The Effect of Holes on Tensile Deformations in Plain Concrete

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Recent research has shown that concrete can exhibit considerable inelastic deformation in tension. This is particularly evident in flexural and uniaxial tensile tests in stiff machines and in tests where strain gradients are present. This paper describes experimental work on the effect of centrally located holes on tensile deformation in thin concrete plates. Tests were conducted in a special uniaxial-tension machine, and strain measurements along the horizontal axes of the plates showed that extensive inelastic deformation occurred in the vicinity of the hole edges. Strains somewhat further away remained "elastic" for most of the loading range, being quite small in specimens with diamond holes. The presence of holes had apparently little effect on tensile strength. Specimen behavior is explained in terms of the energy-release concept of fracture mechanics, it being concluded that inelastic deformation is the result of progressive and extensive microcracking. Such microcracking, in which crack lengths remain relatively short, apparently occurs when the critical strain-energy release rate is retarded, and it seems certain that the strain gradients created by the presence of holes have this retarding effect. These gradients clearly localize cracking in the vicinity of the hole edges and inhibit it considerably elsewhere. With regard to tensile strength, it may be concluded that, as inelastic deformation near hole edges reaches a certain stage, stress redistribution and eventual uniformity take place.

•THE TENDENCY IN RECENT YEARS to exploit an increasingly higher proportion of the ultimate strength of concrete in the design of structures has made research into its basic nature and behavior essential. As a result of such research, it is now fully recognized that strength, deformation, and crack development and propagation are closely interrelated. It has also been discovered that the mechanism of fracture is similar under most types of loading and that crack propagation, which leads to failure, is directly related to internal tension. This discovery together with the fact that tensile strength and deformation are of particular importance in connection with shear strength, flexure, torsion, and design of prestressed concrete and liquid-retaining structures, pavements, and airfields has pointed to the necessity for thorough studies of the behavior of concrete in tension.

It is the purpose of this paper to make a brief review of research on the deformational behavior of concrete in tension and to extend the field of knowledge by presenting results and discussion of experimental work by the writer. This work consisted principally of tests in uniaxial tension of long concrete plates containing centrally located holes. Microcracking and the energy concepts of fracture mechanics are discussed where appropriate.

PREVIOUS RESEARCH ON TENSILE DEFORMATION IN CONCRETE

A number of researchers have investigated tensile deformation in concrete in both uniaxial tension and flexure (1 through 18). They have reported "cracking" strains

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ranging from 40 to 180 $\mu\epsilon$, but the bulk of the research indicates that the range of 90 to 120 $\mu\epsilon$ covers most concretes. Cracking strain is defined here as the strain at which there is significant departure from linearity in the load-strain curve and is known as the "extensibility" of concrete. It may also be considered as the elastic limit of the material.

Evans (3) and Todd (5) both considered that cracking strain had a value of about 100 $\mu\epsilon$ but, whereas Todd claimed that ultimate strain did not much exceed this value, Evans found that it sometimes exceeded 200 $\mu\epsilon$. Flexural tests on plain concrete beams by Blackman, Smith, and Young (7) gave values of 250 $\mu\epsilon$ for ultimate strain, and similar tests by Evans and Kong (14) and Welch (16) some years later produced values of nearly 500 $\mu\epsilon$.

A significant feature of the work of Blackman, Smith, and Young, who conducted tests not only in pure flexure but also in uniaxial and eccentric tension, was the discovery that the presence of a strain gradient affected the magnitude of ultimate strain. This strain varied from 140 $\mu\epsilon$ in uniaxial tension (zero strain gradient) to a maximum of 250 $\mu\epsilon$ in pure flexure. They also discovered that concrete had its lowest tensile strength in uniaxial tension and that the variation of ultimate strain with strain gradient agreed with the difference between uniaxial tensile strength and the modulus of rupture. Their findings were confirmed by Kaplan (11), who also discovered that, as the percentage of coarse aggregate increased, both cracking and ultimate strains decreased. Cracking strain in uniaxial tension, for example, decreased to a value as low as 40 $\mu\epsilon$.

The results reported so far were obtained from the usual short-term tests. Ohno and Shibata (9) have shown, however, that the magnitude of ultimate strain is also a function of rate and duration of loading. In tests conducted at relatively slow rates of loading over periods of several days, they recorded ultimate strains of over 500 $\mu\epsilon$ in uniaxial tension. That such strains could occur in concrete in uniaxial tension was a significant discovery, especially when one realizes that the tests were conducted in a relatively "soft" machine incapable of providing much restraint to the specimens.

The effect of restraint on the behavior of concrete has received particular attention in recent years, and it has come to be recognized that the magnitude of tensile strain is a function of the degree of restraint in the tensile zone. Keeton (19) has shown that tensile strains of nearly 2,100 $\mu\epsilon$ can occur internally in cylinders under concentric compression. Zielinski and Rowe (20) have found that large tensile strains can also occur in similarly loaded prisms and cubes and, as their maximum measurement of 1,170 $\mu\epsilon$ was the average over a gage length of 50.8 mm (2 in.), it is likely that the maximum tensile strain was much larger than this. From these results, it may be concluded that concrete can undergo large tensile strains under certain loading conditions. These conditions undoubtedly create some form of restraint that inhibits or retards failure. Such restraint occurs not only in the usual compressive tests but also in compressive, flexural, and tensile tests conducted in stiff machines.

By using a machine sufficiently stiff to produce the required restraint, Sturman, Shah, and Winter (15) were able to measure tensile strains in flexure as large as 1,780 $\mu\epsilon$. Because their specimens were removed from the machine before failure so that they could be specially examined for microcracking, this strain magnitude was clearly less than that of ultimate strain. Their tests also indicated that the presence of a strain gradient considerably increased the capacity of the concrete to deform, the deformation depending on the magnitude of the gradient. It seemed clear, therefore, that restraint was produced not only by the stiffness of the machine but also by the strain gradient.

Hughes and Chapman (17) were among the first researchers to test concrete in uniaxial tension in an adequately stiff machine. The stiffness of the machine made it possible for them to control the rate of strain, and they were able to record "peaked" load-strain curves. At the peak load, they measured strains of about 100 $\mu\epsilon$ and then, as the load decreased, continued to measure strains that reached as high as 1,840 $\mu\epsilon$ before failure. Similar tests by Evans and Marathe (18) produced values of ultimate strain as high as 2,800 $\mu\epsilon$.

This capacity of the material to deform under certain conditions of restraint possibly explains why it has a higher tensile strength in ordinary flexure than in ordinary

uniaxial tension. In flexure, the extreme "fibers" are initially the most highly stressed and strained, and restraint appears to be provided by the less highly strained adjacent ones. Such restraint apparently makes it possible for relatively large deformation to take place in the extreme fibers as loading increases. Kaplan's bending tests on notched beams (21) indicate that this is what happens and that the deformation is similar to the plastic type in metals. If this happens, there must come a stage in the deformation when the load-carrying capacity of the extreme fibers begins to decrease. The higher stresses are transferred to the adjacent fibers that now begin to deform more extensively, and the process continues, with external load increasing, until the zone of extensive deformation penetrates deep enough for failure to take place. An explanation of this deformational behavior in terms of microcracking and the concepts of fracture mechanics has been given by the writer (22).

TESTS ON SPECIMENS WITH HOLES

Types of Holes

The deformational behavior of concrete in tension that was revealed by tests in which restraint was produced by stiff machines and/or flexural strain gradients prompted the writer to investigate the effect of strain gradients on specimens loaded in uniaxial tension. Such strain gradients can be created by the presence of holes, and it was decided that tests would have most satisfactory results if the holes were centrally located in relatively thin concrete plates. These tests were part of a program of research on the effects of holes in concrete (23).

The holes were of various shapes and sizes, but the results shown diagrammatically here are for circular and diamond shapes of one particular size only. These results are considered sufficient to demonstrate the general behavior of the concrete. A complete report of results for all the shapes and sizes can be found elsewhere (23).

Specimens and Method of Test

The specimens were made from a specially "scaled-down" concrete mix that had a maximum aggregate size of 4.76 mm ($^3/_{16}$ in.). This was found to be the smallest size required for producing a true concrete and ensuring at the same time that gage lengths were sufficiently short for measurement to be as nearly at a "point" as possible. Such measurement was essential in view of the fact that strain in the vicinity of a hole was expected to change rapidly from point to point. Tests of 28 days on cubes and uniaxial tension specimens without holes gave strengths of 48.3 N/mm^2 (7,000 lbf/in.^{2}) in compression and 3.7 N/mm^2 (540 lbf/in.^{2}) in tension.

The specimens were 1,200 mm long by 200 mm wide by 35 mm thick and were tested in a uniaxial tension machine designed by the writer (23). The tensile force was applied through a self-centering, "lazy-tongs" device that gripped the specimens laterally. The clear length of the specimens between the upper and the lower grips was 620 mm. This length ensured that a condition of pure tension was produced at the central, minimum cross section where strain measurements were made. The gripping device was an improved and larger version of one designed by O'Clery and Byrne (24) at Dublin University nearly 10 years ago. The upper grips were connected through a pinned link to a double-flanged, universal coupling of the type used on drive shafts in agricultural machinery and thence to a rod attached to a hydraulically operated center-pull jack. The jack was located at the center of the top plate of the machine. The lower grips were connected to the base plate through a pinned link and universal coupling identical to the ones at the top. Preliminary experiments on specimens without holes indicated that the machine could provide a reliable uniaxial tension test.

Strain was measured by means of polyester-backed, electrical resistance strain gages, 10 mm long by 1.5 mm wide. These gages were placed along the horizontal axis of one face of the specimens, 5 on each side of the hole, and connected to the 10-point switching unit of a strain indicator. They were located on the specimens for which results are given here such that their centerlines were at distances from the hole edges of 2, 10, 20, 40, and 58 mm respectively. The transparency of the gage

backing and the clearly marked centerlines made accurate location relatively simple. The first 2 gages on each side of the hole were grouped relatively close together because of the rapid variation of strain expected along those sections of the horizontal axis near to the hole edges. The specimens were loaded in steps of 0.91 kN (205 lbf), and strain readings were taken at each step until failure.

Test Results

Diagrams of vertical strain distribution along the horizontal axis of typical specimens are shown in Figures 1 and 2. The distribution is shown for each load step until failure, and the points representing measured strains are joined by linear interpolation. Although the strain variation between gage locations was not likely to be linear, this method of interpolation is considered the best one for drawing the diagrams. In any case, the patterns of strain distribution are clearly illustrated by this method.

A study of the diagrams reveals some interesting features of strain behavior. These are briefly described as follows:

1. Strain distribution was such that the existence of strain gradients was clearly evident. These gradients started to increase quite early in the loading range, the strains in the vicinity of the hole edges becoming increasingly larger than those at locations farther away. This behavior began at strain levels well below the cracking strain or elastic limit. The gradients became quite steep as failure approached, particularly in the specimens with the diamond holes.

2. As loading increased, the rate of strain increase in the vicinity of the hole edges became more rapid, particularly after the elastic limit had been passed. The increase in the final load stage was very rapid indeed. Most of the strains at

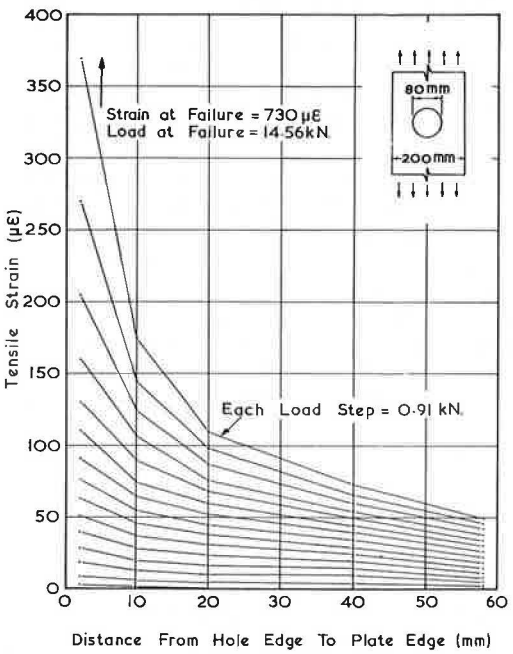

Figure 1. Vertical strain distribution along horizontal axis of specimen with central circular hole.

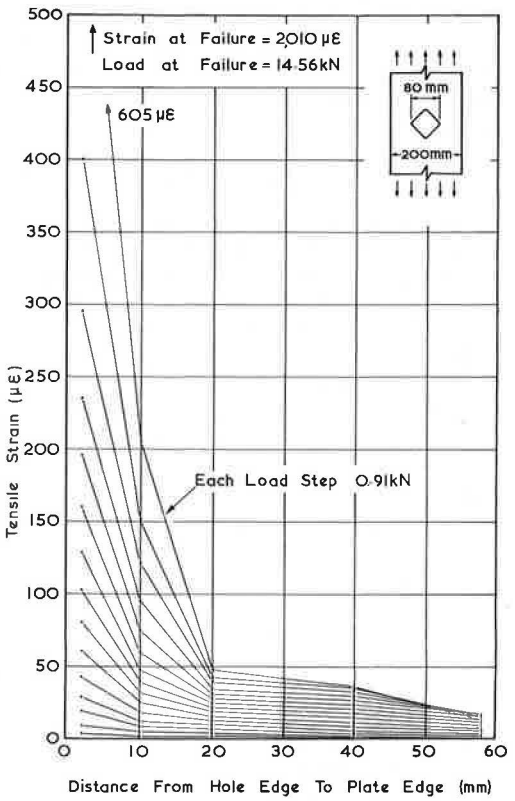

Figure 2. Vertical strain distribution along horizontal axis of specimen with central diamond hole (corner radius = 2.9 mm).

locations farther away increased at reasonably uniform rates and rarely exceeded the range of cracking strains. They were also rather smaller in the specimens with diamond holes than in those with circular ones.

3. The strains at the hole edges reached quite high values in the specimens with diamond holes, the maximum strain exceeding 2,000 $\mu\epsilon$ in 2 specimens. In a substantial proportion of the specimens with diamond holes and a somewhat smaller proportion of those with circular holes, the maximum strain at the hole edges reached values well over 1,000 $\mu\epsilon$. The strains near the specimen edges, on the other hand, were quite small and, as failure approached, showed little or no increase. In some specimens, there was even a strain decrease near failure. This strain relaxation was also evident in some of the specimens with circular holes.

Although results shown in the diagrams are for one size of hole only, it has already been explained that there were various sizes of holes for each shape. The variation of size had a marked effect on strain distribution. In the specimens with the largest size of hole, the strains at locations away from the hole edges became distinctly smaller and smaller as the specimen edges were approached. As the holes became smaller, however, the relative differences among the strains at these locations became correspondingly smaller and, in some cases, the strains at the specimen edges were equal to, or a little larger than, those at locations farther inside. The variation of size had no effect on the magnitude of ultimate strain at the hole edges. In particular, the maximum strains in most of the specimens with diamond holes were very large whatever the size of the hole.

A significant and important result of the research program was the discovery that relatively large holes and the elastic stress concentration associated with them had no appreciable effect on ultimate tensile strength of concrete. The only effect on the specimens was to reduce their central cross section and thus reduce the magnitude of ultimate load. That this behavior was almost certainly related to the deformational characteristics of concrete and its cracking mechanism is discussed in a later section of the paper.

FRACTURE MECHANICS AND MICROCRACKING

An understanding of the behavior exhibited in the tests can best be understood by a consideration of the energy concept of fracture mechanics and its relation to microcracking and crack propagation in concrete. It is appropriate, therefore, that an outline and brief discussion of research in this field be presented before proper analysis of the test results can be made.

In 1920, Griffith (25) suggested that the tensile fracture strength of brittle materials was greatly affected by the presence of small cracks and other discontinuities that existed before, or were formed after, load was applied. He claimed that these flaws acted as tensile stress concentrators and proposed a strength theory based on the energy-release concept. According to his theory, a flaw would extend by cracking when the stress concentration at its tip exceeded a certain value, thus causing the creation of a new crack surface and the transformation of strain energy to surface energy. He suggested that the crack would extend rapidly when the rate of release of strain energy was at least equal to the rate of increase of surface energy due to the formation of new surface area.

The Griffith condition for crack initiation and extension is given by the equation

$$\frac{2\Pi c\sigma^2}{E} = 4T \qquad (1)$$

in which 2c is the length of the crack, σ is the tensile field stress remote from the crack, T is the specific surface energy, and E is the modulus of elasticity.

This approach has been extended by Irwin (26) and Orowan (27) who took into account the irrecoverable work done in materials in which plastic flow occurs. They suggested that the surface energy term in Eq. 1 should be augmented by the work of plastic deformation. The view is now held that, if the augmented surface energy term is replaced by the total energy absorbed in crack initiation and extension, the modified

Griffith theory may be applied to concrete. Irwin has designated the rate of release of strain energy as G and has further designated it as G_c when it reaches the critical value required for unstable crack extension and eventual fracture. Kaplan (21) and Romualdi and Batson (28) have attempted to make quantitative determinations of G_c for concrete but, as Popovics (29) points out, the elusive values of E and T make this an extremely difficult and uncertain exercise. Nevertheless, it seems clear that G_c is a fundamental property of concrete.

In uniaxial tension tests conducted in the usual "soft" machines, fracture occurs fairly soon after the load-strain curve deviates from linearity. An explanation for this has been given by Glucklich (30), who says that, up to a certain stress, the behavior of the concrete is strictly elastic and the stress-strain curve is a straight line. At that stress, the most severe crack existing prior to load begins to extend slowly, and the specimen cross section capable of sustaining load decreases. The stress now begins to increase at a faster rate, the crack extends farther, and the stress-strain curve begins to deviate from linearity. The strain-energy release rate, $2\Pi c\sigma^2/E$, increases even faster and is soon equal to the maximum rate of energy absorbed by the crack surface, with resultant unstable crack extension and fracture.

The abrupt termination of the stress-strain curve after a relatively small deviation from linearity indicates either that there is little inelastic deformation in ordinary uniaxial tension or that the rapidity of the strain-energy release rate prevents observation of such deformation. Flexural tests of plain concrete beams in ordinary machines have shown, however, that concrete can exhibit appreciable inelastic deformation in tension, the magnitude of ultimate strain being many times greater than that of cracking strain, as mentioned earlier. The capacity of concrete to deform in this way is even more clearly demonstrated by the stiff-machine tests in uniaxial tension and flexure that are reported earlier in the paper. It seems certain that the inelastic deformation is due to a process of progressive microcracking (31, 32, 38) in which the critical rate or release of strain energy, G_c, is retarded. This relationship and the near certainty that the microcracking process is intimately related to the restraint imparted by the loading system and/or strain distribution bring us to a consideration of microcracking.

That microcracking occurs in concrete at loads measurably below the ultimate was first suspected by Brandtzaeg (33) some 40 years ago. Ultrasonic, acoustic, microscopic, X-ray, and electrical-resistance-strain-gage techniques (4, 6, 11, 13 through 16, 18, 31, 34 through 38) have since been used to study microcracking, and it has been established that microcracking exists before any load is applied. These cracks are mainly in the form of bond cracks that form along the interface between coarse aggregate and mortar. Up to about 30 percent of ultimate load, the extension of these cracks is negligible but, above this load level, the cracks begin to increase appreciably in length, width, and number. At about 50 to 60 percent of ultimate load, mortar cracks begin to bridge between already existing bond cracks, but the process is still one of controlled crack growth and is confined to short lengths. This is the "prerupture" stage (34). At about 70 to 85 percent of ultimate load, the number and length of mortar cracks begin to increase appreciably, continuous crack patterns develop, and the load-strain curve bends significantly to the horizontal. The "critical" load has now been reached and fracture soon follows. If fracture can be retarded by some form of restraint, microcracking continues progressively, an extensive cracking pattern develops, and considerable inelastic deformation takes place. The load-strain curve also begins to descend and then flattens out.

The behavior described has been established primarily from tests in uniaxial compression and flexure, but research (17, 18, 32, 36) has indicated that similar behavior occurs in concrete in uniaxial tension. The almost identical shapes of the load-strain curves obtained from stiff-machine tests in uniaxial compression (39) and tension (17, 18) give nearly certain confirmation of this similarity of behavior.

What seems to happen in stiff-machine tests is that the strain energy is only partially released into the specimen, the rest being absorbed by the machine. Consequently, the rate of release of strain energy into the specimen is retarded, the energy released is rapidly absorbed by the numerous crack surfaces distributed throughout the material,

and crack extension is soon stabilized. Crack extension is also arrested or controlled when the need for release of strain energy is increased by bond cracks entering a tougher mortar phase or mortar encountering a zone of higher strength such as an aggregate particle or a region of advanced hydration. As load increases, cracking progresses by the growth of other cracks next in order of weakness. These are, in turn, soon arrested; the load continues to increase and others begin to grow. Eventually, the microcracking progresses to a stage where the load-carrying capacity of the concrete decreases and the load-strain curve descends. Fracture, however, does not yet take place because the extensive multiplication of microcracks still provides sufficient crack surfaces for energy absorption. At some stage, the cracking pattern becomes extensive enough for only a few areas of the concrete to be able to sustain load or for no further cracks to form. Critical stresses and crack lengths now occur, G_c can no longer be retarded, rapid crack extension occurs, and fracture takes place. If the extensive multiplication of microcracks occurs within the measuring area of a strain gage, the gage will indicate considerable deformation and this is the reason for the phenomenally large strains measured by Evans and Marathe (18).

Retardation of the strain-energy release rate also seems to occur in the presence of strain gradients. This was indicated by the flexural tests of Sturman, Shah, and Winter (15) that are mentioned earlier in the paper. From their microscopic examinations, they discovered that strain gradients retarded crack extension considerably, especially in the mortar, and caused any appreciable cracking to be localized at and near the extreme fibers. The steeper the gradient, the more noticeable was this effect. It seems clear that this behavior was responsible for the extensive inelastic deformation in tension that their specimens exhibited and that strain gradients must somehow have a restraining effect sufficient to arrest or retard critical cracking.

ANALYSIS AND DISCUSSION OF TEST RESULTS

The effect of holes on stress and strain distribution in homogeneous elastic and elastoplastic materials is well treated in the relevant literature, and the writer has already made a review of research in this field (23). Figure 3 shows the distribution of vertical strain along the horizontal axis of a relatively long plate of homogeneous, elastic material subjected to a uniform tensile stress σ_a in the vertical direction. The plate contains a centrally located circular hole, the diameter of the hole and the width of the plate being the same as those of the specimen shown in Figure 1.

The maximum strain, ϵ_m, occurs at the hole edge such that

$$\frac{\epsilon_m}{\epsilon_a} = 3.74 \qquad (2)$$

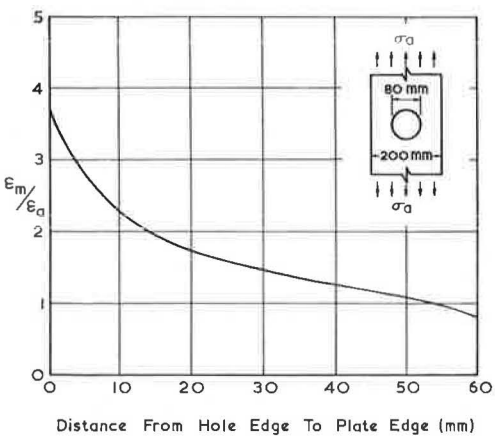

Figure 3. Vertical strain distribution along horizontal axis of elastic plate with central circular hole.

where ϵ_a is the elastic strain proportional to σ_a, the applied stress. The ratio ϵ_m/ϵ_a is known as the strain concentration factor. The corresponding stress concentration factor is σ_m/σ_a.

The strain distribution curve shows that a strain gradient exists from the very commencement of application of σ_a. The writer is not aware of any published work on strain distribution in plates of finite width containing diamond holes. Isida (40) has, however, determined that the maximum vertical strain at the hole edge along the horizontal axis of elastic plates containing diamond holes of the size and corner curvature shown in Figure 2 is such that

$$\frac{\epsilon_m}{\epsilon_a} = 10.1 \tag{3}$$

This value and the strain distribution diagrams obtained from research on plates with circular and V-shaped notches indicate that the elastic strain gradient in plates containing centrally located diamond holes is steeper than in those containing circular ones.

When the elastic limit is reached at the points of maximum strain on the hole edges in elastoplastic materials such as ductile metals, plastic zones form around these points and large strains occur there. As the load increases, more plastic zones develop in the vicinity of these points and considerable increase in strain takes place. The process continues until failure, the strains at the critical points becoming several times greater than those elsewhere. Experimental results (41) show that the strain gradient along the horizontal axis can become very steep near the hole edges as failure approaches. Stress computations from these results and the values of ultimate load indicate that, as the strain gradient increases with plastic deformation, the stress gradient decreases and effectively vanishes near failure. Stress redistribution and eventual uniformity of stress must, therefore, take place.

The strain behavior of the writer's concrete specimens was similar in many respects to that of elastoplastic materials just described, and it seems clear that the existence of strain gradients from the commencement of loading led eventually to the development and localization of inelastic deformation at and near the hole edges. From the earlier discussion, this deformation must almost certainly have been due to progressive microcracking and the retardation of the strain-energy release rate associated with it. The strain behavior must also have been responsible for the fact that the magnitudes reached by ultimate loads showed that the presence of holes had no effect on tensile strength. It may be concluded that, as microcracking became extensive near the hole edges, the load-carrying capacity of the concrete there decreased, and the stresses were redistributed to other regions farther away until uniformity of stress distribution occurred. Stress concentration effects due to the presence of holes, consequently, vanished, and the material displayed tensile strength as if there were no holes.

A significant feature of strain behavior was the fact that the strain gradients started to increase early in the loading range and at strain levels well below the cracking strain or elastic limit. As such increases could only take place in homogeneous materials after the onset of inelastic deformation at the hole edges, it is clear that they were made possible by the heterogeneous nature of concrete and its resultant capacity for microcracking under load. Such microcracking must clearly have occurred early in the most highly stressed regions near the hole edges. The early gradient increases were particularly marked in the specimens with diamond holes, and this behavior can be attributed to the fact that the relatively high elastic stress concentration factors and steep elastic strain gradients were conducive to early microcracking near the hole edges.

As microcracking progressed at and near the hole edges, the rate of strain increase there became more rapid. The rate continued thus until the development of inelastic deformation when it became considerably more rapid. The maximum stress, which exceeded 2,000 $\mu\epsilon$ in some specimens, were phenomenally large. Because they were measured at locations not exactly at the hole edges and the strain gradients near failure were very steep at these locations, the strains at the actual edges must have been even larger still. There were 2 other factors that made it very likely that the strains at the actual edges were significantly larger than those recorded. First, the strains were measured by gages that, although quite small, had finite area and could only measure average and not peak values. Second, the strains increased so rapidly before fracture that it was impossible to measure the actual values at fracture accurately. The writer estimates that the strain at fracture in some specimens was probably as large as 3,000 $\mu\epsilon$ at the actual hole edges.

The relatively small strains at locations some distance away from the hole edges and the particularly small ones at the edges of specimens with diamond holes led to the

inescapable conclusion that the effect of the strain gradients and their early increase must have been such as to inhibit deformation at these locations considerably. Moreover, the reasonably uniform rates of strain increase indicated that the concrete in these regions remained elastic till fracture. The material obviously underwent only a small amount of microcracking and crack extension despite the ultimately high stresses transferred by redistribution as failure approached. Another phenomenon was the strain relaxation in these regions, particularly at the specimen edges, just prior to failure in most of the specimens with diamond holes. The writer postulates that, as large, rapid deformations caused the concrete to "open out" at the hole edges, some form of "clothes-peg" action occurred. Such action would have caused a pinching effect at those locations away from the hole edges that were outside the "pin" of the clothes-peg.

The tendency of the strain distribution curves to have a relatively gentle gradient at locations some distance away from the hole edges in the specimens with the smaller holes was not surprising in view of the well-known findings of research on the effect of holes on homogeneous materials (Fig. 3). The fact that the strains at the edges of some of these specimens were a little larger than those at locations farther inside can be explained by the heterogeneous nature of concrete, some regions of the material being more susceptible to deformation than others. What was remarkable, however, was the fact that the size of the hole had no apparent effect on the magnitude of ultimate strain in the vicinity of the hole edges. Because the hole size and the magnitude of its associated elastic stress concentration factor do have some effect on the magnitude of ultimate strain in these regions in homogeneous materials, it seems evident that such behavior was due to the heterogeneous nature of concrete and its capacity for progressive microcracking. This microcracking was clearly limited to the vicinity of the hole edges and caused extensive inelastic deformation there whatever the size of the hole.

CONCLUSIONS

The research reported and discussed in this paper indicates the following:

1. Although fracture in ordinary uniaxial tension occurs fairly soon after the load-strain curve deviates from linearity, concrete can exhibit extensive, inelastic deformation in tension under certain conditions of loading and strain distribution. This behavior is most noticeable in stiff-machine tests in uniaxial tension and flexure and in tests in which specimens are subjected to strain gradients created either by flexure or by the presence of holes.
2. Extensive inelastic deformation is explicable in terms of the energy concept of fracture mechanics and the microcracking inherent in concrete. If the critical strain-energy release rate is retarded in some way, critical crack extension is arrested and progressive microcracking takes place. The occurrence of such microcracking within the area of a strain gage leads to observations of phenomenally large tensile strains.
3. In stiff-machine tests, the retardation of the critical strain-energy release rate can be explained by the fact that the strain energy is only partially released into the specimen, the rest being absorbed by the machine. It can also be explained to some extent by the increased need for released strain energy when cracks encounter zones of higher strength in the heterogeneous material. It is not clear what causes retardation of the rate in the presence of strain gradients, but microscopic and X-ray examinations have shown that cracks, especially in the mortar, are localized in the regions of largest strain and confined to short lengths.
4. Uniaxial tension tests on long, thin concrete plates show that the strain gradients created by the presence of centrally located holes have a marked effect on the deformational behavior of concrete. Inelastic strains develop and localize at and near points on hole edges where the highest stresses occur in the elastic range. These strains can become very large and reach values several times higher than limiting elastic strain. Strains at locations some distance away from the hole edges, on the other hand, are relatively small and generally remain within the elastic range up to failure. Those at the specimen edges can be very small indeed, and, clearly, the presence of strain gradients inhibits deformation in these regions considerably.

5. The strain relaxation evident at those locations outside the regions of large deformation just prior to failure in most of the specimens with diamond holes can be explained by the writer's clothes-peg theory. It seems that, as the large, rapid deformations occur near failure and cause the concrete to open out at the hole edges, some form of clothes-peg action takes place, resulting in a pinching effect at locations some distance away from the hole. The effect is more marked as the specimen edges are approached.

6. The presence of relatively large holes has apparently no effect on tensile strength of concrete specimens apart from the weakening effect of reduced cross section. This can be explained by the fact that concrete exhibits similar deformational behavior to that of elastoplastic materials containing centrally located holes, uniform stress distribution occurring before failure. The only difference is that "plasticity" in concrete is really the result of brittle microbehavior.

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