A THREE-DIMENSIONAL MATHEMATICAL MODEL TO PREDICT THE DYNAMIC RESPONSE OF AN AUTOMOBILE OCCUPANT

Ronald D. Young, Texas Transportation Institute, Texas A&M University

This paper outlines the development of an analytical model that predicts the response of an automobile passenger in three-dimensional space during vehicle motion, which can also be three-dimensional in nature. The predicted response includes position of the occupant relative to the vehicle, accelerations of various parts of the body, and forces acting on various parts of the body—all as a function of time.

Validation of this passenger model has been achieved for the case of frontal collisions in which the occupant is either totally unrestrained, restrained by a lap belt only, or restrained by a lap belt and a shoulder strap.

•ENGINEERS are currently attempting to reduce the severity of single vehicle accidents by designing and building a safer roadway environment.

To effectively design or evaluate a roadway or its immediate environment for safety, consideration must be given to the dynamic response of the vehicle and occupant during interaction with geometric features such as curves and ditches, or obstacles such as guardrails, bridge rails, median barriers, and signposts. Accordingly, the design of highway safety devices such as breakaway signs, energy-absorbing impact cushions, and earth berms (an earth embankment geometrically designed to safely redirect a vehicle that has left the roadway), depends directly on the dynamic response of vehicle and passenger during collision with these objects.

These considerations are accurately summarized in the following quotation (1):

> Unless the motion time history of the vehicle can be translated into the expected kinematics of the vehicle occupant and further translated into the nature and extent of physical damage, it is not possible to establish performance requirements for roadside structure modifications that will effect a reduction in occupant injuries during single vehicle collisions.

The reported research was aimed at providing an analytical means of supplementing existing technology as related to roadside energy conversion systems. This was accomplished by developing a mathematical model to predict the response of an automobile passenger during violent vehicle motion of a general nature, i.e., a three-dimensional path including simultaneous rotations about the three directions.

DESIGN CONSIDERATIONS

Usual design practice is to first determine the time history and levels of acceleration (g-levels) experienced by the vehicle during a particular maneuver or collision. These are next compared to certain tolerance limits assuming that the occupant is subjected to the same g-level. This assumption is rigorously true only if the occupant is rigidly fastened to the vehicle. In actuality the passenger is unrestrained, lap-belted,

---

Sponsored by Committee on Vehicle Characteristics and presented at the 50th Annual Meeting.

or shoulder-harnessed and movement is not completely restricted, so that this assumption could range anywhere from overly conservative to dangerously inadequate depending on the situation.

Another factor that influences highway safety design problems is the quantitative consideration of contact forces between vehicle occupant and vehicle interior. It is possible for an automobile passenger to suffer fatal injuries from contact forces during a vehicle maneuver that at present may appear completely tolerable from the standpoint of vehicle accelerations alone.

It is felt that an analytical model of a passenger used in conjunction with available biomechanics data on human tolerance limits can be of significant value in approaching highway safety design problems.

REVIEW OF LITERATURE

A survey of the literature has shown that the mathematical modeling of a vehicle occupant has been attempted in recent years. In most cases these efforts were aimed at developing restraint systems for the occupant, but in no instance was the occupant's general dynamic response the prime consideration.

In the early 1960's, a mathematical approach to the occupant restraint problem was made by the aerospace industry (5, 10). The primary concern was the behavior of viscera for fully restrained subjects.

During 1962-63, an analytical study of occupant restraint systems was performed by Cornell Aeronautical Laboratory (CAL) (7). A 7 degree-of-freedom nonlinear mathemathical model of a restrained, articulated body on a test cart, for the case of a frontal collision, was formulated and programmed for an electronic computer. This study also led to the development of an 11 degree-of-freedom passenger model completed in 1966, which is the most sophisticated yet employed in the occupant restraint problem (9).

In 1967, Emori (2) conducted a study whose purpose was "to understand the mechanics of the automobile collision and to establish a logical background for the injury reduction of occupants." The scope of his research precluded the use of the CAL model and a single degree-of-freedom spring mass system for the occupant, and a similar representation of the automobile was used.

Renneker (11) used a 2 degree-of-freedom model of an occupant characterized by hip and torso restraint to study the effect of vehicle forestructure energy absorption on occupany injury.

Martinez and Garcia (6), in 1968, developed a mathematical model to represent the motion of the head and neck during rear-end collisions to study the whiplash phenomenon.

In 1969, Suggs et al. (12) considered the problem of objectionable amplitudes and frequencies in the vibration of seats using a 2 degree-of-freedom representation of the human for the purpose of developing more comfortable seats.

With the exception of the CAL model (9), the foregoing efforts have little in common with the reported research but are acknowledged because they were mathematical simulations of the vehicle occupant.

The CAL model provided the major guidelines for performing this research because, in this writer's opinion, the results of that study reflect an adequate representation of the vehicle occupant for the two-dimensional environment considered. However, the specific equations derived by CAL were not applicable to this study because this study involves a three-dimensional formulation, although the same basic geometrical configuration and concepts were applicable.

MATHEMATICAL FORMULATION

Vehicle Occupant

The vehicle occupant is defined separately from the vehicle, or as an independent system of articulated rigid mass segments in three-dimensional space. Consequently, the vehicle interior can be thought of as a confining environment for the occupant and is discussed in a subsequent section.

Figure 1 shows the centerlines of the 12 rigid mass segments and their connection pattern, chosen as to geometrically resemble the human body. Fixed at the center of mass of each segment n is a right-handed cartesian coordinate system denoted by axes X_n, Y_n, and Z_n. The positive directions of these axes are defined such that when the body is standing upright with arms hanging vertically (downward) X_n will be positive straight ahead, Y_n will be positive to its left, and Z_n will be positive upwards. The orientation of segment n with respect to the space-fixed coordinate system denoted by axes X', Y', and Z' is defined using three angular coordinates commonly referred to as "Eulerian angles" (4).

If all joints of the articulated body shown in Figure 1 were of the ball-and-socket type, then this body would have 39 degrees of freedom, i.e., 36 angular coordinates (3 Eulerian angles for each of the 12 rigid mass segments), plus the 3 translational coordinates $(X'_{T1}, Y'_{T1}, Z'_{T1})$ for the reference point on the body. [Such a point is

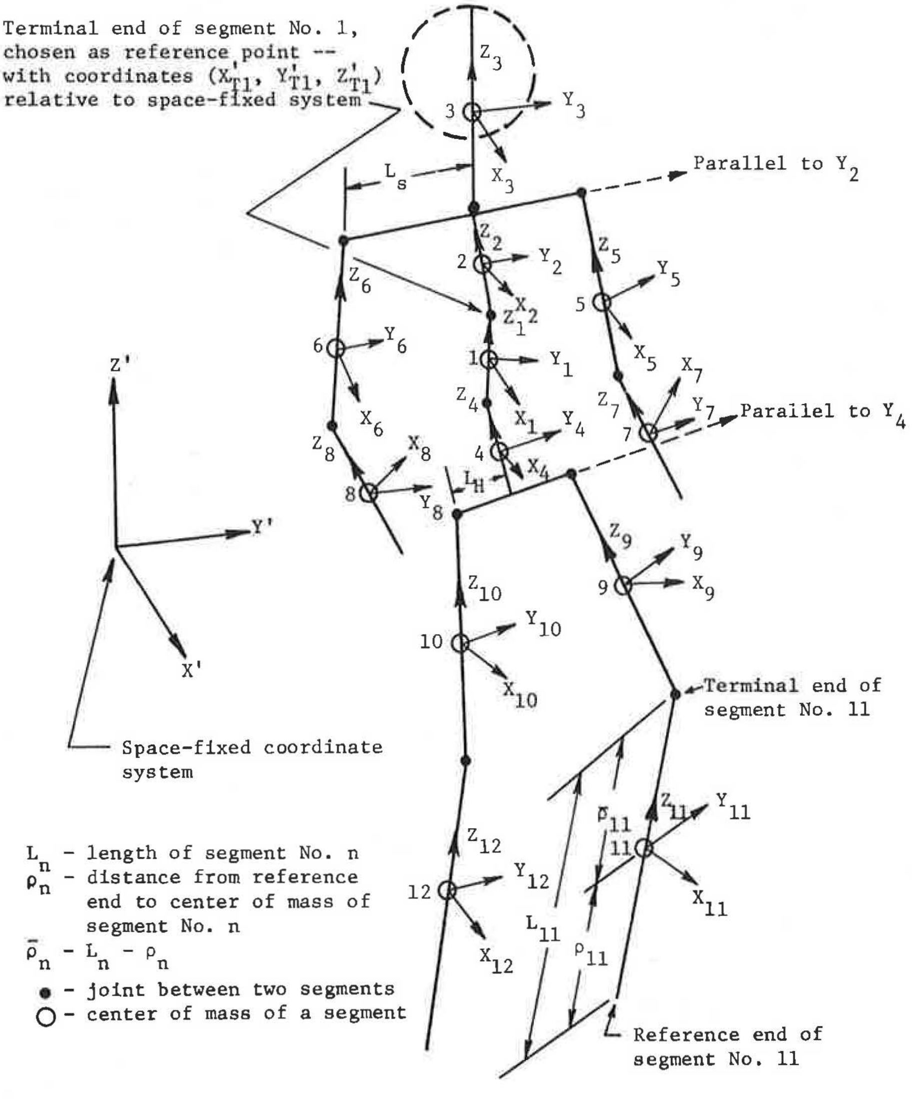

Figure. 1. Articulated body with coordinate systems.

needed to account for translation of the body as a unit (Fig. 1).] However, it was realized that the elbows and knees are pinned in nature; therefore only 1 angular coordinate is required (instead of 3) to specify the orientation of a forearm or a lower leg segment in space. Consequently, the vehicle occupant has 31 degrees of freedom that also correspond to the 31 generalized coordinates used in Lagrange's equations (4) to derive equations of motion for the articulated body.

Lagrange's equations for nonconservative systems (4) were employed, and these may be written as

$$\frac{d}{dt}\left(\frac{\partial U}{\partial \dot{q}_j}\right) - \frac{\partial U}{\partial q_j} + \frac{\partial V}{\partial q_j} = Q_j \qquad (1)$$

where t = time, U = kinetic energy of the system, V = potential energy of the system, q_j = generalized coordinates, $\dot{q}_j$ = generalized velocities, Q_j = generalized forces acting on the system that are not necessarily derivable from a potential function, and j = 1, 2, ..., 31, for this particular problem.

The potential energy, V, is of two types; i.e., potential energy of position (due to gravity) and potential energy due to restoring springs located in each of the two back joints shown in Figure 1. These rotational springs simulate spinal elasticity or the ability of the human spine to recover its initial configuration after bending.

The generalized forces, Q_j, are also of two types; i.e., generalized forces resulting from externally applied loads (passenger-vehicle interaction) and generalized forces resulting from frictional resistance in all joints (viscous damping) to simulate muscle tone. The human body's muscular network can act as a dissipater of rotational kinetic energy that is derived from an external source. Hence, the viscous damping in body joints approximates the tensing of muscles in a panic situation.

Passenger-Vehicle Interaction

The idea of passenger-vehicle interaction is analogous to that of placing an object in a glass box, fastening the lid, and then observing the motion of the object while the box is shaken. One could conclude from such an experiment that the motion of the object is totally dependent on the forces afforded to it by the walls of the box (with the exception of gravity), and that these forces are dependent on the path of the box in space as a function of time. Likewise, before contact forces on the passenger can be computed, it is necessary to define the path of the vehicle.

A tabular record of the vehicle's path in space as a function of time is sufficient for purposes of computing contact forces. This record is fed to the computer program for the passenger model and, if necessary, interpolation between time stations is performed. A record of the vehicle's path can be obtained from another computer program that describes vehicle motion (8, 13) or from full-scale testing.

The Idealized Passenger Compartment—To facilitate the computation of contact forces, the vehicle interior or passenger compartment is idealized by a series of planar surfaces. This greatly simplifies the geometry considerations for predicting contact between the articulated body and its confining environment.

Figure 2 shows the numbering of the points where coordinates are necessary for defining the geometry of the idealized passenger compartment. These points are used to express the equations of the planar surfaces and their inward normal vectors.

The Prediction of Contact—The computer program is written such that the passenger is initially placed inside the vehicle; then each of the various parts of the articulated body are checked for contact with each of the planar surfaces of the vehicle interior as the vehicle moves along its path. The technique used to predict contact employs the use of spheres and lines as well as the planes that define the vehicle interior. A finite number of spheres are strategically located along the segments of the articulated body (Fig. 3) for the purpose of giving size and dimension to the body segments. The proximity of each "contact sphere" to each planar surface of the passenger compartment is calculated by (a) passing a line through the center of the sphere in a direction parallel to the inward normal vector of the planar surface; (b) finding the point of intersection

(X_{vi}, Z_{vi}) - coordinate of point i in X_v - Z_v plane

Note: $Z_{v5} = Z_{v18}$; $Z_{v20} = Z_{v21}$; $Z_{v6} = Z_{v11} = Z_{v19}$.

$Z_{v2} = Z_{v3}$; $Z_{v17} = Z_{v8} = Z_{v13} = Z_{v14}$; $X_{v11} = X_{v12} = X_{v15}$;

$X_{v6} = X_{v7} = X_{v22}$.

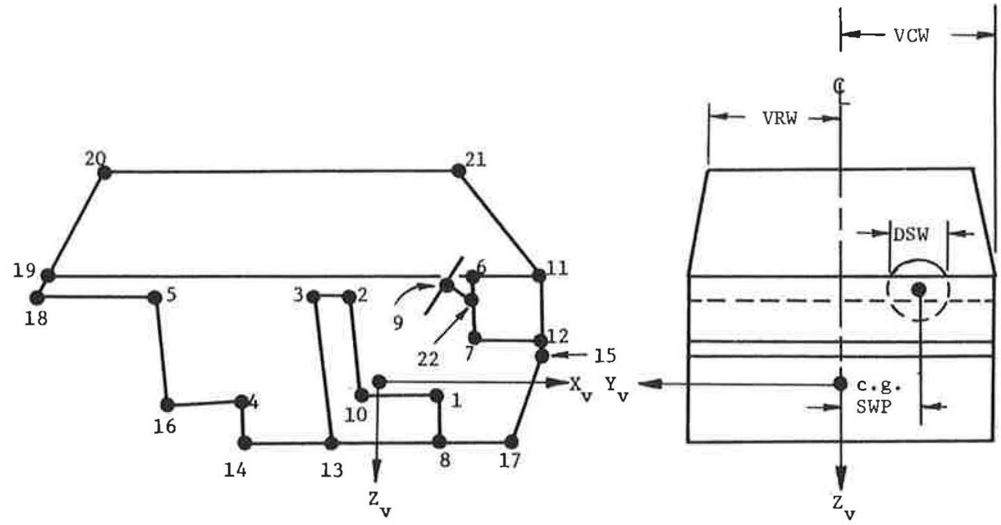

Figure 2. Coordinates and dimensions of the idealized passenger compartment.

of the line with the plane; and (c) calculating the distance between this point of intersection and the center of the contact sphere. Contact or amount of deformation is computed by comparing the radius of the sphere to the distance of its center from the planar surface (item c). Finally, the contact force is computed based on force-deformation data that are input to the computer program.

Lap and Torso Restraint Belts—Other sources of contact forces that the vehicle occupant may be subjected to are the safety belts. The lap belt has its ends anchored at arbitrary points and loops around the pelvic area (contact sphere No. 3). Likewise, the torso belt has its ends anchored at arbitrary points and loops around the upper torso area (contact sphere No. 2).

It is assumed that the centerline of a belt defines a plane that contains the center of its respective contact sphere at all times. This facilitates the definition of the restraining force vector, which by definition also lies in this plane.

Solution of Equations

The equations of motion derived from Eq. 1 comprise a set of 31 differential equations that are categorized as being ordinary, of second order, simultaneous, and nonlinear.

The fact that these differential equations are nonlinear immediately dictates a solution by numerical integration, and the particular approach used was the "Runge-Kutta" method (3) because of its inherent stability. Differential equations to which this method is applicable must be of the form where the highest derivative is expressible as a function of lower derivatives, the dependent variable, and the independent variable. For this reason the 31 differential equations of motion were written in matrix form as

$$[D]\ \{\ddot{q}\} = \{\overline{E}\} \tag{2}$$

from which the column vector of highest derivatives $\{\ddot{q}\}$ is available for integration with respect to time.

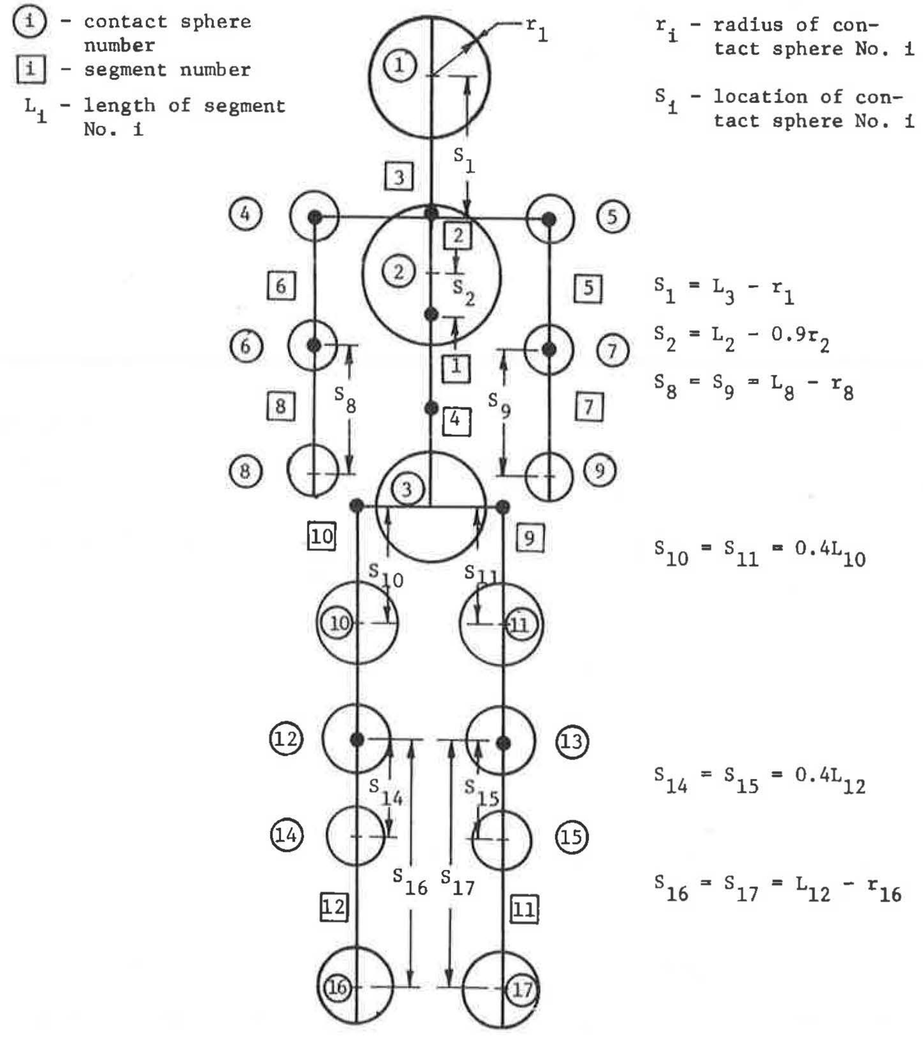

Figure 3. Locations of contact spheres.

The solution of the equations of motion consists of a time history of the following quantities output by the computer program:

1. The coordinates of the end points of each body segment with respect to the vehicle-fixed coordinate system;
2. Acceleration components of the center of mass of each body segment with respect to the segment-fixed coordinate system for that segment (this is total acceleration);
3. Angular acceleration components of each body segment with respect to its segment-fixed coordinate system;
4. Angular velocity components of each body segment with respect to its segment-fixed coordinate system;
5. The force on each body contact sphere plus the identification of whatever vehicle interior surface is being hit;
6. The coordinates of the point of application of the contact force with respect to the center of the contact sphere in segment-fixed coordinates (only for the head, chest, and pelvic area); and
7. The force of restraint applied to the body by the lap and torso restraint belts.

VALIDATION STUDY

An original objective of this project was to validate the passenger model's three-dimensional response capabilities by comparison with existing test data of this nature. To produce conclusive results, any such data should provide the following information:

1. A time history of the vehicle's path in three-dimensional space, preferably numerical instead of photographic (photographic records could be used for application of the passenger model but only after validation);
2. A corresponding time history of the occupant's dynamic behavior, e.g., accelerations, forces, or a photographic record of motion;
3. A quantitative description of the occupant—dimensions, weight, etc.; and
4. Force-deformation properties of the pertinent vehicle surfaces (could be measured).

Unfortunately, test results possessing all these qualities were not to be located and funds for full-scale testing were not available, thus precluding a validation of the general case at this time.

However, suitable test results were available (9) for a partial validation, i.e., the case of a frontal automobile collision.

Test Data

The test data used for comparison were generated at the Biomechanics Research Center of Wayne State University, Detroit, Michigan, under the direction of Cornell Aeronautical Laboratory (CAL), Inc., Buffalo, New York, for the U.S. Public Health Service, March 1967. All experimental results shown in this report were directly from CAL's documentation of these tests (9).

The tests consisted of a dummy seated on a cart capable of controlled deceleration. Mounted on the cart were target assemblies for head, chest, and knee impact. Accelerations of various parts of the dummy and impact forces were measured by instrumentation while the motion of the dummy was recorded on high-speed film. Several cases were run consisting of lap restraint, lap and torso restraint, and no restraint for initial velocities of 10 and 20 mph.

Also measured and documented (9) were the force-deformation characteristics of the targets, seat, and restraint belts plus the dummy's initial position and the amount of friction in each of its joints.

Discussion of Results

Response Comparison for No Restraint at 20-mph Cart Velocity—Figures 4 through 6 show the comparison of dummy kinematics, head forces, and head accelerations for the case of no restraint with 20-mph cart velocity. Agreement between simulated motion and the high-speed film record is excellent. Quantitative comparisons (forces and accelerations) are better than expected because of the idealized vehicle interior (simulation) being geometrically different from the target assemblies used in the test. The simulation utilized a full instrument panel and steering wheel as opposed to isolated targets of about 6 to 8 in. in diameter for the test. This resulted in hand contact in the simulation that was absent during the test. Also, the knee targets were inclined for the test, producing a downward force component, whereas the knees in the simulation contacted a vertical surface (instrument panel) with friction as the only downward force.

Response Comparison for Lap and Torso Restraint at 20-mph Cart Velocity—Figures 7 through 9 show the comparison of dummy kinematics plus head and chest accelerations for the case of lap and torso restraint with 20-mph cart velocity. Agreement between simulated motion and the high-speed film record is good including unsymmetrical body movements as a result of the unsymmetrical torso restraint belt. However, the spring action of the torso belt seems to be excessive in the simulation since the arms are whipped back into the seat as shown in the 0.080-sec frame of Figure 7. This test was subject to the same sources of possible discrepancy as the unrestrained case

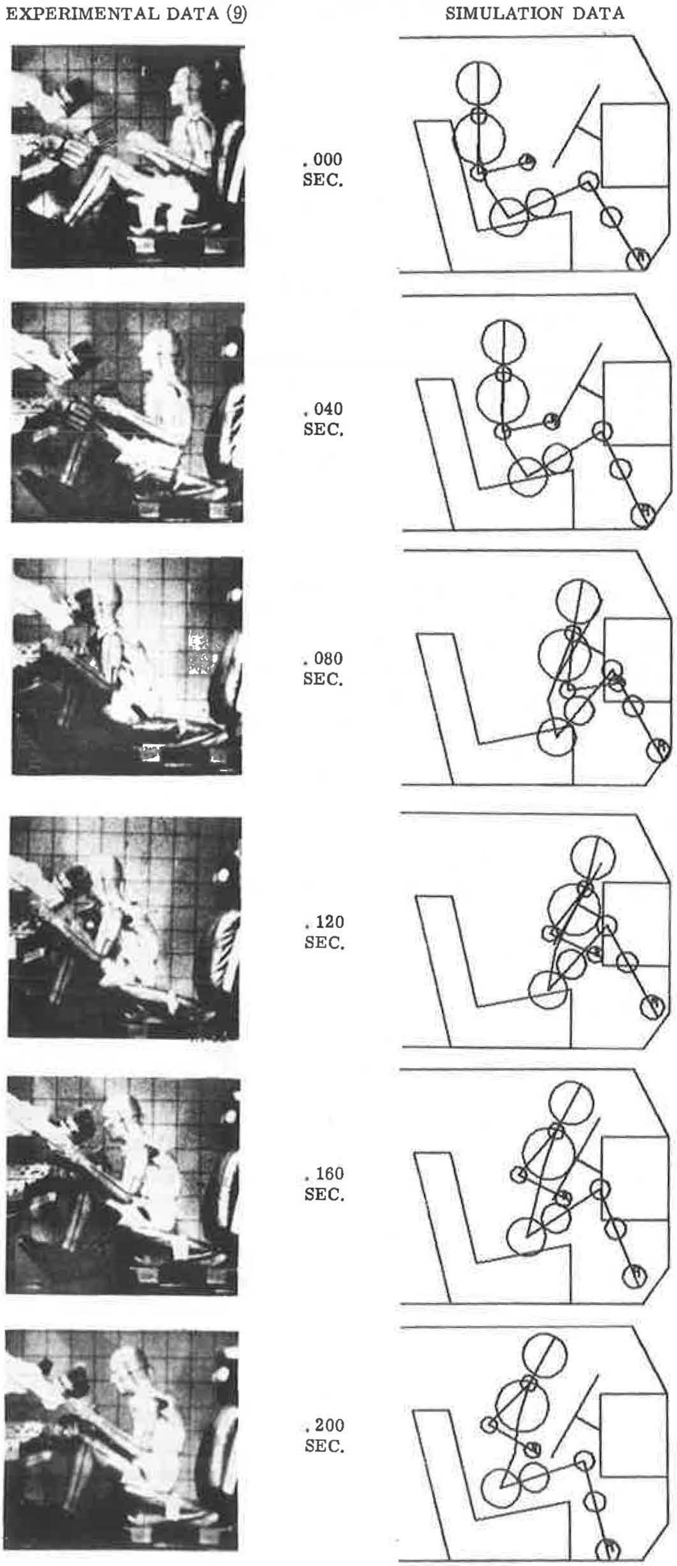

Figure 4. Dummy kinematic comparison, no restraint, 20-mph cart velocity.

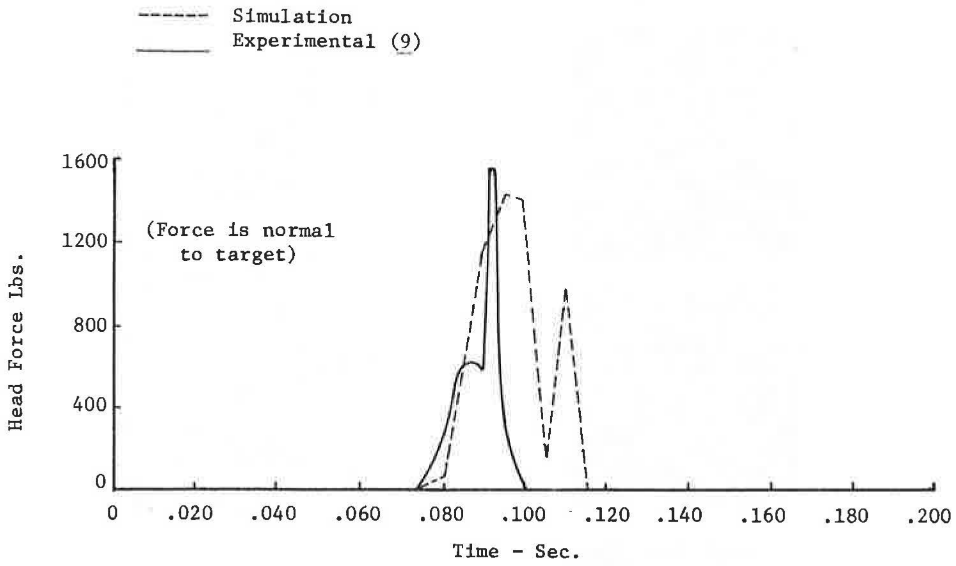

Figure 5. Head force, no restraint, 20-mph cart velocity.

plus an additional one. The anchor points for the ends of the belt were unknown and therefore were estimated for the simulation. This could account for some of the difference in arm kinematics.

Closure—The deceleration pattern used in the test (9) approached a 20-g square wave for a duration of about 0.08 sec. It is interesting to note that the passenger experienced levels of acceleration on the order of 80 g with durations of approximately 0.03 sec for the case of no restraint and levels of approximately 40 g with durations

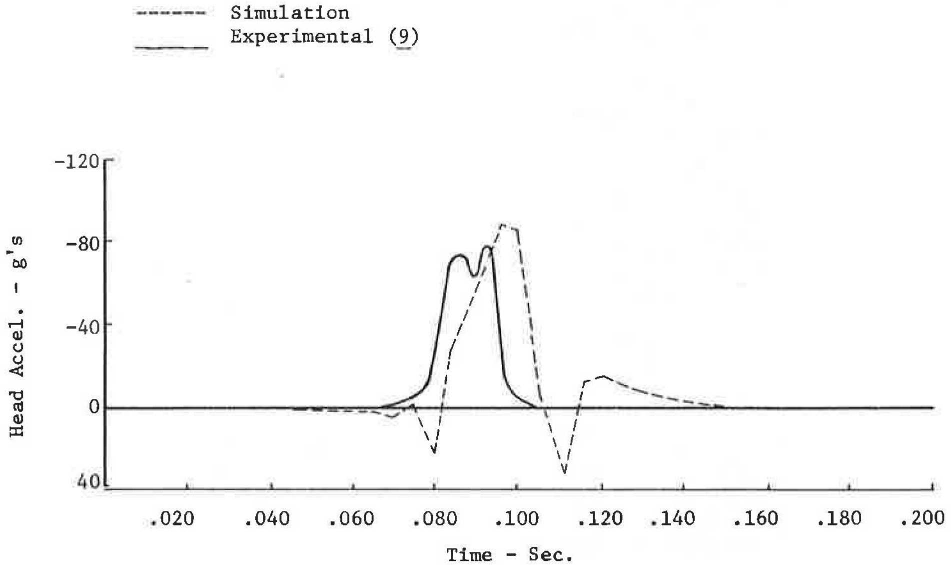

Figure 6. Head acceleration in segment X direction, no restraint, 20-mph cart velocity.

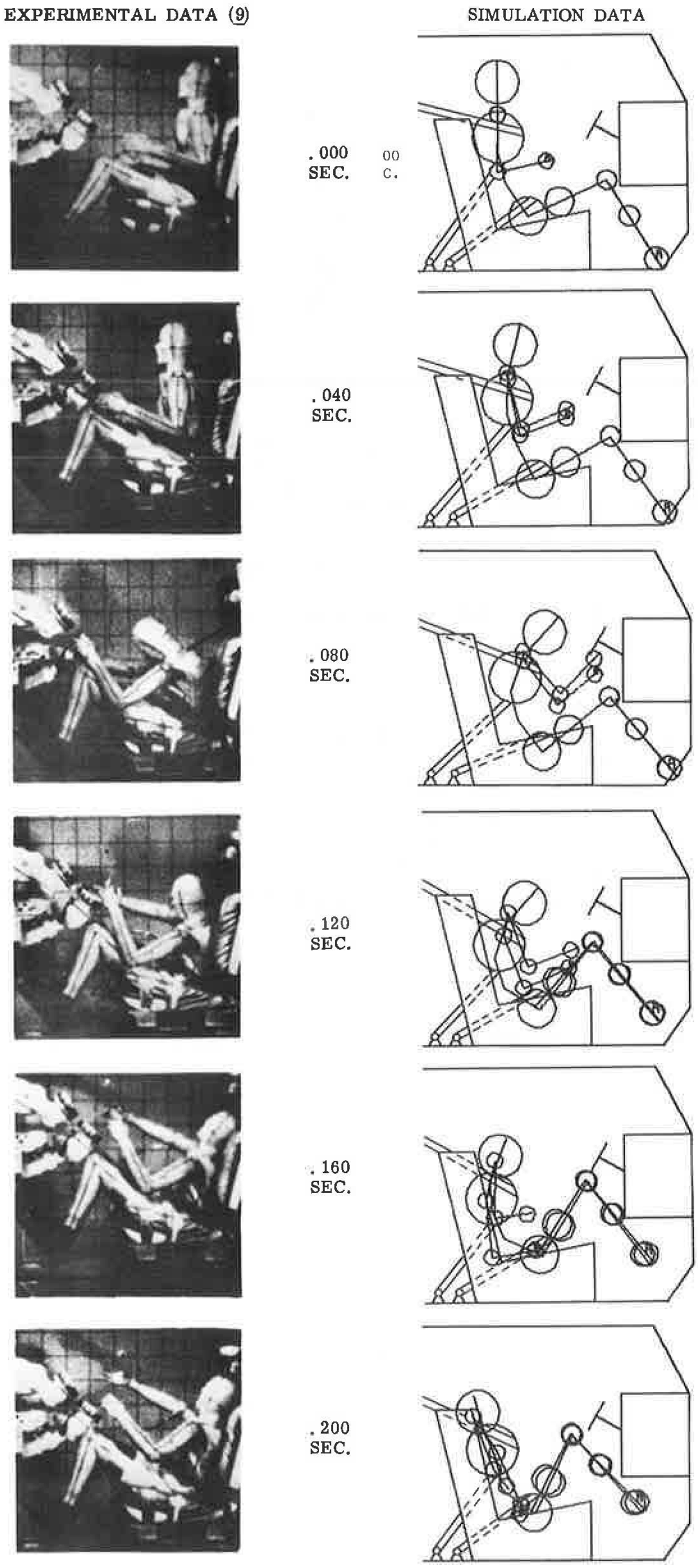

Figure 7. Dummy kinematic comparison, lap and torso restraint, 20-mph cart velocity.

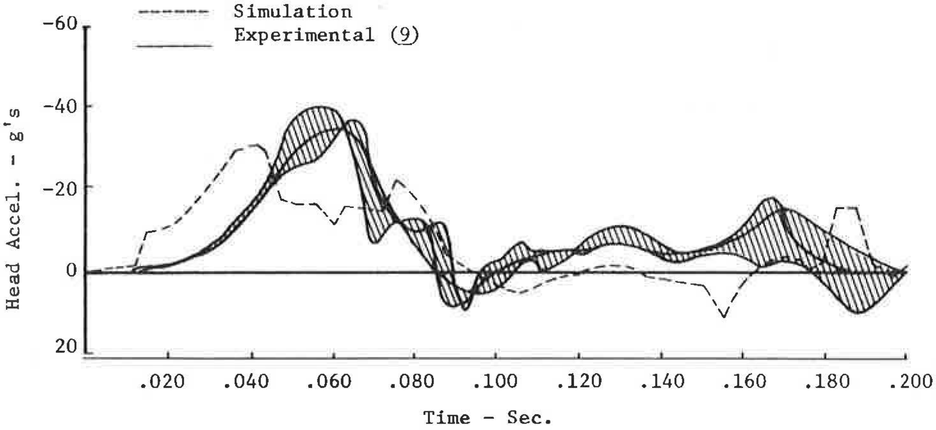

Figure 8. Head acceleration in segment Z direction, lap and torso restraint, 20-mph cart velocity.

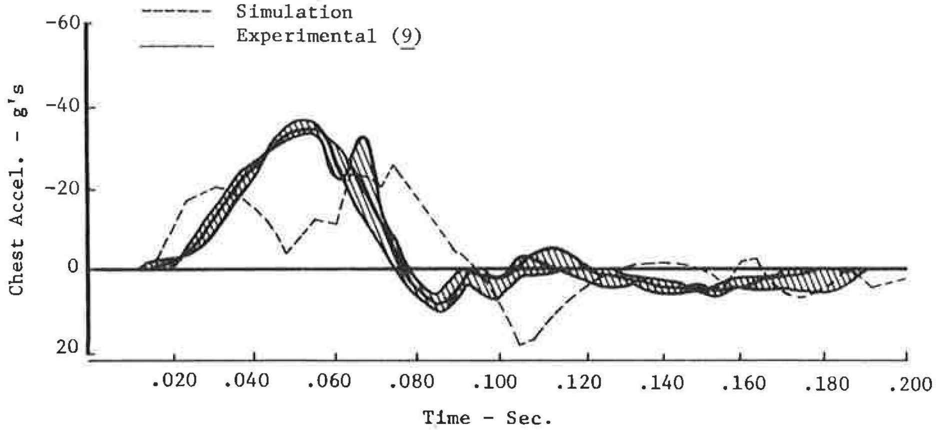

Figure 9. Chest acceleration in segment X direction, lap and torso restraint, 20-mph cart velocity.

of about 0.03 sec for the case of lap and torso restraint. This points to the fact that in some instances the response of the vehicle is no indication of what the passenger actually feels.

CONCLUSIONS

The analytical model described here provides the engineering profession with a useful tool with which to study vehicle and roadway problems, which results in saving lives and reducing occupant injuries. Admittedly, the model was validated for the planar case only; but this in no way precludes its application to three-dimensional motion, especially if qualitative results are being sought.

More specifically, the passenger model reduces the problem of predicting the motion, acceleration, and forces experienced by a vehicle occupant during a collision or violent maneuver of the vehicle to that of specifying the path of the vehicle as a function of time plus the deformation properties of the vehicle interior. (These should reflect the low stiffness property of the human body or dummy, whatever the case may be.) When used with available biomechanics data on human tolerance, the application of the passenger model includes the following:

1. The evaluation of roadway geometry—sideslopes, ditches, terrain involving a variation of vertical and horizontal alignment, etc.; roadside safety features such as the breakaway sign and energy-absorbing impact cushions; and roadside protective barriers such as guardrails, bridge rails, and median barriers;
2. The design of the vehicle interior and restraint systems;
3. The study of the dynamic behavior of a pedestrian when struck by an automobile; and
4. The study of collisions involving more than one vehicle.

ACKNOWLEDGMENTS

The information contained here was developed on Research Project 2-5-69-140, entitled "Evaluation of the Roadside Environment by Dynamic Analysis of the Interaction Between the Vehicle, Passenger and Roadway," which is a cooperative research study sponsored jointly by the Texas Highway Department and the Federal Highway Administration.

The opinions, findings, and conclusions expressed in this paper are those of the author and not necessarily those of the Federal Highway Administration.

REFERENCES

1. Chi, M., and Eicher, J. The Human in the Automobile Crash Environment. Traffic Systems Div., Bureau of Public Roads, unpublished report.
2. Emori, R. I. Analytical Approach to Automobile Collisions. Auto. Engr. Congress, Detroit, SAE Paper 680016, Jan. 8, 1967.
3. Froberg, C. E. Introduction to Numerical Analysis. Addison-Wesley Pub. Co., Inc., Reading, Mass., 1965.
4. Goldstein, H. Classical Mechanics. Addison-Wesley Pub. Co., Inc., Reading, Mass., 1950.
5. Harris, C. M., and Crede, C. E. Shock and Vibration Handbook. McGraw-Hill Book Co., New York, 1961, Vol. 3, Ch. 44.
6. Martinez, J. L., and Garcia, D. J. A Model for Whiplash. Jour. of Biomechanics, Pergamon Press, New York, Vol. 1, No. 1, Jan. 1968, pp. 23-32.
7. McHenry, R. R. Analysis of the Dynamics of Automobile Passenger-Restraint Systems. Proc. of Seventh Stapp Conf., Springfield, Ill., 1963.
8. McHenry, R. R., and Deleys, N. J. Vehicle Dynamics in Single Vehicle Accidents: Validation and Extensions of a Computer Simulation. Cornell Aeronautical Laboratory, Inc., Buffalo, N.Y., CAL No. VJ-2251-V-3, Dec. 1968.
9. McHenry, R. R., and Naab, K. N. Computer Simulation of the Automobile Crash Victim in a Frontal Collision—A Validation Study. Cornell Aeronautical Laboratory, Inc., Buffalo, N.Y., CAL No. YB-2126-V-1R, July 1966.
10. Payne, P. R. The Dynamics of Human Restraint Systems. In Impact Acceleration Stress, National Academy of Sciences, Pub. 977, 1962.
11. Renneker, D. N. A Basic Study of Energy-Absorbing Vehicle Structure and Occupant Restraints by Mathematical Model. Chrysler Corp., No. 670897.
12. Suggs, C. W., Abrams, C. F., and Strikeleather, L. F. Application of a Damped Spring-Mass Human Vibration Simulator in Vibration Testing of Vehicle Seats. Ergonomics, London, Vol. 12, No. 1, Jan. 1969, pp. 79-90.
13. Young, R. D., Edwards, T. C., Bridwell, R. J., and Ross, H. E., Jr. Documentation of Input for the Single Vehicle Accident Computer Program. Texas Transportation Institute, College Station, Research Rept. 140-1, July 1969.