GENERAL COMPUTER PROGRAM FOR ANALYSIS OF AUTOMOBILE BARRIERS

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A computer program developed to predict the behavior of an automobile striking a protective barrier is described. The barrier is idealized as a structural framework of arbitrary configuration and the automobile as a body surrounded by a cushion of springs. Large displacements and inelastic behavior, including hysteresis effects on unloading, are considered in the barrier structure. The automobile slides along the barrier, and the effects of normal forces, friction forces, and wheel drag forces are considered in determining its motion. Input data to the program consist of the configuration of the barrier, the properties of the barrier members and automobile, and the trajectory of the automobile before impact. Output consists of time histories of automobile positions, velocities and accelerations, barrier deflections, and barrier member forces. The paper describes the procedure used to solve the problem, discusses the capabilities and limitations of the program, and presents results for two example barriers.

•THE INTERACTION of an automobile with a barrier system is exceptionally difficult to simulate on a computer. Dynamic effects, extremely large displacements, and inelastic behavior must all be considered. Dynamic loads are not explicitly specified but must be determined by satisfying force equilibrium and displacement compatibility between the automobile and barrier.

The objective of the study described in this report has been the development of a computer program to predict the behavior of a wide variety of automobile barrier systems. Because of the complexity of the problem, it has not been possible to develop a computer program that is applicable to all barriers. Nevertheless, barriers of many different types and configurations can be analyzed. The features and limitations of the program are essentially as follows:

1. The barrier is idealized as a two-dimensional structural framework of arbitrary shape.
2. Seven different types of structural members may be specified in any combination. These are beams, cables, springs, ideal columns, viscous damping links, friction damping links, and posts. Two- and three-dimensional elements are not available but can be simulated by equivalent lattices of bars.
3. Nonlinear modes of behavior are assigned to the structural members, and hysteresis effects in yielding members are taken into account. Bilinear elastic-plastic behavior is assumed for each yielding member, but, by combining members in parallel, a variety of more complex force-deformation characteristics can be represented.
4. The automobile is idealized as a rigid body of arbitrary shape surrounded by a cushion of inelastic springs. The automobile boundary is defined by a series of discrete points at which interaction with the barrier may occur.
5. The barrier must possess a clearly defined interface, or series of interfaces, along which interaction with the automobile points takes place. The automobile slides along the barrier interface. Normal and tangential (friction) forces are transmitted.

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6. The analysis is two-dimensional in the horizontal plane. Out-of-plane effects, which include vertical displacements of both the automobile and the barrier, are not considered.

The computer program should provide valuable information at two different stages in the evaluation of barrier systems, as follows:

1. At the initial design stage, the program can be used to compare design concepts and to assist in the prototype design; and
2. At the testing stage, if it is shown that the program gives good results for the ranges of parameters considered in the tests, the behavior can be predicted for other values of the parameters, and the number of tests can be reduced.

In all cases, however, the results of the computer analyses should be interpreted cautiously. This is because it is necessary to estimate the structural properties of the barrier members before analyses can be carried out, and it will be found that it is frequently difficult to obtain accurate estimates of these properties. Also, the behavior of a barrier system may be governed as much by the design details of the barrier as by its overall configuration. That is, barriers that may be idealized identically for the purposes of analysis may perform quite differently in actual tests because of differences in their structural details. Final evaluations must therefore be obtained from field tests of actual designs.

In this paper, a brief description of the solution procedure is presented, the idealization of the barrier and automobile structures is explained, and two example analyses are discussed. A more detailed description of the technique, with additional examples, is presented elsewhere (1).

SOLUTION PROCEDURE

A dynamic step-by-step analysis is carried out in which linear structural behavior of the barrier and automobile is assumed within any time step. The conditions at the beginning of any step are known, including the following: (a) the positions, velocities, and accelerations of all points on the automobile and barrier; (b) the magnitudes and directions of the normal and friction forces exerted between the automobile and barrier, and the positions at which they act; and (c) the axial forces and bending moments in the barrier members. The problem is to find these quantities at the end of the time step so that the computation can be continued. A simplified description of the procedure is given in the following.

Step 1. The axial forces and bending moments in the barrier members are examined, and it is determined whether each member is in an elastic or yielded condition at the beginning of the step.

Step 2. A stiffness matrix for the barrier structure, in its deformed configuration, is assembled member by member, taking into account the condition of each member. A yielded member is assigned essentially zero stiffness. This stiffness matrix is then modified to account for inertia, viscous damping, and structural stability effects.

Step 3. The solution of the interaction problem is initiated by allowing the automobile and barrier to move as independent dynamic systems, each under constant load. The loads are the interaction forces at the end of the previous time step. The changes in automobile and barrier positions are determined by structural analysis procedures.

Step 4. Because the automobile and barrier have been permitted to move independently, gaps or overlaps will generally have developed at the automobile-barrier interface. That is, the geometric compatibility condition required for solution of the interaction problem will generally be violated. This must ultimately be corrected, but a further error resulting from allowing the automobile and barrier to move independently is first considered. This error results from the fact that, as the vehicle moves along the barrier interface, the positions of the interaction forces on the barrier change, although their magnitudes may remain constant. That is, the dynamic nature of the loading is due not only to changes in the magnitudes of the loads but also to changes in their positions. The effect of change in position is taken into account as follows: The

geometrical relationships at the automobile-barrier interface at the end of Step 3 are examined. New locations on the barrier interface are determined for all automobile points that were in contact with the barrier at the beginning of the time step. New normal and tangential directions to the barrier are also determined at each of these points. The change in barrier load resulting from change in automobile position is therefore determined by removing the normal and tangential forces from the barrier at their positions at the beginning of the time step and by reapplying them in their new positions. The barrier displacements resulting from this load change are determined and added to the displacements computed in Step 3. New automobile positions are also determined, allowing for the slight changes in direction of the normal and tangential forces.

Step 5. The automobile and barrier positions at the end of Step 4 have been determined on the assumption that the interaction forces remain constant in magnitude and, as noted previously, will generally not satisfy the geometric compatibility requirement at the interface. Changes in magnitude of the forces must therefore be determined such that compatibility is reestablished. The geometrical relationships at the automobile-barrier interface at the end of Step 4 are examined, and locations are determined for all automobile points potentially in contact with the barrier. Those automobile points overlapping the barrier interface are assumed to be potential contact points. In addition, those points that were in contact with the barrier at the end of the previous time step are potential contact points, although some of these points may not overlap the barrier interface. The normal and tangential directions for each potential contact point are determined, together with the normal distance from each point to the barrier interface. The magnitudes of these normal distances are a measure of the extent to which compatibility is violated. Changes in the interaction forces must be determined such that these distances are reduced to zero.

To determine the required force changes, a flexibility method of analysis is used. Unit normal forces, with tangential forces of appropriate magnitude, depending on the coefficient of friction and the direction of slip along the interface, are applied to the barrier and automobile at each contact point, and the changes in normal distance are determined. These changes constitute a set of dynamic flexibility coefficients, which can be used to determine the changes in the interaction forces required to reduce the normal overlap distances to zero. These force changes are determined by solving a set of linear simultaneous equations.

The solution of the interaction problem is now nearly complete except that the force changes computed for some automobile points may be such that the total interaction forces at these points are tension rather than compression. If this occurs, the solution of the simultaneous equations must be repeated, with an imposed condition that the force changes at each such point must be equal and opposite to the previous interaction force at the point so that the net force is zero. This step is complete when all interaction forces are either compression or zero. The automobile points with compression forces are the contact points at the beginning of the next time step. If all interaction forces are zero for a specified number of successive time steps, the automobile is assumed to have separated from the barrier.

Step 6. The analysis for the time step is finally completed by summing the position changes of the automobile and barrier calculated in Steps 4 and 5 to obtain the total changes during the time step. The changes in velocity and acceleration of the automobile and barrier can then be determined, and the changes in axial force and bending moment in the barrier members can be calculated. The computations are then repeated from Step 1 for the next time step.

The solution of the interaction problem is essentially a two-cycle iteration procedure and does not ensure exact satisfaction of equilibrium and compatibility at the end of each time step. However, the procedure is such that errors are not accumulated from time step to time step, and thus the errors should not be significant. The accuracy with which the interaction problem is solved increases as the time step is made shorter. However, for practical use of the programs, a compromise must be reached between required accuracy and cost of the analysis because the number of steps, and thus the solution time, increases as the time step is reduced.

BARRIER IDEALIZATION

Barrier Structure

The barrier is idealized as a framework of arbitrary shape lying in the horizontal plane. The geometry of the structure is defined by specifying the coordinates of the joints between members of the framework and by specifying which joints are connected by members. The properties of the members are also specified.

Available Structural Members

Seven different types of members can currently be specified, as follows: beam members, possessing extensional and flexural strength and stiffness; cable members, possessing only tensile extensional strength and stiffness; column members, possessing tensile and compressive extensional strength and stiffness (in tension the strength is governed by yielding and in compression by elastic buckling); springs, possessing tensile and compressive extensional strength and stiffness; friction damping members, which slip extensionally at prescribed forces; viscous damping members, which possess velocity-dependent extensional resistance; and posts, which provide resistance to movement at single joints of the structure.

The members can be assigned nonlinear force-deformation characteristics, as explained in the following sections. In all cases the strength and stiffness properties are assigned by the user of the programs. Members with more complex characteristics than those of the basic members can be constructed by placing two or more basic members, of the same or different types, in parallel.

Beam Members—Beam members are assumed to be of uniform cross section and to have bilinear elastic-plastic properties both flexurally and extensionally. The moment-curvature and force-extension relationships are as shown in Figure 1. After yielding, the member unloads around a hysteresis loop as shown. Shearing deformations are ignored.

Interaction between the bending moment and axial force is ignored. That is, a beam member is treated as a combination of a purely flexural member and a purely extensional member, both of which are parallel and independent. Both members are assumed to be infinitely ductile with negligible strain hardening. However, strain-hardening effects can be included by setting up composite members. The extensional member is assumed to yield over its full length when the axial force exceeds the yield force. The flexual member is assumed to yield by forming localized plastic hinges at either end of the member or at both ends.

Cable Members—Cable members are assumed to have bilinear elastic-plastic properties in tension and to possess virtually no stiffness in compression. The force-extension relationship is therefore as shown in Figure 2. Energy is lost during yielding in tension, and the cable may go slack in compression and subsequently retighten. Infinite ductility with negligible strain hardening is assumed after yielding.

Column Members—Column members are assumed to have bilinear elastic-plastic properties in tension and to buckle elastically in compression at a specified force.

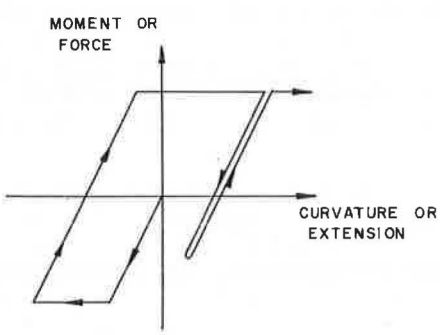

Figure 1. Typical moment-curvature or force-extension relationship.

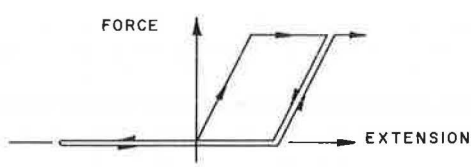

Figure 2. Typical force-extension relationship for a member going slack in compression.

The force-extension relationship is therefore as shown in Figure 3. If a member of a structure may buckle inelastically, or if the buckling load may be increased by inertial resistance, the member must be treated as a series of beam members.

Spring Members—Spring members are assumed to be perfectly elastic, but they have been assigned a "bottoming" feature that permits their stiffnesses to change at specified extensions, as shown in Figure 4. The stiffness may be either increased or decreased at bottoming.

Coulomb Dampers—Coulomb (friction) dampers may be specified to be reversible, to operate in tension only while going slack in compression, or to operate in compression only while going slack in tension. The force-extension relationship for a reversible damper is shown in Figure 1, and the relationship for a damper operating in tension only is shown in Figure 2. A damper operating in compression only has the relationship shown in Figure 2 but with tension and compression reversed.

It is assumed that coulomb dampers can extend indefinitely after slipping. However, bottoming at specified limits of travel can be achieved by placing a spring in parallel with each damper.

Viscous Dampers—Viscous dampers are assigned constant viscous damping coefficients and may be specified to be reversible, to operate in tension only, or to operate in compression only.

Posts—Posts are assigned stiffnesses and yield strengths for displacements in two principal directions at right angles. These stiffnesses are assigned if the post is behaving elastically. Elasto-plastic behavior is assumed to commence when the applied loads on the post are such that the following interaction condition is satisfied:

$$\left(\frac{F_A}{F_{yA}}\right)^2 + \left(\frac{F_B}{F_{yB}}\right)^2 = 1 \tag{1}$$

in which F_A, F_B are the applied loads along the principal directions and F_{yA}, F_{yB} are the specified yield strengths along the principal directions. In the yielded condition an elasto-plastic stiffness corresponding to elastic-perfectly plastic behavior is determined. It should be noted that the forces F_A and F_B may change after a post enters the elasto-plastic range. However, in order to satisfy the yield condition, if one force increases, the other must decrease. If the post reenters the elastic range, unloading along a hysteresis loop is assumed to take place, essentially as shown in Figure 1.

Limiting deflections are also specified, at which the post will fail completely. Failure is assumed to take place when the following condition is satisfied:

$$\left(\frac{\Delta_A}{\Delta_{uA}}\right)^2 + \left(\frac{\Delta_B}{\Delta_{uB}}\right)^2 = 1 \tag{2}$$

in which Δ_A, Δ_B are the deflections along the principal directions and Δ_{uA}, Δ_{uB} are the specified limiting deflections along the principal directions.

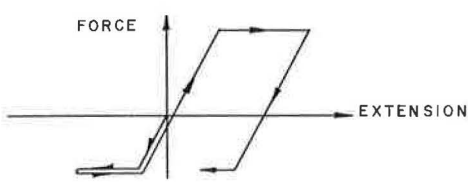

Figure 3. Typical force-extension relationship for a member buckling in compression.

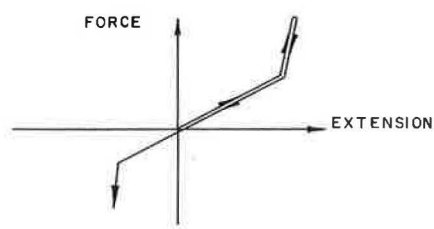

Figure 4. Typical force-extension relationship for a bottoming spring member.

When a post fails it is removed from the structure, and the load it was carrying immediately prior to failure is transferred to the remaining structure. If the failure is sudden, the post load is transferred to the structure suddenly, but, if the failure is more gradual, the load is transferred over a period of time. The failure of a real post is unlikely to be sudden, and thus in the computer program the failure is assumed to extend over the 10 time steps following initiation of failure. The post force is therefore transferred in 10 equal parts, and numerical difficulties that might arise from the sudden application of a shock load are avoided. After failure is initiated, the post contributes no stiffness to the structure, but it is still assumed to contribute mass to the structure.

Posts must be included to represent all attachments of a barrier to the ground or to a rigid object. To simulate connection to a rigid object, the post is made very stiff and strong.

AUTOMOBILE IDEALIZATION

The automobile is idealized as a body of arbitrary shape that possesses mass and rotational inertia. That part of the automobile boundary that may interact with the barrier is defined by specifying a number of points at which contact with the barrier may be made. A discrete nonlinear spring, with a force-deflection relationship of the type shown in Figure 5, is then associated with each point. The springs are assumed to have no mass and no viscous damping. Wheel positions can also be defined, and the brakes can be specified to be either on or off during the analysis. The idealized automobile is therefore shown in Figure 6. The locations of the automobile contact points and wheels are defined with respect to a coordinate system (r, s) as shown.

During the interaction of the automobile with the barrier, both normal and tangential forces are exerted. In the analysis, the normal directions are assumed to be normals to the barrier at the positions where the automobile points are in contact with the barrier. The normal forces are calculated during the solution of the interaction problem, and the tangential forces are assumed to be equal to the normal forces multiplied by coefficients of friction. As the normal forces increase, the springs compress, and thus the automobile points change position relative to the automobile center of gravity. During each analysis step, the increase in displacement of each spring point is computed, and new coordinates are determined.

The wheels may be directed parallel to the automobile axis, or at any angle to this axis. The steer angle is assumed to remain constant during the analysis.

If the brakes are specified to be on, there is a drag force at each wheel equal to a value specified by the user and acting opposite to the direction in which the wheel is traveling relative to the ground at any instant. If the brakes are specified to be off, the wheel will roll freely in the direction the wheel is pointing but will develop drag forces normal to this direction. The drag force in this case is set equal to the full drag force if the wheel moves in a direction greater than 10 deg from the direction in which the wheel is pointing and is reduced linearly for slip angles less than 10 deg.

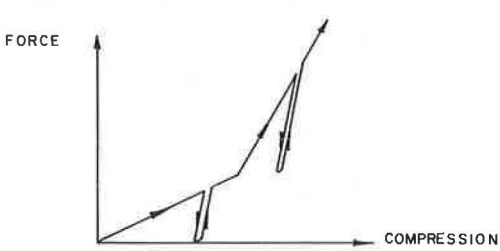

Figure 5. Typical force-compression relationship for automobile boundary spring.

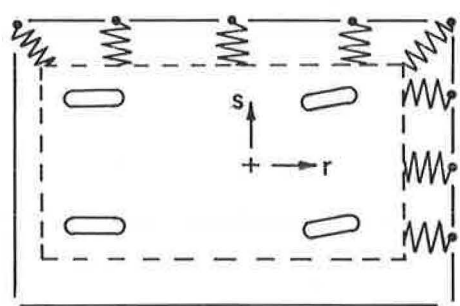

Figure 6. Idealized automobile.

It would be possible to include a sprung mass, representing a passenger, as a part of the idealized automobile, but this feature has not been included in the computer program because it is believed to be a poor idealization of an actual passenger/seat-belt system. Instead, provision has been made for the displacement, velocity, and acceleration time histories to be computed and printed at a number of specified points on the automobile. Because the mass of a passenger is small in relation to that of the automobile, the motion of a passenger will not significantly affect the automobile trajectory. Hence, the computed time history of motion at points on the automobile at which passengers may be seated can be determined and used as input to special-purpose programs for the determination of passenger response.

OUTPUT INFORMATION

A great deal of information on the state of the automobile is determined by the computer program, including positions, velocities, accelerations, details of those automobile points in contact with the barrier, and the magnitudes of the normal and tangential interaction forces. The data printed include the automobile positions and heading angles, forward and sideways velocities and accelerations, velocities and accelerations in the x, y coordinate directions, and resultant velocities and accelerations. Provision is also made for the data to be punched on cards for subsequent reentry into computer plotting routines. This option is valuable because considerable time is required to produce plots by hand.

Information on the deflections of the barrier joints and the forces in the barrier members is also printed, and data on the locations of the barrier joints and automobile points can be punched. The punched data can be used to produce computer plots of deflected shapes of the automobile and barrier.

EXAMPLES OF AUTOMOBILE-BARRIER INTERACTIONS

Two examples are considered in this section; several other example structures are discussed by the author elsewhere (1). The examples considered here are (a) a bridge rail barrier with fragmenting-tube, energy-absorption devices, for which limited test results are available (2); and (b) a multiple-interface ductile beam abutment barrier with dimensions corresponding to a detailed design developed by Cornell Aeronautical Laboratory (3).

The properties selected for the automobile have been chosen largely from information presented elsewhere (4), although a number of arbitrary assumptions have been made. The assumed plan dimensions are shown in Figure 7. The other assumed automobile properties are as follows:

1. Weight = 4,720 lb for the first example, 3,600 lb for the second example;
2. Moment of inertia = 45,000 lb-sec^2 in. for the first example, 34,000 lb-sec^2 in. for the second example;
3. Boundary spring stiffness for each 12 in. of perimeter tributary to any point = 0.5 k/in. for sheet metal, 3.0 k/in. after bottoming, and 4.0 k/in. on unloading;
4. Bottoming distance for boundary springs = 15 in.; and
5. Maximum drag forces between wheel and ground = 27.5 percent of weight for front wheels and 22.5 percent for rear wheels.

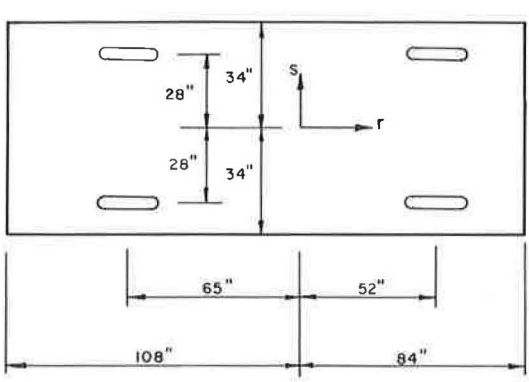

Figure 7. Dimensions of basic automobile.

Fragmenting Tube Bridge Rail

Tubes that fragment on being forced into a die have been investigated as energy-absorbing devices for barrier

structures. The tubes behave as coulomb damping devices with predictable and essentially constant slip forces. A preliminary report on a series of tests of a bridge rail system incorporating fragmenting tubes has been prepared at the Texas Transportation Institute (2). Data for the computer program have been prepared from details given in that report.

The computed motion of the automobile for a typical analysis (impact at 25 deg and 54.8 mph) is shown in Figure 8. The computed time histories of forward and transverse accelerations at the automobile centroid are shown in Figure 9 and compared with accelerometer values determined by Hirsch, Stocker, and Ivey (2). The agreement between theory and experiment is far from exact but is nevertheless encouragingly close considering the complexity of the problem. Both theory and experiment predict two deceleration peaks, the second corresponding to impact of the rear of the automobile on the barrier, and the predicted decelerations are quite close to the measured ones. It should be noted that the test accelerometers were not mounted at the automobile centroid, and that their positions were not known at the time the analyses were carried out. Other analyses (not shown) have demonstrated that the accelerations change substantially from point to point on the automobile. The test results also appear to contain a large oscillation superimposed on a basic deceleration pattern, and this accounts for a large part of the difference between theory and experiment. These oscillations probably result from vibrations of the automobile structure or instrument assembly and would not be experienced by a passenger in the automobile.

The analysis predicted departure of the automobile from the barrier at 28.8 mph at an angle of 14 deg. This is significantly below the test value of approximately 33 mph at an unknown angle. The departure velocity depends substantially on the coefficient of friction between the automobile and barrier, which was assumed to be 0.6 in the analysis. Further investigations are necessary to determine suitable coefficients for use in analysis.

A strength of 12.5 k was assigned to each fragmenting tube. This is larger than the value of approximately 10 k indicated by Hirsch, Stocker, and Ivey (2). Similar automobile deceleration results were obtained with an assumed strength of 10 k, but with

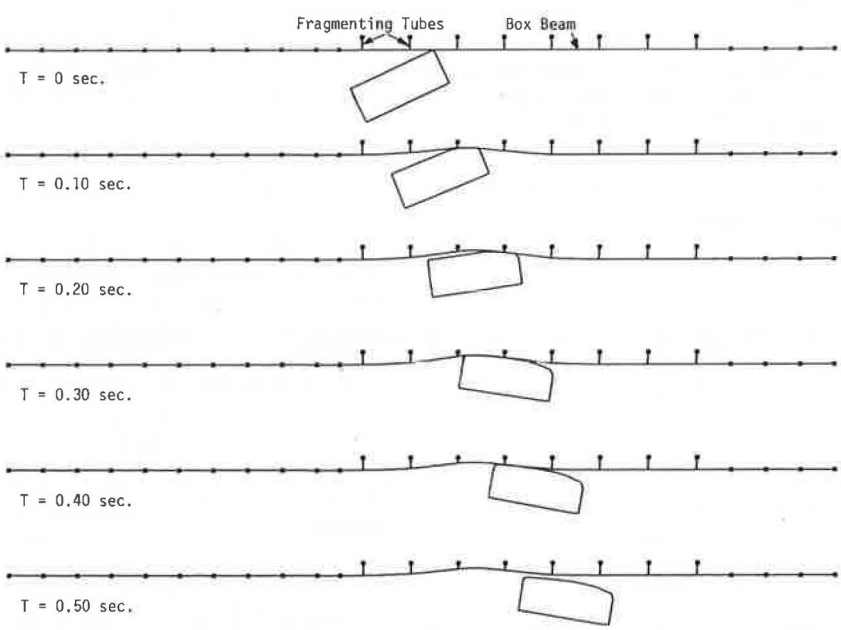

Figure 8. Deflected shapes of fragmenting-tube bridge rail.

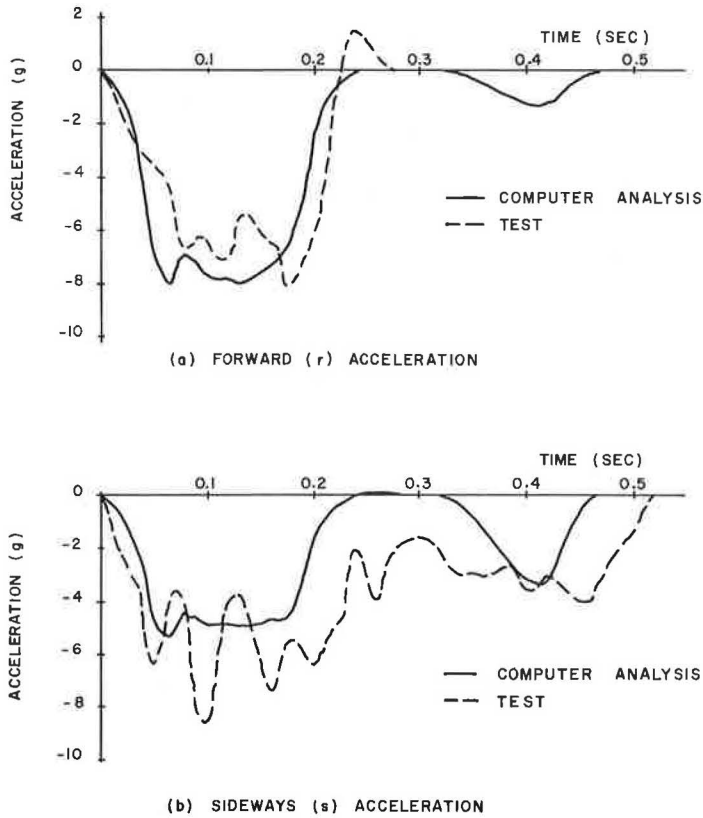

Figure 9. Fragmenting-tube bridge rail (forward and sideways automobile accelerations).

this strength the computed barrier deflections were substantially larger than the measured deflections. The measured and computed amounts of shortening of the five fragmenting tubes in the impact region were as follows:

Tube No.	Test (in.)	Tube Strength of 12.5 k (in.)	Tube Strength of 10 k (in.)
1	0.1	0	0
2	6.0	3.6	4.8
3	14.8	16.0	18.9
4	10.0	12.0	17.0
5	0.1	0.7	3.8

Ductile Beam Abutment Barrier

The geometry of an abutment barrier designed by Cornell Aeronautical Laboratory (3) is shown in Figure 10. The concept of the barrier is that an automobile is decelerated by each of the arch "layers" in turn. The resisting force of the outer layer is not so large that a light automobile is decelerated too rapidly, yet the inner layers

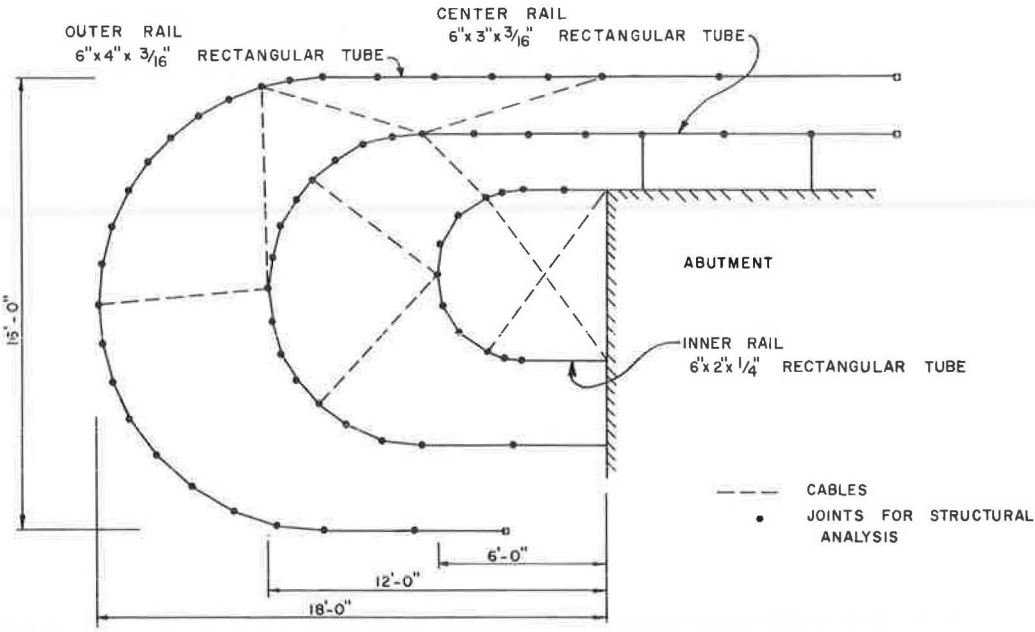

Figure 10. Geometry of ductile beam abutment barrier.

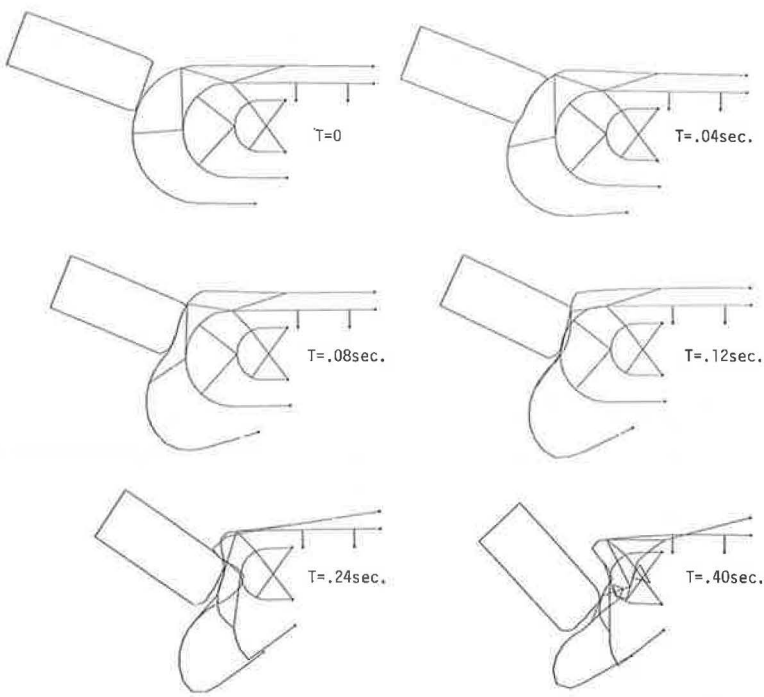

Figure 11. Deflected shapes of ductile beam abutment barrier.

prevent total penetration by a heavy automobile. No test results for this type of barrier are available.

Deflected shapes of the barrier for a head-on impact at 50 mph are shown in Figure 11. Time histories of automobile decelerations are shown in Figure 12. The following points may be noted:

1. The peak decelerations at approximately 0.04 and 0.13 sec correspond to development of the maximum strengths of the outer and first inner barriers respectively. The deceleration at the second peak is large, although the peak is of short duration.
2. After yielding and buckling, the computed resistances of the barrier layers decrease substantially. However, strain hardening was ignored in the analysis, and, if strain-hardening effects are present, the post-yield strength could be significantly increased. Strain-hardening effects can be included in the analysis if desired.
3. The outer barrier layers overlap the inner layers when the displacements are large. In an actual barrier, the design may be such that the layers cannot overlap, and in such a case the behavior might be substantially different. However, provision for barrier-to-barrier contact in the computer program is impractical. The errors introduced by ignoring this contact must be determined by prototype testing.
4. At 0.38 sec the barrier buckled away from the automobile and contact was lost, although the automobile was still moving at 8.0 mph. Ater 15 subsequent steps, contact had not been reestablished, and the computation was terminated. The very large deformations of the barrier toward the end of the analysis should particularly be noted.

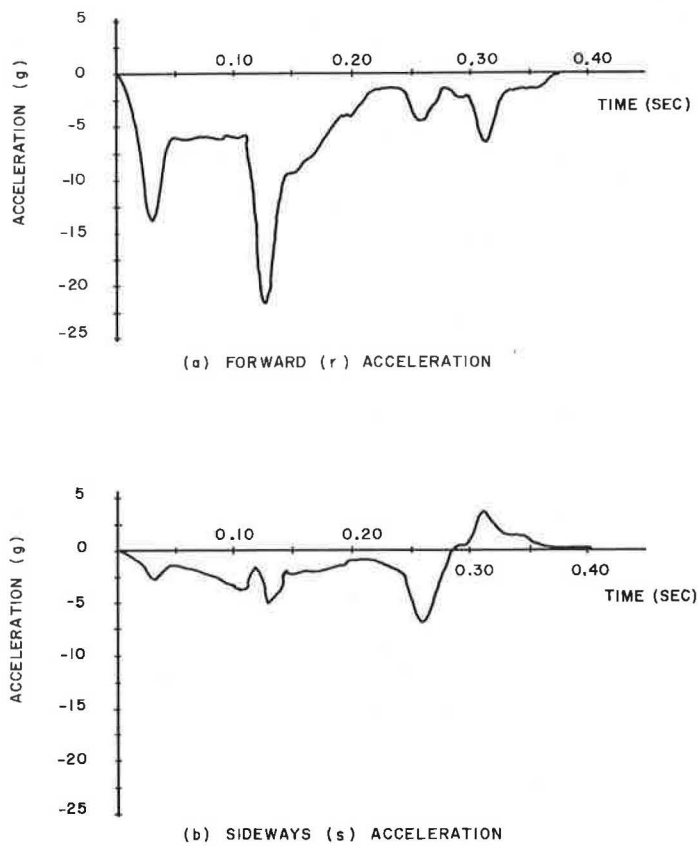

Figure 12. Ductile beam abutment barrier (forward and sideways automobile accelerations).

Solution Times

The computer program is written in FORTRAN IV and operates entirely in core. The author's analyses have been carried out on the CDC 6400 computer at the University of California, Berkeley. The program requires 27300 K decimal for execution and can be run on any machine with adequate capacity, although double precision arithmetic would be essential on machines with short word lengths.

The solution times for the examples considered in this paper were as follows:

Example 1—113 joints in barrier (339 barrier equations to be solved in each time step), 151 structural members; central processor time = 1.74 sec per time step, 365 sec for complete analysis (210 time steps of 0.0025 sec each).

Example 2—69 joints (207 equations, but with larger bandwidth than example 1), 83 members; central processor time = 3.23 sec per time step, 646 sec for complete analysis (200 time steps of 0.002 sec each). The peripheral processor times vary with the problem size and amount of output but are generally small.

CONCLUSION

From the limited series of examples discussed in this paper and elsewhere (1), it appears that the computer program should be a valuable aid for the investigation of barrier systems. The agreement with the few available sets of experimental data is encouraging, the program is fairly easy to use, and the solution technique is surprisingly stable in view of the complexity of the problem. It is obvious, however, that further study, both analytical and experimental, is needed to test the accuracy of the program and to determine its range of application.

A series of barrier structures for which reliable physical data and experimental results are available should be selected and then idealized in different ways for analysis. The range of member types available to the program permits considerable flexibility in the idealization procedure. The effects of varying the idealization should then be studied, and the effects of varying such parameters as automobile size and stiffness, coefficient of friction, and time step should also be investigated for each idealization. The accuracy of the computer results could then be determined, and recommendations on idealization procedures could be made. Until such an investigation has been completed, it is not possible to make detailed recommendations of the use of program or to draw definite conclusions on its accuracy.

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