The Inelastic Deformation of Concrete

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A knowledge of the complete stress-strain curve of concrete is essential for ultimate strength design. Conventional stress-controlled tests give little information on deformation beyond the maximum load. Tests are reported on concrete cylinders and prisms at constant rate of straining using a servomechanism connected to the hydraulic system and automatically recording the load deformation. The effect of specimen size, aggregate type, age, strength, and cyclic loading on the deformational behavior of concrete is studied. The strain at maximum load tends to increase with concrete strength and with age. The ultimate strain at collapse is many times that at maximum load and is independent of strength and height-width ratio of specimen. At equal deformations beyond the maximum load, low-strength concrete and specimens of smaller height-width ratio possess greater load resistance. Age and strength also influence the shape of the stress-strain curve. The elastic modulus based on total strain is about 55 to 80 percent of that obtained from strains in the central part. The presence of the falling branch suggests different cracking and failure mechanisms in the ascending and descending parts, and the significance of internal friction at decreasing loads. The ability to sustain decreasing loads at increasing deformation relieves stress concentrations and ensures plastic flow without violent failure.

The stress-strain relation of a material forms the basis of all design of structural members. With a material such as concrete, which is subjected to time-dependent deformation due to environmental conditions in addition to that due to superimposed load, a knowledge of the complete stress-strain behavior including the post-yield deformation is necessary for an adequate appreciation of the ultimate strength design of concrete members, for the prevention of cracks, and for the design for movements in concrete structures. Although the principles of ultimate strength design of reinforced and prestressed concrete members are well-established, it is still far from clear as to the exact nature of the relationship of the stress-strain characteristics of concrete in concentric compression and in flexural compression.

Very high ultimate strains are generally required if a structural member is to develop its full moment of resistance and exhibit ductile characteristics of failure, and these strains cannot normally be measured when a compression specimen is tested at a constant rate of loading. In a conventional stress-controlled test, the test specimen fails suddenly at a peak strain of about 2,000 microstrains ($10^{-6}$ mm/mm) and gives the impression that concrete behaves like a brittle material with little or no inelastic deformation prior to fracture. The behavior of concrete in flexural compression, on the other hand, shows considerable inelasticity, and ultimate strains in excess of 4,000 microstrains have often been observed in the compression fibers of flexural failure.

This apparent brittle behavior of concrete is due to the flexibility of the hydraulic system and the lack of rigidity of the testing machine, rather than to an inherent property of the material. The possibility of increasing deformation with decreasing loads beyond the maximum was recognized more than 30 years ago ($1, 2$), but it was Whitney ($3$),

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who first reported the complete stress-strain curves in compression and showed that concrete possesses a post-peak deformation that is represented by the falling portion of the stress-strain curve. By applying deformation at a constant rate, strains up to 8,000 microstrains were reported. Because of the increasing deformation with falling loads, the rate of straining has a significant effect on the shape of the stress-strain curve (4, 5, 6). The lack of stiffness of the testing machine is much more significant when testing high-strength concrete, and even with a very stiff machine sudden failures could occur with high-strength concrete (7, 8).

Conventional testing machines with control of load rate can be transformed into control of strain rate by using the machine to deform an elastic member, the deformation of which can be utilized to control the deformation of the test specimen. Such techniques have been successfully used for compression tests (9, 10, 11, 12) and, more recently, for axial tension tests (13), modulus of rupture tests (14), and splitting tests (15). On the other hand, the elastic characteristics of the test frame can be modified by the use of a hydraulic compensator (16). Machines capable of applying a constant rate of deformation have also been developed (17, 18). All these tests show that the specimens remain reasonably intact even when loaded to strains many times that of the stress-controlled test.

CONSTANT STRAIN DEVICE

The major difficulty in obtaining the post-yield deformation arises from the fact that the stiffness of the concrete specimen is of the same order as that of a conventional hydraulic testing machine. In a conventional stress-strain test, the stiffness or the spring rate $K_s$ of the specimen is initially positive, becomes zero at the peak load, and then becomes negative; whereas, the stiffness of the machine $K_m$ remains constant and is represented by a straight line. Over the falling branch of the stress-strain curve the stiffness of the specimen becomes negative, and, if this value is greater than the stiffness of the machine, a region of instability occurs soon after the maximum load that precipitates either a sudden failure or a crack through to a new position of equilibrium (Fig. 1). In using an elastic frame as a load-carrying device during the post-yield deformation stage, the total applied load increases with increase in the total deformation, and the overall stiffness of the system remains always positive.

In the tests reported here this incompatibility of the stiffness characteristics of the specimen and the testing machine has been overcome by the use of a servomechanism and 3 displacement transducers mounted between the platens of the testing machine.

![Figure 1. Instability condition due to stiffness characteristics of test specimen and testing machine.](image-url)
The unit is designed for use with any hydraulically controlled testing machine and, by direct connection to the pressure line, regulates the applied load to the specimen by the operation of an electromagnetic valve. Any desired strain rate of loading can be achieved by comparing the measured strain with that required, which is done automatically using a servoamplifier, the output of which controls the electromagnetic valve. By control of the hydraulic system, the machine is made sufficiently stiff to reduce the loading at the instant of instability to maintain a constant rate of straining, so that the complete load-deformation curve of the specimen can be obtained. The load and deformation regulating console, together with the servomechanism and the hydraulic regulating valve, is shown in Figure 2.

**DESCRIPTION OF TESTS**

The tests reported here were carried out on prisms and cylinders of various sizes. Ordinary portland cement obtained from one delivery was used throughout the tests. Four different types of aggregates, consisting of crushed gravel and crushed rocks of granite, basalt, and limestone, were used. Two different gradings of coarse aggregate were used: a single size of 10 mm and a continuous grading of 5 to 19 mm. The same fine aggregate was used for all the tests and consisted of a washed pit sand with an average fineness modulus of 3.1.

Different mix proportions were generally used. The crushed gravel (5 to 19 mm) and the crushed basalt and crushed limestone tests were all carried out with nominal 1:2:4 mix proportions. The size and grading of the aggregate, the size of the specimen, and the mix proportions are shown in the figures.

All the specimens were cast under the same conditions and were internally vibrated. The specimens were kept in molds for 3 days, were then kept under water for 7 days, and were subsequently stored in a room of constant temperature and humidity. The tests were carried out in a 2,000-kN Amsler hydraulic testing machine with the hydraulic load regulator attached to the pressure system. The displacement of the platens was controlled by 3 differential-transformer type transducers, and, along with the load, was directly fed to an X-Y plotter. In addition, the end-to-end displacement of the specimen was also measured by means of 3 dial gages reading to about $10 \times 10^{-3}$ mm.

All the tests were carried out at a constant strain rate of 2,000 to 2,500 microstrains per minute. At least 3 specimens of any batch were tested at any given age. Because of the variations of concrete in the specimens, small fluctuations in the loading rate and differences in the stress-strain characteristics were inevitable. All but Figure 4 show as near reproductions as possible of the curves recorded through the X-Y plotter. The dial gage readings measuring end-to-end displacements of the specimen generally agreed within 3 percent, and tests with larger differences were discarded. It is emphasized that the strains reported represent the overall average strains measured end-to-end of the specimen. In a few tests, strains over the central part of the specimen were also measured by electric resistance gages on opposite sides. From the large amount of information obtained from the tests, only typically representative data are presented here.
DISCUSSION OF TEST RESULTS

The first cracks observed in the tests were invariably vertical and occurred near the midheight of the specimen. They generally became visible only at loads near the maximum load. As the deformation proceeded, more vertical cracks could be seen forming, and this continued process of cracking was reflected in the lack of smoothness of the descending part of the stress-strain curve. The cracks prior to final collapse generally extended from end-to-end of specimen. The strain at maximum load varied between 2,000 and 4,000 microstrains, and the ultimate strain prior to collapse reached up to 1.5 to 2.0 percent. The strain when the specimen ceased to carry any further load was about 6 to 8 times the strain at maximum load. In spite of the very large strains, the prisms and cylinders remained intact, but cubes showed considerable disintegration (Fig. 3).

CYLINDER AND PRISM TESTS

Figure 4 shows the stress-strain relationships for 150- by 300-mm cylinders of 3 different mixes and made of crushed granite of 10-mm size. The figure is typical of the effect of strength on the stress-strain characteristics of the material. Although the curves for high- and low-strength concretes are similar in shape, there are 3 differences among them worth noting: (a) The strain at maximum load tends to be greater for the stronger concrete; (b) the low-strength concrete shows a much flatter region at the peak load than the high-strength concrete; and (c) at equal deformations the low-strength concrete shows greater load resistance in the falling branch of the curve. The ultimate strains at collapse of the test specimens, however, are not very different for the 3 strengths.

The slopes of both the rising and falling branches of the curves increase with the maximum stress of the specimen. High strength concretes thus tend to become more unstable immediately after the peak load is reached, and this lack of stability increases with increased rate of straining (4, 5).
TABLE 1
RESULTS OF CYLINDER TESTS AT CONSTANT STRAIN

<table>
<thead>
<tr>
<th>Mix</th>
<th>Initial Tangent Modulus (N/mm² x 10⁴)</th>
<th>At Proportional Limit</th>
<th>At Maximum Stress</th>
<th>Cube Strength⁸ (N/mm²)</th>
<th>Cylinder Strength⁸ (N/mm²)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Stress (N/mm²)</td>
<td>Strain (cm/cm x 10⁻⁶)</td>
<td>Stress (N/mm²)</td>
<td>Strain (cm/cm x 10⁻⁶)</td>
<td>fu</td>
</tr>
<tr>
<td>1</td>
<td>0.95</td>
<td>19.8</td>
<td>1,800</td>
<td>24.6</td>
<td>3,500</td>
</tr>
<tr>
<td>2</td>
<td>1.42</td>
<td>27.6</td>
<td>1,950</td>
<td>37.3</td>
<td>3,500</td>
</tr>
<tr>
<td>3</td>
<td>1.71</td>
<td>39.3</td>
<td>2,300</td>
<td>52.4</td>
<td>2,850</td>
</tr>
</tbody>
</table>

³Average of 3 tests.

The main characteristics of the curves shown in Figure 4, together with the associated cube and cylinder strengths, are given in Table 1. The initial tangent moduli shown are based on the end-to-end deformation of the specimen and are seen to be much less than the normal values obtained when strains are measured over the central part of the specimen. The strain distribution in compressively loaded concrete specimens depends not only on the location of the gage length but also on the type of gage used (19). With mechanical gages, surface strains are known to differ by as much as 30 percent from strains measured between embedded inserts; with resistance gages, the measured strain depends on the relation between the gage length and the maximum aggregate size (20). Strains measured with gage lengths of 6 to 8 times the size of aggregate can show variations up to 50 percent over those measured with gage lengths of the same size as the largest aggregate. The overall deformation of a specimen measured between the platens is always greater than that measured over the central portion (19), so that elastic moduli based on end-to-end strain measurements are bound to be significantly lower. There is, in addition, an unknown degree of end restraint that would alter the observed strain values.

In general, the initial tangent modulus measured in these tests for a range of concrete strengths varied from about 55 to 80 percent of the corresponding value, based on strain measurements in the central region of the specimen both at constant strain rate and at constant stress rate of loading. It is, of course, questionable whether elastic moduli measured over the central part in prisms and cylinders are applicable to stress computation in structural members in service, because of the monolithic nature of their construction and the inevitable end restraints produced. This variability of elastic moduli, depending on the size and location of gage, partly explains the large discrepancies in the stresses in reinforced concrete beams computed by the elastic theory.

The stress and strain values at proportional limit given in Table 1 refer to the end of the linear range of the stress-strain curve. It was generally found that the maximum stress of prisms nearly coincided with cylinder crushing strength, and similar results have also been obtained for necked specimens (21).

The results of tests on crushed gravel prisms are shown in Figures 5 and 6; Figure 5 refers to 100 mm cubes, and Figure 6 refers to 200- and 300-mm high prisms of the same cross section. In general, specimens of different height showed more or less the same order of strain capacity prior to collapse, although individual specimens showed wide variations (Fig. 6). At equal deformations, the load capacity during the falling portion of the stress-strain curve depends on the height-width ratio of the specimen and decreases with increasing height of the specimen.

The stress-strain curve for a prism in which additional strain gage measurements were taken in the center of the specimen is shown in Figure 6. Although these strains were recorded in a matter of seconds, there was stress relaxation during this period that increased at higher loads and during the post-peak deformation. This stress relaxation at constant rate of straining points to 2 factors. First, in restrained structural members this confirms a redistribution of stresses and a rapid reduction of peak stresses as the maximum load is approached: The rate of plastic flow is then greater the higher the stress. Second, by analogy, some creep strains are bound to be included in the observed strains in a stress-controlled test, even of short-time duration.
The nonlinearity found in constant stress tests is therefore due partly to internal micro-cracking and partly to time-dependent strains. There is evidence, however, that if the effects of time and creep deformation are taken into account, then the stress-strain relationship at moderate stresses for normal concrete is reasonably linear (22, 23). The creep deformation is very rapid for lean concrete (22).

The effect of age on the stress-strain characteristics of prisms made of crushed basalt aggregate is shown in Figure 7. The 1-day and 3-day tests were carried out immediately after demolding, and the 7-day test on the water-cured surface-dry specimen. These curves show that the strain at maximum load increases between 1 and 3 days and between 7 and 28 days. As the age and strength increase there is a distinct hump at the peak load. The curves show generally the same characteristics as those shown in Figures 5 and 6.

The rate of straining has also a significant influence on the strength and elastic properties of concrete. Not only do strength and elastic modulus increase with increase in rate of straining, but also the observed strain capacity at the maximum load is known to be substantially greater than the corresponding value in the static tests (6, 24, 25).
The fact that all curves shown in Figures 4 through 7 possess 2 distinct regions in their stress-strain characteristics suggests that there may well be 2 distinct mechanisms of cracking and load capacity in the ascending and descending parts of the stress-strain curve. When loaded at a constant stress rate and failure occurs in the region of the maximum load, it has been shown that preexisting and new bond cracks develop during the early part of the loading and that mortar cracks develop at about 70 to 90 percent of the failure load (11). The increase in deformation is then no longer proportional to the increase in load, and the internal cracking and the time-dependent strains cause a progressive deviation of the stress-strain relation from linearity.

In the constant strain test, the ascending part of the curve is linear up to about 70 to 80 percent of the maximum stress and is largely due to the cohesive strength of the material. The descending part, on the other hand, is far from smooth and consists of a series of minute irregular movements. Every time a microcrack forms there is a fractional relaxation of load, followed immediately by stress stabilization, further cracking, and so on. During the post-peak deformation, friction thus plays a predominant part in the load-carrying capacity of the specimen. There is probably increased breakdown of the adhesion between the matrix and the aggregate, and as the internal microcracking proceeds the internal friction becomes more significant. With further internal cracking, the material resembles more and more a granular mass and depends primarily on frictional effects to sustain the decreasing load. Low-strength concretes show the significant effects of internal friction much more than high-strength concrete (Fig. 4).

The descending part of the stress-strain curve represents, in effect, the behavior of the cracked concrete, and, although the specimen is fractured as a material, it can still withstand decreasing loads, though with increasing deformation. This ability to sustain increasing deformation without ceasing to carry the load is an important characteristic of the cracked concrete and contributes to the relief of stress concentrations in structural members, such as around openings (17, 26). In the complete stress-strain curves shown in Figures 4 through 7, the ascending part may be considered to be the characteristic properties of the material, while the falling branch represents the characteristic properties of the structural system of which the cracked specimen forms a part.

**EFFECT OF REPEATED UNLOADING AND RELOADING**

One of the significant characteristics of the falling branch of the stress-strain curve is that a sudden or violent failure cannot occur in a cracked specimen on the descending portion of the curve. It possesses a definite load-carrying capacity, although this decreases with increasing deformation. The stability of the cracked specimen during the post-peak deformation stage was studied by subjecting a number of specimens to
gradually decreasing and increasing load cycles. Typical results are shown in Figures 8 and 9, and some comparative results are shown in Figures 5 and 7. Figure 8 shows the tests on crushed granite cylinders, and Figure 9 shows the results of tests on cubes and prisms of crushed limestone.

All figures show that a cracked specimen subjected to load cycling in the descending part of its stress-strain curve resumes its characteristic deformational behavior at the end of each load cycle. Further, these curves show that, even after considerable cracking beyond the maximum load, the specimens exhibit a certain amount of elastic behavior; they not only possess stability but also are capable of sustaining very large strains without losing their load capacity. The loops formed by the load cycles, which represent the energy loss during each cycle, are generally narrow. Their slope is, in effect, a measure of the elasticity of the cracked concrete specimen and can be considered to represent the elastic modulus of the cracked specimen. This modulus is seen to decrease with decreasing resistance of the fractured specimen. In a way, this decrease in modulus is analogous to the decrease in the modulus of resistance under sustained loading (27). With each cycle, the negative slope of the stress-strain curve also decreases, thus ensuring that the cracked specimen becomes less and less liable to sudden failure.

The load cycles during the ascending part of the stress-strain curve show that concrete is basically an inelastic material and that creep strains occur even under short-term loading, although its 2 basic constituents—the coarse aggregate and the mortar
matrix—are largely elastic. There are permanent residual strains on unloading, even at low load levels. On reloading, however, the slope is nearly constant; and on repeated reloading, the straight-line sections are again nearly parallel. Upon reloading the specimen generally follows its previous deformational behavior. The energy dissipated during the ascending part is generally less than that in the descending part of the curve, and this again confirms the significance of frictional damping in its contribution to load capacity after the member has cracked. The width of the hysteresis loop generally depends on the range of stress and increases with it. The ability to absorb strain energy again depends on the rate of straining and on the strength and strain capacity at the maximum load (24).

**FRICTIONAL DAMPING**

The jagged nature of the descending part of the stress-strain curve and the area of the loop formed by the loading-unloading cycles also indicate a measure of the damping properties of concrete. Damping arises mainly from the inelasticity of the material and is a measure of its capacity to absorb and dissipate energy through internal friction. These tests show that frictional damping can be an important characteristic of concrete. At low loads, energy dissipation occurs through inelastic deformation; at higher loads, damping mainly occurs through energy dissipation into surface energy through crack propagation and rupture of cohesive and frictional bonds within the system. This is further assisted by local yielding and progressive inelastic deformation around the internal discontinuities and the propagating cracks. Friction damping thus occurs mainly at the interfacial boundaries between the matrix and aggregate inclusions and becomes the main load-bearing mechanism in the descending part of the stress-strain curve. It can therefore be very significant in cracked structural members and in members subjected to dynamic loading and can contribute to substantial reductions of the maximum strains within a structure. The amount of energy absorbed appears to depend on the rate of straining (24), although tests on steel specimens show damping capacity to be independent of the rate of loading.

**CONCLUSIONS**

1. The ultimate strain in uniaxial compression is 6 to 8 times the strain at maximum load and is generally independent of concrete strength and the height-width ratio of the specimen.
2. The strain at maximum load varies between 2,000 and 4,000 microstrains and tends to increase with concrete strength and age.
3. A cracked specimen on the falling branch of the stress-strain curve has considerable load capacity although this decreases with increasing deformation. This ability to sustain large strains without ceasing to carry load ensures relief of peak stresses and the absence of sudden and violent failure.
4. At equal deformations beyond the maximum load, low-strength concrete and specimens of small height-width ratio possess greater load resistance.
5. The internal friction due to the continued process of discrete cracking at the interfacial boundaries between the matrix and aggregate inclusions plays a significant role in the load-carrying capacity of the cracked specimen.
6. The initial tangent modulus at constant strain based on end-to-end strain of the specimen is about 55 to 80 percent of the normal value obtained from strains measured in the central section of the specimen.
7. The cracked concrete subjected to repeated cyclic loading on the falling branch exhibits a certain amount of elastic behavior. The energy loss during each cycle is generally low, and the repeated unloading and reloading does not significantly affect the load capacity of the specimen.
8. Permanent residual strains remain on unloading even at low stress levels.
REFERENCES


