Deformation of Concrete and Its Constituent Materials in Uniaxial Tension

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Stress-strain curves for concrete loaded in uniaxial tension are shown for a wide variety of mixes. The data illustrate the influence of water-cement ratio and maximum size, grading, and type of aggregate on the form of the stress-strain curve. In addition, all mixes are found to conform to a dimensionless stress-strain curve plotted in terms of tensile strength and failure strain. Stress-strain curves for cement paste and rock are also shown, and values for the bond strength of various paste-rock combinations are included. Experimental values of the initial tangent modulus of different concretes are compared with analytical values obtained by using equations proposed in the literature, and fairly good agreement is observed.

•THE STRESS-STRAIN BEHAVIOR of concrete loaded in uniaxial compression has been the subject of a great deal of research and is now considered to be fairly well understood. Although many researchers believe that the behavior in uniaxial tension is very similar, the data supporting or repudiating this view are very limited. This paper gives tensile stress-strain curves obtained for a wide variety of concrete mixes and attempts to isolate the influence of various mix parameters on the form of the curve. Tensile stress-strain curves for cement paste and different rocks from which the aggregate was crushed are shown, and paste-rock bond strengths are also included. In conclusion, an attempt is made to explain concrete behavior in uniaxial tension in terms of the tensile properties of the constituents, using some of the different models referred to in the literature to explain its behavior in compression.

TESTS

Testing Technique

Uniaxial tension tests were performed at a loading rate of 25 psi/min on 6 in. square (aggregate maximum size $1\frac{1}{2}$ in.) or 4 in. square (aggregate maximum size $\frac{3}{4}$ or $\frac{3}{8}$ in.) prisms. Four-in. square prisms were used for cement paste, rock, and paste-rock bond tests. Load was applied through tension grips in which the specimen was held solely by friction. Strains were optically recorded by a pair of 8-in. roller extensometers. The test method has been analysed and discussed in detail in a previous paper (1), where coefficients of variation ranging from 4 to 6 percent depending on aggregate maximum size are quoted for concrete. All data subsequently discussed refer to Type 1 cement concretes or pastes cured in water for 28 days and tested in a saturated condition.

Mixes

The mixes used in the test program fall into 2 main groups referred to as Series 1 and 2. Series 1 comprised 36 basalt aggregate mixes in 3 groups of 12 having effective

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Paper sponsored by Committee on Mechanical Properties of Concrete and presented at the 49th Annual Meeting.

water-cement ratios of 0.35, 0.45, and 0.55. Twelve different aggregate gradings, 4 each for aggregate maximum sizes of 1½, ¾, and ⅜ in., are examined, and the percentage of material finer than a No. 4 sieve is indicated where appropriate. The gradings employed are those given in Road Note 4 (2) and Research Report 4 (3). Series 2 comprised 36 mixes employing 5 crushed rock aggregates and an irregular gravel. Effective water-cement ratios were as in Series 1, and 3 gradings, representing each aggregate maximum size, were selected from the range covered in Series 1.

INFLUENCE OF MIX PARAMETERS

Aggregate Maximum Size and Grading

Figures 1, 2, and 3 show the tensile stress-strain curves obtained from the Series 1 mixes. It is evident that both the tensile strength and the failure strain increase as the aggregate maximum size decreases, and that, for a constant aggregate size, both increase as the sand (material finer than a No. 4 sieve) content increases. The variation of both parameters with aggregate maximum size and grading has been related in a previous paper (4) to the mean particle diameter for the grading and will not be discussed further. As the present paper is mainly concerned with values of elastic moduli, the important feature of Figures 1, 2, and 3 is the essentially constant value of the initial tangent modulus at any particular water-cement ratio. Only in the 2 finest ⅜-in. gradings is a slight decrease evident. This is attributable to the relatively higher paste content of these mixes compared with the other ten in each figure. These results imply that the tangent modulus is independent of aggregate size and grading. However, it is likely to be affected to a small extent by changes in the proportion of fine to coarse aggregate, because these 2 materials do not generally have the same elastic modulus.

Water-Cement Ratio

Figures 1, 2, and 3 show that both the tensile strength and the failure strain decrease as the water-cement ratio increases, and appropriate relationships have been developed

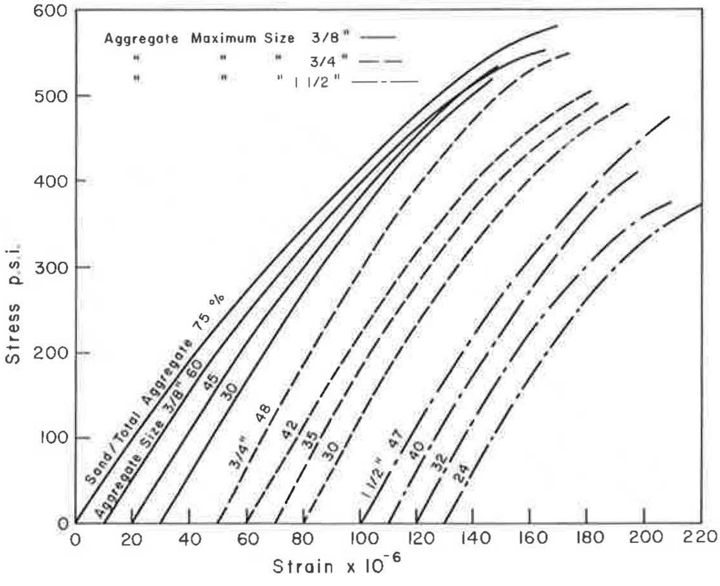

Figure 1. Tensile stress-strain curves for basalt aggregate concretes (Series 1 mixes) of water-cement ratio 0.35.

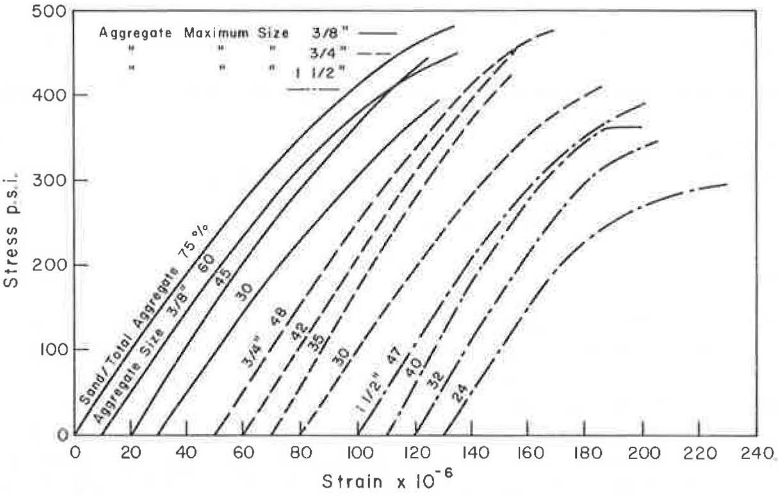

Figure 2. Tensile stress-strain curves for basalt aggregate concretes (Series 1 mixes) of water-cement ratio 0.45.

in the earlier paper (4). As far as the tangent modulus is concerned, a slight decrease is evident as the water-cement ratio increases. However, it is incorrect to try to relate this change to water-cement ratio alone, as both the elastic modulus of the paste and its volumetric proportion in the mix alter when the water-cement ratio changes.

Aggregate Type

Figures 4, 5, and 6 show relevant tensile stress-strain curves obtained from the Series 2 mixes. Once again, relationships involving tensile strength and failure strain

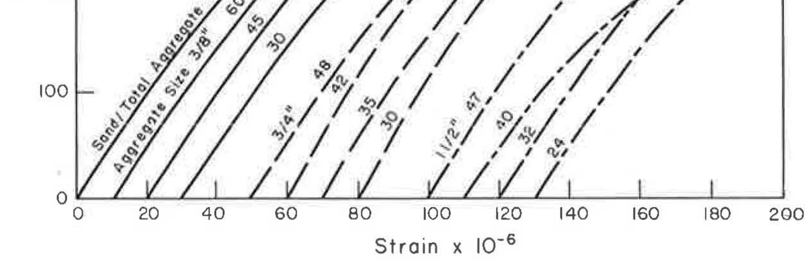

Figure 3. Tensile stress-strain curves for basalt aggregate concretes (Series 1 mixes) of water-cement ratio 0.55.

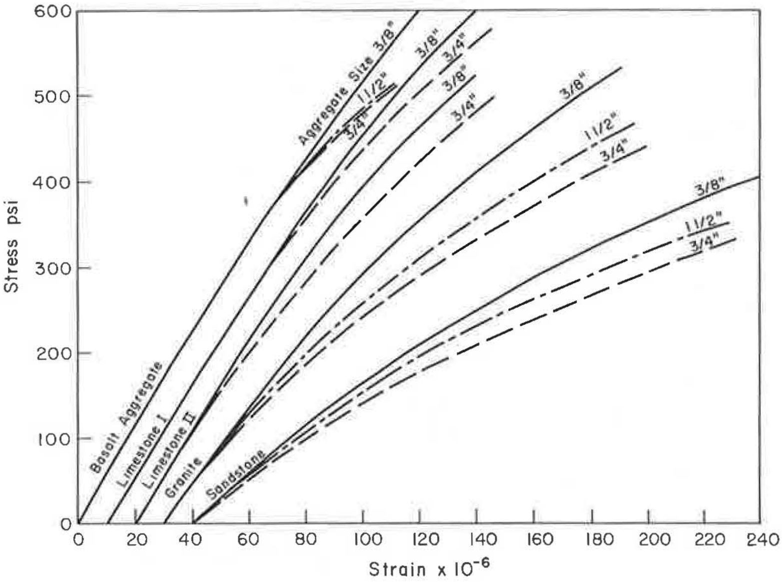

Figure 4. Tensile stress-strain curves for different aggregate concretes (Series 2 mixes) of water-cement ratio 0.35.

are dealt with in the earlier paper (4), and it is sufficient to note that, while the tensile strength increases, the failure strain decreases as the elastic modulus of the aggregate increases. Figures 4, 5, and 6 also show a marked change in the initial tangent modulus of concrete as the elastic modulus of the aggregate alters, while its value is once again seen to be essentially independent of aggregate size and grading for any given aggregate. The relationship is more clearly shown in Figure 7 where points for concretes

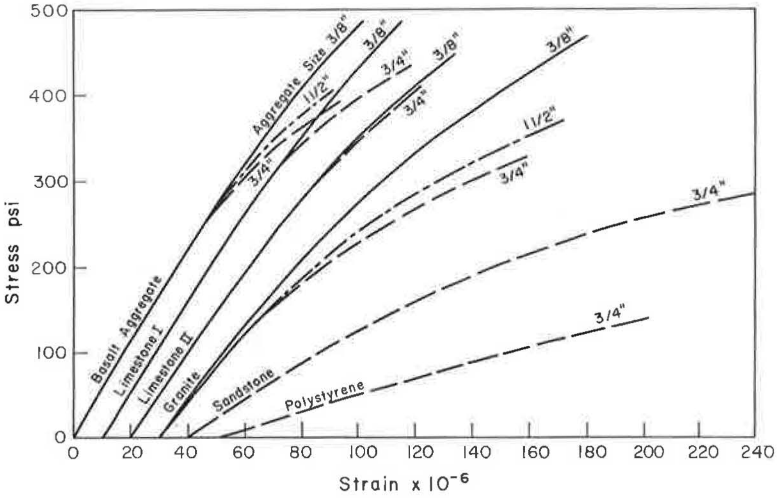

Figure 5. Tensile stress-strain curves for different aggregate concretes (Series 2 mixes) of water-cement ratio 0.45.

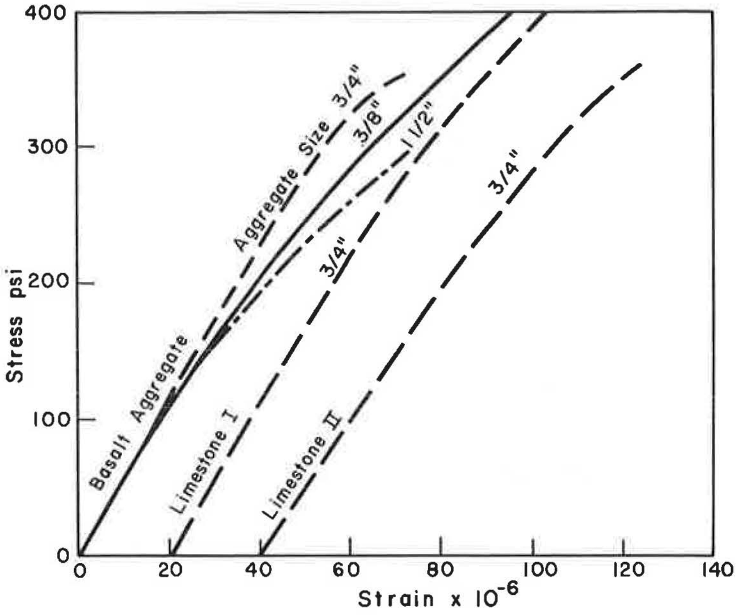

Figure 6. Tensile stress-strain curves for different aggregate concretes (Series 2 mixes) of water-cement ratio 0.55.

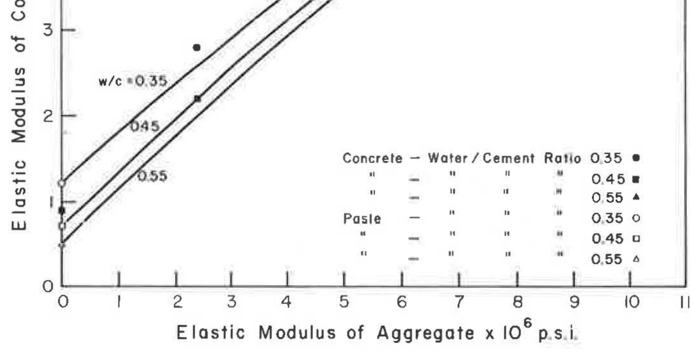

Figure 7. Elastic modulus of an aggregate and the initial tangent modulus of its concretes.

TABLE 1

MATERIAL PROPERTIES OF CONCRETE

Water-Cement Ratio	Paste Content (percent)	Aggregate	Elastic Modulus (10^{-6} psi)		
			Concrete	Aggregate	Paste
0.35	42	Basalt	5.9	10.6	2.84
		Limestone I	5.5	10.3	2.84
		Limestone II	5.3	7.8	2.84
		Granite	4.4	6.2	2.84
		Sandstone	2.8	2.4	2.84
		Voids	1.2	0.0	2.84
0.45	33	Basalt	5.6	10.6	2.05
		Limestone I	5.4	10.3	2.05
		Limestone II	4.9	7.8	2.05
		Granite	4.2	6.2	2.05
		Sandstone	2.2	2.4	2.05
		Voids	0.7	0.0	2.05
		Polystyrene	0.9	?	2.05
0.55	29	Basalt	5.4	10.6	1.63
		Limestone I	5.5	10.3	1.63
		Limestone II	4.8	7.8	1.63
		Voids	0.5	0.0	1.63

containing aggregate of zero elastic modulus are also included. These are calculated by using appropriate values of the elastic modulus of cement paste and the volumetric proportion of paste in the mix given in Table 1. It is interesting to note the close agreement between the 0.45 water-cement ratio paste and a similar concrete made with polystyrene aggregate of very low elastic modulus, whose properties became available by chance from a separate undergraduate project.

DIMENSIONLESS STRESS-STRAIN FUNCTION FOR TENSION

In view of the dimensionless nature of the stress-strain curve for compression observed by Gilkey (5), the preceding stress-strain curves for tension are analysed in a similar manner. The reference parameters are the maximum stress achieved and the corresponding strain. This is the maximum strain achievable, except when special stiff testing apparatus is used to detect the descending portion of the stress-strain curve. As an example, the percentage strain corresponding to 50 percent maximum stress is considered. The results for the Series 1 mixes, in which the variables are water-cement ratio and aggregate grading, are given in Table 2, and those for the Series 2 mixes, in which the variable is aggregate type, are given in Table 3. It is apparent from Tables 2 and 3 that there is no definite change in the tabulated value with mix parameters and that the values are sufficiently similar to be regarded as constant. The similarity of the mean value and standard deviation obtained in each table support this view, and the dimensionless curve shown in Figure 8 is accordingly obtained. This curve makes it possible for the 28-day stress-strain curve for any concrete containing

TABLE 2

STRAIN (PERCENTAGE OF FAILURE STRAIN) AT A STRESS OF 50 PERCENT OF THE TENSILE STRENGTH, SERIES 1 MIXES

Water-Cement Ratio	Basalt Aggregate in All Mixes											
	1½-in. Aggregate Grading				¾-in. Aggregate Grading				⅜-in. Aggregate Grading			
	No. 1	No. 2	No. 3	No. 4	No. 1	No. 2	No. 3	No. 4	No. 1	No. 2	No. 3	No. 4
0.35	36.1	36.1	45.4	38.9	38.8	35.5	38.3	38.0	39.8	42.6	36.9	37.9
0.45	36.8	41.0	41.5	36.7	40.6	44.0	46.0	38.8	42.2	42.3	38.1	41.6
0.55	43.2	35.8	33.7	40.0	44.8	39.5	34.7	41.4	39.0	41.2	40.9	35.9

Note: Mean value 39.4 percent; standard deviation ±3.0.

TABLE 3

STRAIN (PERCENTAGE OF FAILURE STRAIN) AT A STRESS OF 50 PERCENT OF THE TENSILE STRENGTH, SERIES 2 MIXES

Aggregate Type	Water-Cement Ratio for $1\frac{1}{2}$-in. Aggregate Grading No. 2			Water-Cement Ratio for $\frac{3}{4}$-in. Aggregate Grading No. 1			Water-Cement Ratio for $\frac{3}{8}$-in. Aggregate Grading No. 2		
	0.35	0.45	0.55	0.35	0.45	0.55	0.35	0.45	0.55
Basalt	42.9	41.4	36.4	41.8	36.8	40.4	45.0	45.5	43.9
Granite	39.4	35.5		38.5	36.0		39.7	40.5	
Gravel	34.7	31.8		37.8	36.2		39.3	38.9	
Sandstone	40.0			40.3	38.4		36.5		
Limestone I				41.2	37.2	44.5	41.5	44.3	
Limestone II				42.1	42.7	43.7	39.8	42.0	

Note: Mean value 39.9 percent; standard deviation ±3.2.

Type 1 cement to be derived once the tensile strength and the failure strain are known. These parameters can be predicted by using the relationships developed by Johnston (4).

PROPERTIES OF THE CONSTITUENT MATERIALS

Stress-Strain Behavior of Cement-Paste and Aggregate

Stress-strain curves for the 5 rock types and for cement pastes having water-cement ratios equal to those of the concretes of Series 1 and 2 are shown in Figure 9. The data shown in Figure 9 indicate that most rocks and cement pastes, unlike concrete, have linear stress-strain curves. Some difficulty was experienced in testing the white limestone (limestone II) because of the presence of numerous, almost invisible, geological joints and fossils commonly found in this type of rock, so it is doubtful whether the full extent of the curve for this material is shown. With the exception of the curve for limestone II, the data indicate that aggregate tensile strength is greater than that of the strongest corresponding concrete. Therefore, it is to be expected that concrete failure in tension involves negligible aggregate fracture, a fact observed throughout the wide variety of mixes tested.

In contrast to the fairly consistent behavior of concrete, that is coefficients of variation in strength of 4 to 6 percent, the behavior of cement paste was extremely variable. Although the slope of the various stress-strain curves obtained was essentially constant at a particular water-cement ratio, the tensile strength varied considerably, coefficients of variation being about 3 times those for concrete. This suggests that the relatively less variable strength of concrete is attributable to the presence of aggregate particles acting as crack arrestors in the paste, which on its own behaves as a brittle material with very variable strength governed by the presence of randomly oriented flaws.

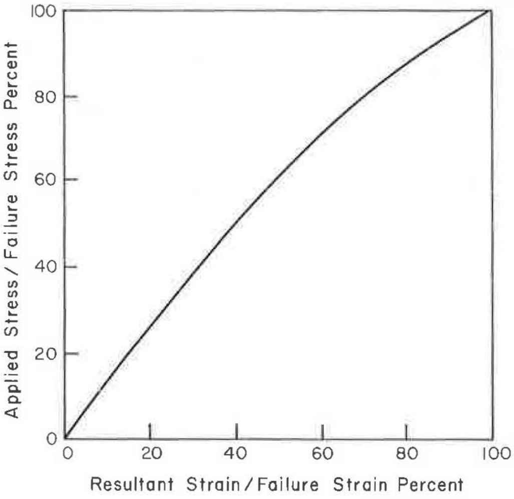

Figure 8. Dimensionless stress-strain function for concrete in uniaxial tension.

Paste-Rock Bond Strength

The first series of bond tests was performed on 4-in. square cross sections of cement paste and sawed rock joined in the orientation shown in Figure 10a. The uni-

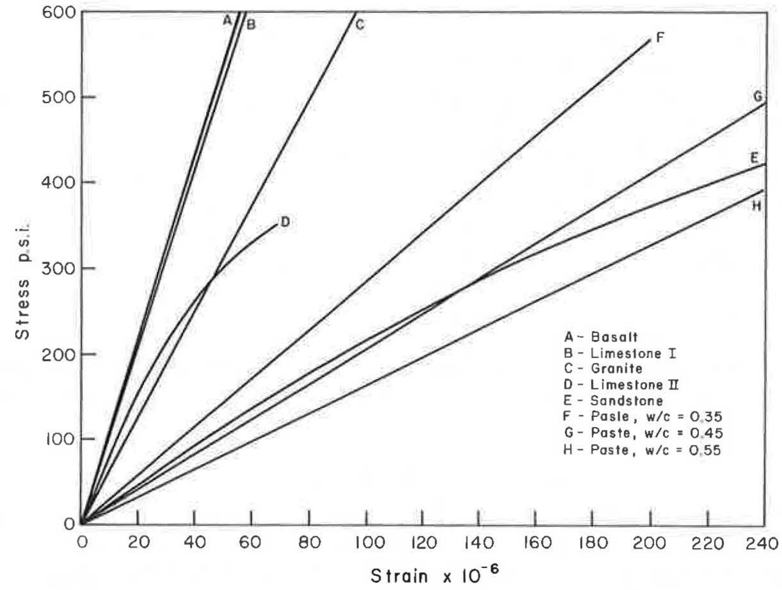

Figure 9. Tensile stress-strain curves for various rocks and cement pastes.

axial tensile bond strengths of various paste-rock combinations are given in Table 4. Results are the mean obtained from 4 tests and, in common with other similar investigations (6, 7), exhibit high coefficients of variation. However, they establish the order of tensile bond strength, although no quantitative relationship to rock type or water-cement ratio is evident in this limited number of results. Alexander (6), from more extensive data, has indicated that bond strength decreases with increasing water-cement ratio and decreasing silica content. However, the strengths given in Table 4 are several orders of magnitude lower than those of Alexander (6). One possible contributory factor is the well-known difference between strengths obtained from midpoint flexural loading and those obtained from uniaxial tensile loading. Moreover, Hsu and Slate (7) have obtained values of the same order as the writer, so it appears that bond strength is very dependent on the method of determination.

The second series of tests was performed by using a 0.45 water-cement ratio paste joined to sawed basalt in all the

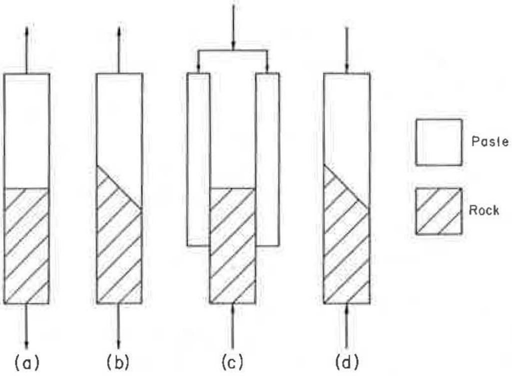

Figure 10. Orientations used for testing the paste-rock bond strength.

TABLE 4

PASTE-ROCK BOND STRENGTHS IN UNIAXIAL TENSION

Rock Type	Bond Strength (psi) W-C Ratio 0.35	W-C Ratio 0.45	W-C Ratio 0.55	Mean
Basalt	212	195	199	202
Limestone I (dolomitic)	234	130	190	185
Limestone II (white)	64	78	71	71
Granite	107	173	233	171
Sandstone	109	112	146	122

various configurations shown in Figure 10, and results are given in Table 5. It is apparent that the paste-rock bond strength is very dependent on orientation, increasing from a minimum in pure tension to a maximum in pure compression when the strength of the weaker material rather than bond becomes the criterion. In addition, Alexander (6) has shown that bond strength depends on the orientation of the interface relative to the direction of casting, and he gives values of 108 and 46 percent for top and bottom surfaces respectively, relative to side or vertical surfaces as 100 percent. These variations are attributable to various degrees of bleeding of water from the cement paste and its accumulation at the interface.

TABLE 5

BOND STRENGTHS FOR VARIOUS LOADING ORIENTATIONS

Loading Orientation	Bond Strength (psi)
Uniaxial tension	186
Equal tension and shear at 45 deg	139
Pure shear	251
Equal compression and shear at 45 deg	924
Uniaxial compression	5,500[a]

[a] Cement paste strength.

ANALYTICAL MODELS

Some of the most significant work in this field has been presented by Hansen (8) in a paper that summarizes the variety of mathematical expressions for the elastic modulus of concrete derived by him and many other workers. The following are 2 simple equations suggested in Hansen's paper for a 2-phase material consisting of elastic particles, subscripted 2, dispersed in a matrix, subscripted 1.

Based on equal strains in each phase,

$$E = V_1E_1 + V_2E_2 \tag{1}$$

and based on equal stresses in each phase,

$$E = \frac{1}{\dfrac{V_1}{E_1} + \dfrac{V_2}{E_2}} \tag{2}$$

In addition, a complex equation for spherical particles evenly distributed in a continuous matrix, obtained by Hashin, has been simplified by assuming that Poisson's ratio for all phases is 0.2. This yields the following result:

$$E = \left[\frac{(1 - V_2)\,E_1 + (1 + V_2)\,E_2}{(1 + V_2)\,E_1 + (1 - V_2)\,E_2}\right] E_1 \tag{3}$$

Equation 1 is the upper bound value for the elastic modulus of a 2-phase material, which Hansen suggests is applicable for $E_1 > E_2$. Equation 2 is the lower bound value, which he suggests is applicable for $E_1 < E_2$, provided that there is no bond between particles and matrix. Equation 3 is an intermediate value, which he suggests is applicable for $E_1 > E_2$ and also for $E_1 < E_2$, provided that there is maximum bond between particles and matrix.

Average values for concrete, obtained for the Series 1 mixes shown in Figures 1, 2, and 3 are compared with values from Eqs. 2 and 3 given in Table 6, as the condition $E_1 < E_2$ applies. It is apparent that Eq. 3 gives good agreement, particularly at the higher water-cement ratios. In view of this a similar comparison is given for

TABLE 6

EXPERIMENTAL AND CALCULATED ELASTIC MODULI FOR SERIES 1 MIXES

Water-Cement Ratio	Experimental	Eq. 2	Eq. 3
0.35	6.1	4.92	5.71
0.45	5.4	4.22	5.38
0.55	5.1	4.05	5.14

Note: Amounts are in 10^{-6} psi.

TABLE 7

EXPERIMENTAL AND CALCULATED ELASTIC MODULI FOR SERIES 2 MIXES

Water-Cement Ratio	Experimental	Eq. 1	Eq. 2	Eq. 3	Eq. 3 Amended
0.35	5.9	7.31	4.90	5.68	5.71
	5.5	7.14	4.88	5.61	5.66
	5.3	5.70	4.48	4.93	5.21
	4.4	4.78	4.13	4.38	4.91
	2.8	2.59	2.57	2.58	3.54
		1.20	0.0	0.76	
0.45	5.6	7.67	4.37	5.34	5.36
	5.4	7.48	4.33	5.26	5.31
	4.9	5.83	3.98	4.62	4.90
	4.2	4.55	3.66	4.08	4.58
	2.2	2.28	2.27	2.27	3.24
	0.9P	0.70	0.0	0.42	
0.55	5.4	7.92	4.02	5.08	5.10
	5.5	7.71	3.98	5.02	5.07
	4.8	5.96	3.66	4.40	4.67
		0.49	0.0	0.29	

Note: Amounts are in 10^{-6} psi.

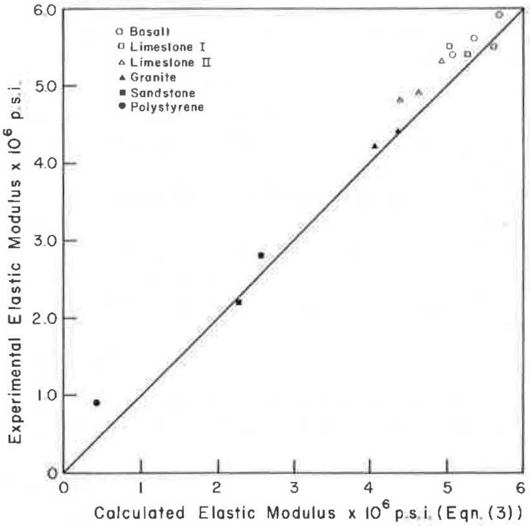

Figure 11. Comparison of experimental and calculated elastic moduli.

the wider variety of mixes of Series 2 in Table 7 and Figure 11. Once again, Eq. 3 gives good agreement while tending to slightly underestimate the experimental values. Although Eq. 1 may apply for $E_1 > E_2$ (see polystyrene concrete), it substantially overestimates the experimental values for $E_1 < E_2$. Eq. 2 is inaccurate for $E_1 < E_2$, yields the trivial answer of zero when E_2 is zero, and gives reasonable values only if $E_1 \simeq E_2$, when both Eqs. 1 and 3 also give similar values. In short, it might be concluded that Eq. 3 represents normal-weight concrete and Eq. 1 represents lightweight concrete. However, one unknown factor in these analyses is the elastic modulus of the fine aggregate. Up to now E_2 has been assumed to be equal to the elastic modulus of the coarse aggregate, but it is, of course, the resultant of the volumetric proportioning of the moduli of the fine and coarse aggregates. By using a value of 11.0 x 10^6 psi given by Hansen (8) for pure quartz sand similar to that used in this investigation, amended values for Eq. 3 have been calculated, and it is evident that they introduce a marked discrepancy when the modulus of the coarse aggregate is noticeably less than that of the fine aggregate. Therefore, in spite of the good agreement with the original values obtained by using Eq. 3, it must be asked whether the aggregate particles behave in such a way that only the modulus of the large particles is of paramount importance, or whether the modulus of the sand is considerably smaller than assumed. Unfortunately, no further data are available to answer this question.

CONCLUSIONS

1. The initial tangent modulus of the stress-strain curve in uniaxial tension is essentially independent of aggregate maximum size and grading, somewhat dependent on water-cement ratio, and very dependent on the elastic modulus of the aggregate. The tensile strength and the failure strain are related to all 3 mix parameters as shown by Johnston (4).

2. The tensile stress-strain curves for any 28-day concrete containing Type 1 cement can be included in a single curve plotted by using dimensionless coordinates, as shown in Figure 8.

3. Cement paste and rock, unlike concrete, have linear stress-strain curves in tension. Cement paste is particularly brittle and variable in its behavior, and it is concluded that the relatively less variable behavior of concrete is attributable to the

crack-arresting action of the aggregate particles. Paste-aggregate bond strengths in both tension and shear are considerably less than concrete strengths, so the progressive curvature of the stress-strain curve occurs as bond is broken down. Unlike compression, when aggregate particles may still transmit stress in the absence of bond, the stress in the aggregate must fall gradually to zero under uniaxial tension, and it is to be expected that the concrete just before failure will behave as if it contained aggregate of zero elastic modulus.

4. The initial tangent modulus of normal-weight concrete seems to be accurately predicted for a wide range of aggregates by using Eq. 3 and by assuming that the modulus of elasticity of the coarse aggregate is representative of the combined fine and coarse aggregates. This finding is contrary to that of Hansen, who concludes, mainly on the basis of compression test data, that Eq. 2 is applicable to normal-weight concrete.

ACKNOWLEDGMENT

The author gratefully acknowledges the financial support contributed by the National Research Council of Canada toward the preparation and presentation of the paper.

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