Optimal Design of Flexible Pavement Sections

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Although several methods are available for the design of flexible pavements, no existing technique explicitly considers the optimal combination of flexible pavement components to minimize the total in-place cost of the pavement system. The purpose of this systems analysis was to develop a rational method for the optimal selection of the arrangement of the various pavement components. This cost minimization must be realized within the boundary conditions imposed by the practical limitations of the design parameters. The design model consists of an objective function and various constraint equations. The total cost of the pavement system is quantitatively described by this objective function, and a minimum-cost solution is obtained for each combination of material costs and design conditions. The various constraining equations quantify the boundary conditions to which the design of a flexible pavement is subject. These physical limitations complete the realism of the mathematical model in describing the real-world situation of flexible pavement design. The design model was solved by a modified linear programming technique. In developing practical solutions to the design model, optimal flexible pavements are designed for cross sections without subbase, cross sections with subbase through shoulders, and cross sections with subbase and subdrains. The design requirements for the various components are predicted on the design parameters of traffic conditions, soil support values, pavement material characteristics, environmental effects, and pavement performance requirements, and on unit costs of pavement components. Substantial cost savings result in the selection of flexible pavement sections by this design procedure.

The primary objective of highway pavement design is to provide an acceptable roadway surface that can withstand the deteriorating effects of traffic and environment for the service life of the facility. In addition, the pavement structure must adequately serve the demands of the road users at an acceptable level of performance. A properly designed, constructed, and maintained pavement is a major factor in providing economical, efficient, safe, convenient, and comfortable highway travel. This goal is an integral part of the total highway transportation program.

Although several design techniques are available for determining reasonable thicknesses of flexible pavements to satisfy the specified design parameters, no present method explicitly considers an optimization of flexible pavement components to minimize the total cost of the pavement system. Of course, this cost minimization must be realized within the boundary constraints imposed by the selected values of the design parameters. The purpose of this systems analysis was to develop a rational method for the optimal design of flexible pavement sections.

The objective of flexible pavement design in this investigation is to select the various pavement components so that the total pavement cost is minimized within the limitations of the various design parameters. Minimum-cost designs are determined for
flexible pavements to satisfy the demands of traffic and environment on the system of pavement structure and soil support. Therefore, this technique affords a practical and economical solution to the problem of designing flexible pavements. This approach to design embodies the essence of sound engineering.

CONCEPTUAL MODEL

A flexible pavement distributes the traffic loads through a system of pavement components to the subgrade. These pavement layers are generally identified as surface, base, and subbase. Several different thickness combinations of the materials comprising the various components may adequately satisfy the structural design of the highway pavement. However, all satisfactory thickness arrangements may not provide an economical solution to the engineering problem of pavement design. In general, only one pavement structure is an optimal selection of the flexible pavement components for the designated design conditions.

The concept for this flexible pavement design procedure is illustrated by the logic diagram in Figure 1. The total pavement system is described by the various design parameters representing traffic conditions, soil support values, pavement material characteristics, environmental effects, and pavement performance requirements. In addition, unit costs of pavement components and alternate cross section designs are considered in the selection of the optimum flexible pavement section.

The structural requirements of flexible pavements are predicated on an estimated number of equivalent 18-kip single-axle load repetitions and on an appropriate measure of the soil support afforded by the subgrade. The elements of pavement performance and environment are also incorporated as initial and terminal serviceabilities and regional factor respectively. The combined effect of traffic loading, soil support, pavement performance, and environment is denoted as a structural number (SN) according to the interim design guide for flexible pavements of the American Association of State Highway Officials (1). Pavement component thicknesses are then selected to reproduce the specified structural number by a linear combination of layer thickness times its coefficient of relative strength. A minimum pavement thickness is equal to the summation of the component thicknesses.

Consideration of significant environmental factors, such as depth of frost penetration, provides another control on the selection of a minimum pavement thickness. This design procedure specifies a minimum pavement thickness \( T_{\text{min}} \) to account for various influencing environmental conditions. This minimum thickness is based on a design procedure that requires a selected design wheel load and a specified soil support value. The greater minimum thickness value becomes the design requirement.

To account for varying design practices, several types of pavement cross sections are available as possible alternatives in this procedure for designing flexible pavements. These arrangements include cross sections without subbase, cross sections with subbase through shoulders, and cross sections with subbase and subdrains. Finally, the unit costs of the pavement components are specified to permit the design of an acceptable pavement structure for the least cost. This cost-effectiveness approach provides both an optimal and a practical solution to the problem of flexible pavement design.

In a real sense, the minimum thicknesses represent design constraints and not design objectives. The design objective is to produce a flexible pavement system at the least total cost within the specified boundary conditions. The in-place unit costs of the component materials depend on the locale in which the flexible pavement is to be constructed. In addition to the traffic loading, soil support, pavement performance, and environmental constraints, practical limitations on layer thicknesses are specified in concurrence with present highway construction practices.

DESIGN MODEL

The logic diagram for this optimal design of flexible pavements is shown in Figure 1. A detailed description of this design technique is presented in the following sections,
Figure 1. Design concept.
which provide the various computational procedures and design features for determining the optimal selection of flexible pavement sections.

Design Parameters

Design parameters represent the various measures of traffic conditions, soil support, pavement material properties, environmental effects, and pavement performance requirements. The results of these evaluations provide the summary quantities that are necessary for the optimal design of flexible pavements.

The initial measure of the stability of the subgrade soil is determined by the standard California bearing ratio (CBR) test. This soil strength is then translated into the soil support value (SSV) as defined by AASHO (1). In this study the following equation was developed to relate soil support values to CBR measures:

\[ SSV = 4.90 \log_{10}(CBR) \]

where SSV is the soil support value and CBR is the California bearing ratio.

The traffic conditions are expressed as the number of 18-kip single-axle load repetitions for the service life of the pavement. These load applications are estimated from an evaluation of the formula

\[ W = 365 \times (TF) \times (DP) \]

where

- \( W \) = total number of equivalent 18-kip single-axle load repetitions during the pavement design period,
- \( TF \) = truck factor (18-kip single-axle load applications per day), and
- \( DP \) = design period (years).

To develop a measure of the truck factor, a correlation was derived between the number of 18-kip single-axle load applications and the percentages of various truck types in the traffic stream. The following expression was obtained from loadometer data collected on highways in Indiana:

\[ TF = \left[ \frac{(ADT_1) + (ADT_2)}{4} \right] \left[ \frac{11.7(TR)(LU) + 0.83(TR)(LU)(CT)}{10,000} \right] \]

where

- \( TF \) = truck factor (18-kip single-axle load applications per day),
- \( ADT_1 \) = average daily traffic volume at the start of the design period (vehicles per day in both directions),
- \( ADT_2 \) = average daily traffic volume at the end of the design period (vehicles per day in both directions),
- \( TR \) = percentage of all trucks,
- \( CT \) = percentage of combination trucks, and
- \( LU \) = truck lane use factor (1.0, 0.9, and 0.8 for two-, four-, and six-lane highways respectively).

The various measures of traffic conditions, soil support, environmental effects, and pavement performance requirements are now combined into a single design parameter defined as the structural number (SN). Two nomographs have been prepared by AASHO to quantify this structural requirement (1). However, the following equation was developed from these nomographs to use in the computer program for this design procedure:


where
\[ W = \text{total number of equivalent 18-kip single-axle load repetitions during the pavement design period,} \]
\[ SN = \text{structural number,} \]
\[ CO = \text{initial pavement serviceability index (4.2 for all highways),} \]
\[ P = \text{terminal pavement serviceability index,} \]
\[ SSV = \text{soil support value, and} \]
\[ RF = \text{regional factor.} \]

The effects of the environment are numerically summarized in the regional factor (2), and the desired pavement performance is specified by selected values for the initial and terminal pavement serviceability indexes. An iterative procedure is used to solve this equation for the structural number of a particular design situation.

Another consideration of environmental influences is determining a minimum thickness as a design against the detrimental effects of frost action and the loss of subgrade strength in the spring break-up period. Design charts developed by Hicks (3) provide correlations between bearing capacity and CBR and between pavement thickness and bearing capacity. Adverse subgrade conditions are represented by using a 4-day soaked value for the selected CBR. The following relationships were prepared from these design charts for 9-kip and 10-kip wheel loads respectively:

\[ T_{\text{min}(9)} = 4.723 + \frac{61.037}{(\text{CBR})^{0.05}} - 45.18 \cdot e^{-\text{CBR}} \]
\[ T_{\text{min}(10)} = 4.423 + \frac{52.706}{(\text{CBR})^{0.90}} - 19.884 \cdot e^{-\text{CBR}} \]

where
\[ T_{\text{min}(9)} = \text{minimum pavement thickness for 9-kip design wheel load (inches),} \]
\[ T_{\text{min}(10)} = \text{minimum pavement thickness for 10-kip design wheel load (inches),} \]
\[ \text{CBR} = \text{California bearing ratio for reduced strength conditions.} \]

The 10-kip wheel load is considered satisfactory for the design of primary highways, whereas the 9-kip wheel load is applicable for flexible pavements on secondary routes. In the computer input for this design model, the highway engineer specifies the design wheel load for either a primary or a secondary highway. This minimum-thickness determination accounts for environmental effects by highway classification and provides another realistic constraint in selecting optimal flexible pavement sections.

The characteristics of each pavement material are described by the in-place density and the coefficient of relative strength. These values depend on the local materials used in the construction of flexible pavements. The evaluation of the pavement material characteristics permits the application of the design model for the prevailing construction practices.

The foregoing descriptions numerically define the various design components of the flexible pavement system. Although the selected equations provide reasonable evaluations of these parameters, other expressions can be used to satisfy local design conditions.
Figure 2. Typical cross section of a flexible pavement without subbase for a four-lane highway, one direction.

Figure 3. Typical cross section of a flexible pavement with subbase through the shoulder for a two-lane highway.

Figure 4. Typical cross section of a flexible pavement with subbase through the shoulder for a four-lane highway, one direction.
Design Sections

Because reasonable variations exist in the design of highway elements, three acceptable cross sections were selected for two-lane and divided multilane highways to provide several alternative designs in the model. These arrangements include the following distinct designs:

1. Cross sections without subbase, $S_1$;
2. Cross sections with subbase, $S_2$, extended through the shoulders for two-lane highways and extended through the right shoulder with subdrain under the left shoulder for divided multilane highways; and

3. Cross sections with subbase and subdrains under both shoulders, $S_3$.

Typical details of these cross-sectional designs are shown in Figures 2, 3, 4, and 5 respectively. The shoulder designs are further detailed in Figure 6 for cross sections with subbases and with subdrains.

Of course, additional cross-sectional arrangements may be incorporated into this design model. Because each section represents a different design, an objective function is required for each cross section to permit the optimal selection of flexible pavement sections. The best design then is the cross section that minimizes the total pavement cost for the specified design parameters.

Optimization Model

The optimal design of flexible pavement sections is depicted by the following objective functions for the three different design sections.

1. Cross sections without subbase:

$$\text{Min. } S_1 = \left( \frac{C_1 D_1 L k_i}{12 \times 2,000} \right) d_1 + \left( \frac{C_2 D_2 L k_i}{12 \times 2,000} \right) d_2$$

$$+ \left( \frac{C_3 D_3 L k_j}{12 \times 2,000} \right) d_3 + E_L + H_L$$

2. Cross sections with subbase through shoulders:

$$\text{Min. } S_2 = \left( \frac{C_1 D_1 L k_i}{12 \times 2,000} + \frac{C_4 A}{12 \times 27} \right) d_1$$

$$+ \left( \frac{C_2 D_2 L k_j}{12 \times 2,000} + \frac{C_4 A}{12 \times 27} \right) d_2$$

$$+ \left( \frac{C_3 D_3 L k_j}{12 \times 2,000} + \frac{C_4 A}{12 \times 27} \right) d_3$$

$$+ \left[ \frac{C_3 (L + A)}{12 \times 27} \right] d_4 + E_L + H_L + M_L - Y_L$$

3. Cross sections with subbase and subdrains:

$$\text{Min. } S_3 = \left( \frac{C_1 D_1 L k_i}{12 \times 2,000} + \frac{C_4 B}{12 \times 27} \right) d_1$$

$$+ \left( \frac{C_2 D_2 L k_i}{12 \times 2,000} + \frac{C_4 B}{12 \times 27} \right) d_2$$

$$+ \left( \frac{C_3 D_3 L k_j}{12 \times 2,000} + \frac{C_4 B}{12 \times 27} \right) d_3$$

$$+ \left[ \frac{C_4 (L + B)}{12 \times 27} \right] d_4 + E_L + H_L + N - Z_L$$
where:

- \( S \) = total cost of pavement system (dollars per longitudinal foot);
- \( C_i \) = unit cost of material \( i \) (dollars per ton for materials 1, 2, 3, 5, and 8; dollars per cubic yard for materials 4 and 6; and dollars per foot for material 7);
- \( D_i \) = density of material \( i \) (pounds per cubic foot);
- \( L \) = pavement width (24 ft for two-lane and one-way section of divided four-lane highways and 36 ft for one-way section of divided six-lane highways);
- \( d_i \) = thickness of material \( i \) (inches); with \( i = 1 \) for bituminous surface, 2 for stabilized base, 3 for compacted aggregate base, 4 for granular subbase, 5 for bituminous shoulder surface, 6 for subdrain granular fill, 7 for subdrain pipe, and 8 for wearing surface;
- \( k_j \) = adjustment factor for increase in width of pavement layers; with \( k_1 = 1.00 \) for first layer, \( k_2 = 1.04 \) for second layer, \( k_3 = 1.08 \) for third layer, and \( k_4 = 1.12 \) for fourth layer;
- \( E_i \) = cost of shoulder (dollars per longitudinal foot), where, for two-lane highways,

\[
E_1 = 20 \times 3.0 \left( \frac{C_5 D_5}{12 \times 2,000} \right) + 31 \times 6.0 \left( \frac{C_3 D_3}{12 \times 2,000} \right)
\]

and for divided multiline highways,

\[
E_2 = 14 \times 3.0 \left( \frac{C_5 D_5}{12 \times 2,000} \right) + 19.75 \times 6.0 \left( \frac{C_3 D_3}{12 \times 2,000} \right)
\]

- \( H_i \) = adjustment for the additional cost of the wearing surface, where, for two- and four-lane highways,

\[
H_1 = \frac{(C_8 - C_1)}{12 \times 2,000} D_1 \times \left( \frac{90}{110} \right) \times 24
\]

and for six-lane highways,

\[
H_2 = \frac{(C_8 - C_1)}{12 \times 2,000} D_1 \times \left( \frac{90}{110} \right) \times 36
\]

- \( A_i \) = width of shoulder subbase for an embankment slope of 6:1 (feet), where, for two-lane highways,

\[
A_1 = \left[ 22 + \frac{2(d_1 + d_2 + d_3)}{3} \right]
\]

and for divided multiline highways,

\[
A_2 = \left[ 14.375 + \frac{(d_1 + d_2 + d_3)}{3} \right]
\]

- \( B_i \) = adjusted width of shoulder subbase when subdrains are provided (feet), where, for two-lane highways, \( B_1 = 5.0 \), and for divided multiline highways, \( B_2 = 5.875 \);
- \( M_i \) = cost of subdrain when used under median shoulder only (dollars per longitudinal foot), where, for two-lane highways, \( M_1 = 0.0 \), and for divided multiline highways, \( M_2 + 1.1 (0.075 C_6 + C_7) \);
- \( N \) = cost of subdrains under both shoulders (dollars per longitudinal foot), where, for all highways, \( N = 2 \times 1.1 (0.075 C_6 + C_7) \);
- \( Y_i \) = adjustment for the amount of subbase material replaced by the shoulder surface and base (dollars per longitudinal foot), where, for two-lane highways,
and for divided multilane highways,
\[ Y_2 = \frac{162 \times C_4}{12 \times 27} \]

\[ Z_1 = \frac{50 \times C_4}{12 \times 27} \]

and for divided multilane highways,
\[ Z_2 = \frac{60 \times C_4}{12 \times 27} \]

Thus, the objective of this optimal selection of flexible pavement components is to minimize the total cost of the pavement system. The various material and layer notations of the design model are graphically described in the figures illustrating the design sections.

To quantify the boundary conditions to which the optimal design of the flexible pavement components is subject, the following constraint equations are necessary to complete the realism of this design model.

1. The selection of layer thicknesses must satisfy the structural number requirement:

\[ a_1 d_1 + a_2 d_2 + a_3 d_3 + a_4 d_4 \geq SN \]

where \( a_i \) = coefficient of relative strength of material \( i \), and \( SN \) = structural number for design. The coefficients of relative strength are given in Table 1 for the four pavement materials used in this design model.

2. The total thickness of the flexible pavement must be at least equal to the minimum thickness required by an influencing environmental consideration:

\[ d_1 + d_2 + d_3 + d_4 \geq T_{min} \]

where \( T_{min} \) = total minimum thickness of flexible pavement to satisfy environmental conditions.

The remaining constraining equations are required to account for the physical limitations inherent in the construction of the various layers of a flexible pavement. The following seven relationships complete the mathematical representation of the concept for the optimal selection of flexible pavement components.

3. The bituminous surface course of a primary highway is at least 3.0 in. in thickness; that is, \( d_1 \geq 3.0 \).

4. If a stabilized base is selected for the pavement system, the minimum thickness is 4.0 in.; that is, \( d_2 = 0 \) or \( d_2 \geq 4.0 \).

5. If a compacted aggregate base is included in the flexible pavement, a minimum thickness of 4.0 is necessary for construction purposes; that is, \( d_3 = 0 \) or \( d_3 \geq 4.0 \).

<table>
<thead>
<tr>
<th>Material Notation</th>
<th>Material Description</th>
<th>Coefficient of Relative Strength ( (a_i) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d_1 )</td>
<td>Bituminous surface</td>
<td>0.44</td>
</tr>
<tr>
<td>( d_2 )</td>
<td>Stabilized base</td>
<td>0.24</td>
</tr>
<tr>
<td>( d_3 )</td>
<td>Compacted aggregate base</td>
<td>0.14</td>
</tr>
<tr>
<td>( d_4 )</td>
<td>Granular subbase</td>
<td>0.08</td>
</tr>
</tbody>
</table>
6. If a granular subbase is specified from the optimal selection, at least a 4.0-in. layer is required; that is, \( d_4 = 0 \) or \( d_4 \geq 4.0 \).

7. Because rutting and shoving of the pavement may result under high load repetitions for excessive thicknesses of bituminous mixtures, the maximum thickness of the bituminous surface is 10.0 in.; that is, \( d_1 \leq 10.0 \).

8. The maximum thickness of the stabilized base is established at 10.0 in. because of large vertical deformations that may result in this base course if excessive thicknesses of bituminous mixtures are used; that is, \( d_2 \leq 10.0 \).

9. An upper limit of 20.0 in. is set for the thickness of the granular subbase to conform with present construction practice in Indiana; that is, \( d_4 \leq 20.0 \).

In summary, the optimal design of flexible pavement components is predicated on determining that minimum-cost combination of layer thicknesses that satisfies the real and practical constraining conditions. The selection of actual in-place construction costs enhances the mathematical representation of the flexible pavement design process and provides further economies in the highway construction industry.

**SOLUTION**

The final step in determining the optimal design of flexible pavement sections is to obtain a solution to the design model. This solution optimizes the objective function and is subject to the set of constraining situations. The design model was programmed for solution on digital computers using FORTRAN IV language.

The optimization process is performed in two stages. In the first phase, the following nine separate arrangements of flexible pavement components are optimized by a linear programming algorithm:

1. Bituminous surface and stabilized base;
2. Bituminous surface, stabilized base, and compacted aggregate base;
3. Bituminous surface, stabilized base, and granular subbase with subbase through shoulders;
4. Bituminous surface, stabilized base, and granular subbase with subdrains;
5. Bituminous surface, stabilized base, compacted aggregate base, and granular subbase with subbase through shoulders;
6. Bituminous surface, stabilized base, compacted aggregate base, and granular subbase with subdrains;
7. Bituminous surface and compacted aggregate base;
8. Bituminous surface, compacted aggregate base, and granular subbase with subbase through shoulders; and

Six of these nine layered combinations of pavement components represent all possible flexible pavement systems for the cross sections with subbase, \( S_2 \), and for the cross sections with subbase and subdrains under both shoulders, \( S_3 \). Only three arrangements of these components are possible for the cross sections without subbase, \( S_1 \); they include combinations 1, 2, and 7.

The other phase of the solution involves the selection of that pavement-component arrangement that minimizes the total cost of the pavement system for the selected unit costs of the pavement materials. This final solution exists for the specified pavement design and material cost parameters. Each flexible pavement section fulfills the design objectives for the least total cost.

**DESIGN EXAMPLES**

To illustrate the application of this design model, two typical examples for the design of flexible pavements are shown in Figure 7. In each case, the computer output provides a listing of the stipulated design data and the material specifications. After these design parameters are summarized, the optimal solution is tabulated in terms of the best design section and the required thicknesses of the pavement components.
Figure 7. Example 1, design of flexible pavement for primary highways.
To permit a cost-effectiveness evaluation, the next two best solutions are generated for the remaining design sections. These alternate suboptimal solutions provide an economic measure of the additional cost for designs other than the optimal cross section.

The first situation involves a two-lane highway with a primary classification. The other example is a four-lane highway with pavement material costs that differ from those corresponding values in the first illustration. The rather significant increases in flexible pavement costs are evident when the alternate suboptimal solutions are compared to the optimal solutions in the two design examples.

Real economies are achieved when engineering designs are formulated to permit the selection of the optimal answer. This design model affords the highway engineer a practical and realistic method for the optimal design of flexible pavement sections.

REFERENCES