Time-Dependent Deformation of Noncomposite and Composite Prestressed Concrete Structures

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This paper reports the results of an investigation of the use of sand-lightweight concrete in prestressed laboratory beams and bridge girders. The study is divided into 3 parts: a materials study of the concrete behavior itself, a laboratory study of the behavior of both noncomposite (5 beams) and composite (4 beams) prestressed beams, and the field measurement of camber of prestressed girders (5 girders) used in the fabrication of a composite bridge in Iowa. In addition, systematic design procedures are presented and verified by the experimental results. The methods described for predicting material behavior and structural response are generalized to apply to prestressed concrete structures of different weight concretes. Continuous time functions are provided for all needed parameters so that the general solutions readily lend themselves to computer solutions. Approximate equations are also included. Design procedures are presented for calculation of strength and elastic properties, and creep and shrinkage of the sand-lightweight concrete of this project at any time, including ultimate values. An indication is given of the calculation of these properties for normal-weight, sand-lightweight, and all-lightweight concrete in general. Design procedures are also given for calculation of loss of prestress and camber at any time, including ultimate values, of noncomposite and composite prestressed structures. Results computed by these methods are shown to be in agreement with the control specimen data, the laboratory beam data, and the bridge girder data.

•ALTHOUGH the behavior of noncomposite and composite prestressed beams of normal-weight concrete has been studied extensively, with most of these referring to noncomposite beams only (1, 2, 3, 4, 5, 6, 7, 8), it appears that no such investigation has been made of composite prestressed members of lightweight concrete.

Sinno (9), in his study of lightweight noncomposite prestressed bridge girders, concluded that hyperbolic functions (used in modified form in this paper) can be used to predict loss of prestress and camber. Branson and Ozell (5) and Sinno (9) observed that camber tends to reach its ultimate value relatively early compared to creep and shrinkage, because of the offsetting effects of loss of prestress on the one hand and camber growth due to creep on the other.

Methods used in this study for predicting loss of prestress and camber are based in part on papers by Branson and Ozell (5), Branson (10), and ACI Committee 435 (11).

DESCRIPTION OF EXPERIMENTAL INVESTIGATION

Materials and Test Specimens

The details of the laboratory beams and bridge girders are shown in Figure 1 and are given in Appendix B. The laboratory beams were designed as follows:

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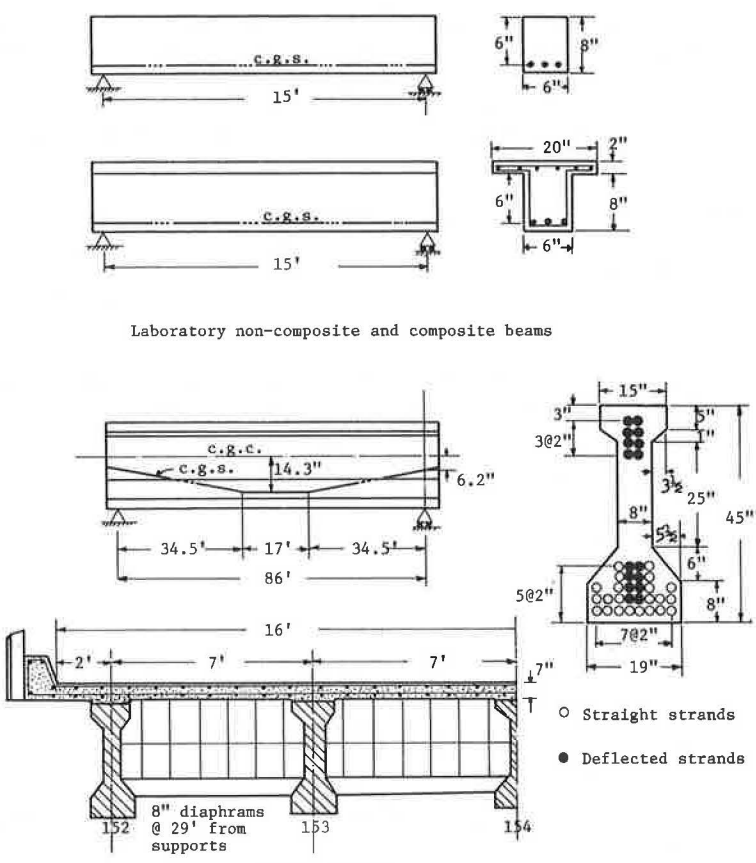
Group A—three noncomposite beams with different prestress moments.
Group B—three beams, two of which are composite beams. The slabs were cast at 4 weeks and 10 weeks after the prestressed beams were cast. The same prestress moment was used for the 3 beams.
Group C—same as Group B but with a different prestress moment.

The laboratory beams (moist-cured for 3 days and prestressed at age 7 days) and bridge girders (steam-cured until prestressed at age 2 or 3 days) are sand-lightweight aggregate concrete (100 percent sand substitution for fines along with lightweight coarse aggregate), while the slabs are normal-weight concrete. The composite bridge deck was cast 9 weeks after the bridge girders were cast.

The mix ingredients per cubic yard of sand-lightweight concrete were cement (Type 1), 705 lb; sand, 1,395 lb; idealite aggregate (60 percent of $^3/_4$ to $^5/_{16}$ in. and 40 percent of $^5/_{16}$ in. to No. 8), 822 lb; water, 35.0 gal; Darex at $^7/_8$ oz per sack, 6.5 oz; and WRDA (used instead of 21.0 oz of pozzolith for lab beams), 50 oz. Two shrinkage specimens and 3 creep specimens (6 by 12 in. cylinders placed under a sustained uniform stress of about 30 percent of the ultimate concrete strength) were cast for each sand-lightweight concrete.



Laboratory non-composite and composite beams

Bridge girders

Figure 1. Laboratory beams and bridge girders.

Instrumentation and Test Data

The principal instrumentation for the laboratory beams included load cells for each strand to measure the prestressing force, dial gages for camber, and a Whittemore strain gage for concrete strains. A level rod and precise level were used to obtain the camber measurements for the bridge girders.

The laboratory specimen experimental data consist of the following:

1. Concrete strength properties, elastic properties, creep and shrinkage data from control specimens, and steel properties;
2. Temperature and humidity data;
3. Steel relaxation data;
4. Initial and time-dependent concrete beam strains (these are used in determining experimental loss of prestress); and
5. Initial and time-dependent camber.

Camber data for the bridge girders were obtained in cooperation with Young's study (12). Material properties of the bridge girder concrete were measured in the laboratory. The concrete and steel properties, and other data, are given in Appendix B.

STRENGTH AND ELASTIC PROPERTIES

A study of concrete strength versus time in this project and elsewhere (13) indicates an appropriate general equation in the form of Eq. 1 for predicting compressive strength at any time.

$$(f_c')_t = \frac{t}{a + bt} (f_c')_{28d} \tag{1}$$

where a and b are constants, $(f_c')_{28d}$ = 28-day strength, and t is time.

The following equations were developed in this study and by Branson and Christiason (13) and were used by ACI Committee 209 (14) for normal-weight, sand-lightweight, and all-lightweight concrete (using both moist- and steam-cured concrete and Types 1 and 3 cement). Equations 2 and 4 refer to the concrete (Type 1 cement) of this project.

For moist-cured concrete, Type 1 cement,

$$(f_c')_t = \frac{t}{4.00 + 0.85t} (f_c')_{28d}; \text{ or } (f_c')_{7d} = 0.70(f_c')_{28d}, \ (f_c')_u = 1.18(f_c')_{28d} \tag{2}$$

For moist-cured concrete, Type 3 cement,

$$(f_c')_t = \frac{t}{2.30 + 0.92t} (f_c')_{28d}; \text{ or } (f_c')_{7d} = 0.80(f_c')_{28d}, \ (f_c')_u = 1.09(f_c')_{28d} \tag{3}$$

For steam-cured concrete, Type 1 cement,

$$(f_c')_t = \frac{t}{1.00 + 0.95t} (f_c')_{28d}; \text{ or } (f_c')_{2.0d} = 0.69(f_c')_{28d}, \ (f_c')_u = 1.05(f_c')_{28d} \tag{4}$$

For steam-cured concrete, Type 3 cement,

$$(f_c')_t = \frac{t}{0.70 + 0.98t} (f_c')_{28d}; \text{ or } (f_c')_{2.0d} = 0.75(f_c')_{28d}, \ (f_c')_u = 1.02(f_c')_{28d} \tag{5}$$

where t is age of concrete in days, and $(f_c')_u$ refers to an ultimate (in time) value. The results of Eqs. 2 and 4 agree with the experimental data of this project, as shown in Figure 2. As shown elsewhere (13, 14), Eqs. 2, 3, 4, and 5 refer to average values only (the references give ranges of variation).

The secant, initial tangent, and computed moduli of elasticity (using Eq. 6) for the laboratory beam and bridge girder concrete are given in Appendix B.

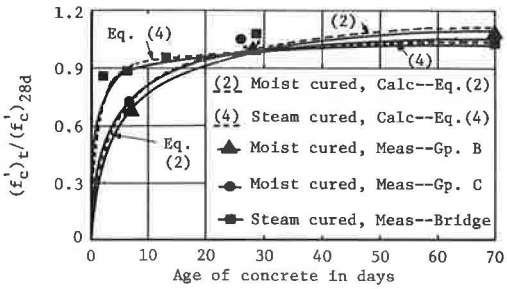


Figure 2. Measured and computed compressive strength versus time curves for the moist-cured laboratory beam concrete and steam-cured bridge girder concrete.

$$E_c = 33\, w^{1.5} \sqrt{f'_c},\ \text{psi}; \quad w \text{ in pcf and } f'_c \text{ in psi} \tag{6}$$

The computed values for the limited number of tests made were from 6 to 15 percent higher than the initial tangent values. However, the computed initial camber of the laboratory beams and bridge girders was in aggrement with the measured results (Table 2). Equation 6 (15) is considered satisfactory for normal-weight, sand-lightweight, and all-lightweight concrete.

CREEP AND SHRINKAGE

The principal variables that affect creep and shrinkage are outlined and discussed in Appendix C. The design approach presented here for predicting creep and shrinkage refers to "standard conditions" and correction factors for other than standard conditions.

Based largely on the data and information from several sources (16, 17, 19, 20, 21, 22, 23, 24, 25, 26) and from this project, the following design procedure, which was developed in this project and by Branson and Christiason (13) and used by ACI Committee 209 (14), is recommended for predicting a creep coefficient and unrestrained shrinkage at any time, including ultimate values. The general values suggested for C_u and $(\epsilon_{sh})_u$ should be used only in the absence of specific creep and shrinkage data for local aggregates and conditions. However, the time-ratio part on the right side, except for C_u and $(\epsilon_{sh})_u$, of Eqs. 7 through 9 have been found (13) to apply quite generally. Branson and Christiason (13) and ACI Committee 209 (14) show that these general values of C_u and $(\epsilon_{sh})_u$ refer to average values only and give ranges of variation.

1. Standard creep equation—4 in. or less slump, 40 percent ambient relative humidity, minimum thickness of member 6 in. or less, loading age 7 days for moist-cured and 1 to 3 days for steam-cured concrete.

$$C_t = \frac{t^{0.60}}{10 + t^{0.60}}\, C_u \tag{7}$$

For the laboratory beam sand-lightweight concrete (moist-cured) of this project, $C_u = 1.75$. The average relative humidity, H, was 40 percent.

For the bridge girder sand-lightweight concrete (steam-cured) of this project, $C_u = 2.15$ for H = 40 percent. H was 70 percent. From Eq. 12 for H = 70 percent, $C_u = 0.80(2.15) = 1.72$.

General value suggested for all weights of structural concrete (both moist- and steam-cured concrete, Types 1 and 3 cement), $C_u = 2.35$ for H = 40 percent. From Eq. 12 for H = 70 percent, $C_u = 0.80(2.35) = 1.88$.

2. Standard shrinkage equations—4 in. or less slump, 40 percent ambient relative humidity, minimum thickness of member 6 in. or less.

a. Shrinkage at any time after age 7 days for moist-cured concrete,

$$(\epsilon_{sh})_t = \frac{t}{35 + t}\, (\epsilon_{sh})_u \tag{8}$$

For the laboratory beam sand-lightweight concrete of this project, $(\epsilon_{sh})_u = 650 \times 10^{-6}$ in./in. The average relative humidity, H, was 40 percent.

General value suggested for all weights of structural concrete (both Types 1 and 3 cement), $(\epsilon_{sh})_u = 0.70(800 \times 10^{-6}) = 560 \times 10^{-6}$ in./in.

b. Shrinkage at any time after age 1 to 3 days for steam-cured concrete,

$$(\epsilon_{sh})_t = \frac{t}{55 + t} (\epsilon_{sh})_u \tag{9}$$

For the bridge girder sand-lightweight concrete of this project, $(\epsilon_{sh})_u = 560 \times 10^{-6}$ in./in. for H = 40 percent. H was 70 percent. From Eq. 13 for H = 70 percent, $(\epsilon_{sh})_u = 0.70(560 \times 10^{-6}) = 392 \times 10^{-6}$ in./in.

General value suggested for all weights of structural concrete (both Types 1 and 3 cement), $(\epsilon_{sh})_u = 730 \times 10^{-6}$ in./in. for H = 40 percent. From Eq. 13 for H = 70 percent, $(\epsilon_{sh})_u = 0.70(730 \times 10^{-6}) = 510 \times 10^{-6}$ in./in.

In Eqs. 7, 8, and 9, t is time in days after loading for creep and time after initial shrinkage is considered. Values from the standard Eqs. 7, 8, and 9 of C_t/C_u and $(\epsilon_{sh})_t/(\epsilon_{sh})_u$ are given in the following:

Item	1 Month	3 Months	6 Months	1 Year	5 Years
C_t/C_u, Eq. 7	0.44	0.60	0.69	0.78	0.90
$(\epsilon_{sh})_t/(\epsilon_{sh})_u$, Eq. 8	0.46	0.72	0.84	0.91	0.98
$(\epsilon_{sh})_t/(\epsilon_{sh})_u$, Eq. 9	0.35	0.62	0.77	0.87	0.97

The lower creep and shrinkage for the concrete of this project, as compared with the average or general values, was probably due to the high concrete strengths attained. The computed (Eqs. 7 and 8) and measured creep and shrinkage for the moist-cured concrete of this project are shown in Figures 3 and 4.

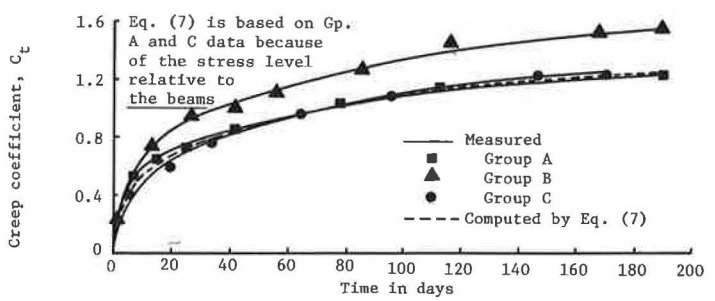


Figure 3. Measured and computed creep coefficient for the sand-lightweight concrete of Groups A, B, and C—slump less than 3 in., loaded at age 7 days, average relative humidity 40 percent, and thickness of specimens 6 in.

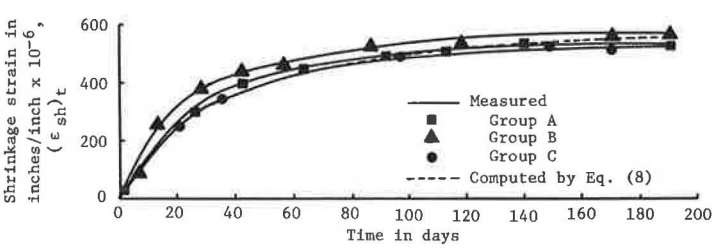


Figure 4. Measured and computed shrinkage strains for the sand-lightweight concrete of Groups A, B, and C—slump less than 3 in., shrinkage from age 7 days, average relative humidity 40 percent, and thickness of specimens 6 in.

3. Correction factors—All correction factors are applied to ultimate values. However, because creep and shrinkage for any period in Eqs. 7, 8, and 9 are linear functions of the ultimate values, the correction factors in this procedure may be applied to short-term creep and shrinkage as well.

For slumps greater than 4 in., see Figure 11.

For loading ages later than 7 days for moist-cured concrete and later than 1 to 3 days for steam-cured concrete, use Eqs. 10 and 11 for the creep correction factors (13). These results are also shown in Figure 5.

$$\text{Creep (C.F.)}_{LA} = 1.25\, t_{LA}^{-0.118} \text{ for moist-cured concrete} \tag{10}$$

$$\text{Creep (C.F.)}_{LA} = 1.13\, t_{LA}^{-0.095} \text{ for steam-cured concrete} \tag{11}$$

where t_{LA} is the loading age in days. Examples are as follows:

t_{LA}	Moist-Cured $(C.F.)_{LA}$	Steam-Cured $(C.F.)_{LA}$
10	0.95	0.90
20	0.87	0.85
30	0.83	0.82
60	0.77	0.76
90	0.74	0.74

For shrinkage considered from other than 7 days for moist-cured concrete and other than 1 to 3 days for steam-cured concrete, determine the differential in Eqs. 8 and 9 for any period starting after this time. For shrinkage of moist-cured concrete from 1 day (used to estimate differential shrinkage in composite beams, for example), use Shrinkage C.F. = 1.20.

For greater than 40 percent ambient relative humidity, use Eqs. 12 and 13 for the creep and shrinkage correction factors (13, 25, 27).

$$\text{Creep (C.F.)}_{H} = 1.27 - 0.0067\,H, \quad H \geqq 40 \text{ percent} \tag{12}$$

$$\text{Shrinkage (C.F.)}_{H} \left.\begin{aligned} &= 1.40 - 0.010\,H, \quad 40 \leqq H \leqq 80 \text{ percent} \\ &= 3.00 - 0.030\,H, \quad 80 \leqq H \leqq 100 \text{ percent} \end{aligned}\right\} \tag{13}$$

where H is relative humidity in percent. Examples are as follows:

H	Creep $(C.F.)_H$	Shrinkage $(C.F.)_H$
40 or less	1.00	1.00
50	0.94	0.90
60	0.87	0.80
70	0.80	0.70
80	0.73	0.60
90	0.67	0.30
100	0.60	0.00

For minimum thickness of members greater than 6 in., see Figure 11 for the creep and shrinkage correction factors as a function of length of drying and loading periods. For most design purposes, this effect (as shown in Appendix C) can be neglected for creep of members up to about 10 to 12 in. minimum thickness, and for shrinkage of members up to about 8 to 9 in. minimum thickness.

This method of treating the effect of member size was based on information from other sources (13, 16, 28) and this project. For large-thickness members, refer to the method of Hansen and Mattock (28) and others for relating size and shape effects for creep and shrinkage to the volume-surface ratio of the members.

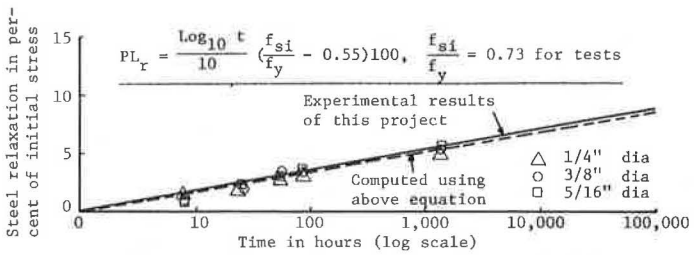


Figure 5. Measured and computed steel relaxation for relaxation tests.

Other correction factors for creep and shrinkage, which are usually not excessive and tend to offset each other, are described in Appendix C. For design purposes, these may normally be neglected.

LOSS OF PRESTRESS AND CAMBER

Relaxation Tests

Relaxation measurements were made for 3 different diameter 7-wire prestressing strands. The results agreed well with the equation suggested by Magura et al. (29) as shown in Figure 5.

It should be noted, however, that the relaxation of steel stress in a prestressed member takes place under decreasing steel strain (due to creep and shrinkage), rather than at constant length as in a relaxation test. The loss of prestress due to steel relaxation is also affected by slab-casting (level of stress in steel is raised) in the case of composite beams. Because of these effects and the practice of overtensioning to counteract the relaxation that takes place between the time of tensioning and release (this practice was assimilated in the laboratory beam tests, where it is shown in Figure 5 that about 2 percent relaxation takes place in 24 hours, for example), it is felt that about 75 percent of the steel relaxation in a constant-length relaxation test should be used in prestressed concrete loss calculations.

Antill concluded (30) that steel relaxation is probably insignificant beyond 100,000 hours (11.4 years), and that this ultimate value might be taken as twice the value at 1,000 hours (1.4 months). The relaxation equation recommended in this paper is the same time-function (log t) as that of Magura et al. (29) except that it is reduced by 25 percent in magnitude and has incorporated Antill's idea (30) that the ultimate value be taken as twice the value at 1,000 hours. This results in an ultimate steel relaxation for prestressed concrete of 7.5 percent, as shown in Term 4 of Eq. 14 and in the other equations.

Computed Loss of Prestress and Camber

The discussion in this section is based on or related to previous studies by the authors and others (5, 6, 7, 10, 11, 29, 30, 31, 32).

Noncomposite Beams at Any Time, Including Ultimate Values—The loss of prestress, in percentage of initial tensioning stress, is given by Eq. 14.

$$PL_t = \Bigg[\overbrace{(n\, f_c)}^{1} + \overbrace{(n\, f_c) C_t \left(1 - \frac{\Delta F_t}{2 F_o}\right)}^{2} + \overbrace{(\epsilon_{sh})_t\, E_s/(1 + npk_s)}^{3} + \overbrace{\frac{f_{si}}{100}\, 1.5 \log_{10} t}^{4} \Bigg] \frac{100}{f_{si}} \tag{14}$$

Term 1 is the prestress loss due to elastic shortening = PL_{el}; $f_c = \frac{F_i}{A_t} + \frac{F_i e^2}{I_t} - \frac{M_D e}{I_t}$; and n is the modular ratio at the time of prestressing. Frequently F_o, A_g, and I_g are used instead of F_i, A_t, and I_t, where $F_o = F_i (1 - n p)$. Only the first 2 terms for f_c apply at the ends of simple beams. For continuous members, the effect of secondary moments due to prestressing should also be included.

Term 2 is the prestress loss due to concrete creep. The expression, $C_t\left(1 - \frac{\Delta F_t}{2 F_o}\right)$, was used by Branson and Ozell (5) and by ACI Committee 435 (11) to approximate the creep effect resulting from the variable stress history. A later section on required calculations and summary of general parameters gives approximate values of $\Delta F_t/F_o$ (in form of $\Delta F_s/F_o$ and $\Delta F_u/F_o$) for this secondary effect (expression in parentheses) at 3 weeks to 1 month, 2 to 3 months, and ultimate values.

Term 3 is the prestress loss due to shrinkage (32). The expression, $(\epsilon_{sh})_t E_s$, somewhat (approximately 1 percent loss differential for the bridge girder ultimate value in the example here) overestimates on the safe side Term 3. The denominator represents the stiffening effect of the steel.

Term 4 is the prestress loss due to steel relaxation and assumes maximum value = 7.5 percent (at or above 10^5 hours = 11.4 years). In this term, t is time after initial stressing in hours. This expression applies only when f_{si}/f_y is greater than or equal to 0.55, in which f_y is the 0.1 percent offset yield strength.

The camber is given by Eq. 15. It is suggested that an average of the end and midspan loss be used for straight tendons (laboratory beams here) and 1-point harping, and the midspan loss for 2-point harping (bridge girders here).

$$\Delta_t = \overbrace{(\Delta_i)_{F_o}}^{1} - \overbrace{(\Delta_i)_D}^{2} + \overbrace{\left[-\frac{\Delta F_t}{F_o} + \left(1 - \frac{\Delta F_t}{2 F_o}\right) C_t \right] (\Delta_i)_{F_o}}^{3} - \overbrace{C_t (\Delta_i)_D}^{4} - \overbrace{\Delta_L}^{5} \quad (15)$$

Term 1 is the initial camber due to the initial prestress force after elastic loss, F_o. Appendix D gives common cases of prestress moment diagrams with formulas for computing camber, $(\Delta_i)_{F_o}$. Here $F_o = F_i(1 - n f_c/f_{si})$, where f_c is determined as in Term 1 of Eq. 14. For continuous members, the effect of secondary moments due to prestressing should also be included.

Term 2 is the initial dead load deflection of the beam; $(\Delta_i)_D = K M L^2/E_{ci} I_g$. Appendix A gives the K and M formulas. I_g is suggested instead of I_t for practical reasons.

Term 3 is the creep (time-dependent) camber of the beam due to the prestress force. This expression includes the effects of creep and loss of prestress, that is, the creep effect under variable stress. ΔF_t refers to the total loss at any time minus the elastic loss. It is noted that the term, $\Delta F_t/F_o$, refers to the steel stress or force after elastic loss, and the prestress loss in percent, PL (as used here), refers to the initial tensioning stress or force. The two are related as

$$\frac{\Delta F_t}{F_o} = \frac{1}{100}(PL_t - PL_{el})\frac{f_{si}}{f_o}$$

and can be closely approximated by

$$\frac{\Delta F_t}{F_o} = \frac{1}{100}(PL_t - PL_{el})\frac{1}{1 - n p}$$

Term 4 is the dead load creep deflection of the beam.

Term 5 is the live load deflection of the beam.

Unshored and Shored Composite Beams at Any Time, Including Ultimate Values—Subscripts 1 and 2 are used to refer to the slab (or effect of the slab such as under

slab dead load) and precast beam respectively. The loss of prestress, in percentage of initial tensioning stress, for unshored and shored composite beams is given by Eq. 16:

$$PL_t = \Bigg[\overbrace{(n\, f_c)}^{1} + \overbrace{(n\, f_c) C_{s_2} \left(1 - \frac{\Delta F_s}{2 F_o}\right)}^{2} + \overbrace{(n\, f_c)(C_{t_2} - C_{s_2}) \left(1 - \frac{\Delta F_s + \Delta F_t}{2 F_o}\right) \frac{I_2}{I_c}}^{3}$$

$$+ \overbrace{(\epsilon_{sh})_t\, E_s/(1 + npk_s)}^{4} + \overbrace{\frac{f_{si}}{100}\, 1.5 \log_{10} t}^{5} - \overbrace{(m\, f_{cs})}^{6} - \overbrace{(m\, f_{cs}) C_{t_1} \frac{I_2}{I_c}}^{7} - \overbrace{PG_{DS}}^{8} \Bigg] \frac{100}{f_{si}} \tag{16}$$

Term 1 is the prestress loss due to elastic shortening. Term 1 of Eq. 14 gives the calculation of f_c.

Term 2 is the prestress loss due to concrete creep up to the time of slab-casting. C_{s_2} is the creep coefficient of the precast beam concrete at the time of slab-casting.

Term 2 of Eq. 14 has comments concerning the reduction factor $\left(1 - \frac{\Delta F_s}{2 F_o}\right)$.

Term 3 is the prestress loss due to concrete creep for any period following slab-casting. C_{t_2} is the creep coefficient of the precast beam concrete at any time after slab-casting. The reduction factor, $1 - [(\Delta F_s + \Delta F_t)/2 F_o]$, with the incremental creep coefficient, $(C_{t_2} - C_{s_2})$, estimates the effect of creep under the variable prestress force that occurs after slab-casting. The reduction factor term was modified from previous references. The expression, I_2/I_c, modifies the initial value and accounts for the effect of the composite section in restraining additional creep curvature (strain) after slab-casting.

Term 4 is the prestress loss due to shrinkage. Term 3 of Eq. 14 has comments.

Term 5 is the prestress loss due to steel relaxation. In this term, t is time after initial stressing in hours. Term 4 of Eq. 14 gives the maximum value and limitations.

Term 6 is the elastic prestress gain due to slab dead load, and m is the modular ratio at the time of slab-casting.

$$f_{cs} = \frac{M_{s,Di}\, e}{I_g}, \quad M_{s,Di}$$

refers to slab or slab plus diaphragm dead load, and e and I_g refer to the precast beam section properties for unshored construction and the composite beam section properties for shored construction.

Term 7 is the prestress gain due to creep under slab dead load. C_{t_1} is the creep coefficient for the slab loading, where the age of the precast beam concrete at the time of slab-casting is considered. For shored construction, the term, I_2/I_c, is dropped.

Term 8 is the prestress gain due to differential shrinkage. $PG_{DS} = m\, f_{cd}$, where $f_{cd} = \frac{Q\, y_{cs}\, e_c}{I_c}$, and f_{cd} is the concrete stress at the steel cgs. The nomenclature in Appendix A gives additional descriptions of terms. Because this effect results in a prestress gain, not loss, and is normally small (Table 3), it may usually be neglected.

The camber of unshored and shored composite beams is given by Eqs. 17 and 18 respectively.

Unshored Construction—

$$\Delta_t = \overbrace{(\Delta_i)_{F_o}}^{1} - \overbrace{(\Delta_i)_2}^{2} + \overbrace{\left[-\frac{\Delta F_s}{F_o} + \left(1 - \frac{\Delta F_s}{2 F_o}\right) C_{s_2} \right] (\Delta_i)_{F_o}}^{3}$$

$$+ \overbrace{\left[-\frac{\Delta F_t - \Delta F_s}{F_o} + \left(1 - \frac{\Delta F_s + \Delta F_t}{2F_o}\right)(C_{t_2} - C_{s_2})\right](\Delta_i)_{F_o}\frac{I_2}{I_c}}^{4} - \overbrace{C_{s_2}(\Delta_i)_2}^{5}$$

$$- \overbrace{(C_{t_2} - C_{s_2})(\Delta_i)_2\frac{I_2}{I_c}}^{6} - \overbrace{(\Delta_i)_1}^{7} - \overbrace{C_{t_1}(\Delta_i)_1\frac{I_2}{I_c}}^{8} - \overbrace{\Delta_{DS}}^{9} - \overbrace{\Delta_L}^{10} \qquad (17)$$

Term 1 is the initial camber due to the initial prestress force after elastic loss, F_o. Term 1 of Eq. 15 gives a further explanation.

Term 2 is the initial dead load deflection of the precast beam. $(\Delta_i)_2 = K M_2 L^2/E_{ci} I_g$. Term 2 of Eq. 15 has a further explanation.

Term 3 is the creep (time-dependent) camber of the beam due to the prestress force up to the time of slab-casting. Term 3 of Eq. 15 and Terms 2 and 3 of Eq. 16 give further explanations.

Term 4 is the creep camber of the composite beam due to the prestress force for any period following slab-casting. Term 3 of Eq. 15 and Terms 2 and 3 of Eq. 16 give further explanations.

Term 5 is the creep deflection of the precast beam up to the time of slab-casting due to the precast beam dead load. C_{s_2} is the creep coefficient of the precast beam concrete at the time of slab-casting.

Term 6 is the creep deflection of the composite beam for any period following slab-casting due to the precast beam dead load. Term 3 of Eq. 16 has a further explanation.

Term 7 is the initial deflection of the precast beam under slab dead load. $(\Delta_i)_1 = K M_1 L^2/E_{cs} I_g$. The nomenclature in Appendix A contains K and M formulas. When diaphragms are used, additions to $(\Delta_i)_1$ are required:

$$(\Delta_i)_{1D} = \frac{M_{1D}}{E_{cs} I_g}\left(\frac{L^2}{8} - \frac{a^2}{6}\right)$$

where M_{1D} is the moment between diaphragms, and a is L/4, L/3, and so on for 2 symmetrical diaphragms at the quarter points, third points, and so on respectively.

Term 8 is the creep deflection of the composite beam due to slab dead load. C_{t_1} is the creep coefficient for the slab loading, where the age of the precast beam concrete at the time of slab-casting is considered. Term 3 of Eq. 16 gives comments concerning I_2/I_c.

Term 9 is the deflection due to differential shrinkage. For simple spans, $\Delta_{DS} = Q y_{cs} L^2/8E_{cs} I_c$, where $Q = D A_1 E_1/3$. The nomenclature in Appendix A has additional descriptions of terms. The factor 3 provides for the gradual increase in the shrinkage force from day 1, and also approximates the creep and varying stiffness effects (7). This factor 3 is also consistent with the data here and elsewhere. Table 4 gives numerical values used here. In the case of continuous members, differential shrinkage produces secondary moments (similar to the effect of prestressing but opposite in sign, normally) that should be included (35).

Term 10 is the live-load deflection of the composite beam, in which the gross-section flexural rigidity, $E_c I_c$, is normally used.

Shored Construction—

$$\Delta_t = \text{Eq. 17} \qquad (18)$$

with Terms 7 and 8 modified as follows:

Term 7 is the initial deflection of the composite beam under slab dead load. $(\Delta_i)_1 = K M_1 L^2/E_{cs} I_c$. Appendix A gives K and M formulas.

Term 8 is the creep deflection of the composite beam under slab dead load – $C_{t_1}(\Delta_i)_1$. The composite-section effect is already included in Term 7.

It is suggested that the 28-day moduli of elasticity for both slab and precast beam concretes, and the gross 1 (neglecting the steel), be used in computing the composite moment of inertia, I_c, in Eqs. 16, 17, and 18.

Special Case of "Ultimate" Loss of Prestress and Camber—For computing ultimate values of loss of prestress and camber, Eqs. 19, 20, 21, 22, and 23 correspond term-by-term to Eqs. 14, 15, 16, 17, and 18 respectively.

Loss of prestress for noncomposite beams, as per Eq. 14:

$$PL_u = \left[\overbrace{(n\, f_c)}^{1} + \overbrace{(n\, f_c) C_u \left(1 - \frac{\Delta F_u}{2\, F_o}\right)}^{2} + \overbrace{(\epsilon_{sh})_u E_s/(1 + npk_s)}^{3} + \overbrace{0.075\, f_{si}}^{4} \right] \frac{100}{f_{si}} \tag{19}$$

Camber of noncomposite beams, as per Eq. 15:

$$\Delta_u = \overbrace{(\Delta_i)_{F_o}}^{1} - \overbrace{(\Delta_i)_D}^{2} + \overbrace{\left[- \frac{\Delta F_u}{F_o} + \left(1 - \frac{\Delta F_u}{2\, F_o}\right) C_u \right] (\Delta_i)_{F_o}}^{3} - \overbrace{C_u (\Delta_i)_D}^{4} - \overbrace{\Delta_L}^{5} \tag{20}$$

Loss of prestress for unshored and shored composite beams, as per Eq. 16:

$$PL_u = \left[\overbrace{(n\, f_c)}^{1} + \overbrace{(n\, f_c)(\alpha_s C_u)\left(1 - \frac{\Delta F_s}{2\, F_o}\right)}^{2} + \overbrace{(n\, f_c)(1 - \alpha_s) C_u \left(1 - \frac{\Delta F_s + \Delta F_u}{2\, F_o}\right) \frac{I_2}{I_c}}^{3} \right.$$

$$+ \overbrace{(\epsilon_{sh})_u\, E_s/(1 + npk_s)}^{4} + \overbrace{0.075\, f_{si}}^{5} - \overbrace{(m\, f_{cs})}^{6} - \overbrace{(m\, f_{cs})(\beta_s C_u) \frac{I_2}{I_c}}^{7}$$

$$\left. - \overbrace{PG_{DS}}^{8} \right] \frac{100}{f_{si}} \tag{21}$$

Camber of unshored composite beams, as per Eq. 17:

$$\Delta_u = \overbrace{(\Delta_i)_{F_o}}^{1} - \overbrace{(\Delta_i)_2}^{2} + \overbrace{\left[- \frac{\Delta F_s}{F_o} + \left(1 - \frac{\Delta F_s}{2\, F_o}\right) \alpha_s\, C_u \right] (\Delta_i)_{F_o}}^{3}$$

$$+ \overbrace{\left[- \frac{\Delta F_u - \Delta F_s}{F_o} + \left(1 - \frac{\Delta F_s + \Delta F_u}{2\, F_o}\right)(1 - \alpha_s) C_u \right] (\Delta_i)_{F_o} \frac{I_2}{I_c}}^{4} - \overbrace{\alpha_s C_u (\Delta_i)_2}^{5}$$

$$- \overbrace{(1 - \alpha_s) C_u (\Delta_i)_2 \frac{I_2}{I_c}}^{6} - \overbrace{(\Delta_i)_1}^{7} - \overbrace{\beta_s C_u (\Delta_i)_1 \frac{I_2}{I_c}}^{8} - \overbrace{\Delta_{DS}}^{9} - \overbrace{\Delta_L}^{10} \tag{22}$$

Camber of shored composite beams, as per Eq. 18:

$$\Delta_u = \text{Eq. 22} \tag{23}$$

except that the composite moment of inertia is used in Term 7 to compute $(\Delta_i)_1$, and the ratio, I_2/I_c, is eliminated in Term 8.

It is noted that Eqs. 14 through 23 could be greatly shortened by combining terms and substituting the approximate parameters given in Eqs. 24, 25, and 26. They are presented in the form of separate terms, however, in order to show the separate effects or contributions to the behavior (such as the prestress force, dead load, creep, and shrinkage that occur both before and after slab-casting). The grossly approximate equations are as follows:

For noncomposite beams,

$$\Delta_u = \Delta_i + \Delta_i C_u \left(1 - \frac{\Delta F_u}{2 F_o}\right), \quad \Delta_i = (\Delta_i)_{F_o} - (\Delta_i)_D \tag{24}$$

For composite beams,

$$PL_u = \left[n f_c \left(1 + \frac{C_u}{2}\right) - n f_{cs} + (\epsilon_{sh})_u E_s + 0.075 f_{si}\right] \frac{100}{f_{si}} \tag{25}$$

$$\Delta_u = \Delta_i + \Delta_i C_u (I_2/I_c), \quad \Delta_i = (\Delta_i)_{F_o} - (\Delta_i)_2 - (\Delta_i)_1 \tag{26}$$

Required Calculations and Summary of General Parameters

Continuous time functions are provided for all needed material parameters (and for different weight concretes, moist- and steam-cured), so that the equations here readily lend themsleves to computer solutions. Certain other read-in data (such as the effect of behavior before and after slab-casting—α_s, β_s, m, and $\Delta F_s/F_o$) are also included. The parameters related to material properties are summarized later so that, for composite beam hand calculations, for example (in addition to the section properties, prestress force, F_o, and concrete stresses, f_c, f_{cs}), the only calculations needed for computing prestress loss and camber are the initial camber, $(\Delta_i)_{F_o}$, $(\Delta_i)_2$, and $(\Delta_i)_1$; Δ_{DS}; and Δ_L.

The following loss of prestress ratios at the time of slab-casting and ultimate are suggested for most calculations:

1. $\Delta F_s/F_o$ for 3 weeks to 1 month between prestressing and slab-casting = 0.11 for normal-weight, 0.13 for sand-lightweight, and 0.15 for all-lightweight;
2. $\Delta F_s/F_o$ for 2 to 3 months between prestressing and slab-casting = 0.15 for normal-weight, 0.18 for sand-lightweight, and 0.21 for all-lightweight; and
3. $\Delta F_u/F_o$ = 0.22 for normal-weight, 0.25 for sand-lightweight, and 0.29 for all-lightweight.

Note that these are defined as the total loss (at slab-casting and ultimate) minus the initial elastic loss divided by the prestress force after elastic loss. The different

TABLE 1

AVERAGE MODULAR RATIOS

Modular Ratio	Normal-Weight (w = 145)		Sand-Lightweight (w = 120)		All-Lightweight (w = 100)	
	MC	SC	MC	SC	MC	SC
At release of prestress, n =	7.3	7.3	9.8	9.8	12.9	12.9
3 weeks between prestressing and slab-casting, m =	6.1	6.2	8.1	8.3	10.7	10.9
1 month between prestressing and slab-casting, m =	6.0	6.2	8.0	8.2	10.5	10.7
2 months between prestressing and slab-casting, m =	5.9	6.1	7.9	8.2	10.2	10.6
3 months between prestressing and slab-casting, m =	5.8	6.0	7.7	8.0	10.2	10.5

values for the different weight concretes are due primarily to different initial strains (because of different E's) for normal stress levels.

Table 1 gives average modular ratios based on f'_{ci} = 4,000 to 4,500 psi for both moist-cured (MC) and steam-cured (SC) concrete and Type 1 cement for both 250 and 270 K prestressing strands. Up to 3 months, f'_c = 6,360 to 7,150 psi (using Eq. 2) for MC; and at 3 months, f'_c = 6,050 to 6,800 psi (using Eq. 4) for SC.

$E_s = 27 \times 10^6$ psi for 250 K strands, $E_s = 28 \times 10^6$ psi for 270 K strands, α_s refers to the part of the total creep that takes place before slab-casting ($\alpha_s = \frac{t^{0.60}}{10 + t^{0.60}}$, as per Eq. 7), and β_s [equal to the average Creep $(C.F.)_{LA}$ from Eqs. 10 and 11] is the creep correction factor for the precast beam concrete age when the slab is cast (under slab dead load). Equations 7, 8, and 9 and the correction factors here give suggested values of C_u and $(\epsilon_{sh})_u$.

The following may be substituted for normal-weight, sand-lightweight, and all-lightweight concrete (moist- and steam-cured, and Types 1 and 3 cement):

Time Between Prestressing and Slab-Casting	α_s	β_s
3 weeks	0.38	0.85
1 month	0.44	0.83
2 months	0.54	0.78
3 months	0.60	0.75

Sample Calculations

The following numerical substitutions for ultimate loss of prestress at midspan, using Eqs. 21 and 25, and ultimate midspan camber, using Eqs. 22 and 26, with the general parameters given here, are made for the sand-lightweight, steam-cured composite bridge girders (with slab moist-cured) of this project.

Parameters and Terms for Interior Girders—Span = 86 ft; girder spacing = 7 ft; 2-point harping at 0.4 L pt. from end, 3 (midspan) = 14.3 in.; e (end) = 6.2 in.; f_{si} = 190,000 psi; F_i = 867 k; A_s = 4.56 in.^{2}; A_g = 520 in.^{2}; p = 0.00883; I_g = 108,500 in.^{4}; M_D (precast beam) = 410 ft-k; I_c = 334,100 in.^{4} (using slab width divided by a factor of E_{stem}/E_{slab} = 3.42/3.41 = 1.00); and $M_{S,Di}$ (slab plus diaphragm moment at midspan span) = 630 ft-k.

Moduli of elasticity (using Eqs. 2, 4, and 6 for concrete): $E_s = 28 \times 10^6$ psi, as suggested for 270 K grade strands here. Slab $E_c = 3.41 \times 10^6$ psi, for f'_c = 3,500 psi, w = 145 pcf (Table 7). Precast beam (description of m and n is given in general parameters section for concrete properties): $E_{ci} = E_s/n = 28 \times 10^6/9.8 = 2.86 \times 10^6$ psi; and $E_{cs} = E_s/m = 28 \times 10^6/8.2 = 3.42 \times 10^6$ psi.

Using F_i, A_t, and I_t, as per Term 1 of Eq. 14 or 16 or 21, f_c = 2,467 psi; as per Term 6 of Eq. 16 or 21, f_{cs} = 1,006 psi. These concrete stresses refer to the midspan section. As per Term 1 of Eq. 15 or 17 or 22, for camber, $F_o = F_i(1 - n\, f_c/f_{si})$ = 758 k, using f_c = 2,467 psi.

From the general parameters section, $n = E_s/E_{ci}$ = 9.8; for 2-month period between prestressing and slab-casting, $m = E_s/E_{cs}$ = 8.2; α_s = 0.54; β_s = 0.78; and $\Delta F_s/F_o$ = 0.18. $\Delta F_u/F_o$ = 0.25.

From Eqs. 7 and 9, for H = 70 percent, C_u = 1.88 and $(\epsilon_{sh})_u = 510 \times 10^{-6}$ in./in.

From Eq. 8, for differential shrinkage, $(\epsilon_{sh})_u = 1.2(560) = 670 \times 10^{-6}$ in./in.

Initial camber and deflection, and differential shrinkage deflection: $(\Delta_i)_{F_o}$ = 4.09 in., as per Term 1 of Eq. 15 or 17 or 22; $(\Delta_i)_2$ = 1.74 in., as per Term 2 of Eq. 15 or 17 or 22; and $(\Delta_i)_1$ = 2.26 in., as per Term 7 of Eq. 17 or 22. This deflection is due to the slab and diaphragm dead load. Δ_{DS} = 0.49 in., as per Term 9 of Eq. 17 or 22.

Solutions for Interior Girders—Ultimate loss of prestress at midspan using Eq. 21 is

$$PL_u = \overset{(1)}{12.7} + \overset{(2)}{11.7} + \overset{(3)}{2.8} + \overset{(4)}{6.5} + \overset{(5)}{7.5} - \overset{(6)}{4.3} - \overset{(7)}{2.0} - \overset{(8)}{1.6} = 33.3 \text{ percent}$$

Ultimate midspan camber using Eq. 22 minus Δ_L is

$$\Delta_u = \overset{(1)}{4.09} - \overset{(2)}{1.74} + \overset{(3)}{3.05} + \overset{(4)}{0.80} - \overset{(5)}{1.77} - \overset{(6)}{0.48} - \overset{(7)}{2.26} - \overset{(8)}{1.06} - \overset{(9)}{0.49} = 0.14 \text{ in.}$$

Ultimate loss of prestress at midspan using the approximate Eq. 25 is

$$PL_u = 24.6 - 5.2 + 7.5 + 7.5 = 34.4 \text{ percent}$$

Ultimate midspan camber using the approximate Eq. 26 is

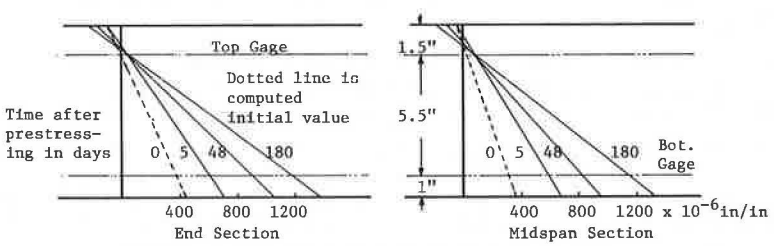
$$\Delta_u = 0.09 + 0.05 = 0.14 \text{ in.}$$

where $\Delta_i = 4.09 - 1.74 - 2.26 = 0.09$ in.

Also given in Tables 2 and 3 are the prestress loss and camber results by the more reliable Eqs. 14 through 17 and 19 through 22, and the approximate Eqs. 24 through 26, for the laboratory beams and bridge girders. Although the agreement is good (note the camber is near zero due to the slab effect) by these methods, the approximate method may be suitable in many cases (Tables 2 and 3) for rough calculations only. Also, the calculations needed by the approximate methods are not significantly fewer than those by the other methods. The more reliable equations should be preferable for computer use.

Experimental Loss of Prestress and Camber Results

The loss of prestress at the end and midspan for the laboratory beams was determined from the measured concrete strains. However, this measured loss does not include the steel relaxation loss, because steel relaxation is a "stress relaxation at constant length—or nearly so in the case of a prestressed concrete beam" phenomenon.



Initial plus time-dependent strain distribution diagrams from concrete strains measured on the sides of the beams

Typical experimental prestress loss determined for end section at 180 days, where f_{si} = 172 ksi, $E_s = 27 \times 10^3$ ksi, and observed concrete strain at cgs = $1{,}001 \times 10^{-6}$ in./in.

Item	Percent
Loss from measured strains $(1{,}001 \times 10^{-6})(27 \times 10^3)(100)/172$	15.7
Increase in measured loss due to lateral distribution (determined as 2.5 percent of 15.7)	0.4
Measured loss due to steel relaxation (75 percent of value shown in Fig. 5)	5.5
Total experimental loss of prestress	21.6

Figure 6. Typical measured strain distribution diagrams for the end and midspan sections of Beam B1, and example of experimental prestress loss determined for the end section at 180 days after prestressing.

Separate relaxation tests were made, and the results are shown in Figure 5. From these and other tests, the relaxation equation given in Term 4 of Eq. 14 was determined. An example of the experimental determination of prestress loss for a typical laboratory beam is shown in Figure 6.

Experimental and computed loss of prestress versus time curves for the laboratory beams are shown in Figure 7, and the computed curves for the bridge girders are shown in Figure 8. Measured and computed midspan camber versus time curves for the beams and girders are shown in Figures 9 and 10. The general Eqs. 14 through 17 with experimental parameters were used in all comparisons with test results in Figures 7, 9, and 10. These results are given in Tables 2 and 3 at release of prestress (camber only), just before slab-casting (3 and 9 weeks for the beams and 9 weeks for the girders, after prestressing), and at 180 days for the beams and 560 days for the

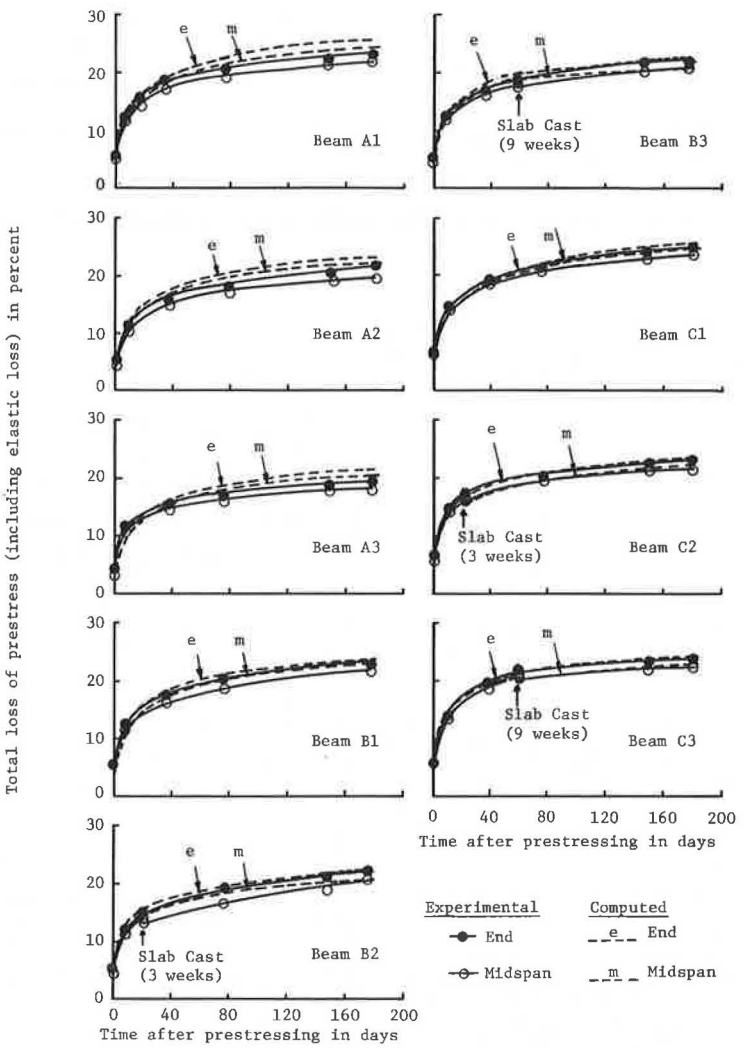


Figure 7. Experimental and computed loss of prestress (using general Eqs. 14 and 16 with experimental parameters) for the laboratory beams.

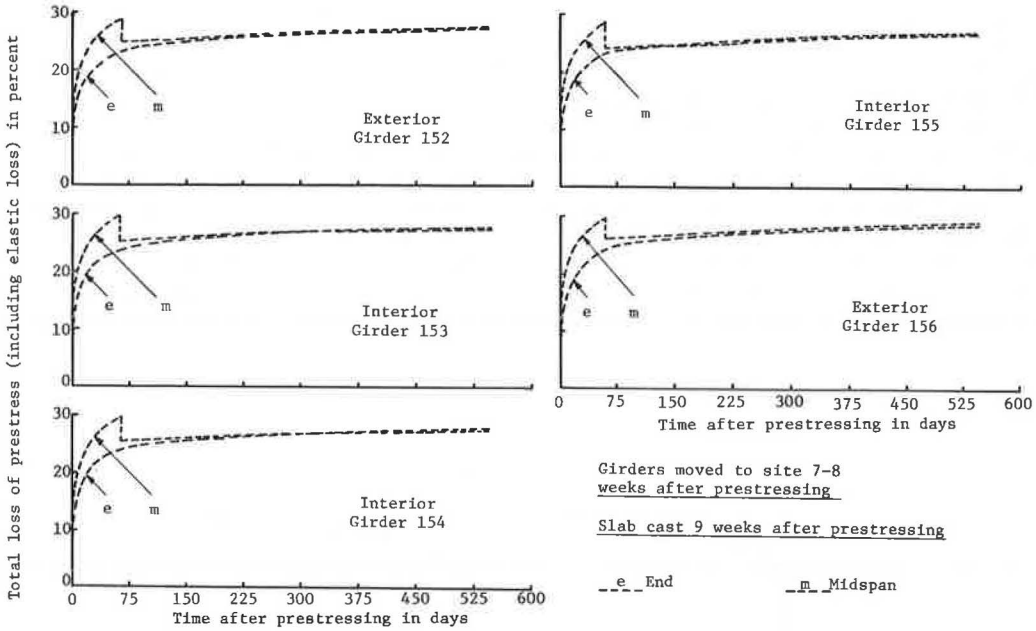


Figure 8. Computed loss of prestress (using general Eq. 16 with experimental parameters) for the bridge girders.

TABLE 2

EXPERIMENTAL AND COMPUTED LOSS OF PRESTRESS FOR LABORATORY BEAMS AND COMPUTED LOSS OF PRESTRESS FOR BRIDGE GIRDERS

No.	Time Between Prestress and Slab-Cast[a] (days)	Computed Loss Just Before Slab-Cast		Experimental Loss at 180 Days[b]		Computed Loss by General Eqs. 14 and 16 With Exp. Parameters[c]				Computed Ultimate Loss[d]					
										Gen. Eqs. 14 and 16 With Exp. Parameters		Ult. Eqs. 19 and 21 With Gen. Parameters		Approx. Eq. 25 With Gen. Parameters	
		Mid	Ratio	End	Mid	End	Ratio	Mid	Ratio	End	Mid	End	Mid	End	Mid
Laboratory Beams															
A1	—	—	—	23.5	22.0	25.5	1.09	24.6	1.12	31.7	30.5	36.9	35.4	—[e]	—
A2	—	—	—	21.0	19.5	23.2	1.10	22.3	1.14	28.9	27.8	33.5	32.1	—	—
A3	—	—	—	19.0	18.5	21.4	1.13	20.4	1.10	26.7	25.5	32.0	30.6	—	—
B1	—	—	—	21.6	21.0	24.0	1.11	22.9	1.09	29.8	28.6	34.6	33.1	—	—
B2	21	15.0	1.07	21.9	20.5	22.2	1.02	20.7	1.01	26.5	25.0	28.9	27.2	31.0	29.4
B3	63	19.4	1.10	21.4	20.0	22.6	1.06	21.1	1.05	26.8	25.2	29.4	27.6	31.0	29.4
C1	—	—	—	25.0	24.0	25.7	1.03	24.7	1.03	31.9	30.8	37.2	35.7	—	—
C2	21	16.4	0.97	23.0	21.4	23.7	1.03	22.4	1.05	28.2	26.7	30.9	29.3	33.1	31.6
C3	63	21.1	1.01	23.6	22.3	24.4	1.03	23.0	1.03	28.7	27.2	31.7	30.0	33.1	31.6
Bridge Girders															
152	65	28.4	—	—	—	27.3	—	28.2	—	29.4	29.6	30.4	34.0	30.5	35.0
153	65	29.4	—	—	—	28.0	—	28.6	—	30.2	30.0	30.3	33.3	30.5	34.4
154	65	29.4	—	—	—	28.0	—	28.6	—	30.2	30.0	30.3	33.3	30.5	34.4
155	60	28.4	—	—	—	27.2	—	27.0	—	29.3	28.7	30.3	33.3	30.5	34.4
156	60	29.8	—	—	—	28.4	—	29.2	—	30.5	31.0	30.4	34.0	30.5	35.0

Note: All losses are expressed in percentage of initial stress. The ratios in the table are computed-experimental. The note to Table 4 gives a description of the experimental parameters. The section on sample calculations gives a description of the general parameters.

[a]The laboratory beams and bridge girders were prestressed at age 7 days and 2 to 3 days respectively.

[b]Figure 6 shows an example of the experimental prestress loss determination. The 180 days and 560 days in footnote c refer to days after prestressing.

[c]180 days for laboratory beams and 560 days for bridge girders.

[d]Because the laboratory beam concrete strengths at release were well beyond the range specified for the general parameters, the n and m values in the general parameter columns were computed separately for the laboratory beams. Where general parameters are used, a correction factor is applied for relative humidity only.

[e]No approximate equation was given in the paper for noncomposite beam loss of prestress. Equation 25 refers to composite beams only.

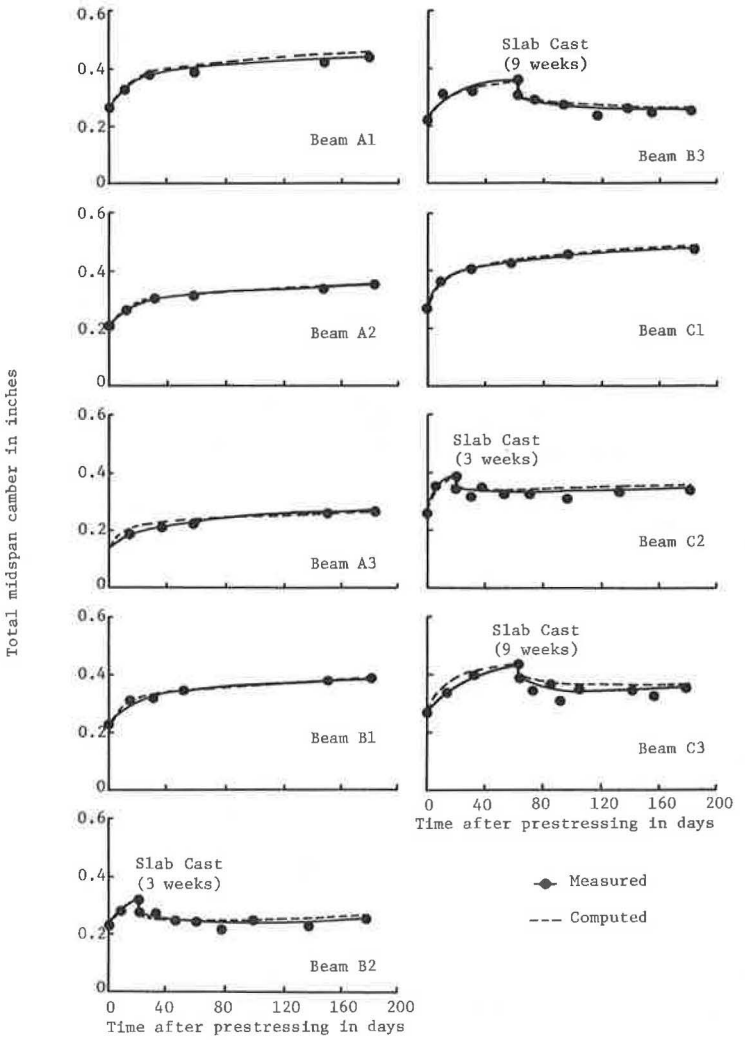


Figure 9. Measured and computed camber (using general Eqs. 15 and 17 with experimental parameters) for the laboratory beams.

TABLE 3

MEASURED AND COMPUTED MIDSPAN CAMBER FOR LABORATORY BEAMS AND BRIDGE GIRDERS

No.	Initial Camber			Time Between Prestress and Slab-Cast[a] (days)	Camber Just Before Slab-Cast			Computed Camber by General Eqs. 15 and 17 With Exp. Parameters[b]			Computed Ultimate Camber[c]		
	Meas	Comp	Ratio		Meas	Comp	Ratio	Meas	Comp	Ratio	Gen. Eqs. 15 and 17 With Exp. Parameters	Ult. Eqs. 20 and 22 With Gen. Parameters	Approx. Eqs. 24 and 26 With Gen. Parameters
							Laboratory Beams						
A1	0.27	0.25	0.93	—	—	—	—	0.44	0.46	1.04	0.54	0.68	0.77
A2	0.20	0.19	0.95	—	—	—	—	0.35	0.35	1.00	0.42	0.52	0.59
A3	Bad D.	0.15	—	—	—	—	—	0.27	0.26	0.96	0.31	0.38	0.44
B1	0.22	0.22	1.00	—	—	—	—	0.39	0.39	1.00	0.46	0.58	0.66
B2	0.23	0.22	0.96	21	0.32	0.32	1.00	0.25	0.27	1.08	0.28	0.26	0.29
B3	0.23	0.22	0.96	63	0.36	0.35	0.97	0.26	0.27	1.04	0.28	0.28	0.30
C1	0.27	0.27	1.00	—	—	—	—	0.47	0.49	1.04	0.57	0.73	0.75
C2	0.27	0.27	1.00	21	0.39	0.39	1.00	0.34	0.36	1.06	0.38	0.37	0.39
C3	0.27	0.27	1.00	63	0.44	0.44	1.00	0.35	0.37	1.06	0.39	0.39	0.39
							Bridge Girders						
152	2.05	2.14	1.04	65	3.10	3.06	0.98	0.50	0.45	0.90	0.43	0.51	0.53
153	2.05	2.22	1.08	65	3.10	3.13	1.02	0.25	0.19	0.76	0.16	0.14	0.14
154	2.10	2.22	1.06	65	3.05	3.13	1.03	0.20	0.19	0.95	0.16	0.14	0.14
155	1.90	2.14	1.13	60	2.95	3.04	1.03	-0.02	0.04	—	0.01	0.14	0.14
156	1.85	2.27	1.23	60	2.92	3.16	1.08	0.30	0.52	1.74[d]	0.50	0.51	0.53

Note: All camber values are in inches. The ratios in the table are computed-measured. The note to Table 4 gives a description of the experimental parameters. The section on sample calculations gives a description of the general parameters.

[a]The laboratory beams and bridge girders were prestressed at age 7 days and 2 to 3 days respectively. The 180 days and 560 days in footnote b refer to days after prestressing.

[b]180 days for laboratory beams and 560 days for bridge girders.

[c]Because the laboratory beam concrete strengths at release were well beyond the range specified for the general parameters, the n and m values in the general parameter columns were computed separately for the laboratory beams.

[d]Camber has been reduced from about 3 in. before slab-casting to less than ½ in. after 1 year (Fig. 10). This ratio is large for the near-zero camber even though the difference in camber is 0.22 in.

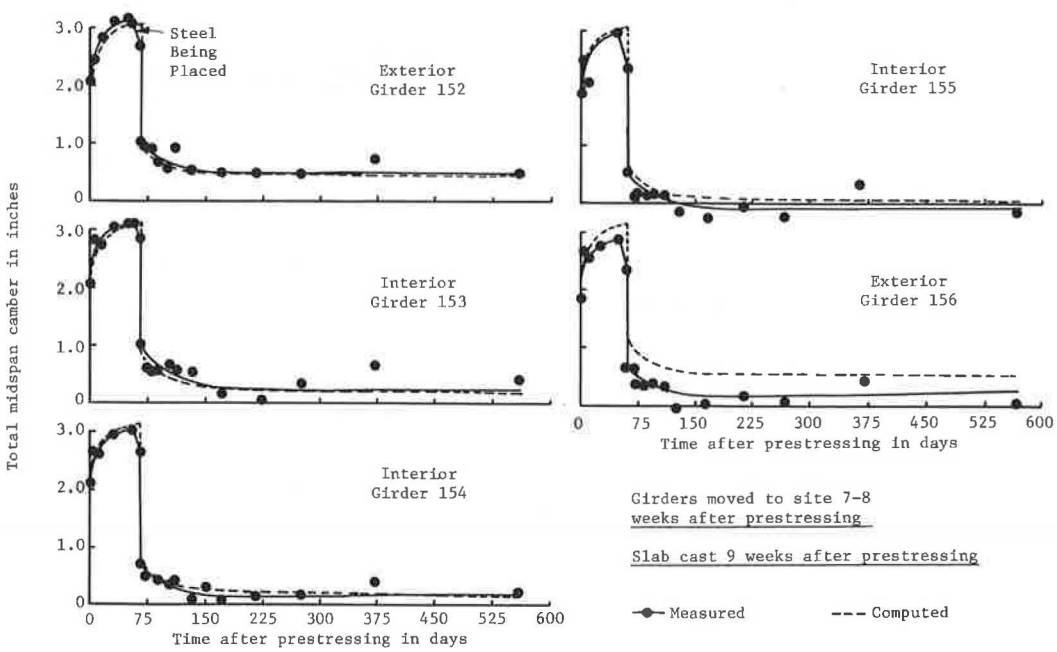


Figure 10. Measured and computed camber (using general Eq. 17 with experimental parameters) for the bridge girders.

TABLE 4

COMPUTED ULTIMATE LOSS OF PRESTRESS AT MIDSPAN, BY TERMS, FOR LABORATORY BEAMS AND BRIDGE GIRDERS, USING GENERAL EQS. 14 AND 16 WITH EXPERIMENTAL PARAMETERS

No.	Elastic Loss	Creep Loss Before Slab-Cast	Creep Loss After Slab-Cast	Shrink Loss	Relax Loss	Elastic Gain Due to Slab	Creep Gain Due to Slab	Gain Due to Differential Shrink	Total Loss, Eqs. 14 and 16
				Laboratory Beams					
A1	5.2	8.0	—	9.8	7.5	—	—	—	30.5
A2	4.1	6.3	—	9.9	7.5	—	—	—	27.8
A3	3.2	4.8	—	10.0	7.5	—	—	—	25.5
B1	4.5	6.9	—	9.7	7.5	—	—	—	28.6
B2	4.5	2.9	1.2	9.7	7.5	-0.4	-0.2	-0.2	25.0
B3	4.5	4.0	0.9	9.7	7.5	-0.4	-0.2	-0.8	25.2
C1	5.4	8.3	—	9.6	7.5	—	—	—	30.8
C2	5.4	3.5	1.5	9.6	7.5	-0.4	-0.2	-0.2	26.7
C3	5.4	4.8	1.1	9.6	7.5	-0.4	-0.2	-0.6	27.2
				Bridge Girders					
152	11.5	9.8	2.1	4.5	7.5	-3.7	-1.5	-0.6	29.6
153	12.0	10.3	2.2	4.5	7.5	-4.2	-1.7	-0.6	30.0
154	12.0	10.3	2.2	4.5	7.5	-4.2	-1.7	-0.6	30.0
155	11.5	9.6	2.2	4.5	7.5	-4.3	-1.7	-0.6	28.7
156	12.3	10.3	2.3	4.5	7.5	-3.8	-1.5	-0.6	31.0

Note: The table is arranged in the order of terms in Eq. 16. All losses are expressed in percentage of initial stress. The experimental parameters used in the calculations for this table are shown in Table 7 (strength and elastic properties) and elsewhere in this paper for the sand-lightweight concrete of this project. The slab shrinkage is shown here only. The correction factors given here for age of loading, humidity, and member thickness (8 in. for bridge girders) are used where appropriate with experimental parameters. The resulting creep and shrinkage factors used are as follows:

Item	Laboratory Beams (40 percent relative humidity)	Bridge Girders (70 percent relative humidity)
Precast beam creep	$C_u = 1.75$	$C_u = 1.62$
Precast beam shrinkage (x 10^{-6} in./in.)	$(\epsilon_{sh})_u = 650$	$(\epsilon_{sh})_u = 352$
Slab shrinkage (from day 1), used in computing differential shrinkage (x 10^{-6} in./in.)	$(\epsilon_{sh})_u = 470$	$(\epsilon_{sh})_u = 330$

The section on sample calculations gives a comparison with the general parameter results.

TABLE 5

COMPUTED ULTIMATE MIDSPAN CAMBER, BY TERMS, FOR LABORATORY BEAMS AND BRIDGE GIRDERS, USING GENERAL EQS. 15 AND 17 WITH EXPERIMENTAL PARAMETERS

No.	Initial Camber Due to Prestress	Initial Deflection Due to Beam Dead Load	Creep Camber up to Slab-Cast	Creep Camber After Slab-Cast	Dead Load Creep Deflection up to Slab-Cast	Beam Dead Load Deflection After Slab-Cast	Elastic Deflection Due to Slab Dead Load	Creep Deflection Due to Slab Dead Load	Deflection Due to Differential Shrink	Total Camber, Eqs. 15 and 17
				Laboratory Beams						
A1	0.30	-0.05	0.37	—	-0.09	—	—	—	—	0.53
A2	0.24	-0.05	0.31	—	-0.09	—	—	—	—	0.41
A3	0.19	-0.05	0.25	—	-0.09	—	—	—	—	0.30
B1	0.27	-0.05	0.34	—	-0.10	—	—	—	—	0.46
B2	0.27	-0.05	0.14	0.07	-0.04	-0.02	-0.05	-0.02	-0.01	0.29
B3	0.27	-0.05	0.19	0.05	-0.05	-0.01	-0.04	-0.02	-0.04	0.30
C1	0.32	-0.05	0.40	—	-0.09	—	—	—	—	0.58
C2	0.32	-0.05	0.16	0.08	-0.03	-0.02	-0.04	-0.02	-0.01	0.39
C3	0.32	-0.05	0.22	0.06	-0.05	-0.01	-0.04	-0.02	-0.04	0.39
				Bridge Girders						
152	3.71	-1.56	2.33	0.65	-1.42	-0.36	-1.96	-0.78	-0.18	0.43
153	3.87	-1.64	2.39	0.68	-1.49	-0.38	-2.21	-0.87	-0.19	0.16
154	3.87	-1.64	2.39	0.68	-1.49	-0.38	-2.21	-0.87	-0.19	0.16
155	3.72	-1.57	2.28	0.71	-1.40	-0.37	-2.26	-0.91	-0.19	0.01
156	3.96	-1.68	2.38	0.73	-1.50	-0.39	-2.01	-0.81	-0.18	0.50

Note: The table is arranged in the order of terms in Eq. 17. All values are in inches. The note to Table 4 gives a description of the experimental parameters. The section on sample calculations gives a comparison with the general parameter results.

girders. The test period for the laboratory beams was terminated after 6 months in order to conduct load-deflection tests.

The computed ultimate values are also given in Tables 2 and 3 using the general Eqs. 14 through 17 with experimental parameters determined for the sand-lightweight concrete of this project, and using the ultimate-value Eqs. 19 through 26 with general parameters given for normal-weight, sand-lightweight, and all-lightweight concrete. For the general parameters, the same creep and shrinkage factors are suggested for all 3 concretes, with different modular ratios and prestress loss ratios ($\Delta F_s/F_o$ and $\Delta F_u/F_o$) for each. The computed ultimate values for loss of prestress and camber are given term-by-term in Tables 4 and 5 using the general Eqs. 14 through 17 with experimental parameters.

DISCUSSION AND CONCLUSIONS

The experimental and computed loss of prestress and camber for the sand-lightweight concrete structures of this project are shown in Figures 5 through 10 and Tables 2 through 5. Results both by general Eqs. 14 through 17 (for values at any time, including ultimate) with experimental parameters and by Eqs. 19 through 22 and 24 through 26 (for ultimate values) with general parameters (given here) are included. These results serve to substantiate the generalized procedure presented for predicting loss of prestress and camber of noncomposite and composite prestressed structures. The approximate Eqs. 24 through 26 may be suitable for rough calculations only in some cases.

Results computed by the material parameter, Eqs. 2, 4, 7, 8, and 9, are compared with the data of this project in Figures 2 through 4. Equations 2 through 9 are generalized for different weight concretes. The procedure for predicting creep and shrinkage is one of providing standard functions, with suggested ultimate values for different weight concretes, and correction factors for pertinent conditions other than "standard" (13). These conditions are briefly described in the text and in Appendix C. The ultimate values suggested should be used only in the absence of specific information pertaining to local aggregates and conditions.

Continuous time functions are provided for all needed material parameters (and for different weight concretes, moist- and steam-cured), so that the prestress loss and camber equations readily lend themselves to computer solutions. Certain other read-in data (such as for the effect of behavior before and after slab casting—α_s, β_s, m, and $\Delta F_s/F_o$) are also included, along with a summary of parameters convenient for hand calculations. By using these parameters, the calculations needed in the approximate Eqs. 24 through 26 are not significantly fewer than those needed in the more reliable Eqs. 14 through 23.

It is noted that Eqs. 14 through 23 could be greatly shortened by combining terms, but they are presented in the form of separate terms (results are given in Tables 4 and 5 and in the section on sample calculations) in order to show the separate effects or contributions to the behavior (such as prestress force, dead load, creep, and shrinkage that occur both before and after slab-casting).

The following specific observations and conclusions are made relative to the results shown in Figures 5 and 7 through 12 and given in Tables 1 through 4 and other parts of the paper.

1. The ultimate steel relaxation percentage recommended for regular 7-wire strand to be used in prestressed concrete structures is 7.5 [Fig. 5 and its results and discussion, Term 4 of Eq. 14, and other research (29, 30)].
2. The computed initial camber agreed well in most cases with the measured initial camber, as given in Table 2.
3. The computed prestress loss for the laboratory noncomposite beams was slightly higher (from 0.3 to 2.8 percent prestress loss differential after 6 months) than the experimental results (Fig. 7 and Table 2). The direct application of laboratory creep data for uniformly loaded specimens to beams with nonuniform stress distribution appears to slightly overestimate the creep effect relative to loss of prestress of noncomposite beams. The same overprediction was not found in the case of camber,

apparently because the F/A stress component, which is a dominant factor in loss of prestress results, does not contribute directly to camber. The camber results and other prestress loss results (for composite beams) shown in Figures 7, 9, and 10 and given in Tables 2 and 3 are considered to be in very good agreement. For these cases (noncomposite beam camber and composite beam loss and camber), offsetting creep (and shrinkage in the case of composite beams) effects occur.

4. As shown in Figures 7 and 8 and as given in Table 2, the difference in the end and midspan prestress loss was quite small for the laboratory beams, and relatively large for the bridge girders before slab-casting. After slab-casting, the prestress loss in the bridge girders was only slightly different at end and midspan.

5. The loss of prestress for the sand-lightweight concrete bridge girders was of the order of 27 to 29 percent at 560 days after prestressing and 29 to 31 percent ultimately (Fig. 8 and Table 2). It was determined that loss percentages for bridges under similar conditions using normal-weight concrete will normally be somewhat lower than these (of the order of 25 percent), and those for bridges using all-lightweight concrete will normally be somewhat higher than these (of the order of 35 percent or higher). Higher losses for the lighter concretes, for example, are due primarily to the lower modulus of elasticity (higher elastic strains for a given stress level) and not necessarily to greater creep and shrinkage behavior.

6. Slab-casting causes an elastic deflection (downward) and prestress gain and a time-dependent deflection and prestress gain due to creep and differential shrinkage. Loss of prestress due to creep and camber growth under the prestress force and precast beam dead load is also reduced by the effect of the hardened slab (as opposed to the case of no composite slab). These results are given in Tables 4 and 5 and in the section on sample calculations. The composite slab reduces the ultimate loss of prestress at midspan of the bridge girders about 11 percent (41 - 30 = 11 percent). The camber curves nearly level off at about 3.0 in. just before slab-casting (Fig. 10 and Table 3). After slab-casting and up to ultimate, the camber is reduced to near zero.

7. The effect of the 3-week and 9-week slab-casting schedules for the laboratory beams had only a small effect on loss of prestress (Fig. 7) and a more noticeable effect on camber (Fig. 9). When considering a 3-week slab (slab cast 3 weeks after prestressing) for the bridge girders, as compared with the actual 9-week slab, the ultimate loss of prestress at midspan was about 2 percent less and the ultimate midspan camber about 0.10 in. less for the 3-week slab. These results serve to point out the relatively small beneficial effect of casting the deck slab as early as possible [also indicated by Corley et al. (6)]. It is noted that there are also offsetting effects in the case of the effect of slab-casting schedules. An earlier slab tends to reduce total creep deformation (causing upward camber) by forming an earlier composite section, but it also reduces differential shrinkage deformation (causing downward deflection).

8. The different individual contributions to prestress loss and camber, as illustrated by the different terms in Eqs. 14 through 23, are sensitive to the stiffness, creep, and shrinkage concrete properties. However, the net results of these equations tend toward more correct solutions than the individual terms because of offsetting effects. This is especially true in the case of composite beams and is less the case for noncomposite beams (Tables 2 and 3, and also the comparison of ultimate-value results with experimental parameters and general parameters).

9. The inclusion of all terms in Eqs. 14 through 23 appears to incorporate all significant effects in the reliable prediction of prestress loss and camber. These effects can be seen in the term-by-term data given in Tables 4 and 5 and in the section on sample calculations. In the sample calculations for the bridge girders using the general parameters, for example, the 7 terms (omitting Term 8, differential shrinkage) for loss of prestress varied from 1.6 to 12.7 percent, and the 9 terms for camber varied from 0.48 to 4.09 in. The results by the approximate Eqs. 24 through 26 and the more reliable equations were in reasonably good agreement (Tables 2 and 3 and the section on sample calculations) for the structures of this project.

10. All of the bridge girder data shown in Figure 10 indicated an increase in camber of about 0.4 in. between 300 to 370 days (starting in April). This appears to be due to higher temperatures and is consistent with the observations of Delarue (33).

11. The systematic procedures described in this paper for predicting time-dependent behavior are deterministic in nature. Probabilistic methods are also needed for estimating variability of behavior.

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Appendix A

NOTATION

1 = subscript denoting cast-in-place slab of a composite beam or the effect of the slab as due to slab dead load.
2 = subscript denoting precast beam.
A = area of section.
A_g = area of gross section, neglecting the steel.
A_s = area of prestressed steel.
A_t = area of transformed section.
a = empirical constant in Eq. 1 (also used in Term 7 of Eq. 17 as the distance from end of beam to the nearest of 2 symmetrical diaphragms, and in Appendix D from end to harped point in 2-point harping).

b = empirical constant in Eq. 1.
C = creep coefficient defined as ratio of creep strain to initial strain.
C.F. = correction factor.
C_s = creep coefficient at time of slab-casting.
C_t = creep coefficient at any time.
C_{t_1} = creep coefficient of the composite beam under slab dead load.
C_{t_2} = creep coefficient due to precast beam dead load.
C_u = ultimate creep coefficient.
c = subscript denoting composite section (also used to denote concrete, as E_c).
cp = subscript denoting creep.
D = differential shrinkage strain (also used to denote dead load).
DS = subscript denoting differential shrinkage.
d = effective depth of section.
E_c = modulus of elasticity of concrete such as at 28 days.
E_{ci} = modulus of elasticity of concrete at the time of transfer of prestress.
E_{cs} = modulus of elasticity of concrete at the time of slab-casting.
E_s = modulus of elasticity of prestressing steel.
e = eccentricity of steel cgs.
e_c = eccentricity of steel at center of beam (see Appendix D; also used in Eq. 16 to denote eccentricity of steel in composite section).
e_o = eccentricity of prestressed steel at end of beam (see Appendix D).
F = prestress force after losses.
F_i = initial tensioning force.
F_o = prestress force at transfer (after elastic losses).
ΔF = loss of prestress due to time-dependent effects only, such as creep, shrinkage, steel relaxation (the elastic loss is deducted from the tensioning force, F_i, to obtain F_o).
ΔF_s = total loss of prestress at slab-casting minus the initial elastic loss that occurred at the time of prestressing.
ΔF_t = total loss of prestress at any time minus the initial elastic loss.
ΔF_u = total ultimate loss of prestress minus the initial elastic loss.
f_c = concrete stress such as at steel cgs due to prestress and precast beam dead load in the prestress loss equations.
f_{cd} = concrete stress at steel cgs due to differential shrinkage.
f_{ci} = concrete stress at the time of transfer of prestress.
f_{cs} = concrete stress at steel cgs due to slab dead load (plus diaphragm and dead load when applicable).
f_o = stress in prestressing steel at transfer (after elastic loss).
f_{si} = initial or tensioning stress in prestressing steel.
f_y = yield strength of steel (defined here as 0.1 percent offset).
f'_c = compressive strength of concrete.
$(f'_c)_t$ = compressive strength of concrete at any time.
$(f'_c)_{7d}$ = compressive strength of concrete at 7 days (similarly for 2.0 days, or 1 to 3 days, and 28 days).
$(f'_c)_u$ = ultimate (in time) compressive strength of concrete.
H = relative humidity in percent.
I = moment of inertia (second moment of area).
I_1 = moment of inertia of slab.
I_2 = moment of inertia of precast beam.
I_c = moment of inertia of composite section with transformed slab (slab is transformed into equivalent precast beam concrete by dividing the slab width by E_{c_2}/E_{c_1}).
I_g = moment of inertia of gross section, neglecting the steel.
I_t = moment of inertia of transformed section.
i = subscript denoting initial value.
K = deflection coefficient. For example, for beams of uniform section and uniform load,

$K = 1/4$, for cantilever beam,
$K = 5/48$, for simple beam,
$K = 8/185$, for hinged-fixed beam (one end continuous), and
$K = 1/32$, for fixed-fixed beam (both ends continuous).

k_s = $1 + e^2/r^2$, where $r^2 = I_g/A_g$.
L = span length (also used as a subscript to denote live load).
LA = subscript denoting loading age.
M = bending moment. When used as the numerical maximum bending moment for beams of uniform section and uniform load,
(-) $M = q L^2/2$, for cantilever beam,
(+) $M = q L^2/8$, for simple beam,
(-) $M = qL^2/8$, for hinged-fixed beam (one end continuous), and
(-) $M = q L^2/12$, for fixed-fixed beam (both ends continuous).

M_1 = maximum bending moment under slab dead load.
M_2 = maximum bending moment under precast beam dead load.
M_{1D} = bending moment between symmetrically placed diaphragms.
$M_{S,Di}$ = bending moment due to slab or slab plus diaphragm dead load.
m = modular ratio of the precast beam concrete, E_s/E_{cs}, at the time of slab-casting.
n = modular ratio, E_s/E_{ci}, at release of prestress.
PG = prestress gain in percentage of initial tensioning stress or force.
PG_{DS} = prestress gain due to differential shrinkage.
PL = total prestress loss in percentage of initial tensioning stress or force.
PL_{el} = prestress loss due to elastic shortening.
PL_r = prestress loss due to steel relaxation.
PL_t = total prestress loss in percent at any time.
PL_u = ultimate prestress loss in percent.
p = steel percentage, A_s/A_g.
Q = differential shrinkage force = $D A_1 E_1/3$. The factor 3 provides for the gradual increase in the shrinkage force from day 1, and also approximates the creep and varying stiffness effects (7, 30).
q = uniformly distributed load.
r = radius of gyration, $r^2 = I_g/A_g$.
s = subscript denoting time of slab-casting (also used to denote steel).
sh = subscript denoting shrinkage.
t = time in general, time in hours in the steel relaxation equation, and time in days in other equations here.
t_{LA} = age of concrete when loaded, in days.
u = subscript denoting ultimate value.
w = unit weight of concrete in pcf.
y_{cs} = distance from centroid of composite section to centroid of slab.
α = ratio of creep coefficient at any time to ultimate creep coefficient.
α_s = ratio of creep coefficient at the time of slab-casting to C_u.
β = creep correction factor for the precast beam concrete age when loaded.
β_s = creep correction factor for the precast beam concrete age when slab cast.
Δ = maximum camber (positive) or deflection (negative).
Δ_i = initial camber, deflection.
$(\Delta_i)_1$ = initial deflection under slab dead load.
$(\Delta_i)_{1D}$ = initial deflection due to diaphragm dead load.
$(\Delta_i)_2$ = initial deflection under precast beam dead load.
$(\Delta_i)_D$ = initial dead load deflection.
$(\Delta_i)_{F_o}$ = initial camber due to the initial prestress force, F_o.
Δ_{DS} = differential shrinkage deflection.
Δ_L = live load deflection.
Δ_t = total camber, deflection, at any time.
Δ_u = ultimate camber, deflection.
$(\epsilon_{sh})_t$ = shrinkage strain in microinches per inch at any time.
$(\epsilon_{sh})_u$ = ultimate shrinkage strain in microinches per inch.

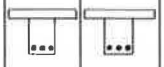
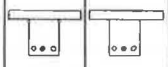
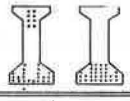
Appendix B

DETAILS OF DESIGN AND TESTS OF BEAMS AND GIRDERS

The details of the laboratory beams and bridge girders are given in Table 6, and the concrete properties, temperature, and humidity data are given in Table 7.

TABLE 6

[e]DETAILS OF LABORATORY BEAMS AND BRIDGE GIRDERS

Beam Group	[a]All Beams are 6" x 8", d=6", Span=15', [b]Slabs are 20" x 2"									L=86', 7"slab
	Group A			Group B			Group C			Bridge Girder
Beam No.	A1	A2	A3	B1	B2	B3	C1	C2	C3	152-156
Beam			c 							end mid 
Prestressing in Strand dia	2-3/8 1-5/16	3-5/16	1-3/8 1-5/16	3-5/16	3-5/16	3-5/16	2-3/8 1-5/16	2-3/8 1-5/16	2-3/8 1-5/16	30-1/2
A_s in^2	0.2176	0.1734	0.1377	0.1734	0.1734	0.1734	0.2176	0.2176	0.2176	4.56
$p = A_s/A_g$	0.00453	0.00361	0.00287	0.00361	0.00361	0.00361	0.00453	0.00453	0.00453	0.00883
Des.Pre.For.F_i,k	38.0	30.0	24.0	30.0	30.0	30.0	38.0	38.0	38.0	867.0
Meas.Pre. F_i,kip	37.0	29.6	23.4	30.0	29.9	29.9	38.0	37.9	37.9	867.0
[d]Concrete Stresses at release of prestress,psi	t=+388 b=-1932	t=+311 b=-1541	t=+244 b=-1224	t=+313 b=-1563	t=+312 b=-1555	t=+312 b=-1555	t=+395 b=-1975	t=+394 b=-1970	t=+394 b=-1970	t=-429 t=-107 b=-2633 b=-2955

[a] ○ 3/8" Strand, ● 5/16" Strand, Measured stress in all strands of lab. beams = (172±4) ksi. Measured stress in all strands of bridge girders = 190 ksi.

[b]Six gage WWF, 6" by 6" (A_s=0.058 in^2 per ft width), slab steel placed in center of slab. No. 3 U-Stirrups in form of ties for composite slab are spaced at 6" cc. in end quarter span and at 22 1/2" cc. in middle half of beam.

[c]Strands placed so that lateral eccentricity is eliminated.

[d]These stresses are computed using the Measured F_i: t = top fiber stress, b = bottom fiber stress. These initial stresses refer to the prestressed section in all cases. The stresses in the case of laboratory beams refer to the end section only. The rectangular (6" by 8") beam dead load, extreme fiber stress at midspan = 218 psi.

[e]The ultimate strength and yield strength (0.1% offset) were: for the laboratory beam steel 250 ksi and and 235 ksi, respectively, and for the bridge girder steel 270 ksi and 250 ksi respectively.

TABLE 7

[a-g]CONCRETE PROPERTIES, TEMPERATURE AND HUMIDITY DATA

Property	Concrete Batch								
	Gp.A Lt.Wt	Gp.B Lt.Wt	Gp.C Lt.Wt	Slab B2 N.Wt	Slab C2 N.Wt	Slab B3 N.Wt	Slab C3 N.Wt	[f]Bridge Lt.Wt	Bridge [g]Slab N.Wt
f'_c (7 days) psi	6700	5500	6150	--	--	--	--	5600	
f'_c (28 days) psi	9350	8150	8750	4800	4140	5100	4300	6100	3500
Unit Wt (Wet) pcf	124.0	124.0	125.0	--	--	--	--	--	--
U. Wt (Dry-7d) pcf	123.0	123.5	123.5	153	152	152	153	122.0	145
Meas. Air Ent. %	4.0	6.0	6.0	--	--	--	--	--	--
Slump in	2.0	2.5	2.5	2.5	2.5	3.0	2.5	--	--
[c]Modulus of Elasticity at 7 Days psi x 10^6	--	--	a. 3.20	--	--	--	--	a. 3.04	--
	--	--	b. 3.33	--	--	--	--	b. 3.10	--
	3.68	3.35	c. 3.55	--	--	--	--	c. 3.32	--
[c]Modulus of Elasticity at 28 Days psi x 10^6	--	--	a. 3.28	--	--	--	--	--	--
	--	--	b. 3.58	--	--	--	--	--	--
	4.35	4.09	c. 4.23	4.33	3.97	4.41	4.05	3.47	3.41

[a]Lab. temp: 61-85 deg. F., avg. temp. 78 deg. F. Lab. relative humidity: 25-61%, avg. rel. hum. 40%. Avg. rel. hum. for central Iowa (from U.S. Weather Bur.): Jan.-79%, July-66%, Mean Annual 71%. For Spr-Sum-Fall, use 70%.

[b]Stress levels for creep tests were approx. design stresses for lab. beams:

Mix	Strength, f'_c, at 7 days	Stress Level for Creep Tests	% of 7d-f'_c
Gp. A	6700 psi	2010 psi	30%
Gp. B	5500	1375	25
Gp. C	6150	1845	30

[c]The modulus of elasticity values are as follows: a. Measured secant (to 0.5 f'_c) mod. of el., b. Measured initial tangent mod. of el., c. All values underlined are computed using $E_c = 33\ \sqrt{w^3}\ f'_c$, psi.

[d]Computed values of modulus of elasticity at release for bridge girders:

Girder No.	Age at Release	Strength at Rel.	[c]Mod. of El. at Rel.
152	2 days	5160 psi	3.19 x 10^6 psi
153	2	4670	3.04
154	2	4685	3.05
155	3	5130	3.19
156	3	4440	2.96

[e]Computed mod. of el. of pres. units at time of slab casting, [c]E_c x 10^6psi: Gp.B --4.09, 4.30; Gp.C--4.23, 4.44; Girders 152,153,154--3.50; Girders 155,156--3.40.

[f]Concrete specimens for data in this column obtained from casting yard for Bridge Girders 155 and 156. Measurements made in laboratory.

[g]"Design" values were used for bridge slab concrete.

Appendix C

PRINCIPAL VARIABLES AFFECTING CREEP AND SHRINKAGE

Presented here is a summary of the principal variables that affect creep and shrinkage (15, 16, 17, 18, 19) in most cases. The corresponding nominal correction factors, based on the standard conditions herein, are given earlier and shown in Figure 11 (13, 14, 16, 31). The results shown in Figure 11 and equations for these curves were developed by Branson and Christiason (13). The variables considered are minimum thickness of member, water-cement ratio in the form of slump and cement content, mix proportions in the form of percentage of fines and air content, environmental humidity, and time of initial loading and time of initial shrinkage.

The following comments refer to the nominal correction factors for creep and shrinkage (Fig. 11), which are normally not excessive and tend to offset each other. For design purposes, in most cases, these (except possibly the effect of member size and slump, as discussed in the text and in the following) may normally be neglected.

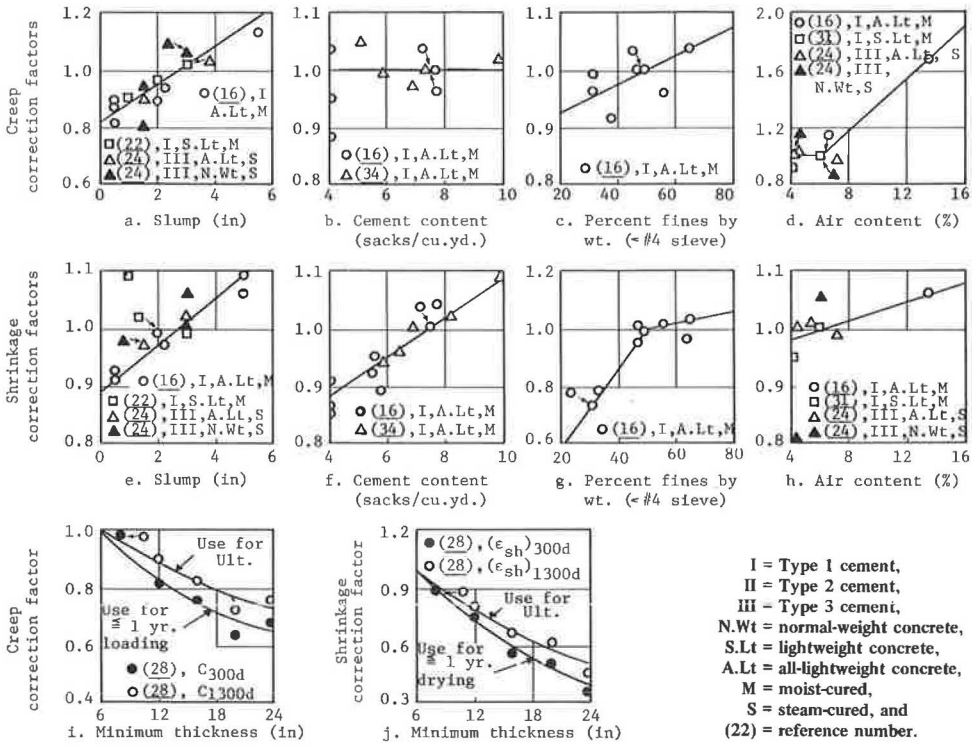


Figure 11. Nominal creep and shrinkage correction factors for the parameters shown (13).

Creep Correction Factors

Slump: C.F. = 0.95 for 2 in., 1.00 for 2.7 in., 1.02 for 3 in., 1.09 for 4 in., and 1.16 for 5 in. Tends to be offset by effect of member thickness. May be marginal but normally can be neglected.

Cement content (sacks/cu yd): C.F. = 1.00. No correction factor required for concrete of, say, 5 to 8 sacks per cu yd at least.

Percent fines (by wt): C.F. = 0.95 for 30 percent, 1.00 for 50 percent, and 1.05 for 70 percent. Normally negligible.

Air content (percent): C.F. = 1.00 for 6 percent or less, 1.09 for 7 percent, and 1.17 for 8 percent. Tends to be offset by effect of member thickness. May be neglected for, say, up to 7 percent air.

Minimum thickness of member: C.F. = 1.00 for 6 in. or less and 0.82 for 12 in. Tends to be offset by effect of slumps greater than 3 in. and air contents greater than 6 percent. Can normally be neglected for members up to about 10 to 12 in.

Shrinkage Correction Factors

Slump: C.F. = 0.97 for 2 in., 1.00 for 2.7 in., 1.01 for 3 in., 1.05 for 4 in., and 1.09 for 5 in. Tends to be offset by effect of member thickness. Normally can be neglected.

Cement content (sacks/cu yd): C.F. = 0.87 for 4 sacks, 0.95 for 6 sacks, 1.00 for 7.5 sacks, and 1.09 for 10 sacks. Normally negligible for, say, 5 to 8 sacks per cu yd at least.

Percent fines (by wt): C.F. = 0.86 for 40 percent, 1.00 for 50 percent, and 1.04 for 70 percent. May be marginal but normally can be neglected.

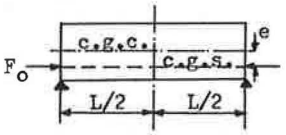
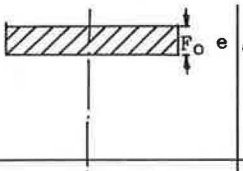
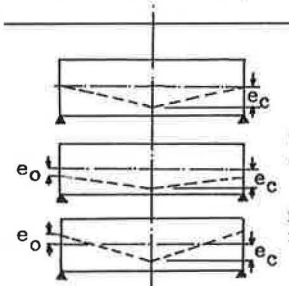
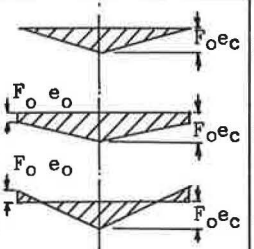
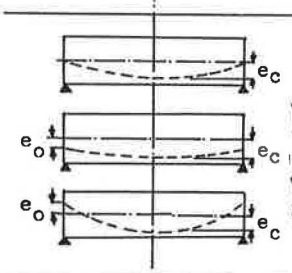
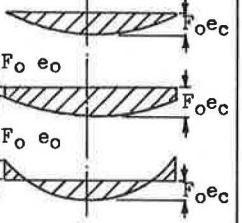
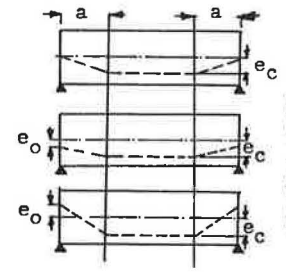
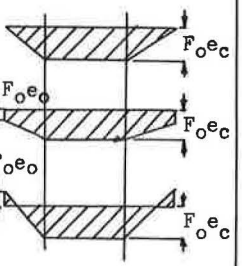
Air content (percent): C.F. = 0.98 for 4 percent, 1.00 for 6 percent, and 1.03 for 10 percent. Normally negligible.

Minimum thickness of member: C.F. = 1.00 for 6 in. or less and 0.84 for 9 in. Tends to be offset by effect of slumps greater than 3 in. Can normally be neglected for members up to about 8 to 9 in. minimum thickness.

Appendix D

PRESTRESS MOMENT DIAGRAMS AND CAMBER FORMULAS

The following are common cases of prestress moment diagrams with formulas for computing camber.

Prestressed Beam	F_o e Moment Diagram	Midspan Camber Due to F_o e Moments
		$(\Delta_i)_{F_o} = F_o e\, L^2/8\, E_{ci} I_g$
		$(\Delta_i)_{F_o} = F_o e_c L^2/12 E_{ci} I_g$ $(\Delta_i)_{F_o} = \frac{F_o(e_c-e_o)L^2}{12\, E_{ci}\, I_g} + \frac{F_o e_o L^2}{8 E_{ci} I_g}$ $(\Delta_i)_{F_o} = \frac{F_o(e_c+e_o)L^2}{12\, E_{ci}\, I_g} - \frac{F_o e_o L^2}{8 E_{ci} I_g}$
		$(\Delta_i)_{F_o} = 5\, F_o\, e_c\, L^2/48\, E_{ci}\, I_g$ $(\Delta_i)_{F_o} = \frac{5F_o(e_c-e_o)L^2}{48\, E_{ci}\, I_g} + \frac{F_o e_o L^2}{8 E_{ci} I_g}$ $(\Delta_i)_{F_o} = \frac{5F_o(e_c+e_o)L^2}{48\, E_{ci}\, I_g} - \frac{F_o e_o L^2}{8 E_{ci} I_g}$
		$(\Delta_i)_{F_o} = \frac{F_o\, e_c}{E_{ci} I_g}\left(\frac{L^2}{8} - \frac{a^2}{6}\right)$ $(\Delta_i)_{F_o} = \frac{F_o(e_c-e_o)}{E_{ci}\, I_g}\left(\frac{L^2}{8} - \frac{a^2}{6}\right) + \frac{F_o e_o L^2}{8 E_{ci} I_g}$ $(\Delta_i)_{F_o} = \frac{F_o(e_c+e_o)}{E_{ci}\, I_g}\left(\frac{L^2}{8} - \frac{a^2}{6}\right) - \frac{F_o e_o L^2}{8 E_{ci} I_g}$