Pile-Driving Formulas

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The basis for the fundamental dynamic pile-driving formulas is presented first. Then modifications to account for energy losses are described. Some of the more popular pile-driving formulas are discussed with emphasis on their inherent assumptions. Finally, a solution is described that is based on the motions experienced by all parts of the hammer-cap block-pile system after hammer impact occurs. This solution is commonly known as the wave equation. The assumptions on which this solution is based are discussed, and some of its advantages are described.

*A GREAT DEAL HAS BEEN WRITTEN about dynamic pile-driving formulas. One can find numerous formulas listed in soil mechanics and foundation engineering texts. Furthermore, one frequently sees reports in technical journals of new investigations by researchers who have found none of the existing formulas to their liking and have developed a new one that better suits the results of their personal experience. Yet most of the texts on this subject caution the engineer to be wary of the use of dynamic pile-driving formulas inasmuch as they cannot be relied on to predict a pile's ultimate capacity with a reasonable degree of accuracy.

Why, then, is there this continued interest in these formulas. The reason is, of course, that there is a practical need for a dynamic formula. The majority of pile foundations are installed in soil profiles of such a variable nature that the only convenient method for determining whether or not a pile has reached sufficiently dense material is by observing the ease or difficulty with which the pile penetrates the ground. Another reason for this continued interest is that no single dynamic formula has been found that will consistently predict ultimate pile capacities that agree with load tests. This second consideration is based on the often invalid assumption that no change occurs in the soil's ability to resist further pile penetration from the time of driving to time of load testing.

In August 1960, Smith (1) presented a practical means for calculating the response of a pile to the impact of a hammer by means of finite-difference equations making use of electronic digital computers. The method of solution is such that one can conveniently account for all of the significant factors that influence pile penetration resulting from a hammer blow. During the past few years a considerable amount of research related to wave-equation solutions has been done at the Texas Transportation Institute (2, 3, 4) and has included comparisons of wave-equation solutions with data obtained from instrumented piles during driving. These investigations have confirmed a good correlation between experimental and wave-equation solution results. It is the writers' opinion that, of solutions currently available, this one is the only reliable method for calculating the response of any particular cap block-cushion, pile, and soil system resulting from the impact of a pile hammer. However, at the present time a variety of dynamic pile-driving formulas are being used for this purpose in building codes and specifications. The intent of this paper is first to review basic forms of the dynamic pile-driving formulas and the assumptions on which they are based. Then, a parameter study will be presented comparing 3 typical pile-driving formula solutions with corresponding wave-equation solutions.

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The main requirement of any dynamic pile-driving formula is to correlate driving resistance, usually recorded in blows per inch, with soil resistance encountered at the time of driving. The main limitation is that a dynamic formula can only calculate the soil resistance at the time of driving. Any changes in the soil's ability to resist further pile penetration some period of time after driving must be evaluated by static methods and added to, or subtracted from, the resistance encountered at the time of driving.

Most of the dynamic pile-driving formulas are based on the fundamental energy equation wherein the kinetic energy of the hammer's ram at impact is equated to the work done on the pile, that is, the product of the distance the pile moves and the soil force resisting this movement. However, several complicating factors must be accounted for; these include the following:

1. The resisting soil force is not constant during the period of time the pile is penetrating because the soil has some elasticity and damping characteristics.
2. The temporary elastic compressions of the cap block, cushion, pile, and soil absorb energy that does not contribute to making the pile penetrate.
3. An impact energy loss occurs because the cap block and cushion have coefficients of restitution less than unity.
4. A pile is a long slender object, and each incremental part at any instant of time will experience a different motion from that of the other parts.

The dynamic pile-driving formula most commonly used in the United States is the Engineering-News formula.

\[ R = \frac{2E}{S + 0.1} \]

where

- \( R \) = safe pile working load, lb;
- \( E \) = hammer energy, ft-lb; and
- \( S \) = pile set, in./blow.

This is one of the simplest formulas because it does not attempt to account for the first, third, and fourth complicating factors listed earlier, and furthermore it presumes that the elastic compression losses are the same for any cap block, cushion, pile, and soil. This formula is written in such a way that it has a theoretical factor of safety of six.

The second dynamic formula used for comparison is the Eytelwein formula.

\[ R = \frac{2E}{S + 0.1(P/W)} \]

where

- \( R \) = safe pile working load, lb;
- \( E \) = hammer energy, ft-lb;
- \( S \) = pile set, in./blow;
- \( P \) = pile weight, lb; and
- \( W \) = ram weight, lb.

It is slightly more complicated than the Engineering-News formula inasmuch as it considers the ratio of the pile's total weight to the hammer ram's weight in such a way that the first, second, and fourth complicating factors are not accounted for, but the combined coefficient of restitution of the cap block and cushion is assumed to be zero. It also has a theoretical factor of safety of six.

The third dynamic formula used for comparison is the Hiley formula.

\[ R_u = \frac{12eE}{S + (\mu)^2} \left( \frac{W + \mu^2P}{C_1 + C_2 + C_3} \right) \]

where

- \( R_u \) = ultimate pile capacity, lb;
- \( E \) = hammer energy, ft-lb;
- \( e \) = hammer efficiency;
- \( S \) = pile set, in./blow;
C₁ = compression of cap block and cushion, in.;  
C₂ = compression of pile, in.;  
C₃ = soil quake, in.;  
W = ram weight, lb;  
P = pile weight, lb; and  
µ = coefficient of restitution of cap block and cushion.

C₁, C₂, and C₃ are based on Chellis (6) as follows:

<table>
<thead>
<tr>
<th></th>
<th>Steel</th>
<th>Concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td>C₁</td>
<td>0.16 (Rᵤ/A)</td>
<td>0.5 (Rᵤ/A)</td>
</tr>
<tr>
<td>C₂</td>
<td>0.0008 L (Rᵤ/A)</td>
<td>0.008 L (Rᵤ/A)</td>
</tr>
<tr>
<td>C₃</td>
<td>0.1</td>
<td>0.1</td>
</tr>
</tbody>
</table>

where

\[ A = \text{pile area, in.}^2; \text{and} \]
\[ L = \text{distance from head of pile to center of soil resistance, ft.} \]

This formula is more complicated than the other two because it attempts to account for the second and third complicating factors in a rational manner based on the physical properties of the materials used. However, it does not attempt to account for the first and fourth complicating factors. A more complete description of pile-driving formulas has been given by Cummings (5) and Chellis (6).

The mathematical model for wave-equation solutions can be readily designed to account for all 4 complicating factors. The basis is described by Smith (1).

Solutions for the 3 dynamic pile-driving formulas and for the wave equation are shown in Figures 1, 2, 3, and 4. Symbols used in Figure 4 are defined as follows.

\[ \mu = \text{coefficient of restitution of cap block or cushion;} \]
\[ \text{M/AL} = \text{Micarta-aluminum;} \]
\[ \text{Q-point} = \text{soil quake at tip of pile, in.}; \]
\[ \text{Q-side} = \text{soil quake at side of pile, in.}; \]
\[ \text{J-point} = \text{soil damping constant at point of pile, sec/ft;} \text{and} \]
\[ \text{J-side} = \text{soil damping constant at side of pile, sec/ft.} \]

When comparing the 3 dynamic formulas with the wave-equation solutions, one should note that both the Engineering-News and Eytelwein formulas have built-in theoretical factors of safety of six. This means their intention is to correlate a safe pile working load with driving resistance. On the other hand, the Hiley formula and wave-equation solutions both correlate the ultimate soil resistance with the driving resistance.

A parameter study intended to bracket the range of commonly used steel and concrete piles driven for support of land structures has been selected as a means of comparing the 3 dynamic formulas with the wave-equation solution. The parameters used are as follows:

<table>
<thead>
<tr>
<th>Pile material</th>
<th>Steel, concrete</th>
</tr>
</thead>
<tbody>
<tr>
<td>Pile size, lb-ft</td>
<td>20, 160</td>
</tr>
<tr>
<td>Steel</td>
<td>12 in.²</td>
</tr>
<tr>
<td>Concrete</td>
<td>150</td>
</tr>
<tr>
<td>18 in.²</td>
<td>338</td>
</tr>
<tr>
<td>Pile length, ft</td>
<td>30, 120</td>
</tr>
<tr>
<td>Mode of soil resistance, percent</td>
<td>100</td>
</tr>
<tr>
<td>Point</td>
<td></td>
</tr>
<tr>
<td>Friction</td>
<td>100</td>
</tr>
<tr>
<td>Hammer size, ft-lb</td>
<td>15,000</td>
</tr>
<tr>
<td>No. 1 Vulcan</td>
<td></td>
</tr>
<tr>
<td>No. 2/0 Raymond</td>
<td>32,500</td>
</tr>
</tbody>
</table>
Figure 1. Solutions for the Engineering-News formula.

**TABLE 1**

<table>
<thead>
<tr>
<th>Pile Type</th>
<th>Pile Size (lb-ft)</th>
<th>Pile Length (ft)</th>
<th>Vulcan 1</th>
<th>Friction</th>
<th>Raymond 2-0</th>
<th>Friction</th>
</tr>
</thead>
<tbody>
<tr>
<td>Steel</td>
<td>20</td>
<td>30</td>
<td>1.9</td>
<td>1.4</td>
<td>2.4</td>
<td>1.7</td>
</tr>
<tr>
<td>Steel</td>
<td>20</td>
<td>120</td>
<td>1.8</td>
<td>1.0</td>
<td>2.2</td>
<td>1.5</td>
</tr>
<tr>
<td>Steel</td>
<td>160</td>
<td>30</td>
<td>2.6</td>
<td>3.0</td>
<td>3.8</td>
<td>3.7</td>
</tr>
<tr>
<td>Steel</td>
<td>160</td>
<td>120</td>
<td>2.6</td>
<td>3.0</td>
<td>4.1</td>
<td>4.1</td>
</tr>
<tr>
<td>Concrete</td>
<td>150</td>
<td>30</td>
<td>1.8</td>
<td>1.5</td>
<td>2.2</td>
<td>1.7</td>
</tr>
<tr>
<td>Concrete</td>
<td>150</td>
<td>120</td>
<td>1.8</td>
<td>1.7</td>
<td>2.4</td>
<td>1.8</td>
</tr>
<tr>
<td>Concrete</td>
<td>338</td>
<td>30</td>
<td>1.8</td>
<td>2.2</td>
<td>2.4</td>
<td>2.5</td>
</tr>
<tr>
<td>Concrete</td>
<td>338</td>
<td>120</td>
<td>1.9</td>
<td>2.3</td>
<td>2.4</td>
<td>2.9</td>
</tr>
</tbody>
</table>

**Engineering-News Formula Safe Resistance**

Steel 20 30 1.9 1.4 2.4 1.7 5 20 1.5 1.0 2.1 1.2
Steel 20 120 1.8 1.0 2.2 1.7 5 20 1.0 0.7 1.3 0.7
Steel 160 30 2.6 3.0 3.8 3.7 5 20 2.7 2.3 3.9 2.7
Steel 160 120 2.6 3.0 4.1 4.1 5 20 2.5 2.3 3.9 2.7
Concrete 150 30 1.8 1.5 2.2 1.7 5 20 1.5 1.0 1.7 1.1
Concrete 150 120 1.8 1.7 2.4 1.8 5 20 1.6 1.2 1.9 1.2
Concrete 338 30 1.8 2.2 2.4 2.5 5 20 1.9 1.7 2.2 1.7
Concrete 338 120 1.9 2.3 2.4 2.9 5 20 2.0 1.6 2.6 2.0

**Eytelwein Formula Safe Resistance**

Steel 20 30 1.4 0.6 1.8 0.7 5 20 1.0 0.4 1.4 0.4
Steel 20 120 1.3 0.7 1.5 0.7 5 20 0.9 0.3 0.9 0.3
Steel 160 30 2.7 2.9 3.9 3.6 5 20 2.1 1.4 3.0 1.8
Steel 160 120 5.4 9.0 8.5 11.8 5 20 3.1 3.5 4.9 4.4
Concrete 150 30 1.8 1.4 2.2 1.6 5 20 1.3 0.7 1.0 0.6
Concrete 150 120 1.8 4.7 4.3 4.9 5 20 2.1 1.6 2.4 1.8
Concrete 338 30 2.8 3.5 3.2 4.1 5 20 1.9 1.7 2.2 1.7
Concrete 338 120 7.1 12.7 8.6 16.0 5 20 4.2 5.3 5.4 5.9

**Hiley Formula Ultimate Resistance**

Steel 20 30 0.8 1.0 0.9 1.1 5 20 0.8 0.9 0.9 0.9
Steel 20 120 1.1 1.3 1.0 1.0 5 20 1.1 1.2 0.9 0.9
Steel 160 30 0.9 1.6 1.3 1.9 5 20 1.0 1.3 1.3 1.5
Steel 160 120 1.2 2.3 1.5 2.6 5 20 1.3 1.9 1.7 2.0
Concrete 150 30 1.1 1.5 1.3 1.6 5 20 1.0 1.2 0.8 1.1
Concrete 150 120 1.9 2.7 2.1 2.5 5 20 1.7 2.2 1.7 1.8
Concrete 338 30 1.3 2.6 1.6 2.6 5 20 1.3 1.9 1.4 1.8
Concrete 338 120 2.0 3.6 2.2 4.3 5 20 2.1 3.1 2.5 3.1
Figure 2. Solutions for the Eytelwein formula.
Figure 3. Solutions for the Hiley formula.
Figure 3. Continued.
Figure 4. Solutions for the wave equation.
Figure 4. Continued.
The apparent factors of safety at the time of driving for the 3 dynamic formulas, based on the assumption that the wave-equation solutions are correct, are given in Table 1. In addition to the 5 parameters, Table 1 includes a sixth, that is, moderate driving resistance, considered to be 5 blows/in., and hard driving resistance, considered to be 20 blows/in. One can readily see that there are certain combinations of parameters that cause the dynamic formulas to yield a desirable factor of safety, such as two. For other combinations of parameters, these formulas can be either unsafe or else extremely conservative resulting in uneconomical solutions.

It is believed that practicing engineers will find the graphic results of these solutions a convenient reference, especially those for the wave-equation solution. Because the parameters cover the range of piling systems commonly used, an engineer may readily obtain an approximate wave-equation solution for any particular system by interpolation from these graphs. This will enable him to estimate the optimum combination of hammer, cap block, cushion, and pile for his application. If he desires a more exact solution including maximum tension and compression stresses in the pile, there are several places where he can obtain wave-equation solutions if inconvenient to set up his own program.

REFERENCES