Dynamic Behavior of Embedded Foundation-Soil Systems

YONG S. CHAE, Department of Civil Engineering, Rutgers,
The State University

Development of theoretical solutions to the problem of dynamic interaction between foundation and underlying soil is discussed with a view to idealizations and uncertainties associated with the theories, with a particular emphasis on the effect of embedment on the interaction. An experimental investigation to study the dynamic behavior of embedded foundation-soil systems is described. Based on small-scale model tests on footings of various sizes and shapes, evaluation is made of the effect of embedment depth, mass ratio, and foundation shape on the dynamic response of the system. It is found that the amplitude of motion of the embedded foundation is greatly reduced by the additional shearing resistance along sides of the foundation and a shift in the contact pressure distribution. Embedment is found, however, to have no appreciable effect on the resonant frequency of the system. The system with a higher mass ratio produces a greater amplitude for a given depth of embedment, but the effect of mass ratio becomes smaller with increasing embedment depth. The effects of embedment depth, mass ratio, and foundation size are expressed in terms of two newly defined dimensionless parameters, the amplitude reduction coefficient and the embedment factor, by means of which the dynamic response of embedded foundations can be adequately determined.

•IN RECENT YEARS, there has been an increasing demand for knowledge of the dynamic behavior of foundation-soil systems that can be used in foundation designs for facilities that incorporate dynamic loading. These facilities range from industrial machine foundations and radar towers to highway and airport construction.

A properly designed highway or airport pavement must first of all meet the general requirements for all foundations. The loads imposed by traffic have to be transferred to soil layers capable of supporting them without failing in shear (bearing capacity), and the deformations of the soil layers should be such that they do not suffer excessive permanent settlement. Therefore, in the design and analysis of pavement-soil systems the most critical quantities considered are the bearing capacity and deformation characteristics of the underlying soil, which, in turn, are dependent on the material constants of the soil (the modulus of elasticity, shear modulus, Poisson's ratio, coefficient of subgrade reaction, etc.). The success with which the road or runway is built thus depends on the accuracy with which the material properties of the soil are determined.

At present roads and runways are largely designed on the basis of knowledge of the performance of other roads under similar traffic and subgrade conditions and by testing soil under static loading, such as unconfined compression, plate loading, density, CBR, and others. The use of static loading, however, does not represent the true nature of actual loading resulting from traffic, which is of a dynamic nature. Because the applied load is alternating and repetitive in nature, the material properties under such a load must be investigated in terms of dynamic response and behavior of pavement-soil systems. Such a dynamic testing, in a reversed process, will also provide a rapid, nondestructive testing technique for assessing the performance of a road.

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Past research indicates that theoretical values of the stresses and deformations generated by moving vehicles in multilayered elastic systems depend on the relative values of the dynamic elasticity of the layers forming the road and their thicknesses. It is, therefore, essential to obtain information concerning the dynamic material properties of the common road-making materials under repeated loading of the form applied by traffic. The ultimate objective of the measurement of dynamic properties would be then to develop a method of pavement design based on the dynamic behavior of pavement-soil systems.

DEVELOPMENT OF DYNAMIC ANALYSIS

An analytical solution that has been gaining wide acceptance among researchers and engineers in recent years is the theory that treats the system as an oscillator resting on a semi-infinite, homogeneous, isotropic, elastic body (elastic half-space). The theory was first developed by Reissner (9), who presented a solution for the case of vertical excitation of an oscillator on a circular base, assuming a uniform contact pressure between an oscillator and the elastic body. Reissner's theory has since been extended by Sung (12), who treated the problems for various types of contact pressure distribution, and by Arnold, Bycroft, and Warburton (1), who considered cases for different modes of vibration. The elastic half-space theory was used by Richart and Whitman (11) to compare with a very extensive series of field tests conducted at the Army Engineer Waterways Experiment Station (5). The comparison showed a reasonably good agreement between the theory and the field experiment.

More recently, it has been demonstrated by Lysmer and Richart (8) that the theory for footings resting on an elastic half-space leads to solutions whereby the dynamic behavior of a footing resting on an actual foundation can be represented by a "lumped parameter" system consisting simply of a mass-spring-dashpot. In such "lumped" systems, the mass represents all of the inertia present in the actual system, while the springs and dashpots respectively represent all of the deformability and damping present in the actual system. The key step is evaluation of parameters of the equivalent lumped system. Once this has been done, the response of the actual system can be readily obtained by the use of available mathematical solutions of the lumped system.

DETERMINATION OF DYNAMIC PROPERTIES OF SOILS

Whether the theoretical solutions are used directly (elastic half-space) or indirectly in the form of the lumped parameter system, however, the ability of this theory to predict dynamic response of real foundations depends largely on the accuracy with which the material properties of soil are determined. There are a number of experimental methods to determine the dynamic properties. These methods may be divided into two general categories: (a) those in which tests are performed on a small sample of soil usually in triaxial compression, and (b) those in which the testing on soils is considered to be in situ. The "resonant column" method and the "amplitude ratio" method are performed on soil specimens encompassed in a triaxial chamber. The "wave propagation (seismic)" method and the "elastic half-space" method, in which a small- or large-scale vibrator is set on the soil mass, are performed on soils in situ or in simulated in-situ conditions. Detailed discussion of various methods, test results, and comparisons were made by Chae (3).

The elastic half-space method has been widely used by many researchers in the past as a means of determining the material constants of soil. The accuracy that the method yields, however, is dependent on how closely the actual foundation conditions agree with the idealized condition assumed in the theory, as the effect of certain parameters are either ignored or simply idealized in the theory. The analysis and determination of dynamic response require, therefore, consideration of the following additional parameters: (a) the magnitude of dynamic force applied, (b) the type of dynamic contact pressure distribution between the foundation and soil and its variation with frequency, (c) the shape of the foundation, and (d) the embedded depth of foundation.

EVALUATION OF PARAMETERS NOT CONSIDERED IN THEORY

In the past some progress has been made in evaluating the effect of some of these parameters. The following information briefly summarizes the work done to date.

Magnitude of Dynamic Force Applied

Determination of the dynamic load requires careful consideration. The magnitude, direction, and point of application of various forces applied determine the mode of vibration. The effects of this parameter were studied by Fry (5), Jones (7), and Chae (2). They have shown that the effect of increasing the input force is to decrease the resonant frequency, and the decrease is greater with the systems having higher mass ratios. This variation of resonant frequency with the magnitude of input force is not indicated in the theory. This effect may be explained by the fact that a change in the magnitude of input force causes a change in the pressure distribution under the footings, and the resonant frequency is in turn dependent on the pressure distribution. This point was elaborated by Richart (10) in connection with his concept of an "effective radius" to be used as a tool for the transformation of pressure distribution caused by the variation in input force.

Type of Dynamic Contact Pressure Distribution

The elastic half-space theory assumes the pressure distribution at the base of the footing to be constant regardless of the exciting frequency. The investigation conducted by Chae (2) has shown, however, that it varies with the frequency. The test results on an Ottawa sand show that under a constant total soil reaction the dynamic pressure appears to shift toward the edge of the footing with increasing frequency, changing the shape of distribution from a parabolic type to that corresponding to a rigid base. Even with a rigid base resting on an ideal elastic medium the effect of changing the frequency is to change the intensity of pressure across the base of the footing. The variation of dynamic pressure resulting from a change in frequency is a consequence of the change in amplitude and phase relations between the reaction pressures on each ring, caused by a change in the wave velocity traveling in the soil. Thus, the dynamic soil pressure against the footing is a function of the velocity of the footing moving into the soil and the velocity of the elastic waves in the soil that dissipate this input energy.

Shape of Foundation

In theory, the effect of the footing shape is ignored. The theory is derived on the basis of circular footings. In the case of the footing shape being other than a circle, an equivalent radius based on an equal area is used. This means that a footing of any shape, whether it be circle or rectangle, would yield the same response as long as the area of the footing remains the same. An investigation has been conducted at Rutgers (4) using small-scale models for the vertical mode of vibration. There were six circular, six square, and ten rectangular model footings tested in a soil tank filled with a New Jersey beach sand.

The test results and analysis thereof have revealed that the resonant frequency increases and the maximum amplitude of displacement decreases with a decrease in mass ratio for all shapes of footings, as predicted by the elastic half-space theory. However, the resonant frequency of circular footings is less sensitive to a change in the mass ratio than the square or rectangular footings, and the displacement of rectangular footings is less sensitive to a change in the mass ratio than the circular footings. It has also been found that the use of an equivalent radius based on equal area in computing the resonant frequency of square and rectangular footings appears valid because the predicted results compared favorably with the experimental results. The use of an equivalent radius based on equal area appears valid for the determination of mass ratio in computing maximum amplitude of square footings, but it is not valid for rectangular footings. A suggestion is made to use an equivalent radius, based on a perimeter ratio, in computing the dynamic response of rectangular foundations.

EFFECT OF FOOTING EMBEDMENT ON DYNAMIC BEHAVIOR

Whereas the theory treats the footing to be resting on the surface of elastic half-space, actual foundations usually are partially embedded. A mathematical model of such an embedded footing is shown in Figure 1. In this figure, m_o, r_o, and D are the mass, radius, and diameter of the footing respectively; D_f is the depth of embedment; G, ρ, and μ are the shear modulus, mass density, and Poisson's ratio of the soil; and Q_1 is the amplitude of input force. For the system of a footing resting on the surface, the resonant frequency f_o and maximum amplitude $A_{o,max.}$ can be expressed as follows in the simplified elastic half-space theory:

$$f_o = \frac{(G/\rho)^{1/2}}{2\pi r_o} \cdot \frac{(B - 0.36)^{1/2}}{B} \tag{1}$$

$$A_{o,\,max.} = \frac{Q_1}{K} \quad \frac{B}{0.85(B - 0.18)^{1/2}} \tag{2}$$

in which B = mass ratio, defined as $m_o \cdot (1 - \mu)/4\rho r_o^3 = b(1 - \mu)/4$, and K = spring constant, defined as $4Gr_o/1 - \mu$. In the case of the lumped parameter system, the resonant frequency and maximum amplitude can be readily obtained once the parameter mass, spring constant, and damping are evaluated. From the interrelationship among the material constants G or K and the dynamic response f_o or $A_{o,max.}$, the effect of each parameter on the other can be determined for a given system having a mass ratio B.

There have been some attempts to investigate analytically, mostly based on the finite element technique, the effect of embedment on dynamic behavior under various modes of vibration. However, no major results have been published as of the time of this writing. The results of experimental work by the Corps of Engineers (5) have shown that the effect of partial embedment has been, in general, to increase the resonant frequency and to decrease the amplitude of motion at resonance. This indicates changes in the dynamic properties of soil, whether they be expressed in terms of the equivalent spring constant and damping or in terms of the elastic moduli.

The reduction in motion of an embedded rigid foundation is due primarily to the additional shear resistance along sides of the footing and to a shift in the contact pressure distribution caused by the surcharge around the footing. Thus, the effect of embedment depth (or the amount of surcharge), weight, and size of the footing.

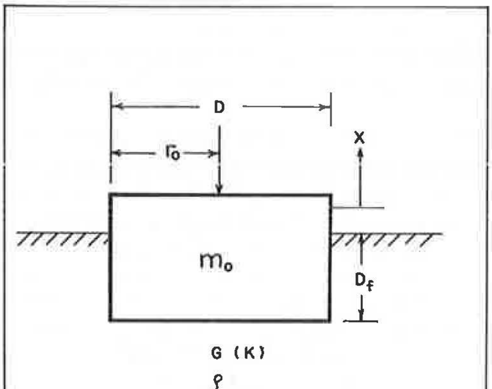

Figure 1. Mathematical model of embedded footing.

EXPERIMENTAL INVESTIGATIONS

Experimental investigations were based on the behavior of model footings embedded in a sand bed and subjected to steady-state vibration. There were five footings of various shapes, sizes, and weights yielding eight different test systems. The test setup was the same as that reported by Chae (4) in connection with previous research. A brief description follows.

Sand Bin and Loading Frame

A sand bin was constructed of 8-in. three-cell cinder blocks. It had inside dimensions of 4.8 by 4.8 ft and was 4.0 ft in height. The holes in the blocks were filled with mortar, and the wall was reinforced to ensure adequate resistance to the lateral earth pressure and the lateral component of force from the loading frame.

The sand used was beach sand from Seaside Heights, New Jersey, and 93 percent of the sand was retained between the No. 20 and No. 60 sieves. The sand was compacted in six layers, each approximately 8 in. thick, using a vibratory compactor. A density test was performed during and after the compaction and the density was computed to be 110.2 pcf and to have a void ratio of 0.51.

The loading frame was designed so that its resonant frequency would not be within the predicted testing range and to make the structure as rigid as possible. The loading frame, shown in Figure 2, consisted of vertical columns and a cross beam to which the oscillator was attached. The height of the oscillator in relation to the footing was adjustable as necessary.

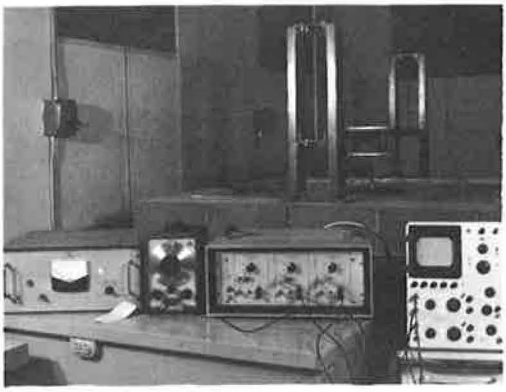

Figure 2. Overall experimental setup.

Footings

The footings were made of 1-in. thick steel plates cut to various sizes and shapes. Three circular footings, a square footing, and a rectangular footing were used. Footings of circular shape were dimensioned to provide area to perimeter ratios of 2, 3, and 4 for the purpose of comparison in analyzing the results. The square and rectangular footings were designed with area to perimeter ratios of 3 and 4 respectively to aid in the study of the effect of shape of a footing on the dynamic response. The footing properties are given in Table 1. To obtain a more diversified scope of testing, provisions were made to use the various footings as extra weight on each other by bolting one or more footings together. The mass ratio could then be varied for a given shape of footing by adding as many weights as desired.

Embedment of the footing was achieved by placing sand surcharge around the thick aluminum tube that was rigidly bolted to the perimeter of the footing. The embedment depth was varied by the depth of surcharge around the footing. The tube was found to be very rigid laterally so that the shear was fully mobilized along the perimeter.

Overall Test Setup and Procedure

Figure 2 shows the overall setup in the laboratory. Figure 3 shows a schematic diagram of the instrumentation sequence in the overall test setup. The vibrator used was a constant-force electromagnetic exciter. The exciter received its alternating

TABLE 1

PROPERTIES OF MODEL FOOTINGS

Shape of Footing	Dimension (in.)	Weight (lb)	Area (sq in.)	Area to Perimeter Ratio	Mass Ratio, b	Equivalent Mass Ratio, B
Circular	D = 8	17.3	50.3	2	4.3	0.72
		35.9			8.8	1.47
	D = 12	37.1	113.1	3	2.7	0.45
		88.4			6.4	1.07
	D = 16	62.4	201.8	4	1.9	0.32
		139.1			4.3	0.72
Square	12 by 12	45.4	144.0	3	2.2	0.37
Rectangular	12 by 24	90.5	288.0	4	1.5	0.25

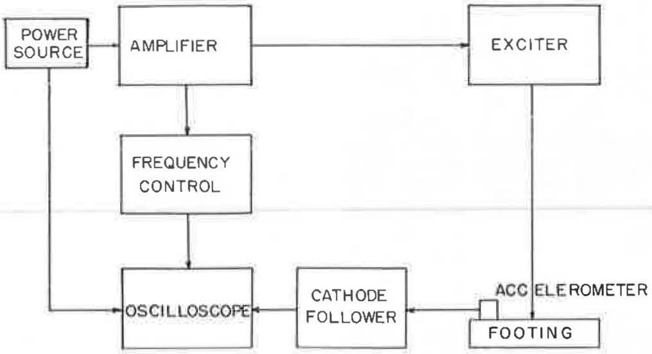

Figure 3. Schematic diagram showing instrumentation sequence.

current from an amplifier that was connected to an outside power source. The frequency was varied by a wide-range frequency meter. The accelerometer was used to measure the footing acceleration and consequently the footing displacement. The acceleration range of the accelerometer was 0.001 to 1,000 g at a frequency range of 0.4 to 2,000 cps. The accelerometer produced a signal with a high impedence and consequently it was connected to a cathode follower. The Tektronix oscilloscope provided linear dual-beam displays with a wide range of sweep rates combined with a high input sensitivity.

After the equipment was connected as shown in Figures 2 and 3, the input current was set at a desired level and kept constant so that the amplitude of input force remained constant. The frequency was then varied in much smaller increments. The peak-to-peak value of footing acceleration and phase angle between the input force and acceleration were recorded at each frequency. The displacement was then computed from the acceleration and frequency measured.

ANALYSIS OF TEST RESULTS

The experimental results for the resonant frequency are given in Table 2, and the results for the maximum amplitude are plotted in Figures 4, 5, and 6 for the circular footings, in Figure 7 for the square footing, and in Figure 8 for the rectangular footing. In these figures variation of the maximum amplitude is shown as a function of embedment depth, which was varied up to 10 in.

It is observed in these figures that embedment has, as expected, a significant effect on the maximum amplitude of motion. This can readily be explained in terms of change in the material constant G or stiffening spring constant K in the lumped-parameter system. These figures also show the general trends, as far as the maximum amplitude is

TABLE 2

EXPERIMENTAL RESULTS FOR RESONANT FREQUENCY

Footing	Mass Ratio	Depth of Embedment (in.)					
		0	2	4	6	8	10
Circular							
D = 8 in.	4.3	131	129	128	121	121	—
	8.8	132	131	125	125	121	—
D = 12 in.	2.7	135	136	134	129	128	—
	6.4	127	125	128	125	122	—
D = 16 in.	1.9	134	129	126	123	124	123
	4.3	120	116	117	115	116	114
Square	2.2	129	132	129	132	129	122
Rectangular	1.5	130	131	121	130	131	130

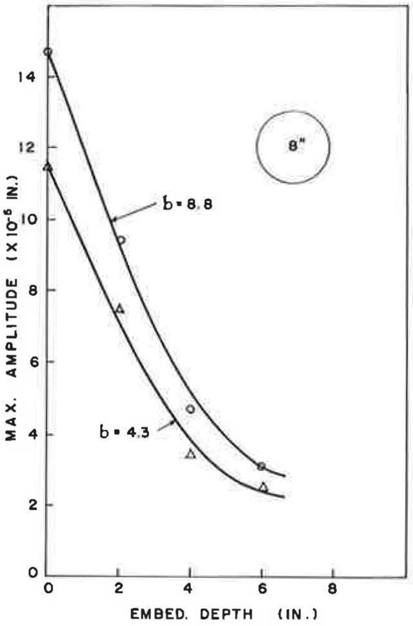

Figure 4. Maximum displacement amplitude versus embedment depth for 8-in. diameter footing.

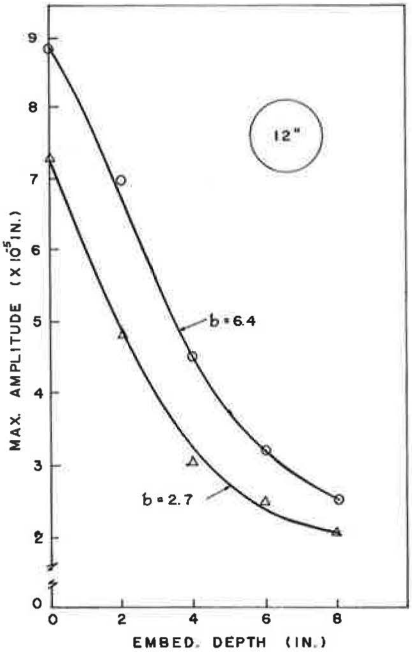

Figure 5. Maximum displacement amplitude versus embedment depth for 12-in. diameter footing.

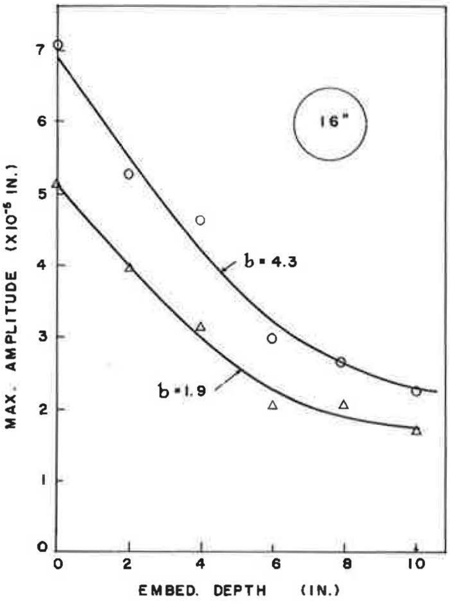

Figure 6. Maximum displacement amplitude versus embedment depth for 16-in. diameter footing.

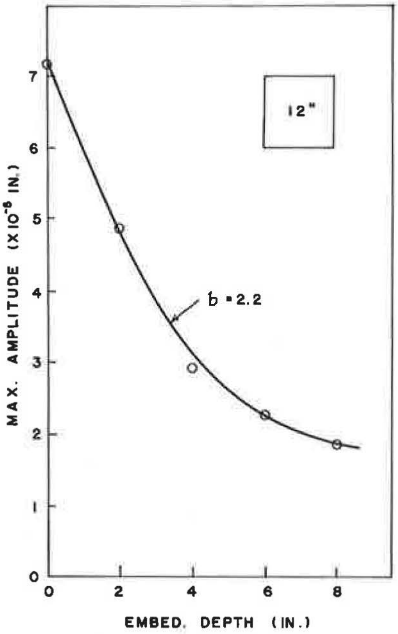

Figure 7. Maximum displacement amplitude versus embedment depth for square footing.

concerned, as one would expect from the elastic half-space theory for a constant force oscillator. The maximum amplitude is greater with higher mass ratio for a given system having the same input force, and this is true with the embedded footings. The effect of mass ratio seems, however, to get smaller as the embedment depth is increased. Another point of interest is that the rate of decrease becomes smaller with increasing embedment depth and it appears that beyond a certain depth of embedment further reduction in amplitude may not be obtained.

It is noted in Table 2 that embedment has no appreciable effect on the resonant frequency. Theoretically, an increase in the value of spring constant (stiffening) should increase the resonant frequency as long as the mass remains the same. With embedded footings, however, the mass of the system appears to increase because of the surcharge neutralizing the stiffening effect of the spring.

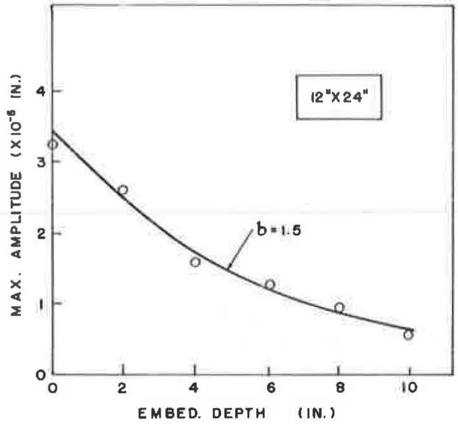

Figure 8. Maximum displacement amplitude versus embedment depth for rectangular footing.

A comparison of Figure 7 with Figure 5, and Figure 8 with Figure 6 shows that the effects of embedment for square and rectangular footings are similar to those for circular footings having the same mass ratios. The mass ratio of a noncircular footing can be computed in terms of an equivalent radius based either on equal area or on a perimeter ratio as defined by Chae (4).

An interesting feature of the experimental results is exhibited when the amount of amplitude reduction at each embedment depth is compared to the maximum amplitude of motion of the footing resting on the surface of soil. To show this, two new dimensionless quantities, designated as the amplitude reduction coefficient R_f and the embedment factor N, are introduced and are defined as

$$R_f = \frac{A_{max.}}{A_{o,\, max.}} \qquad (3)$$

$$N = \frac{D_f}{D} \qquad (4)$$

Figures 9, 10, and 11 show the amplitude reduction coefficient plotted against the embedment factor for the three circular footings. These figures show patterns generally similar to those for the amplitude-embedment depth curves. The reduction coefficient decreases as the embedment factor increases, and the rate of decrease becomes smaller with an increasing embedment factor. It is of interest to note, however, that the mass ratio does not significantly change the reduction coefficient for a given embedment factor.

To elaborate the fact just pointed out, the test results shown in Figures 9, 10, and 11 are combined and plotted in Figure 12. It is evident in this figure that the relationship between the reduction coefficient and the embedment factor can be derived independently of the mass ratio. This implies that, for any footing-soil systems, the reduction of amplitude due to embedment can be determined once the amplitude of the footing resting on the surface $A_{o,\, max.}$ and the embedment depth D are known. In practical situations the embedment factor ranges up to 0.5. For this range the reduction in amplitude is up to about 60 percent, and the reduction coefficient may be

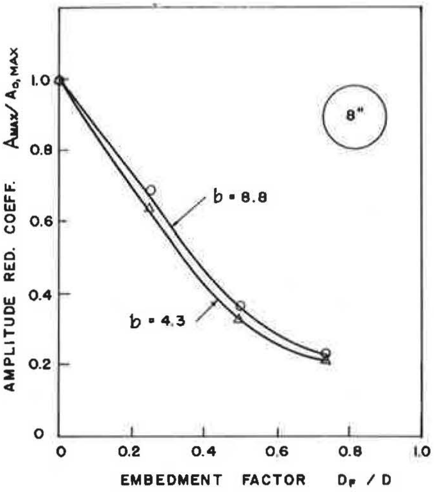

Figure 9. Amplitude reduction coefficient versus embedment factor for 8-in. diameter footing.

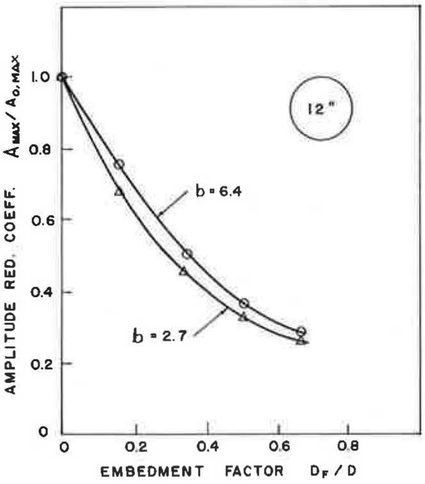

Figure 10. Amplitude reduction coefficient versus embedment factor for 12-in. diameter footing.

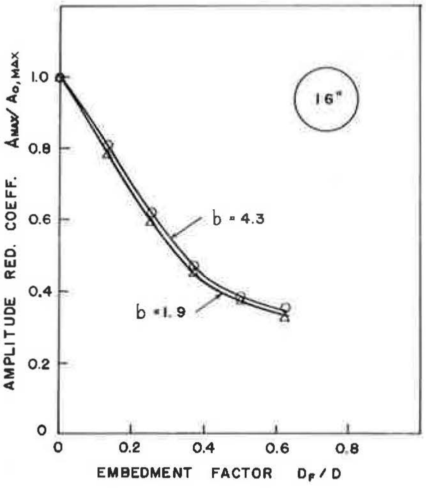

Figure 11. Amplitude reduction coefficient versus embedment factor for 16-in. diameter footing.

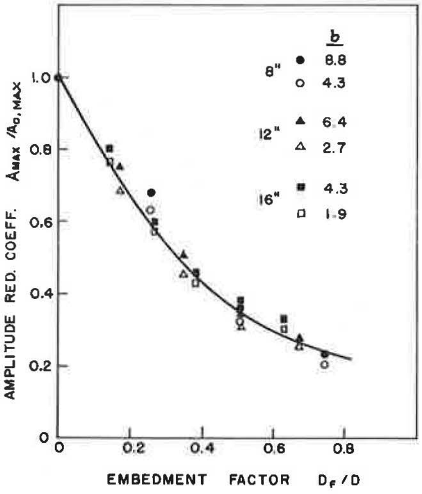

Figure 12. Amplitude reduction coefficient versus embedment factor for all circular footings.

expressed, for a quick approximate calculation, as a function of the embedment factor as

$$R_f = 1.0 - 1.25\left(\frac{D_f}{D}\right) \tag{5}$$

Then the maximum amplitude of an embedded footing can be determined from the following relationship:

$$A_{max.} = R_f \cdot A_{o,\,max.} = \left[1.0 - 1.25 \left(\frac{D_f}{D} \right) \right] A_{o,\,max.} \tag{6}$$

CONCLUSIONS

The essential conclusions to be derived from the experimental investigation may be summarized as follows:

1. The maximum amplitude of motion of an embedded footing is greatly reduced by the additional shearing resistance along sides of the footing. The reduction gets generally greater with increasing embedment depth. The rate of decrease becomes smaller, however, as the depth of embedment is increased. Embedment does not significantly change the resonant frequency resulting from a greater mass associated with the footing vibration.
2. The maximum amplitude is greater for the system having a higher mass ratio. The effect of mass ratio becomes smaller, however, with increasing embedment depth.
3. The effect of embedment for square and rectangular footings is similar to that for circular footings having the same mass ratios.
4. The amplitude reduction coefficient, which is the ratio of amplitude of the footing embedded to that at the surface, is independent of the mass ratio when plotted as a function of the embedment factor, which is the ratio of embedment depth to the diameter of the footing.
5. The amplitude reduction coefficient may be empirically expressed in terms of embedment factor.

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