kalankamary 73-77

Elastic Properties of Pavement Components by Surface Wave Method

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The present study explores the feasibility of using vibration methods in determining the Rayleigh wave velocity and thereby the elastic properties of a three-layer pavement. When the intermediate layer in a three-layer composite pavement is stiffer than the surface layer, the existing procedure to determine the Rayleigh wave velocity of the former cannot be applied. A correction procedure is proposed in this study; the Rayleigh wave velocity and the thickness, determined accordingly, are both within 9 percent of the actual values.

•A KNOWLEDGE of the elastic properties and thickness of the pavement layers that make up the pavement is required for design, maintenance, and repair of highway pavements. In recent years, nondestructive methods have become of increasing interest in highway engineering practice (1, 2). A technique based on the measurements of surface waves was proposed by Jones (2, 3) where the elastic properties of two- or three-layered pavements were determined by using the theory of layered systems. Jones (2) dealt with the case of an intermediate layer that has a modulus of elasticity slightly less than that of the surface layer but considerably greater than that of the underlying medium. The present study, however, treats a three-layer system in which the surface layer has a modulus of elasticity less than that of the intermediate layer; a typical example is an asphalt-cement layer overlying the soil-cement base.

ELASTIC WAVES IN LAYERED SYSTEM

Road construction is regarded as being composed of layers of homogeneous, elastic materials and of infinite horizontal extent. In Figure 1, layer H_1 represents the asphalt surfacing, layer H_2 is the soil-cement base, and the uniform semi-infinite medium is the soil under the pavement.

Miller and Pursey (4) have shown that a vibrator on a circular base operating normal to the surface of a semi-infinite elastic solid radiates 67.4 percent of the power as a surface wave. The surface wave here is the Rayleigh wave, which has its maximum particle displacement normal to the surface. When vibrations of the Rayleigh wave type are propagated in a layered medium their velocity depends on the frequency of the vibrations and the thickness, density, and elastic properties of strata. Accordingly, the approach taken in this investigation has been to carry out the proper dynamical measurements on the layered system and to exploit the properties of surface waves to determine certain unknowns either pertaining to material constants or of geometrical origin.

Single-Layer Overlying the Semi-Infinite Medium

The derivation of the wave equation for the case of one surface layer over a semi-infinite medium involves the computation of a sixth-order determinant, which can yield more than one velocity at each frequency (5). The solution giving the lowest velocity

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Paper sponsored by Committee on Theory of Pavement Design.

refers to the principal mode of propagation of the Rayleigh wave, and this particular mode can be shown to correspond to the fundamental flexural branch of a free plate. The evaluation of phase velocity from the sixth-order determinant can be simplified; for instance, when the wavelength is small compared with the thickness of the layer (i.e., $L/H \to 0$), the phase velocity of surface waves tends to correspond to the Rayleigh wave velocity appropriate to the top medium.

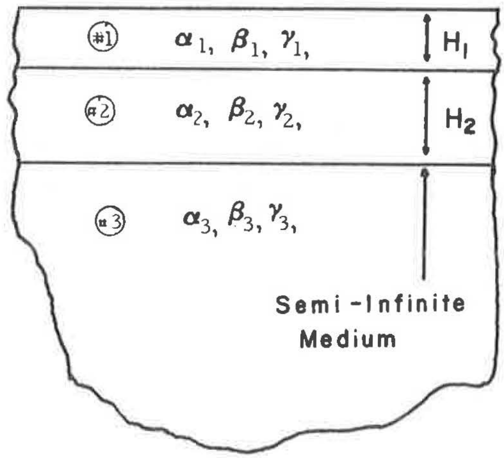

Figure 1. Section of a three-layer pavement: asphalt surface (layer 1), soil-cement slab (layer 2), and subgrade (semi-infinite medium).

"Free Plate" Approximation

A single-layer pavement over a conventional subgrade can be approximately treated as an elastic plate, the surface of which is free of stresses (3). The solutions obtained by Lamb (6) for the propagation of the longitudinal and flexural waves in a free plate are represented by $P = 0$ and $Q = 0$ respectively, where

$$P = b_1^2 \cosh 1/2\, r_1H_1 \sinh 1/2\, s_1H_1 - \frac{4r_1s_1}{k^2} \sinh 1/2\, r_1H_1 \cosh 1/2\, s_1H_1 \tag{1}$$

$$Q = b_1^2 \sinh 1/2\, r_1H_1 \cosh 1/2\, s_1H_1 - \frac{4r_1s_1}{k^2} \cosh 1/2\, r_1H_1 \sinh 1/2\, s_1H_1 \tag{2}$$

in which

$$r_1^2 = k^2\left(1 - \frac{c^2}{\alpha_1^2}\right);\ s_1^2 = k^2\left(1 - \frac{c^2}{\beta_1^2}\right);\ b_1^2 = 1 + \frac{s_1^2}{k^2};$$

c = phase velocity, $k = \frac{2\pi}{L}$, L = wavelength;

H_1 = thickness of the layer; and

$\alpha_1, \beta_1, \gamma_1$ = compressional, shear, and Rayleigh wave velocity in the layer.

Therefore, if experimental measurements have been made at wavelengths short enough to define γ of the layer, its thickness can be determined by a nonlinear curve-fitting procedure (to satisfy $Q = 0$).

Two Surface Layers of Comparable Moduli of Elasticity

The propagation of Rayleigh waves in a system composed of two surface layers over a semi-infinite medium requires the solution of a tenth-order determinant (2, 6), and the computational work becomes almost prohibitive. However, the evaluation of phase velocity from the dispersion equation can again be simplified for the case of very small wavelength ($L/H \to 0$).

In a plate-subgrade system, when the plate is much stiffer than the underlying subgrade the propagation of the vibration becomes sensibly independent of the properties of the subgrade. Accordingly, in the present problem, when the phase velocity exceeds α_3 the propagation will depend almost entirely on the properties and thickness of the materials in the two layers, and the two surface layers may be regarded as a composite layer.

In practice the vibrations are excited and measured at the surface of the upper layer so that Eq. 2—the Lamb solution of flexural vibrations—with the appropriate parameters of the top layer is the solution that will apply. As the wavelength of the surface vibration exceeds $2H_1$, however, the experimental dispersion relation is seen to deviate from the Lamb solution. A simple explanation of the observed deviation is that the surface waves are influenced by the second layer as well. Maxwell and Fry (8) strongly support this viewpoint when they assume that the surface waves are normally conditioned by the material from the surface to a depth equal to one-half of the length of the surface waves. Founded upon this hypothesis, the following empirical relationship is proposed to resolve the phase velocity of the composite plate into two component velocities appropriate to the individual layers:

$$c\,L/2 = c_1 H_1 + (L/2 - H_1)\,c_2 \qquad (3)$$

where

$$2H_1 \leq L \leq 2(H_1 + H_2)$$

and

c = phase velocity (surface wave) of the composite plate at wavelength L;
c_1 = flexural wave velocity appropriate to the first layer at wavelength L; and
c_2 = flexural wave velocity appropriate to the second layer at wavelength L.

When a number of points relating the wavelength and the phase velocity in the second layer have been obtained, extrapolation to zero wavelength can be accomplished by use of the Lamb flexural wave curve that offers the best fit to the data.

EXPERIMENTAL PROCEDURE

A two-layer composite pavement was constructed on a semi-infinite subgrade where the test slab consisted of a 0.32-ft thick soil-cement slab (8 ft by 6 ft, cement 10 percent by weight) overlaid by 0.27-ft thick dense-graded hot plant mix. To justify the assumption of an infinite slab in the horizontal direction, the edges of the slab were tapered.

Beam specimens, 3 by 3 by 11¼ in., were molded from both soil-cement and asphalt mixtures. Elastic constants of both materials were determined from the fundamental transverse vibration and torsional resonant frequency test (ASTM Designation C 215-58T).

Compressional Wave Velocity

Seismic tests (1) were made to determine compressional wave velocities in the soil-cement and the asphalt pavement.

Surface Wave Velocity

Vibration tests were conducted to determine the wavelength and thereby the phase velocity of the surface wave. The details of the equipment (the electromagnetic vibrator, cartridge pick-up, and preamplifier) and the technique used to detect the wavelength using a dual channel oscilloscope can be seen elsewhere (7).

RESULTS AND DISCUSSION

Single Surface Layer of Higher Elastic Moduli Than the Underlying Subgrade

Attention herein is confined to a stiff slab (soil-cement base) over a relatively soft subgrade for which flexural wave dispersion curves conform adequately to the free plate approximation. When the measurements are made to a sufficiently high frequency, to define γ_2 for the slab material, the dispersion relation can be inverted to give the thickness of the slab (curve 1 in Fig. 2). The Rayleigh wave velocity for the soil-cement has been determined as 4,650 ft/sec, which compares favorably with the value computed from the resonant frequency tests (4,600 ft/sec). The thickness of the soil-cement slab is found to be 0.31 ft, which compares favorably with the actual thickness, 0.32 ft.

The Rayleigh wave velocity determined in conjunction with the compressional wave velocity by seismic method and the charts of Knopoff (5, p. 34) make possible the

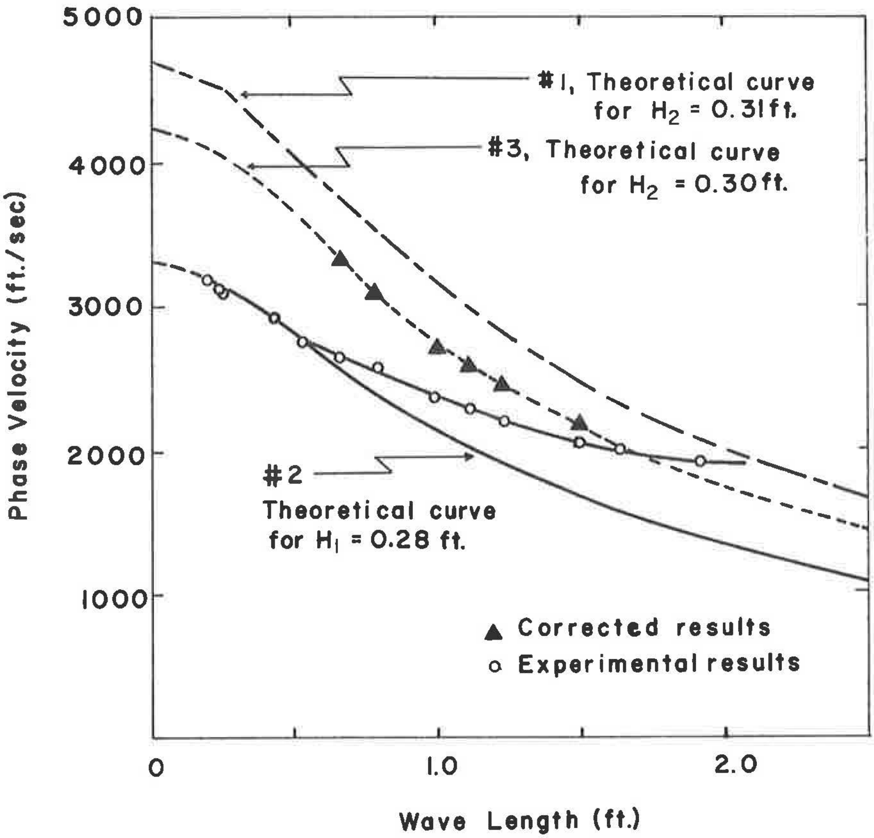

Figure 2. Phase velocity of flexural vibrations in a composite layer: theoretical solution from solving $\Omega = 0$.

determination of Poisson's ratio (0.23). The fact that the Poisson's ratio computed by an independent method—namely from transverse vibration and torsional resonant frequency—is in excellent agreement (0.22 vs. 0.23) validates the applicability of the surface wave technique.

Two Surface Layers of Comparable Moduli of Elasticity

Experimental results of phase velocity and wavelength at the surface of the three-layer construction are given by the open points in Figure 2. The results obtained at short wavelengths ($L \leq 2H_1$) permit the relation to be extrapolated to zero wavelength to provide a value of γ_1 in the asphaltic layer of 3,300 ft/sec. With this γ_1, the thickness, according to the Lamb solution, that best fits the experimental points is 0.28 ft, which is in excellent agreement with the actual thickness of 0.27 ft (curve 2 in Fig. 2).

As expected, when the wavelength exceeds $2H_1$, the experimental points lie to the right of the theoretical curve pertaining to the surface layer alone. The phase velocity for the soil-cement layer is calculated by the correction equation (Eq. 3) and given by solid points in Figure 2. The relation obtained by these data points may be extrapolated to get an approximate value of γ_2. Now that the Rayleigh wave velocity and thickness (thickness is normally known in new or old pavements) are approximately known, a theoretical curve that matches the computed data can be found by a judicious trial procedure (curve 3 in Fig. 2). The agreement between this curve and the Lamb curve, which fits the experimental data resulting from direct measurements, is remarkable.

Rayleigh wave velocity and the thickness of the soil-cement layer obtained by the procedure using the empirical correction equation are 4,200 ft/sec and 0.30 ft respectively, which compare well with the actual values (4,600 ft/sec and 0.32 ft).

It may be noted here that, by performing the vibration experiment alone, it is difficult to determine the relative stiffness of the first layer. The microseismic procedure described by Phelps and Cantor (1) is proposed for positive identification. Accordingly, if the slope of the travel-time graph is not changed, the top layer is stiffer than the bottom layer and the equation proposed by Jones (2) should be used for final correction. If the slope of the travel-time graph tends to change, however, the second layer is taken to be stiffer than its top counterpart and Eq. 7 is proposed in the final analysis.

CONCLUSIONS

The theory of wave propagation in layered media presented and the experimental technique developed provide an effective means of determining the Rayleigh wave velocity appropriate to the top slab in a two-layer pavement or to the top slab of a three-layer pavement. In the three-layer pavement, when the intermediate layer is stiffer than the top layer, the correction method proposed in the present study is satisfactory to determine the Rayleigh wave velocity and the thickness of the intermediate layer.

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