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A Nonlinear Theory for Predicting the Performance of Flexible Highway Pavements

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A nonlinear theory is proposed for the structural analysis of flexible highway pavements. The proposed theory uses a nonlinear incremental approach based on finite element theory. The nonlinear problem is solved by applying each load repetition to the solid continuum in small increments. This procedure, in effect, reduces the nonlinear problem to that of successively solving a large number of elastic problems. A plastic load concept using cumulative plastic strain is developed that makes possible placing all of the nonlinear effects in the load vector. Using this approach, the system stiffness matrix needs to be decomposed only once. As the external load is incrementally increased or decreased and the modulus changes, essentially all that has to be done to calculate the new displacements of the system is to solve an upper triangular matrix by back-substitution. The plastic load approach using the concept of cumulative plastic strains is perfectly general, and the theory is applied to problems involving the application of large numbers of wheel load repetitions and viscoelastic creep loadings.

•IN THE DESIGN of flexible highway pavement systems, apparently the most important structural considerations are rutting of the surfacing due to the accumulation of permanent shear deformations and fatigue cracking of the surfacing which can also lead to rutting (1, 2). At the present time, the structural components of flexible pavement systems are usually designed by either empirical or semi-rational methods based on ultimate strength or elastic theory (3). The presently used design methods cannot rationally account for either important variations of elastic and plastic material properties with increasing numbers of load repetitions or the occurrence of cumulative deformations (rutting) in the pavement system.

Several linear elastic (4-8) and nonlinear elastic (9) theories have been proposed and, at least to some extent, verified for predicting the elastic response of pavement systems subjected to a single wheel load. Very few theories, however, have been proposed and verified for predicting the response of pavement systems under time-dependent creep or repeated wheel loads. Although not directly related to pavement systems, interesting theories and related model studies that deserve mentioning have been presented by Majerus and Tamekuni (10) and by Gallagher, Padlog, and Bijlaard (11).

Monismith and Secor (12) performed a comprehensive investigation to verify a linear viscoelastic theory that used the correspondence principle and characterized the material properties by a 4-element spring and dashpot model. They attempted to verify the theory using an idealized model pavement system consisting of a thin asphaltic concrete slab resting on a bed of closely spaced springs. Even after modifying the linear theory to account for differences in tensile and compressive material properties, the calculated center deflections of the model were found to be considerably less than the measured deflections shortly after the application of a creep loading.

Barksdale and Leonards (2) developed a method using material properties obtained from repeated load triaxial tests for predicting both elastic and permanent deflections and stresses in 3- and 4-layer pavement systems subjected to repeated wheel loads.

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The linear theory used consisted of applying the correspondence principle to elastic layered theory and inverting the solution using the collocation inversion technique. A comparison of both the elastic and permanent surface deflections measured at the AASHO Road Test with those calculated showed reasonable agreement considering the many uncertainties involved in making a comparison of this nature. The disadvantages of this theory are, however, that the elastic modulus cannot be varied with number of wheel load applications, elastic and plastic properties cannot be varied within each layer with changes in stress state, and the plastic material properties must be evaluated by a trial-and-error process.

A general nonlinear theory that can be used to calculate both the elastic and plastic response of layered pavement systems is proposed in this paper. This theory is suitable for use as the basis for a rational, flexible pavement design procedure. A cumulative strain concept is combined with the finite element approach to give a very powerful and general method for the structural analysis of flexible pavement systems. This theory is extended to include repeated loadings and nonlinear viscoelastic creep problems. The problem of a simplified model pavement subjected to a single step function loading is used to show the validity of the proposed nonlinear, viscoelastic theory.

THEORETICAL DEVELOPMENT

The elastic finite element method was originally developed in the aircraft industry as a generalization of matrix structural methods of analysis. A detailed description of the finite element method applied to linear problems has been presented by Zienkiewicz and Cheung (13). Complicated nonlinear elasticity problems can often be reduced by iterative techniques to the problem of successively solving a large number of related linear elastic problems (11, 13, 14, 15, 16, 17). Iterative methods that have been used include direct iteration, in which the total loading is applied in one increment, or incremental methods, where the load is applied in small increments and a constant or variable modulus is used for each iteration (13). When iterative methods are used to solve nonlinear problems, careful attention must be given to whether or not the iterative solution has converged to within an engineering degree of tolerance of the mathematically correct answer.

Linear Elastic Finite Element Theory

As a first approximation, the layered pavement system problem can be idealized as one of axial symmetry. An elastic axisymmetric solid continuum may be represented as an assemblage of a finite number of discrete ring-shaped elements. Each adjacent element in the solid that comes together at a common point is interconnected by frictionless pins called nodes. By requiring that equilibrium of forces and compatibility of element displacements at all the nodes in the system are satisfied under the application of external loads, the continuum problem is reduced to that of solving a large system of simultaneous linear equations. Fortunately, the system of equations usually has a large number of zero terms, which greatly reduces the number of calculations required to solve the system.

A finite element computer program for solving elastic axisymmetric problems was developed and later modified for nonlinear material properties. Rectangular, ring-shaped elements were used in the program to approximate the continuum, and the displacement components were assumed to vary linearly over each element. The corresponding stiffness of each element was numerically evaluated using 3-point Gaussian quadrature in both directions. After assembling the system stiffness matrix and load vector, the resulting system of simultaneous linear equations was solved using Choloski decomposition, taking into consideration the bandwidth of the system stiffness matrix. In formulating the load vector, distributed surface loadings were lumped at the nodes using consistant energy concepts. Inertia forces were neglected throughout this investigation. A comparison was made for the elastic half-space problem between deflections and stresses calculated using the proposed finite element theory, those obtained using a finite element program written by Wilson (18), and those calculated using elastic half-space theory. Good agreement was shown to exist between the stresses and deflections calculated using the different methods.

Incremental Nonlinear Elasticity Theory

The incremental approach can be used to solve nonlinear problems by dividing the total load on the structure into small, equally spaced increments. By following this approach the task of solving nonlinear problems can be reduced to one of solving a relatively large number of linearly elastic ones. Essentially what is done using the proposed procedure is to obtain for each load level, using an iterative procedure, a modulus for each element that is compatible with the corresponding stress state or effective stress. Effective stress-strain laws and a detailed explanation of the method are given elsewhere (19).

The effective stress path followed in this investigation using an iterative procedure subsequently described is shown in Figure 1. An initially guessed elastic modulus E^e is used to calculate the stress state for the first load increment which results in a movement along the stress path from o to a in Figure 1. A new modulus is then determined using the initially calculated stress state based on E^e. A new stress state is then calculated using the total modulus E just calculated. Repeating this iterative procedure would cause a gradual movement from point a to a point near b provided a sufficient number of similar iterations are carried out for the first load increment. The next load increment is then added to the previous increment. The modulus defined by line o-b is used to calculate the first approximate stress state for the second load increment resulting in a movement from b to c on the effective stress path. The correct plastic load increment, $\bar{\epsilon}^p$, for the new load level can be determined using the final plastic load for the first load increment and similar triangles oab and ozc, when the indicated stress path is followed. The same iterative procedure described for the first load increment is repeated again for the second load increment resulting in a movement from c to the vicinity of d. The same procedure is carried out with the addition of each successive load increment until the total desired load is applied to the solid.

The plastic strain, $\bar{\epsilon}^p$, associated with any deviation from the elastic strain, $\bar{\epsilon}^e$, calculated using the initial elastic modulus can be treated exactly like an initially known strain state applied to the solid, such as that due to temperature effects or erection errors. A fictitious load matrix can then be calculated which places all the plastic

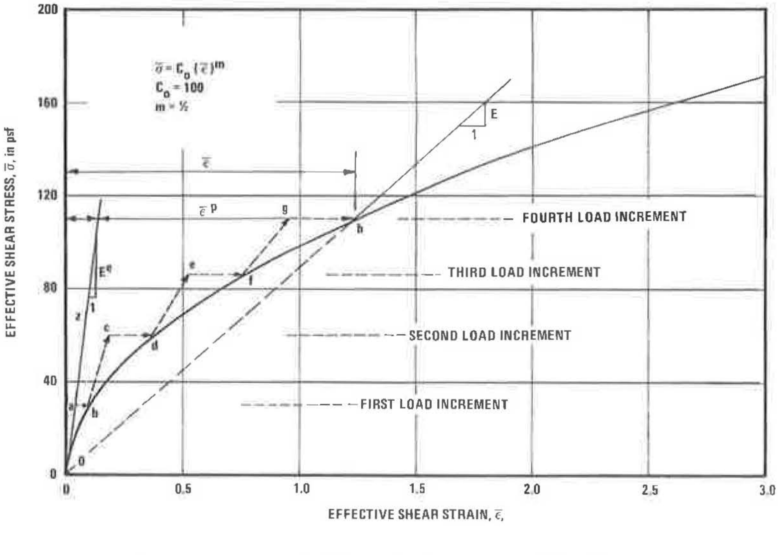

Figure 1. Effective stress path followed using proposed iterative procedure.

effects in the loading. Handling the plastic effects in this way makes possible the solution of nonlinear problems without ever changing the system stiffness matrix from the one initially calculated for the first load increment. The stiffness matrix therefore needs to be decomposed or inverted only one time and successively solved for each iteration and load increment using a different total load vector.

If Poisson's ratio remains constant, it is not necessary when calculating the fictitious load matrix to evaluate the associated volume integral during the entire iterative procedure (19). The contribution to the load matrix due to the plastic strains in each element can be evaluated for each successive iteration by simply multiplying the previously calculated elastic stiffness matrix of each element times the appropriate node displacement vector and a ratio of the total modulus to the plastic modulus. Both elastic and plastic deviations from an initially assumed elastic modulus can be placed in the load vector. Therefore, the elastic stiffness matrix, which involves a complicated volume integral, needs to be evaluated only one time during the solution of a nonlinear problem.

The nonlinear theory just presented was incorporated into the elastic finite element computer program described in the previous section. The nonlinear theory and computer program were initially verified for the problem of a long, hollow cylinder subjected to a uniform internal pressure (20). The stresses obtained using the nonlinear program were found for all degrees of nonlinearity studied to be within less than 5 percent of the values obtained by Nadai (20). The number of load increments required to give good convergence was found to depend upon the degree of material nonlinearity.

Repeated Wheel Load Applications

The proposed nonlinear finite element theory can be readily extended for highway pavements to include large numbers of repeated wheel loadings. Assume that the moving wheel loads can be replaced by a series of stationary, repeated loadings. This idealization, in effect, neglects the influence of inertia forces. Furthermore, assume that only a relatively few wheel loadings are applied very near the edge of the pavement so that an axisymmetric loading can be used to represent the loading and pavement structure. Finally assume a normal surface loading and small deflection theory. A discussion of these assumptions and their implications is given elsewhere (19).

The material properties for the nonlinear analysis of highway pavements subjected to large numbers of wheel applications can be obtained from repeated load triaxial tests. Repeated load tests would be performed on the surface, base, and subgrade materials using an appropriate range of deviator stresses and confining pressures. The stress state and deflections at the end of the first load cycle can be calculated by applying and then removing the wheel load to the pavement system in small increments. During unloading the permanent cumulative strain in each element can be treated in the same way the fictitious plastic strain was handled in the earlier theory. Previous laboratory studies (21, 22) have shown that the average loading modulus and the accumulation of permanent strain during each load repetition changes relatively slowly with increasing number of load applications. From an engineering standpoint, the material properties can therefore be assumed either to remain constant for a certain number of load repetitions or else to vary in some predetermined manner. The number of load repetitions during which the modulus and cumulative strain would be assumed constant would be determined from the results of the repeated load triaxial tests. Fortunately, this interval would, in general, tend to increase logarithmically with the increase in the number of load repetitions. Using the plastic strain concept, this approach would require, at most, reevaluating the system stiffness matrix and decomposing it each time the material properties are changed. This approach not only permits the use of nonlinear material properties but also makes it possible to vary both the elastic and plastic material properties with the number of load repetitions. This approach, therefore, greatly generalized the previous work done by Barksdale and Leonards (2).

Nonlinear Viscoelastic Creep Problems

The problem now is to illustrate how the nonlinear finite element theory can be readily used to predict the response of nonlinear pavement systems subjected to stationary creep loadings. Time-dependent problems may be solved by using finite element techniques either by marching forward in small increments of time (11, 23, 24, 25) or else by directly applying the correspondence principle (2, 10). Applying the correspondence principle to problems involving step loadings and inverting the transformed solution often results in considerably less numerical calculations than solving the same problem using a step-by-step method. The correspondence principle, however, could not, in general, be used to solve nonlinear problems involving arbitrary variations of loading with time. Use of the correspondence principle in solving linear viscoelastic pavement problems has already been described (2). This same approach can also be used in solving nonlinear viscoelastic creep problems with only the minor modifications described in this section.

Appropriate tension and compression creep tests would be performed at stress levels selected so as to cover the range of anticipated stress states to which the material would be subjected under actual loading. For each type of test, a linear n-element Kelvin model could then be readily fitted to the experimental data obtained for each stress level by using the collocation method proposed by Schapery (26). Using the Kelvin model, the operational creep compliance $\bar{D}_c(p)$, as a function of transformed time p, could be calculated for all stress levels for a particular type of test. This operation, in effect, would generate a family of curves in the transformed $\bar{D}_c(p)$-p plane relating operational creep compliance, stress level, and time. Each curve would correspond to a stress level at which a test was performed. At each desired instant of transformed time, a relationship could be obtained between the stress state and the operational creep compliance by cross-plotting these results. This procedure would give the relationship between stress state and operational compliance at a certain instant of operational time which is analogous to the information obtained from a time-independent, nonlinear stress-strain curve. A relationship having this form between stress state and operational creep compliance for given values of operational time p can be readily used in the proposed method of nonlinear stress analysis. This nonlinear viscoelastic approach is equivalent to assuming that the material behaves linearly for small incremental changes in stress state and is similar in overall concept to the proposed time-independent, nonlinear incremental theory.

The transformed time-dependent response $\zeta(p)$ to a creep loading could be readily calculated at a finite number of transformed times by applying the correspondence principle and using the material properties evaluated as described above. The transformed solutions could then be inverted back to the real time plane by applying a collocation inversion method (27).

NONLINEAR VISCOELASTIC MATERIAL CHARACTERIZATION

This section describes the evaluation and characterization of the nonlinear viscoelastic material properties of a sand asphalt. Specimens of sand asphalt were subjected to both uniaxial tensile and triaxial compression creep states of stress to evaluate the effects of stress state and time on the material properties. All tests were performed in constant-temperature chambers at a temperature of 77 ±0.5 F. The material properties were then characterized in a form suitable for use in the proposed nonlinear viscoelastic theory. All samples tested had 6½ percent by total weight of a 120-150 standard penetration grade asphalt cement. The aggregate used was a fine, well-graded crushed granite having a maximum grain size of ⅜ in.

Uniaxial Tensile Creep Tests

The tensile creep properties of the sand asphalt specimens were determined by means of a uniaxial creep test. The 1 x 1 x 5-in. specimens used in this series of tests were cut from 1 x 6 x 6-in. samples prepared by static compaction. Axial tensile step loadings of 2.0, 3.9, 5.6, and 7.5 psi were applied to specimens in order to cover the approximate range of stress levels estimated before the development of the theoretical analysis. The average density of the tensile specimens was 141.7 pcf (±1.4 percent).

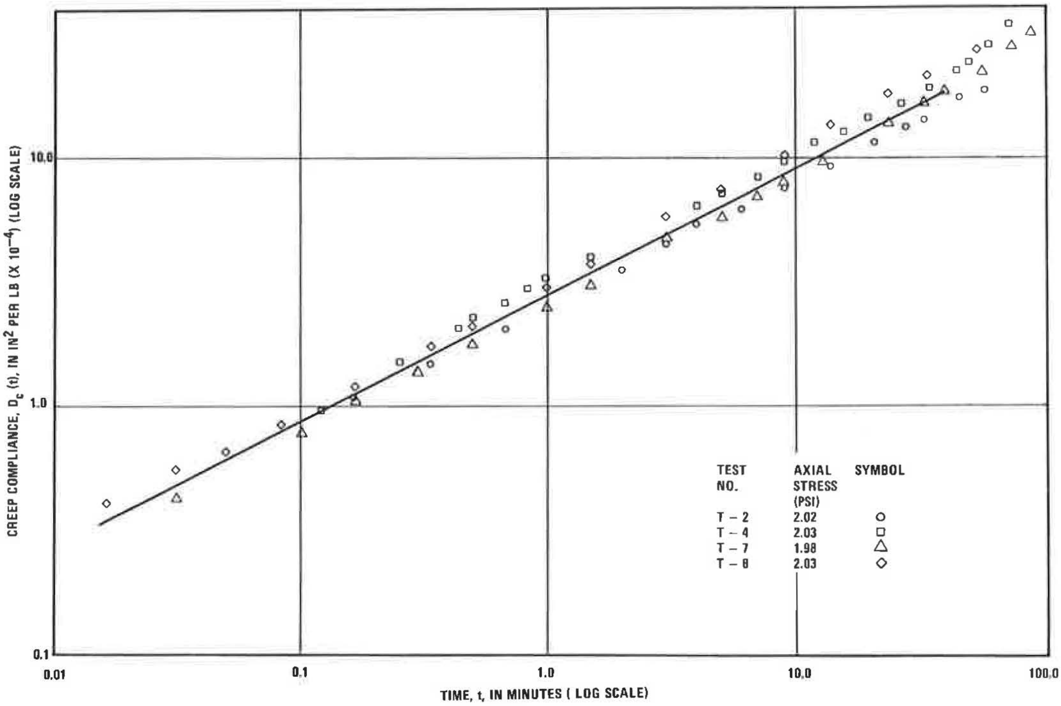

Figure 2. Experimental tensile creep compliance curve for sand asphalt at 77 F and an axial stress of 2.0 psi.

The tensile specimens were orientated with their long axis vertical in an aluminum loading frame. Each specimen was subjected to a creep loading by placing circular lead weights on a load hanger.

Typical test data and the corresponding average creep compliance curve as a function of time for the tests performed at a stress level of 2.0 psi are shown on a log-log plot in Figure 2. The creep compliance is just the reciprocal of the creep modulus and is equal to the time-dependent strain divided by the constant stress. The creep compliance as a function of time was found for a given stress level to be linear over almost 2 log cycles of time. All test data were corrected for the small deformations that occurred in the wood end tabs and loading system. The approximate influence of stress level on creep compliance is shown in Figure 3. Some of these curves have been slightly shifted so as to give consistent results for each successive stress level. These results, although based on a limited number of tensile creep tests, indicate that, at times greater than about 0.5 min, the creep compliance increases as the stress level goes from 2.0 to 7.6 psi. On the other hand, for times less than about 0.5 min, the creep compliance becomes smaller as the stress level is increased.

The viscoelastic material characterization theory described earlier was used to develop a mathematical set of models from the experimentally measured creep material properties suitable for use with the proposed nonlinear viscoelastic theory. A 16-element Kelvin model was fitted to each of the 4 tensile creep response curves shown in Figure 3. A mathematical model was thus obtained for the operational creep compliance as a function of transformed time for each of the 4 stress levels at which tensile creep tests were performed. The relationship between axial stress level and operational creep compliance at 11 values of p spaced approximately evenly between 0.01 and 100 was then obtained by cross-plotting the results from the operational creep compliance curves for each stress level. Typical results of this operation, which give the relationship between axial stress level and operational creep compliance at a given

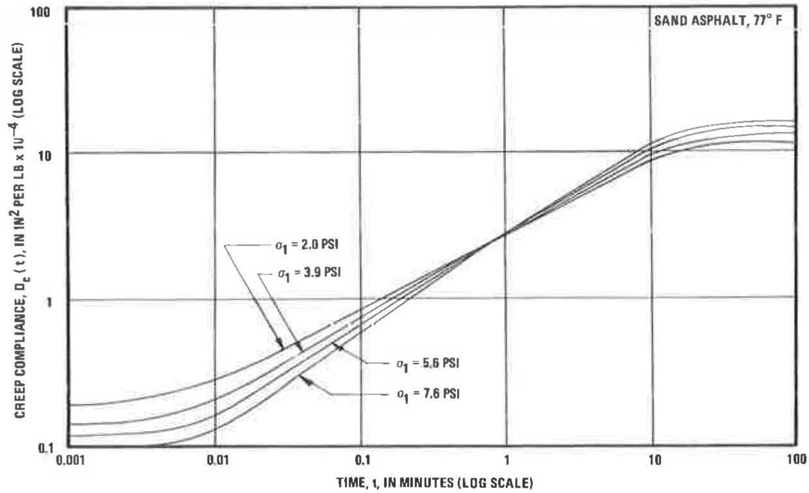

Figure 3. Influence of axial stress on tensile creep compliance.

instant of operational time p, are shown in Figure 4 for p = 0.1. This operational time corresponds to a real time of roughly 5 min. These relationships between axial stress level and operational creep compliance are in a form suitable for use in the proposed nonlinear viscoelastic theory. These curves would be, for example, the nonlinear viscoelastic equivalent to a plot of the modulus of elasticity as a function of axial stress obtained from a uniaxial tension test performed at a constant strain rate.

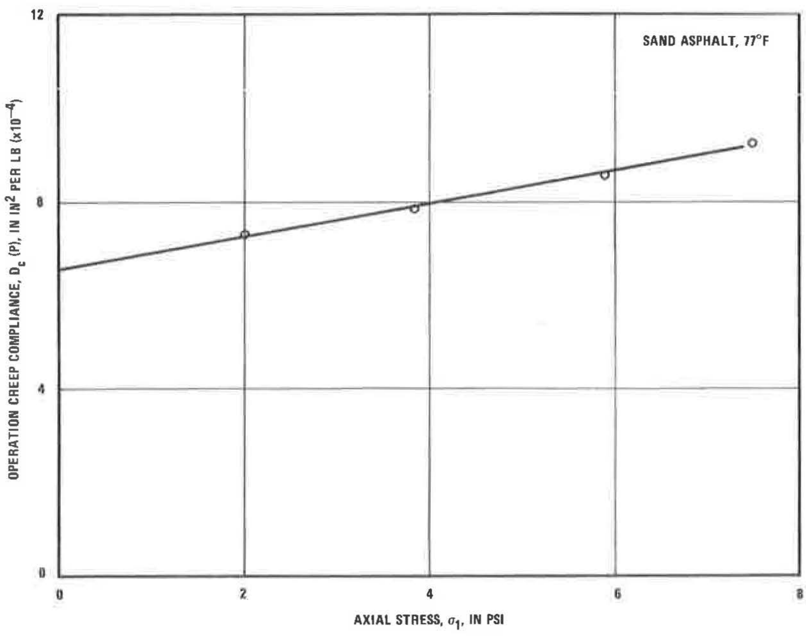

Figure 4. Variation of operational tensile creep compliance with axial stress for p = 0.1.

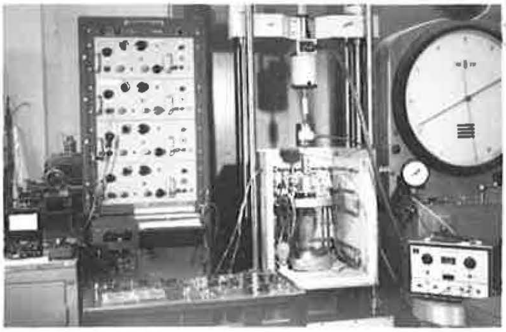

Figure 5. Compressive creep test apparatus.

For all stress levels, lateral deformations at the mid-height of the specimen were measured in one plane by using either two Collins LVDT displacement transducers wired in series or else two 1/10,000-in. Federal dial indicators. Values of Poisson's ratio were calculated from the lateral deflection measurements made at the center of the specimens from the relationship

$$\nu = \frac{1}{2}\left[1 - \frac{1}{\epsilon_a}\frac{\Delta V}{V}\right] \tag{1}$$

where ν = Poisson's ratio, ΔV = volume change, V = initial volume, and ϵ_a = axial strain. This approximate relationship was derived from the theory of linear elasticity, assuming the radial and tangential strains were equal. The change in volume that occurred during the creep tests was estimated by assuming that the end cross-sectional areas did not change and the deformed shape of the specimen could be approximated by plane surfaces. At a given instant of time the calculated values of Poisson's ratio were found to vary quite widely from one test to the next. However, when average values were plotted against the logarithm of time, Poisson's ratio was found to decrease almost linearly from a value of 0.32 at 0.5 min to approximately 0.11 at the end of 10 min.

Compression Creep Tests

The compressive creep properties of statically compacted cylindrical sand asphalt specimens approximately 2.9 in. in diameter and 6.0 in. in height were evaluated for confining pressures of 0, 15.0, and 35.0 psi and 3 deviator stresses in order to determine the effect of stress state on the material properties. The average density of the compressive specimens was 144.2 pcf (±0.9 percent). The compressive creep tests were performed between 20 and 50 days after compaction of the specimen. All tests were performed using a triaxial test cell that was placed inside a constant-temperature chamber as shown in Figure 5. A step function loading was applied to the specimen by means of a pneumatically operated Bellofram connected in series with a load cell and a small aluminum loading head. The Bellofram was activated by electrically opening a quick-release valve.

Lateral displacements at the mid-height of the specimen were measured throughout the tests by means of a lateral deflectometer. Also, final deflection profiles for a number of the specimens were measured using vernier calipers at the end of the creep tests. These data were used in estimating the end deformations and the deflected shape of the sample at selected times during the test. Poisson's ratio was calculated using the volume change relationship given in Eq. 1. The deflected sample profile was assumed to be a parabola with lateral deformations occurring at the top but no deformations at the bottom. These approximate calculations indicated that Poisson's ratio gradually increased with time. Apparently, however, this change is not very large. The average value calculated from 12 tests that included all confining pressures indicated a value of 0.50 for Poisson's ratio, assuming it is time-independent. The calculated variation from the average value, however, was found to be relatively large in both directions.

A typical measured compressive creep compliance response for the test performed at a confining pressure of 15 psi and a deviator stress of 19.0 psi is shown in Figure 6, together with the data from each individual test. All curves were corrected for the deformations occurring in the plexiglass end caps and the base of the triaxial cell. The creep compliance, as a function of the logarithm of time, was found at very small times to gradually increase at an increasing rate with time. By an elapsed time of

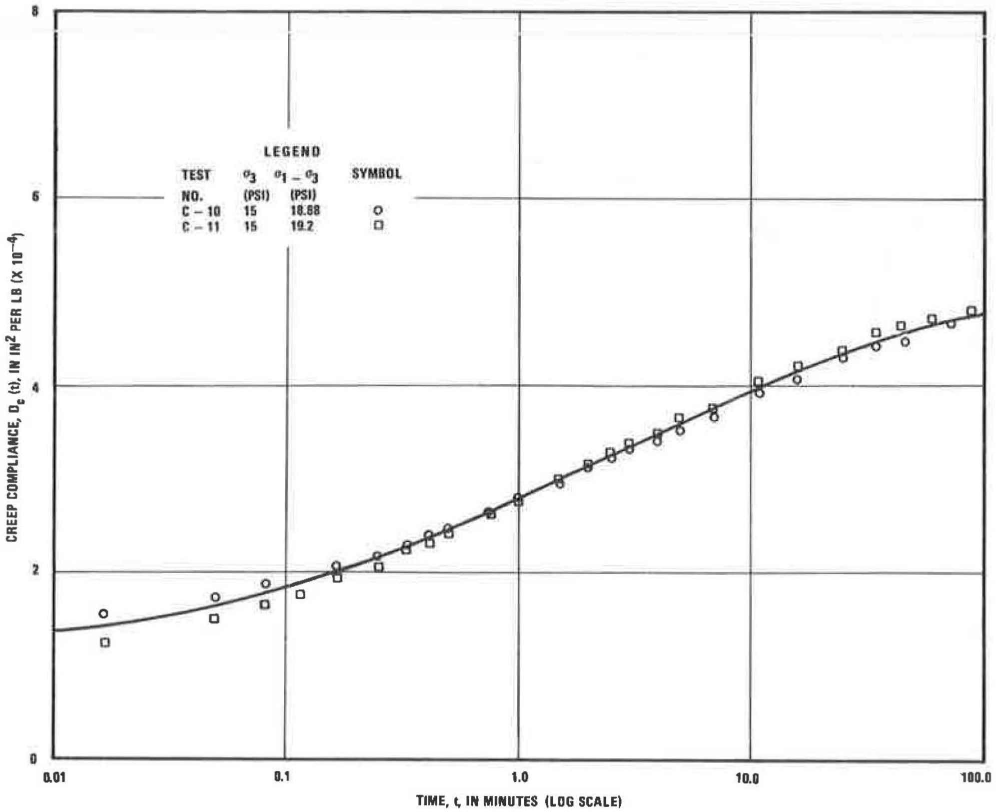

Figure 6. Experimental compressive creep compliance curves for sand asphalt at 77 F and a confining pressure of 15 psi.

approximately 0.1 min, the increase in creep compliance had become almost directly proportional to the logarithmic increase of time. The creep compliance, after remaining almost proportional to the logarithm of time for 2 to 3 log cycles, then started either to asymptotically approach an equilibrium value or else to get larger at a rapidly increasing rate, indicating approaching failure. Since failure of the specimen under a constant creep loading was not being investigated, the tests were discontinued after approximately 100 min if failure had not occurred before this time. Failure or indications of impending failure were observed during the 100-min testinng period only in specimens subjected to deviator stresses greater than 45 psi.

An 11-element Kelvin model was fitted to the experimental compressive creep compliance curves following the same procedure used in reducing the tensile test data. Using the Kelvin model representations, the operational creep compliance was calculated and plotted as a function of operational time for each creep test. The relationship between operational creep compliance, confining pressure, and deviator stress was obtained by cross-plotting the operational creep compliance curves at each desired instant of operational time. Typical results obtained for operational times $p \leq 6$ (which correspond roughly to real times ≥ 0.1 min) are shown in Figure 7 for $p = 0.04$. For this time range the operational creep compliance was found to apparently decrease as the confining pressure and deviator stress were increased, although a more detailed investigation is needed. The test results indicate that, at constant confining pressures, as the deviator stress is increased the rate of decrease in the operational creep compliance becomes smaller. For operational times $p > 6$ the operational creep modulus was found to increase as the confining pressure was increased and decrease as the

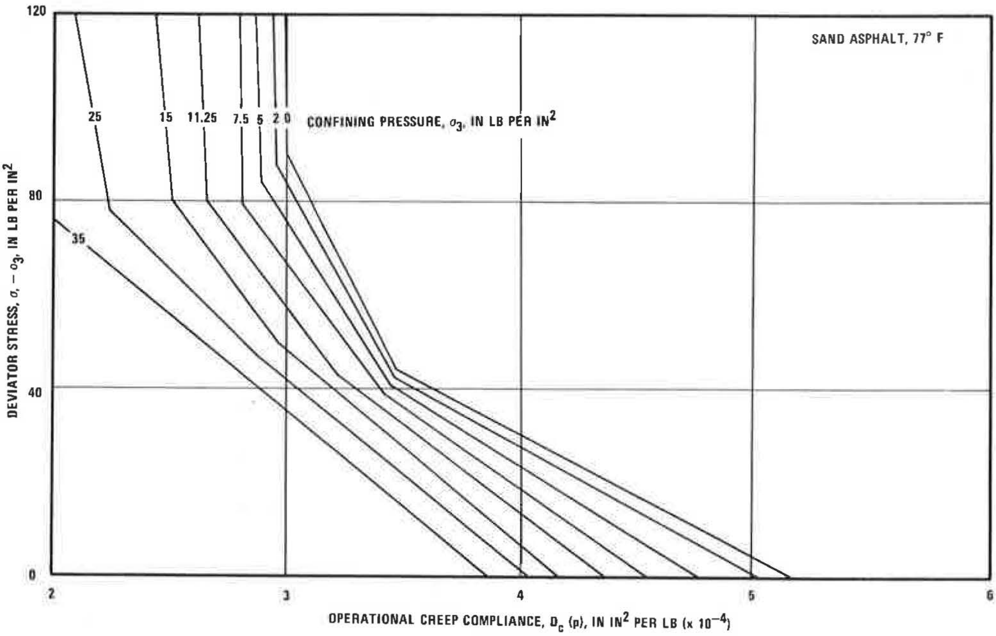

Figure 7. Influence of deviator stress and confining pressure on the compressive operational creep compliance for p = 0.04.

deviator stress increased. The same values of operational time p were used in reducing both the tensile test data and compression test data so that the material characterization could later be used in the nonlinear viscoelastic analysis.

Discussion of Tests

In performing a creep test, a certain finite rise time is required for the total applied load to become fully supported by the specimen. During this rise time the stress on the sample is increasing, and the sample is influenced to an unknown degree by impact effects. For these reasons the results of the creep tests should be considered valid only for times greater than about 10 times the rise time (28). The average measured rise time for the tensile creep tests was found to be 0.10 sec and for the compressive creep tests, 0.16 sec. Therefore, in characterizing the tensile and compressive creep properties for use together in a theoretical analysis, these properties should be considered valid only for times greater than about 1.6 sec after the specimens began to carry load.

In all of the creep tests performed, the axial specimen deformation was determined by averaging the axial deformation occurring on each side of the sample at equal distances from the center. This averaging procedure compensates for the effects of any bending due to eccentricity of load and gives more reproducible results than if only one value of deflection is measured at some distance out from the center. The recorder-transducer systems were calibrated periodically as close to the test setup position as possible by applying a known set of deformations to the transducers and measuring the corresponding movement of the recorder pen. This method of calibration eliminates any variations in transducer and recorder calibration factors, the effects of wire resistances, and ground loop errors that may occur in the electrical system. For both the tensile and compressive creep tests, the measured axial deformations were corrected for deformations occurring in the end caps and support system.

No matter how carefully a series of tests is performed, some experimental errors are always introduced. Assuming that all measurements were made correctly, most of the possible errors, such as eccentricity of loading, strain concentrations, piston friction, and deformations in the system that were not corrected for, would tend to give an apparent modulus somewhat lower than the actual modulus and shorter apparent times to failure. Piston friction was found to be apparently small, and approximate corrections were applied to account for at least part of the deformations occurring in the loading system. Although care was taken in performing the tests, eccentricity of loading was probably one of the more important experimental errors, particularly for the tensile creep tests.

Both the tensile and compressive creep test results show a relatively large scatter in calculated values of Poisson's ratio. Probably one important cause of this scatter is that the change in shape of the sample did not occur symmetrically about its mid-height, as is assumed in the reduction of the data. Furthermore, some scatter is probably partly due to the small deformations involved and the approximate equation used to reduce the data. The tensile test results may also have been influenced by anisotropy in a cross-sectional plane normal to the long axis of the specimen due to compaction. Large cracks parallel to the axis of the sample were observed in the compression specimens that failed. Cracks may also open up at lower stress levels, causing the calculated Poisson's ratio to be greater than 0.50. No matter what the cause, Poisson's ratio is apparently sensitive to the details of the test and deserves further study.

MODEL PAVEMENT STUDY

The only manner by which at least a partial evaluation of a theory can be made is to compare the predicted response of either a model or prototype pavement with experimentally measured response values. For this investigation, a small-scale idealized pavement model was chosen for verifying the proposed theory. The model selected consisted of a circular sand asphalt slab 1 in. thick and 18½ in. in diameter resting on a bed of linearly elastic springs. The slab was subjected to an approximately uniform step loading applied at its center over a radius of 1 in. The surface deflections predicted using the proposed nonlinear viscoelastic theory were then compared with the experimentally measured values.

Model Tests

The circular asphalt slab was prepared using the same aggregate gradation and 6½ percent asphalt cement content as was used in preparation of the samples from which the material properties in creep were evaluated. The asphalt slab was statically compacted to the desired thickness by a 2-stage compaction procedure using, in each stage, the full capacity of a 450,000-lb constant strain rate testing machine and a specially designed mold and loading assembly.

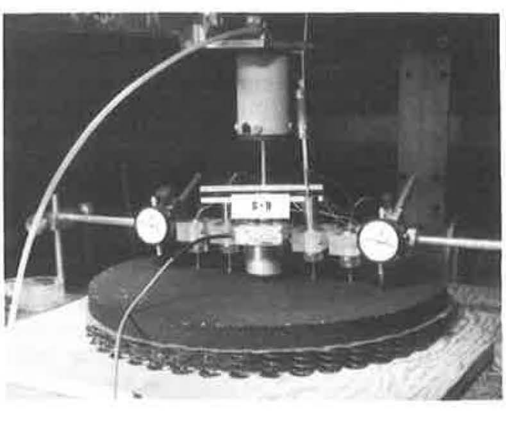

Figure 8. Model pavement represented by an 18½-in. diameter sand asphalt slab resting on a spring base.

The sand asphalt slab used in the model study was supported on 113 springs placed in 5 concentric rings. The stiffness of each spring was determined by calibration to be linear with their stiffnesses varying between 15.0 and 17.0 lb/in. In order to minimized the effect of variation in spring stiffness, the springs were grouped together so that the maximum variation of spring stiffness from the average within any one concentric ring of springs was less than +0.5 lb/in.

A total of 4 model tests were performed at a temperature of 77 F (±1 F) in a large constant-temperature room. The pavement model is shown in Figure 8. A 2-in. diameter step load of 33.5 lb was applied at the center of the model through a $^3/_8$-in. thick rubber cushion that was glued on the bottom of an aluminum loading head. The rubber cushion was used to reduce impact effects and to obtain as uniform a contact pressure as possible over the loaded surface. The step load was applied using the pneumatic apparatus previously described. The average load rise time for the 4 model tests was determined to be 0.15 sec. The deflection creep response to the step loading was measured across a single diameter of each slab by using 5 Collins self-excited LVDT transducers and 2 dial indicators. All transducers and dial indicators were zeroed just before the step load was applied to the specimen. Therefore, the reference datum for all measurements was the slab profile just before application of the step loading.

The average measured deflected surface profiles along a diameter for all 4 tests at times of 5 sec, 1 min, and 10 min after the application of the step load are shown in Figures 9 through 11. All points on the surface of the slab moved downward at the instant of load application, and the deflected surface profile had a bowl-shaped appearance at each successive instant of time. The central portion of the slab moved downward quite rapidly at first with a decreasing rate of movement with increasing time throughout the test. The outside edge of the slab, however, after an initial downward movement lasting less than 1 min, tended thereafter to move slightly upward but not enough to result in a net deflection above the reference datum.

Theoretical Prediction of Slab Response

The time-dependent response of the slab was calculated using the previously described nonlinear viscoelastic theory and the experimentally evaluated material properties. The transformed p-multiplied deflections $p\zeta(p)$ were calculated for the same 11 approximately equally spaced values of p between 0.01 and 100 used in characterizing the tensile and compressive operational creep compliance data. In the nonlinear viscoelastic finite element analysis, the $18\frac{1}{2}$-in. diameter slab was represented as an assemblage of 225 rectangular, ring-shaped finite elements, as shown in Figure 12.

Since there was no bond between the sand asphalt slab and the spring base, tensile forces could not physically be transmitted from the slab to the base. Therefore, the asphalt slab was assumed to rest on 3 to 5 concentric circles of linear springs depending on whether or not the slab was theoretically in contact with the particular ring of

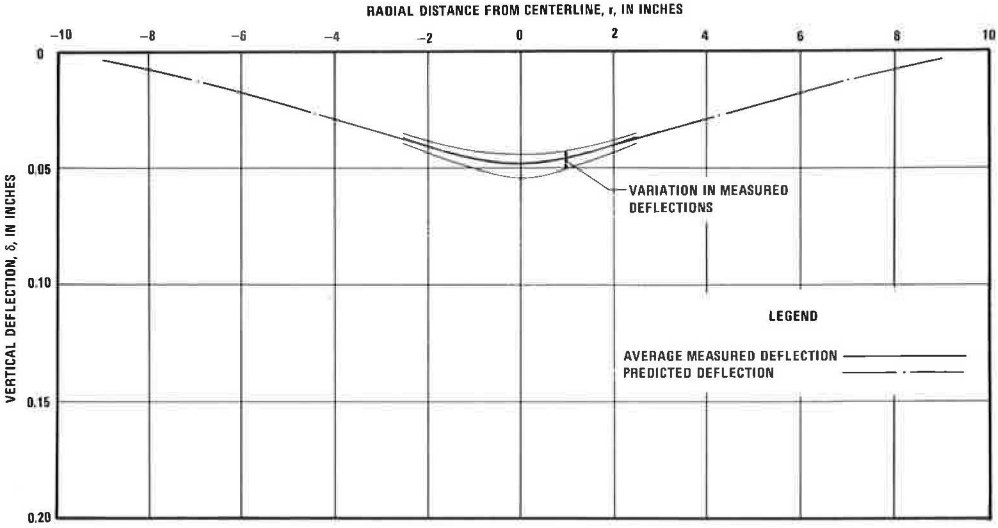

Figure 9. Comparison of measured and predicted surface deflection profiles of the model pavement after 5 sec.

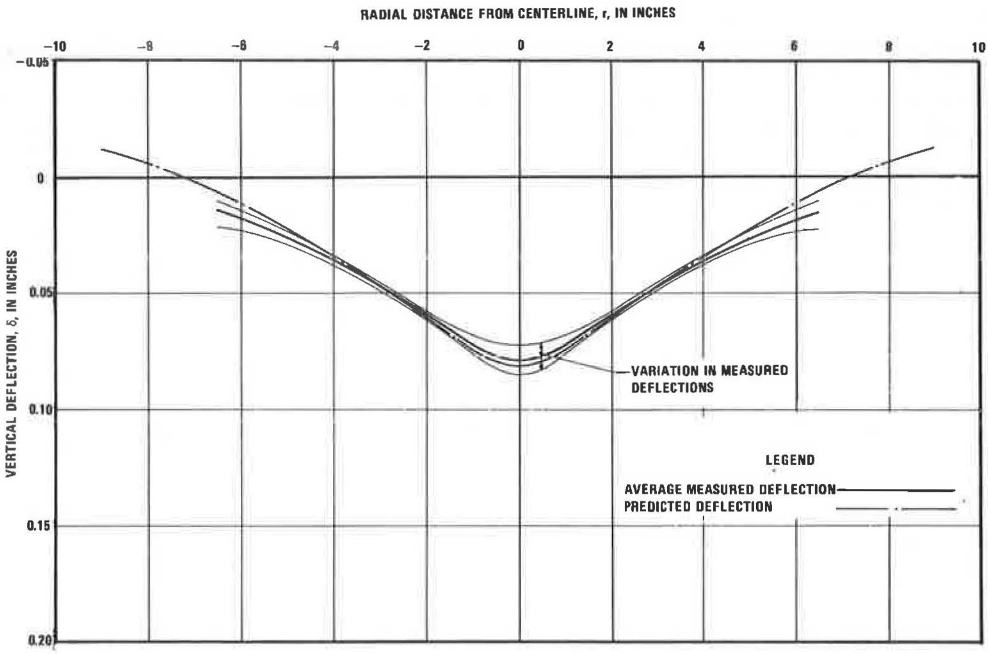

Figure 10. Comparison of measured and predicted surface deflection profiles of the model pavement after 1 min.

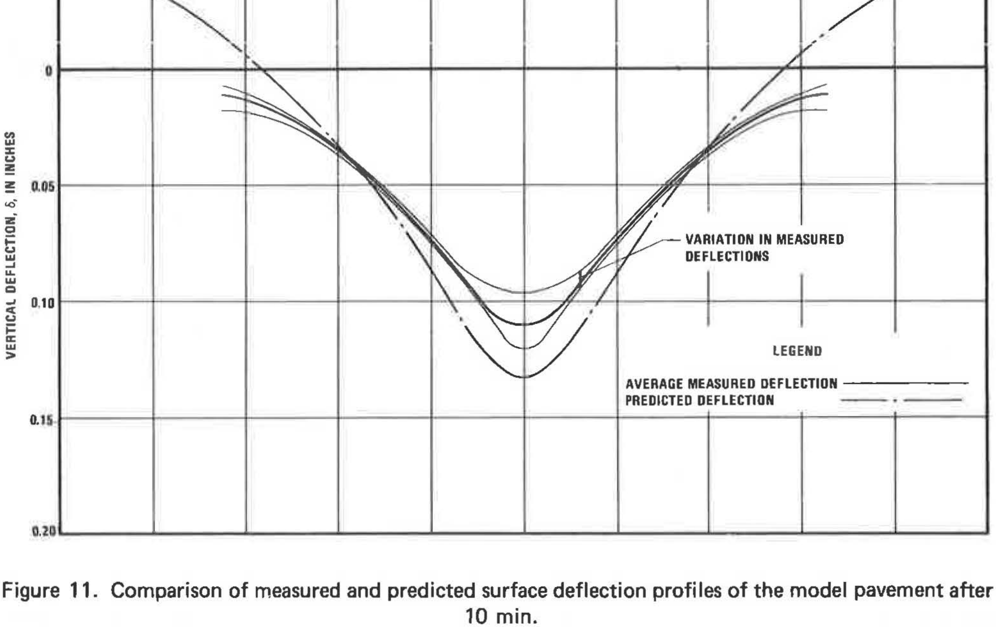

Figure 11. Comparison of measured and predicted surface deflection profiles of the model pavement after 10 min.

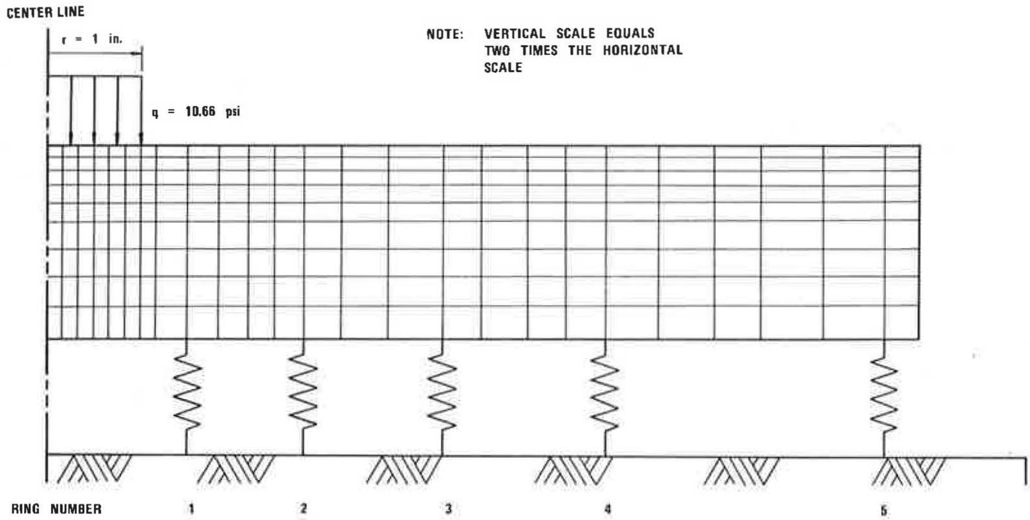

Figure 12. Finite element idealization of the model pavement system.

springs at each instant of operational time. The presence of the 5 concentric groups of linear springs was handled in the theoretical analysis by treating the force on each node connected to the springs like any other external node force. In this way the spring effect for each concentric ring of springs can be handled by simply adding the total spring stiffness for each ring into the correct position in the system stiffness matrix.

If under the imposed loading the volume of an infinitesimal element located at the center of each finite element increased, the entire element was assumed to be in a state of tension. For a tensile stress state, the effective stress law proposed by Nadai (20) was used in selecting the appropriate tensile creep compliance, using the uniaxial tensile creep test data. If in applying the volume change criteria a compressive state of stress was found to exist in an element, the two larger principal stresses (tension is defined as positive) were averaged to obtain an approximate confining pressure. For the desired instant of operational time, the operational creep compliance was selected from the corresponding set of material properties, such as those shown in Figure 7, for instance, using the compressive deviator stress and average confining pressure. For computational purposes, all material property data used in the nonlinear computer analysis were approximated by as many straight line segments as necessary to ensure a good fit. Using the procedure outlined above, for each load level and iteration the stress state at the center of each element was calculated and used to obtain the appropriate operational creep compliance.

In a preliminary study to determine the required number of load increments and iterations per load increment to use in order to ensure convergence, the nonlinear finite element analysis for the asphalt slab was performed at the operational time $p = 0.04$, using 9 load increments and a total of 3 iterations for each increment of load. The same problem was then solved using 18 load increments and 3 iterations per increment. This study indicated that the theoretically calculated slab deflections changed less than 1 percent, whereas the stresses in the slab changed by as much as 15 percent. Furthermore, this and the rest of the study indicated that performing several iterations within a load increment apparently had little effect on the results except in a few elements that were on the borderline between the tensile and compressive zones in the slab. Based on this limited study, 14 load increments with either 2 or 3 iterations for each increment were chosen for use in the remaining theoretical calculations. For other material properties, problem geometrics, and load configurations, the requirements for convergence would, in general, be different. After the p-multiplied values of response were calculated for each value of operational time, the results were inverted back to the real time plane using a 5- to 7-term Dirichlet series (2).

To determine if the creep properties evaluated from the compacted specimens were actually representative of those of the 18½-in. diameter slab, 1 x 1 x 2-in. specimens were cut from near the center after testing the slab. A limited number of tensile and compressive creep tests were performed on these small specimens. The compressive creep properties were found to be almost identical to those measured using the cylindrical specimens, and the tensile creep properties of the slab were found to agree reasonably well with those measured from the 6-in. tensile specimens.

A very good agreement between the theoretical profiles and the measured ones exists up to an elapsed time of about 4 min. By the end of 10 min, the calculated and measured profiles still compare favorably. The theoretical profiles did indicate that the edges of the slab should lift up off the spring base at a time slightly less than 1 min, whereas the measured deflections indicated that the outside edges of the slab did not lift up at all. This discrepancy is probably at least partly due to the fact that the measurements on the model were taken using the deflected profile of the slab just before the loads were applied as the zero reference. During several of the model tests, the outer two rows of springs were observed not to be in contact with the slab.

GENERAL DISCUSSION

The fact that the calculated center deflections for times greater than approximately 1 min became increasingly larger than the measured deflections can be explained if the measured stiffness of the sand asphalt is assumed to be less than the actual stiffness. Almost all experimental errors, such as eccentricity of load, system and end cap deformation, and strain concentrations, would tend to give an experimentally measured stiffness of the sand asphalt less than the actual stiffness. The tensile test specimens failed at times between approximately 8 and 100 min, with the time to failure decreasing as the stress level increased. The time required to cause tensile failure at the same stress level was found to be greatly influenced by seemingly minor details of either the test or the method of specimen preparation or possibly both. All the tensile specimens failed at times of less than 2 hours. However, when subjected to the applied loading for as long as 10 hours, the model pavement did not visibly fail. One possible explanation for this behavior is that at relatively large times the actual stiffness of the slab is indeed greater than the experimentally measured apparent stiffness of the tensile specimens.

The theoretical study shows that, for the model pavement studied using 14 load increments and 3 iterations for each increment, in general the modulus of each element changes only a very small amount from the first to the final iteration. An exception to this occurs when an element is apparently just on the verge of going from a tensile state to a compressive state of stress. In this case, which apparently occurs in only a very few elements, the modulus tends to oscillate between the appropriate tensile and compressive value. Considering all factors for the numerical solution of the problem solved in this investigation, it is concluded that apparently a total of 2 iterations for each load level is sufficient. Based on the work of others and on the very limited number of problems solved during this investigation, it is intuitively felt that as many load increments as possible should be used with only a total of one or two iterations made within each load increment. For moderately nonlinear material properties, 10 to 20 load increments may often be found to give good results. For strongly nonlinear material properties, however, as many as 20 to 40 or more iterations may be required. It should be emphasized that in either instance convergence of the numerical calculations should be studied for each specific problem.

In applying the theory to the idealized model pavement it is assumed that as the external load is incrementally increased the stress state acting on each element either remains the same or always changes in the same direction. This assumption means that unloading of a given element during the incremental application of the external load is assumed to never occur. For most pavement problems, application of the load should not result in a condition of unloading in most of the elements. A few elements may, however, undergo some unloading and introduce an unknown but probably small error into the analysis. The theory developed is perfectly general and a condition of unloading can be readily included using the plastic strain concept.

Tremendous progress has been made in the last few years toward developing numerical step-by-step and iterative methods of stress analysis suitable for solution using high-speed digital computers. These numerical approaches can be used to solve time-dependent problems whose materials exhibit both nonlinear and anisotropic properties. This rapid advancement of theory has resulted in a very definite need for reevaluating presently used methods of material testing and characterization. In particular, a great need now exists for more research on the effect of stress state on material properties and the development of realistic effective stress-strain laws for the materials used in pavement construction, which often undergo high volume changes and exhibit quite different properties in tension and compression.

SUMMARY AND CONCLUSIONS

The problem of developing a rational theory that can be used in predicting the performance of flexible highway pavement systems is very complicated because of the large number of wheel load repetitions, complex material properties and pavement geometry, and cyclic environmental changes that occur during the life of the pavement. The proposed incremental nonlinear finite element theory offers a quite general approach to the problem of predicting both elastic and cumulative stresses, strains, and deflections in highway pavement systems. The proposed plastic strain concept incorporated in the finite element theory results in a practical method for handling the variation of both elastic and plastic material properties with increasing numbers of wheel load applications. The material properties for use with this theory would be evaluated from either repeated-load triaxial tests or other suitable dynamic tests. Variations of the material properties with tensile and compressive stress states, confining pressures, deviator stresses, environmental changes, and changes with age or number of load applications can all be readily considered using the proposed nonlinear theory. The most important assumptions made in this theory are that (a) inertia forces in highway pavements as a first approximation can be neglected, (b) the surface loading is normal to the pavement, and (c) axisymmetry of the load and pavement structure is maintained.

This study and other investigations indicate that the rate of convergence using the incremental nonlinear approach is dependent on the type of loading, the geometry of the solid, the manner in which the load is applied, and the degree of material nonlinearity. For these reasons, the convergence of the iterative numerical calculations for nonlinear problems should always be carefully studied for each specific problem investigated.

The nonlinear theory developed in this investigation and also other numerical methods are presently available for the comprehensive analysis of complex structural mechanics problems. These relatively new theories which can handle complicated nonlinear material properties have resulted in a very definite need for reevaluating presently used methods of material characterization. An important need now exists for a detailed study of the influence of stress state on material properties, and for developing a realistic effective stress-strain law for the class of materials used in pavement construction.

A nonlinear engineering theory suitable for predicting the structural response of flexible pavement systems subjected to repeated loads was presented and verified for the case of a hollow cylinder and an idealized model pavement system subjected to a creep loading. The next step is to verify the proposed theory for either a full-scale pavement test section subjected to repeated wheel loads or else a full-scale laboratory test pavement subjected to either stationary or moving repeated loads.

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