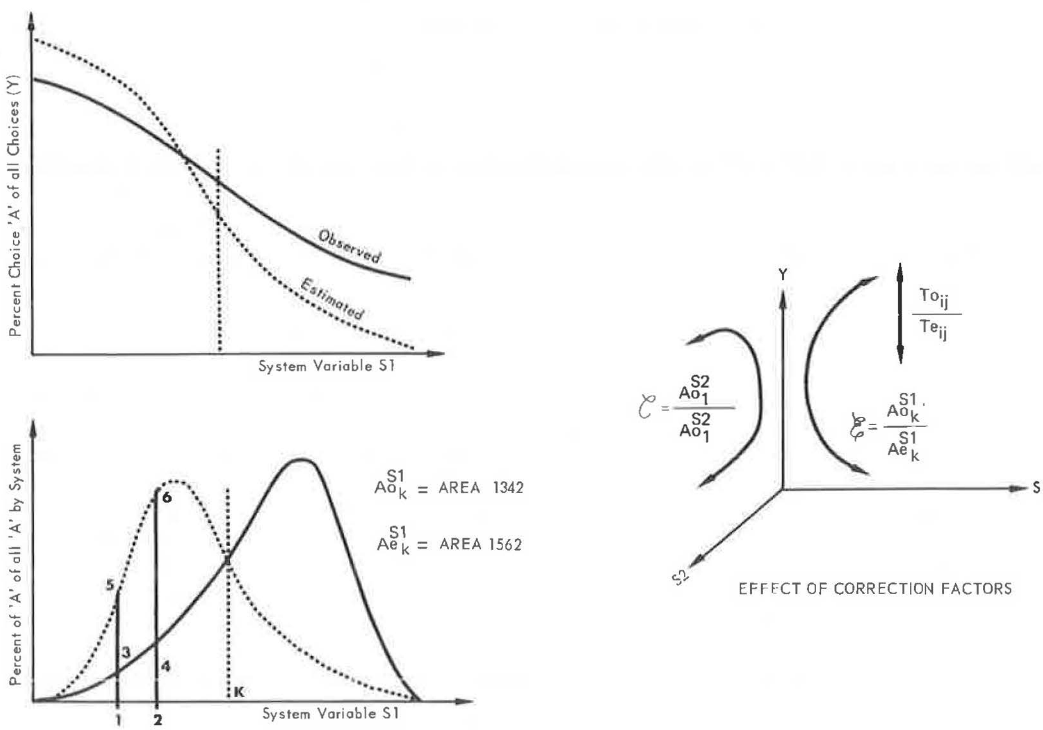

Figure 2. General model calibration procedure.

where

C_l = the trip-length correction factor in region of the point 1 along axis S2, and
k = the point used for the S1 trip-length correction.

This then leads to the most general case of an n-dimensional relationship for the percent choice A.

$$Y(X_1, X_2, \ldots X_n)_{k_1, k_2, \ldots k_n} = F_{1\,k_1} \times F_{2\,k_2} \times \ldots \times F_{n\,k_n} \left[Y(1)_{k_1}(X_1, X_2, \ldots X_n) + S_{X_1, X_2 \cdots X_n}\right] \quad (7)$$

Stratified curves lose most of their meaning beyond four independent variables. The method is such that if all the criteria are not satisfied, then new refined curves are used to estimate trips and the new trips and trip lengths are checked for acceptability. Because the trips are not evenly distributed over the system variables or over the variations of the data entering the model, the technique is an approximation that will approach the correct answer. The rate at which the model will calibrate is proportional to the number of independent variables used to explain (a) the choice model, (b) the regularity of the surface, and (c) the closeness of the first curve set to the true answer.

If it is possible to investigate each surface in all dimensions by parts, then a calibrated model may be obtained in a few iterations.

The technique just described has been applied at varying degrees of sophistication to five cities; the results of three studies are presented in the following section.

MODEL RESULTS

Stratified curve models and the technique just outlined were used recently for modal-split and auto-occupancy models in three cities. The Baltimore models investigated each set of curves individually and then collectively. The remaining two cities, Columbus, Ohio, and Detroit, Michigan, employed only the known number of trips associated with each pair of O-D variables as well as the total trips distributed over the system variables. The modal-split models (except NHB) employed the following form:

Percent of transit (Columbus): Median family income, employment density, and equivalent time difference (1);

Percent of transit (Baltimore): Median family income, parking cost, and equivalent time difference;

Percent of transit (Detroit): Median family income, employment density, travel cost difference, and equivalent time difference; and

Percent of auto driver (Baltimore and Detroit): Median family income, parking cost, and total highway travel time.

TABLE 2

TRIP CORRECTION RATIOS AND OBSERVED TRIPS

Production Variable (median family income)	Attraction Variable (parking cost)				Total
	j = 1	j = 2	j = 3	j = 4	
i = 1	1.055 9,181	1.196 238	1.360 279	0.714 152	1.058
i = 2	1.005 175,135	1.038 2,858	1.162 5,765	1.095 3,500	1.011
i = 3	1.034 1,040,691	1.985 24,990	0.966 32,012	1.000 22,552	1.032
i = 4	1.021 250,884	1.038 6,354	0.991 12,920	1.015 8,800	1.021
Total	1.029	1.073	0.985	1.011	1.028

Note: First row of figures is observed/estimated trips; second row is observed auto-driver work trips.

TABLE 3

MODEL CALIBRATION RESULTS FOR THREE CITIES

Purpose	Transit			Auto Driver	
	Baltimore	Columbus	Detroit	Baltimore	Detroit
Trips:					
Work	99,535	36,593	176,421	255,078	1,648,684
	100,050	36,389	176,533	252,037	1,613,553
Non-HB	3,643	17,409	32,901	37,167	1,315,590
	4,163	17,478	33,880	36,778	1,282,797
Other	13,516	16,164	116,346	87,481	2,587,463
	13,338	16,705	122,251	87,717	2,610,144
School	30,904		125,734	4,073	110,882
	30,905		126,605	5,695	112,474
Misc.		20,201			5,662,609
		20,763			5,618,968
Total	147,598	90,367	450,426	384,699	
	148,457	91,334	459,169	382,227	
CBD attractions:					
Work	32,253	19,747	48,826	24,768	67,735
	31,938	19,951	47,234	25,915	66,673
Non-HB	938	7,575	6,547	1,705	173,961
	919	7,400	7,424	1,760	160,851
Other	4,719	4,041	44,411	3,321	44,884
	4,392	4,132	41,570	3,652	45,028
School	937		1,362	187	3,992
	949		3,019	163	3,880
Misc.	38,847	11,146	101,136	29,981	290,572
	38,198	10,671	99,248	31,490	276,432
Total		42,509			
		42,154			
R^2 interchange:					
Work	0.42	0.66	NA	0.72	NA
Non-HB	0.13	0.38	NA	0.25	NA
Other	0.21	0.15	NA	0.64	NA
School	0.65		NA	0.10	NA
Misc.		0.63			

Note: Percent samples were: Baltimore, 5 percent; Columbus, 25 percent; Detroit, 4 percent. The first row of figures is the observed trips; the second, the estimated trips. R^2 = (explained variation)^2/(total variation)^2.

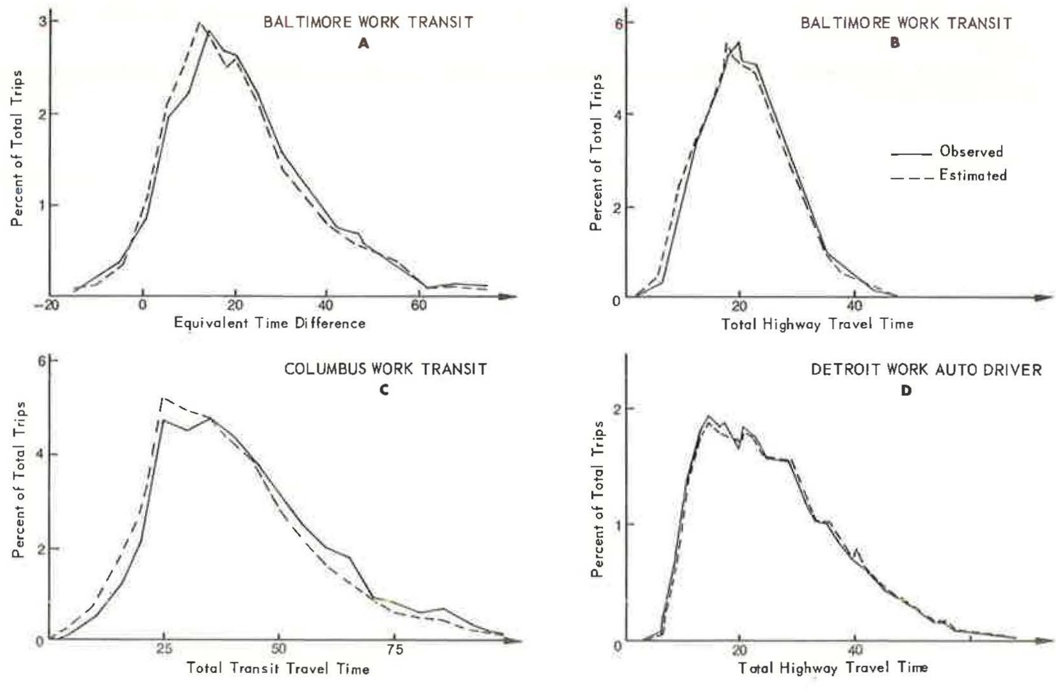

Figure 3. Trip-length-frequency comparison.

The equations all have a production variable, an attraction variable, and at least one continuous system variable. All variables were stratified except equivalent time difference and total highway travel time, which were continuous variables.

A few of the results obtained from these curves are given in Tables 2 and 3 and shown graphically in Figure 3. The results indicate the refinement that may be obtained in a model even when only a few variables are selected. The predictive accuracy of the Detroit work-auto-driver model by strata is given in Table 2. The trips are based on a 3 percent sample; therefore, the number of samples range from a low of five to a high of over 31,000. The precision is in line with the number of samples; the stratum with the largest number of samples varies from the observed by only 3.4 percent. The future use of the model has reliable curves to predict the total trips, provided the range of the variables is not exceeded.

This stratum accuracy check (a trip correction factor) established the number of trips between areas with certain characteristics and, therefore, gives a certain assurance that productions and attractions at the zonal level are approximately correct.

The next factor to investigate is the trip-length distribution along the system variable. The Baltimore work-transit trips provide an excellent example of a quick and inexpensive check on trip distribution.

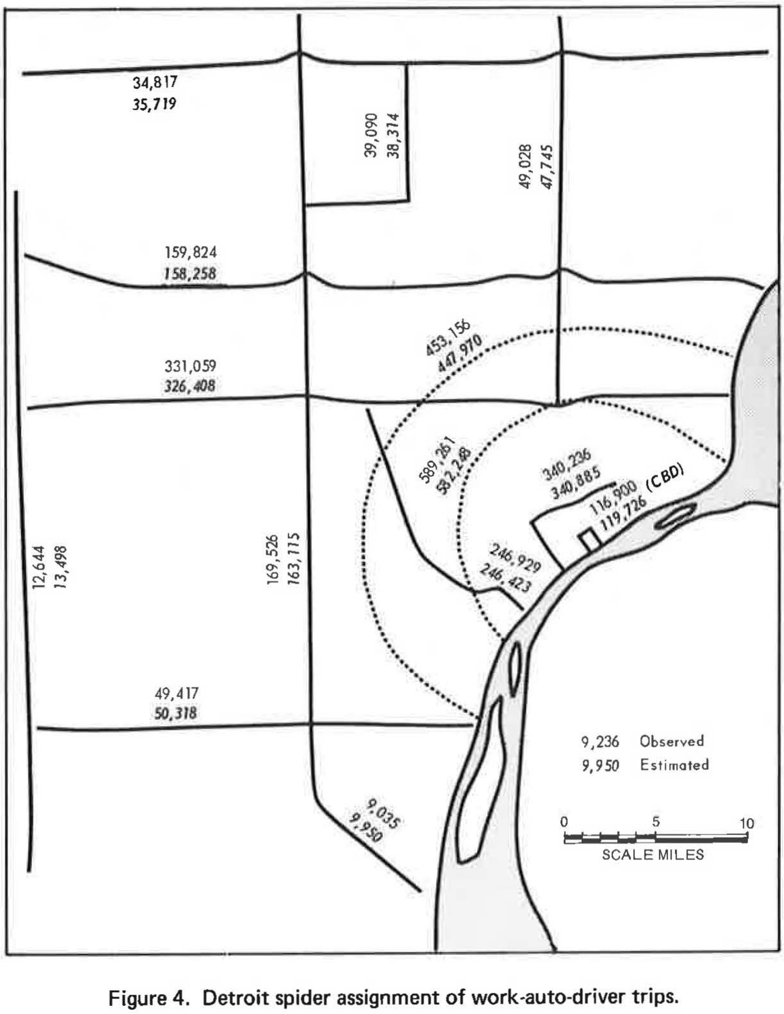

Figure 4. Detroit spider assignment of work-auto-driver trips.

The predicted trips were distributed over the model system variable; the result of this is shown in Figure 3A. A simple check on the correct trip O-D pair is a trip-length distribution over a variable that is not network dependent. The highway network (total highway travel time) was selected because it was less network dependent than transit network and because a spider network or centroid-to-centroid distance matrix was not available. The estimated Baltimore transit trips fit the distributions well for both variables, and the averages have approximately the same precision. The estimated Columbus transit trips distributed over the total transit travel time (Fig. 3C) coincide quite well with the observed distribution. The work-auto-driver trips distributed over the total highway travel time (Fig. 3D) gave an identical fit of the observed and estimated trips. The total hours of auto travel for work agreed within 0.5 percent. The trip-length distributions of Figures 3A and 3D show that a rotational correction is not required in either case; the transit distribution in Figure 3C indicates the transit trips are well distributed over the transit travel time. The distribution shown in Figure 3B is independent of the transit system and may approach a network-independent variable; this indicates the accuracy of the O-D pair selection.

The final check on the choice models is the correct O-D pair selection for each trip. This check may take many forms depending on what facilities and data are available. The first check may be distribution of trips over a network-independent variable such as centroid-to-centroid distance. A second test may be the interchange R^2 values as given in Table 3. The explained agreement ranges from a low of about 10 percent to a high in excess of 65 percent for both transit and auto-driver models. The Baltimore auto-driver model has a very high degree of accuracy in predicting interchange movements, in excess of 70 percent. A final method is the assignment of trips to a spider network as shown in Figure 4 for the Detroit auto-driver model. The assignment for individual links in the CBD gave answers that were equally precise.

CONCLUSION

Stratified curves used for the construction and refinement of choice models have a great many advantages for the transportation planner. The model-builder may inspect each curve or combination of curves for rational behavior. There is now available a practical, fast, and inexpensive method to calibrate and refine choice models. The technique is applicable to interchange models that will allow analysis of interchange movements if necessary. The technique has been used successfully in five cities for modal-split and auto-occupancy models.

REFERENCES

1. Quarmby, D. A. Choice of Travel Mode for the Journey to Work: Some Findings. Journal of Transport Economics and Policy, Vol. 1, No. 3, Sept. 1967.
2. Lisco, T. E. The Value of Commuter's Travel Time: A Study in Urban Transportation. Univ. of Chicago, Dept. of Economics, unpublished PhD dissertation, June 1967. (An abridgment of this paper appears in Highway Research Record 245, 1969, p. 16.)
3. Pratt, R. H., and Deen, T. B. Estimation of Sub-Modal Split Within the Transit Mode. Highway Research Record 205, 1967, pp. 20-30.
4. Hill, D. M., and Von Cube, H. G. Development of a Model for Forecasting Travel Mode Choice in Urban Areas. Highway Research Record 38, 1963, pp. 78-96.
5. Spielberg, F. Auto Occupancy Projection Using a Modal Split Model. Unpublished.