DEFLECTION AND CURVATURE AS CRITERIA FOR FLEXIBLE PAVEMENT DESIGN AND EVALUATION

Y. H. Huang, University of Kentucky

The excessive plastic deformation of the subgrade and the fatigue cracking of asphalt surfaces are two major causes of pavement failures. One of the methods to preclude such failures is to consider the pavement as a three-layer elastic system by limiting the vertical compressive strain on the surface of the subgrade and the horizontal tensile strain at the bottom of the asphalt-bound layer. The disadvantage of this method is that these strains cannot be easily measured in the field, so the adequacy of the method cannot be properly evaluated. The purpose of this study is to find the relationships among deflection, curvature, compressive strain on the subgrade, and tensile strain in the asphalt layer so that deflection and curvature, instead of compressive and tensile strains, can be used as criteria for pavement design and evaluation. These relationships are expressed as dimensionless ratios and presented in tabular forms. The application of these tables to pavement design and evaluation is illustrated. The study shows that, for thinner pavements, curvature is a better indicator of the compressive strain on the subgrade and, for thicker pavements, deflection is the better indicator. The tensile strain in the asphalt layer is related to the curvature and the thickness of the asphalt layer, essentially independent of the thickness of untreated base courses and the properties of paving materials.

•THE application of rational methods to flexible pavement design and evaluation has received a great deal of attention in recent years. One of the methods that has been used most frequently is the three-layer linear elastic theory originally developed by Burmister (1). It is generally agreed that pavement behavior under the repeated application of transient wheel loads is essentially linearly elastic in the sense that the deflection is not only nearly proportional to the wheel load but also almost completely recoverable after each load application. Comparisons of the stresses and strains computed by the layered theory with those measured in prototype pavements, as undertaken by Whiffin and Lister (2) in England, Gusfeldt and Dempwolff (3) in West Germany, and Klomp and Niesman (4) and Nijboer (5) in The Netherlands, have more or less indicated the validity of the theory. If the stresses and strains in a pavement can be predicted theoretically, it is possible to design the pavement so that the stresses and strains at certain critical points will not exceed the allowable values. This rational method has long been used for the design of structures and has also been recommended recently by Shell International Petroleum for the design of flexible pavements.

In the design and evaluation of pavements, two major modes of failure must be considered, namely, rutting and fatigue. Rutting is due to permanent deformations in the pavement, particularly in the subgrade, resulting from a combination of consolidation and shear failure. There are no rational methods of practical significance for predicting the rut depth or the magnitude of permanent deformations. However, by keeping the stresses and strains on the surface of the subgrade to a low level, rut depth can be

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reduced to a tolerable limit so that it will neither cause the pavement to crack nor seriously impair its riding quality. Fatigue is caused by the repetitive application of wheel loads that induce fluctuating stresses and strains in the asphalt layer. Laboratory investigations of asphalt mixtures under bending stresses show that under a given number of load repetitions there exists a limiting tensile strain below which fatigue will not occur (6, 7). If this limiting strain under a given number of load repetitions can be determined in the laboratory, it is possible to design the pavement so that the maximum tensile strain at the bottom of the asphalt layer will not exceed the limiting value.

Peatti (8) suggested the use of the vertical compressive stress on the surface of the subgrade in conjunction with the horizontal tensile strain at the bottom of the asphalt layer as the two design criteria for flexible pavements. A relationship between the permissible value of the vertical stress on the subgrade and the CBR value of the soil was developed from an analysis of road structures known to be satisfactory in practice. He indicated that a relationship connecting the vertical strain at the top of the subgrade with the CBR value of the soil might also be derived, and that designs based on vertical strain as a criterion would be very similar to those developed from the vertical stress criterion. Skok and Finn (9) used the three-layer elastic theory to compute the maximum vertical stress on the subgrade and the maximum horizontal tensile stress and strain in the asphalt layer of the AASHO and WASHO test roads. They found that the rut depth and pavement serviceability were closely related to the vertical stress, and that the initial occurrence of cracking was related to the maximum horizontal stress and strain. Dorman (10) analyzed the U.S. Corps of Engineers CBR design charts and found that, irrespective of the construction, the allowable vertical compressive strain on the surface of the subgrade was practically a constant. Later, in cooperation with Metcalf (11), he developed a series of design charts based on the consideration of the vertical compressive strain on the subgrade and the horizontal tensile strain at the bottom of the asphalt layer. The permissible compressive strain on the subgrade, corresponding to a given number of equivalent 18,000-lb axle load applications, was obtained from an empirical correlation with the results of the AASHO Road Test, and the permissible tensile strain of the asphalt layer was obtained from laboratory fatigue data.

The foregoing literature review, even though quite brief, clearly indicates that the use of three-layer elastic theory by limiting the vertical compressive strain on the subgrade and the horizontal tensile strain at the bottom of the asphalt layer is a valid method of pavement design. However, the specification of these strains as design criteria suffers from the disadvantage that they cannot be easily measured in the field. It is desirable to use the surface deflection or curvature as the design criterion, rather than the vertical strain on the surface of the subgrade and the horizontal strain at the bottom of the asphalt layer, because the surface deflection and curvature can be easily measured. This is particularly true in the evaluation of existing pavements. If the relationships among deflection, curvature, compressive strain on the subgrade, and tensile strain in the asphalt layer under a design wheel load can be established theoretically, the permissible deflection or curvature corresponding to any given permissible compressive and tensile strains can then be determined. It is the purpose of this paper to investigate these relationships and present them in tabular forms. The application of these tables to the practical design and evaluation of flexible pavements is also presented.

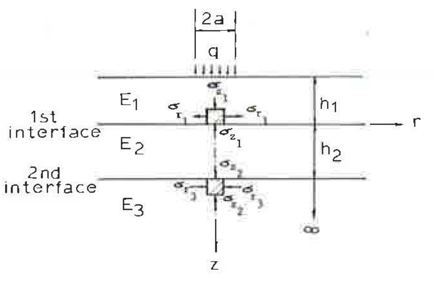

Figure 1. Three-layer elastic system.

DESCRIPTION OF METHOD

In the analysis, the pavement is considered a three-layer elastic system (Fig. 1). Layer 1, which is composed of asphalt-treated surface, binder, and base courses, has a thickness h_1 and an average modulus of elasticity E_1. Layer 2, which consists of untreated granular base and subbase courses, has a thickness h_2 and an average modulus E_2. These two layers are placed on a subgrade of infinite thickness with a modulus of elasticity E_3. A Poisson's ratio of

0.5 is assumed for all three layers. As is usually done in pavement design, it has been assumed that a wheel load having a uniform pressure q is applied on the surface over a circular area of radius a. In the discussions that follow, the surface course includes all the asphalt-treated courses on the upper portion of a pavement, while the base course refers to the untreated granular base and subbase only.

The maximum vertical compressive strain on the subgrade and the maximum horizontal tensile strain at the bottom of layer 1 occur at the axis of symmetry and can be determined by Eqs. 1 and 2 respectively.

$$\epsilon_z = \frac{q\left(\sigma_{z_2} - \sigma_{r_3}\right)}{E_3} \tag{1}$$

$$\epsilon_r = \frac{q\left(\sigma_{z_1} - \sigma_{r_1}\right)}{2E_1} \tag{2}$$

where

ϵ_z = vertical strain on the surface of the subgrade, or layer 3;
ϵ_r = horizontal strain at the bottom of layer 1;
q = applied pressure;
E_1 and E_3 = moduli of elasticity of layers 1 and 3 respectively;
σ_{z_1} and σ_{z_2} = vertical stress factors at the first and second interfaces respectively (Fig. 1); and
σ_{r_1} and σ_{r_3} = interface radial stress factors for layers 1 and 3 respectively.

Values of $\sigma_{z_2} - \sigma_{r_3}$ and $\sigma_{z_1} - \sigma_{r_1}$ can be obtained from Jones's tables (12).

Because values of surface deflections for a Poisson's ratio of 0.5 are not available, a computer program was developed for determining the surface deflections at the center of the loaded area as well as at other radial distances from the center. The surface deflection can be determined from Eq. 3.

$$w = \frac{qa}{E_3} F_w \tag{3}$$

where

w = surface deflection;
a = radius of loaded area; and
F_w = deflection factor, which can be determined from Eq. 3a.

$$F_w = \frac{1.5}{\left(\frac{E_1}{E_2}\right)\left(\frac{E_2}{E_3}\right)} \int_0^{\infty} J_0(mr)\, J_1(ma) \left[\frac{\text{Numerator}}{\text{Denominator}}\right] \frac{dm}{m} \tag{3a}$$

where

r = radial distance at which deflection is computed;
m = a parameter of integration; and
J_0 and J_1 = Bessel functions of the first kind, order 0 and 1 respectively.

The expressions for the numerator and denominator are functions of the thickness and modulus of each layer as well as the parameter m. They can be found in Burmister's original paper (1) and elsewhere (13). Because the expressions are quite long, they are not presented here. To compute the maximum deflection, r can be set to zero or $J_0(mr)$ to 1.

The infinite integral in Eq. 3a was evaluated numerically using Gaussian quadratures. The zeros of one of the Bessel functions were found, and the integration between two zeros was carried out using an 8-point formula. It was found that when h_1/a or h_2/a was relatively large, the integrand diminished very rapidly and the use of an 8-point formula could not give the desired precision. For this reason, a 32-point formula was employed for the first interval of integration and the 8-point formula for all subsequent intervals. The integration was continued until the deflection converged to a specified tolerance.

The curvature can be determined from the differential deflection, Δ, for a given chord length, ℓ. When $\ell = 2a$, as has been assumed in preparing the tables in the Appendix, the differential deflection can be determined by

$$\Delta = \frac{qa}{E_3} F_C \qquad (4)$$

where

$$F_C = \frac{1.5}{\left(\frac{E_1}{E_2}\right)\left(\frac{E_2}{E_3}\right)} \int_0^{\infty} [1 - J_0(ma)] \left[\frac{\text{Numerator}}{\text{Denominator}}\right] \frac{dm}{m} \qquad (4a)$$

It is generally assumed that the deflection profile is a sine curve, so the maximum curvature is

$$\frac{1}{R} = \frac{\pi^2 \Delta}{\ell^2} \qquad (5)$$

Combining Eqs. 4 and 5, and letting $\ell = 2a$, we get

$$a\left(\frac{1}{R}\right) = \frac{\pi^2 q}{4E_3} F_C \qquad (6)$$

To determine the relationships among deflection, curvature, compressive strain on the subgrade, and tensile strain in the asphalt layer, the following dimensionless ratios were used:

$$\frac{w}{a^2\left(\frac{1}{R}\right)} = \text{deflection-curvature ratio} = \frac{4}{\pi^2}\frac{F_W}{F_C} \qquad (7a)$$

$$\frac{w}{a\epsilon_z} = \text{deflection-compressive strain ratio} = \frac{F_W}{\sigma_{z_2} - \sigma_{r_3}} \qquad (7b)$$

$$\frac{a\left(\frac{1}{R}\right)}{\epsilon_z} = \text{curvature-compressive strain ratio} = \frac{\pi^2}{4}\frac{F_C}{\sigma_{z_2} - \sigma_{r_3}} \qquad (7c)$$

$$\frac{w}{a\epsilon_r} = \text{deflection-tensile strain ratio} = \frac{2F_W}{\sigma_{z_1} - \sigma_{r_1}}\frac{E_1}{E_2}\frac{E_2}{E_3} \qquad (7d)$$

$$\frac{a\left(\frac{1}{R}\right)}{\epsilon_r} = \text{curvature-tensile strain ratio} = \frac{\pi^2}{2}\frac{F_C}{\sigma_{z_1} - \sigma_{r_1}}\frac{E_1}{E_2}\frac{E_2}{E_3} \qquad (7e)$$

All terms on the right side of Eqs. 7 either are given or can be determined from Eqs. 3a, 4a, or Jones's tables, so these ratios can be computed.

DISCUSSION OF RESULTS

The relationships among deflection, curvature, compressive strain on the subgrade, and tensile strain in the asphalt layer, as expressed by their ratios, are tabulated in the Appendix. In order to save space, not all these ratios are tabulated. Although the deflection-tensile strain ratios are not shown, they can be easily obtained by multiplying the curvature-tensile strain ratios by the deflection-curvature ratios. Also included are the deflection factor, F_W, and the curvature factor, F_C, as determined from Eqs. 3a and 4a respectively.

In the computation, a wide range of thicknesses and modulus ratios was used. The values of h_1/a range from 0.31 to 2.5 and those of h_2/a from 0.63 to 5. The contact radius, a, for a conventional 18,000-lb design axle load is about 6 in., so the thicknesses of the asphalt layer range from 2 to 15 in. and those of the granular bases and subbases from 4 to 30 in. Some values of h_1/a or h_2/a are awkward because of the necessity to match them with values in Jones's tables in which the stresses are given in terms of h_1/h_2 and a/h_2.

Relationship Between Deflection and Curvature

A study of the deflection-curvature ratios tabulated in the Appendix shows that the ratios generally increase with increasing E_1/E_2, E_2/E_3, or h_1/a, indicating that the change in curvature is much more rapid than the change in deflection. In other words, curvature is more sensitive to the change in surface thickness and modulus ratios than to the deflection. Furthermore, the thickness of the untreated base, or layer 2, has practically no effect on the deflection-curvature ratio, as is shown in Figure 2 for a typical case with $E_1/E_2 = 20$ and $E_2/E_3 = 2$. Figure 2 also shows that when the surface is thin, say 2 in. or less, the thickness of surface course has very little effect on deflection-curvature ratios. These theoretical findings are important because they can explain why, under certain circumstances, both deflection and curvature can be used for pavement evaluation with no differences. Dehlen (14) investigated the deflections and curvatures of roads in South Africa and showed that, for the 1- to 2-in. premix surfacings, both deflection and curvature could be used equally well as an indicator of pavement conditions. This conclusion is valid only when the pavements are of the same type with the same materials, the only difference being the thickness of granular base. If the pavements are of different types with a wide variety of surface thickness and modulus ratios, the deflection-curvature ratio will no longer be a constant, and the evaluation based on deflection will surely be different from that based on curvature.

Relation of Compressive Strain to Deflection and Curvature

The relation of the compressive strains on the subgrade to the deflections and curvatures on the surface is quite erratic. The deflection-compressive strain ratios and the curvature-compressive strain ratios vary significantly with the change in thickness and modulus ratios, and there are no general trends to be traced. For a given permissible compressive strain, the only way to find the corresponding permissible deflection and curvature is to go through the table in the Appendix. When h_2/a is less than or equal to 1.25, which is equivalent to a base course about 8 in. thick, only the curvature-compressive strain ratio, but not the deflection-compressive strain ratio, is tabulated because the former does not change very much with the change in modulus ratios and is, therefore, a better indicator of the compressive strain. It is well-known that the moduli of elasticity of the pavement component layers are quite difficult to determine because they change appreciably

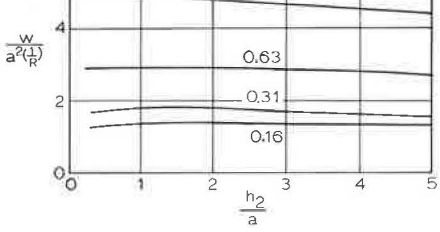

Figure 2. Effect of thickness of deflection-curvature ratio ($E_1/E_2 = 20$, $E_2/E_3 = 2$).

with loading and environment conditions, such as temperature, moisture content, rate of loading, and state of stress. Consequently, a measurement that does not change with the change in modulus ratios is more reliable and should be used whenever possible. When h_2/a is equal to or greater than 2.5a, the reverse is true; i.e., the deflection is a better indicator of the compressive strain, so the deflection-compressive strain ratio, instead of the curvature-compressive strain ratio, is tabulated.

Relation of Tensile Strain to Deflection and Curvature

A study of the data in the Appendix indicates that both the deflection-tensile strain and the curvature-tensile strain ratios are practically independent of the base course thickness or h_2/a. Therefore, a typical h_2/a of 1.25 can be used to illustrate the factors that affect these ratios.

Figure 3 shows the effect of surface course thickness and modulus ratios on deflection-tensile strain ratios. The deflection-tensile strain ratio increases with increasing E_2/E_3 because of the more rapid decrease in tensile strains at the bottom of the asphalt layer as compared to the decrease in deflections. The effect of E_1/E_2 is erratic when h_1/a is less than 0.6, which is equivalent to a surface course of 4 in. or less. In this practical thickness range, the deflection-tensile strain ratio decreases with the increase in surface course thickness unless E_1/E_2 is exceedingly large. If fatigue is done to excessive tensile strains at the bottom of asphalt layers, for a given permissible tensile strain the permissible deflection should decrease with the increase in surface thickness. This is in agreement with the method used by the California Division of Highways (15) that tolerable deflections to preclude fatigue failures decrease with increasing thickness. However, the strong dependence of the deflection-tensile strain ratio on the modulus ratios indicates that deflection is not a good measure of fatigue.

Figure 4 shows the effect of surface course thickness and modulus ratios on curvature-tensile strain ratios for $h_2/a = 1.25$. Unless E_1/E_2 is exceedingly small, the curvature-tensile strain ratio is practically independent of modulus ratios and varies only with the surface course thickness. This is in conformity with the well-known beam theory that the strain in a beam under pure bending depends only on the curvature and the thickness. However, unlike the beam theory, the strain is not proportional to the thickness because the pavement is considered as a continuum in which the effect of vertical stress is also considered. The independence of the curvature-tensile strain ratio on the modulus ratios indicates that the tensile strain at the bottom of an asphalt layer can be easily determined by measuring the curvature. Knowing the curvature and the thickness of the surface course, the tensile strain can be estimated by using Figure 4.

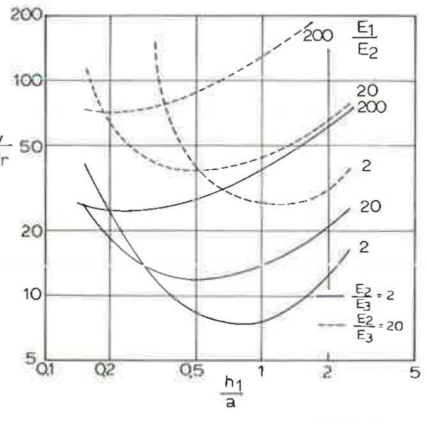

Figure 3. Effect of surface course thickness and modulus ratios on deflection-tensile strain ratio (h_2/a = 1.25).

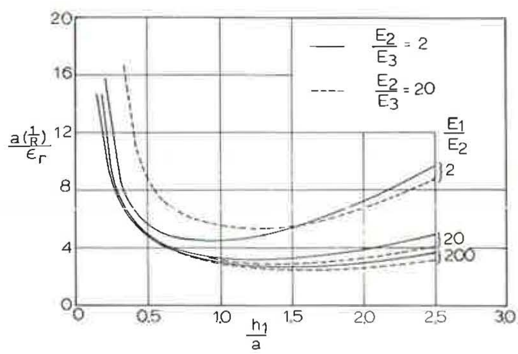

Figure 4. Effect of surface course thickness and modulus ratios on curvature-tensile strain ratio (h_2/a = 1.25).

APPLICATIONS TO PAVEMENT DESIGN AND EVALUATION

The data tabulated in the Appendix are based on a single wheel load. If a single wheel load is actually applied to a pavement and the deflection and curvature are measured, the corresponding compressive and tensile strains in the pavement can be determined from the tables. Because the deflection and curvature are generally measured by placing a Benkelman beam (16) or a curvature meter (17, 18) between a set of dual tires, the deflection and curvature thus measured would be somewhat different from those based on a single wheel load. Thus, a correction factor is needed to convert the deflection or curvature based on a single wheel load to that based on a dual wheel load. By simply multiplying the deflection or curvature under a single wheel load by its respective correction factor as tabulated in the Appendix, the corresponding deflection or curvature for a dual wheel load is obtained. The correction factors are based on the three-layer theory by considering each tire having a radius $a/\sqrt{2}$ and spaced at $3a/\sqrt{2}$ apart. The curvature for dual tires is also based on a chord length of 2a.

Pavement Evaluation

In the evaluation of existing pavements, it is necessary to know the thickness of the surface and base courses and the modulus ratios. The thickness of the pavement components can be measured or obtained from construction records, and the modulus ratios for pavements of conventional types can be reasonably estimated. The Shell vibration machines (19) have been used in various countries for determining the in situ moduli of surface and base courses as well as the subgrade, and typical values have been suggested (7). In pavement evaluation, it is not necessary to know the modulus of each layer; only their ratios are needed. The relationships among deflection, curvature, compressive strain on the subgrade, and tensile strain in the asphalt layer, as expressed by their ratios, can then be determined from the table in the Appendix. Once the deflection and curvature are measured in the field, the corresponding compressive and tensile strains can be computed. By comparing these strains with the permissible strains, the adequacy of the pavement can be evaluated. Table 1 gives the permissible compressive strains on the subgrade and the tensile strains in the asphalt layer corresponding to different equivalent 18,000-lb axle load applications, as suggested by Dormon and Metcalf (11).

For predicting rutting or plastic deformation of the subgrade, it is desirable to measure the deflection and curvature during the summer when the asphalt layer has a smaller modulus and the compressive strain on the subgrade is relatively high; whereas for predicting fatigue, measurements should be made during the winter because the asphalt layer will crack only when the temperature is low. The permissible strains given in Table 1 were determined at low temperatures. However, by conducting the same measurements both in the summer and in the winter, it is possible to establish the correlation between summer and winter measurements so that the deflection and curvature determined during one season can be converted to the other for design and evaluation purposes.

Because the values of h_1/a, h_2/a, E_1/E_2, and E_2/E_3 in actual cases may not be the same as those tabulated in the Appendix, interpolations of data are usually needed. A three-point Lagrange interpolation formula (20) based on logarithmic scales may be used, or a curve may be plotted through the three given points. As only two points are available for E_2/E_3, a straight-line interpolation based on logarithmic scales is suggested.

As an illustrative example, suppose an existing pavement having a 4-in. asphalt surface course and an 8-in. untreated base course is to be evaluated. The deflection profile under an 18,000-lb single axle load with dual tires of 80-psi pressure is measured by a Benkelman beam. If the radius

TABLE 1

PERMISSIBLE STRAINS UNDER DIFFERENT LOAD APPLICATIONS

No. of Load Applications	Compressive Strain on Subgrade	Tensile Strain in Asphalt Layer
10^5	1.05×10^{-3}	2.3×10^{-4}
10^6	6.5×10^{-4}	1.45×10^{-4}
10^7	4.2×10^{-4}	9.2×10^{-5}
10^8	2.6×10^{-4}	5.8×10^{-5}

of curvature determined from the deflection profile for a chord length of 12 in. is 1,000 ft, what are the compressive strain on the subgrade and the tensile strain in the asphalt layer?

First, replace the dual wheel load by a single wheel load with a contact radius of 6 in., so $h_1/a = 0.667$ and $h_2/a = 1.333$. If it is assumed that $E_1/E_2 = 5$ and $E_2/E_3 = 2$, by successive interpolations the following values are obtained: $[a(1R)]/\epsilon_z = 2.46$, $[a(1/R)]/\epsilon_r = 4.31$, and the correction factor for curvature = 0.62. Converting the radius of curvature to single wheel load, R = 0.62 × 1,000 = 620 ft. Substituting R = 620 ft and a = 0.5 ft into the above ratios, $\epsilon_z = 3.28 \times 10^{-4}$ and $\epsilon_r = 1.87 \times 10^{-4}$. Compared with the permissible strains in Table 1, it can be concluded that the pavement can sustain more than 10,000,000 load applications without rutting and less than 1,000,000 applications without fatigue. The latter conclusion is obviously not correct because, for predicting fatigue, curvatures must be measured during the winter. If the radius of curvature measured during the winter is 2,000 ft, by assuming $E_1/E_2 = 20$, $E_2/E_3 = 2$ and a correction factor of 0.74, it can be found that $\epsilon_r = 8.2 \times 10^{-5}$, so the pavement can also sustain 10,000,000 applications without fatigue.

Pavement Design

The use of the table in the Appendix for pavement design differs from that for pavement evaluation in two major respects: (a) it is necessary to know the modulus of each layer, not just the modulus ratios, and (b) a trial-and-error process must be employed to determine the thickness of surface and base courses required.

The thickness of surface course is generally governed by fatigue, i.e., by considering the tensile strain at the bottom of the asphalt layer. For ease of illustration, it is assumed that $E_1/E_2 = 200$ and $E_2/E_3 = 2$, so no interpolations of modulus ratios are needed. If $E_3 = 10{,}000$ psi, q = 80 psi, a = 6 in., and the permissible tensile strain is 9.2×10^{-5}, the thickness of the surface course can be easily determined.

As a first step, it is assumed that the pavement has the following thickness-radius ratios: $h_1/a = 0.625$ and $h_2/a = 1.25$. From the Appendix, the curvature-tensile strain ratio is 4.01, so $a(1/R) = 4.01 \times 9.2 \times 10^{-5} = 3.69 \times 10^{-4}$, which, when substituted into Eq. 6, results in a permissible curvature factor, F_c, of 0.0187. The actual curvature factor for this design as obtained from the Appendix is 0.0177, which is smaller than the permissible value, and the design is considered satisfactory. It should be noted that the thickness of the base course has only little effect on the thickness of the surface course required.

The thickness of the base course is generally governed by rutting, i.e., by considering the compressive strain on the surface of the subgrade. Using a lower modulus ratio of 20 for E_1/E_3, the curvature-compressive strain ratio obtained from the Appendix is 1.80. If the permissible compressive strain is 4.2×10^{-4}, the corresponding permissible curvature factor is 0.0383, which is much smaller than the actual value of 0.0856. The design is, therefore, unsatisfactory, and a thicker base course should be used.

Next assume h_2/a to be 2.5. For such a thick base course, it is preferable to use the deflection rather than the curvature as a design criterion. The deflection-compressive strain ratio obtained from the Appendix is 7.92, so $w/a = 7.92 \times 4.2 \times 10^{-4} = 3.326 \times 10^{-3}$, which, when substituting into Eq. 3, results in a permissible deflection factor, F_w, of 0.416. The actual deflection factor for this design is 0.5610, so a still thicker base should be used.

The final design adopted consists of a 4-in. surface course ($h_1/a = 0.667$) and a 20-in. base course ($h_2/a = 3.333$). By proper interpolations of the data in the Appendix, it is found that for $E_1/E_2 = 20$, $w/(a\epsilon_z) = 10.49$, and that for $E_1/E_2 = 200$, $a(1/R)/\epsilon_r = 3.84$. The permissible F_w and F_c are 0.5508 and 0.0179 respectively, which are slightly greater than the actual values of 0.5167 and 0.0148 as determined by interpolations.

This design procedure is very similar to the conventional method of using directly the permissible compressive and tensile strains as design criteria. However, the conversion of strains into deflections and curvatures has the apparent advantage that the latter can be measured in the field, so the method of design can be evaluated.

Although the method presented is based on linear theory, it can also be applied to nonlinear materials if their moduli of elasticity are determined by a method of successive approximations as suggested by Monismith et al. (21). In predicting pavement deflections from laboratory tests, Monismith et al. used a Poisson's ratio of 0.5 for stress computations and 0.35 for deflection computations because these were the only data readily available. The inclusion of the deflection factors in the Appendix is a supplement to Jones's tables and makes available the maximum deflections for a Poisson's ratio of 0.5.

CONCLUSIONS

Based on the three-layer elastic theory, the relationships among deflection, curvature, the compressive strain on the surface of the subgrade, and the tensile strain at the bottom of the asphalt layer, in terms of various dimensionless ratios, are investigated. These dimensionless ratios together with the deflection and curvature factors are presented in tabular forms for a wide range of layer thicknesses and modulus ratios so that the permissible deflection and curvature corresponding to any given permissible compressive and tensile strains can be determined. This makes possible the use of deflection and curvature, instead of compressive and tensile strains, as criteria for pavement design and evaluation. The advantage of using deflection and curvature is that they can be easily measured in the field. The method is particularly suited for the evaluation of existing pavements. By simply measuring the deflection or curvature of a pavement under a given design wheel load, the compressive strain on the surface of the subgrade and the tensile strain at the bottom of the asphalt layer can be estimated. The adequacy of the pavement to carry the design wheel load can be evaluated by comparing the estimated compressive and tensile strains with established permissible values.

A study of the data results in the following conclusions. It should be borne in mind that deflections and curvatures are not exact measurements because they vary a great deal depending on environmental conditions. These conclusions are valid only under general conditions, sometimes in a qualitative sense, and should not be interpreted with exactitude.

1. The deflection-curvature ratio is independent of the base course thickness. The contention that both deflection and curvature can be used equally well as a measure of pavement conditions is valid only when the pavements evaluated are of the general types and composed of the same materials, the only difference being the base course thickness.
2. Curvature is a better indicator of the compressive strain on the subgrade, when the base course is thin, say 10 in. or less. For thicker base courses, deflection is the better indicator.
3. The thickness of base course has very little effect on the deflection-tensile strain ratio, but the thickness of surface course has a tremendous effect. Unless the surface course is exceedingly thick and strong, the deflection-tensile strain ratio decreases with the increase in surface course thickness. If the tensile strain at the bottom of the surface course is a factor controlling fatigue, the permissible deflections should decrease with the increase in surface thickness.
4. Curvature is definitely related to the tensile strain in the asphalt layer. The curvature-tensile strain ratio depends primarily on the surface course thickness, independent of the base course thickness. The use of curvature, instead of deflection, as a criterion for controlling fatigue is highly desirable because the curvature-tensile strain ratios are not affected by the modulus ratios.

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Appendix

DEFLECTION AND CURVATURE AND THEIR RELATION TO COMPRESSIVE AND TENSILE STRAINS

$\frac{h_1}{a}$	$\frac{E_1}{E_2}$	$\frac{E_2}{E_3}$	Deflection		Curvature		$\frac{w}{a^2(\frac{1}{R})}$	$\frac{a(\frac{1}{R})}{\epsilon_z}$	$\frac{a(\frac{1}{R})}{\epsilon_r}$	Deflection		Curvature		$\frac{w}{a^2(\frac{1}{R})}$	$\frac{w}{a\epsilon_z}$	$\frac{a(\frac{1}{R})}{\epsilon_r}$
			F_w	C.F.*	F_c	C.F.*				F_w	C.F.*	F_c	C.F.*			
0.3125			h_2/a = 0.625							h_2/a = 2.500						
	2	2	1.1829	0.64	0.3519	0.54	1.36	2.02	9.82	0.9103	0.62	0.2714	0.48	1.35	7.73	7.55
		20	0.5853	0.86	0.0754	0.71	3.14	1.55	37.60	0.2601	0.86	0.0295	0.52	3.57	10.50	12.51
	20	2	0.9963	0.75	0.2349	0.68	1.72	1.86	7.45	0.7880	0.74	0.1887	0.64	1.69	7.28	7.21
		20	0.4711	0.91	0.0441	0.79	4.34	1.37	8.44	0.2232	0.90	0.0208	0.66	4.35	10.31	8.12
	200	2	0.6240	0.88	0.0749	0.80	3.38	1.55	7.34	0.5358	0.88	0.0652	0.78	3.33	7.50	7.31
		20	0.2912	0.95	0.0126	0.81	9.36	1.16	7.60	0.1741	0.95	0.0077	0.56	9.14	11.65	7.58
			h_2/a = 1.250							h_2/a = 5.000						
	2	2	1.0359	0.64	0.2962	0.51	1.42	2.65	8.75	0.8289	0.59	0.2655	0.47	1.28	22.33	7.08
		20	0.3986	0.88	0.0417	0.63	3.87	1.63	37.49	0.1741	0.80	0.0268	0.48	2.86	25.22	8.16
	20	2	0.8958	0.75	0.2065	0.66	1.76	2.28	7.40	0.7101	0.72	0.1833	0.63	1.57	19.60	7.09
		20	0.3379	0.91	0.0285	0.73	4.80	1.43	8.68	0.1495	0.86	0.0187	0.63	3.51	22.84	7.52
	200	2	0.5920	0.88	0.0704	0.80	3.40	1.59	7.34	0.4754	0.87	0.0622	0.78	3.11	15.48	7.23
		20	0.2453	0.96	0.0102	0.76	9.72	1.02	7.65	0.1133	0.94	0.0065	0.62	8.10	20.97	7.41
0.6250			h_2/a = 0.625							h_2/a = 2.500						
	2	2	1.0140	0.69	0.2627	0.56	1.56	2.04	5.05	0.8243	0.66	0.2229	0.51	1.50	8.60	4.55
		20	0.4530	0.89	0.0443	0.71	4.14	1.39	7.00	0.2324	0.88	0.0238	0.56	3.95	11.64	5.57
	20	2	0.6587	0.86	0.0914	0.75	2.92	1.54	4.12	0.5610	0.85	0.0801	0.73	2.84	7.92	4.10
		20	0.3060	0.95	0.0146	0.83	8.47	1.09	4.27	0.1819	0.95	0.0091	0.76	8.07	12.51	4.36
	200	2	0.3443	0.95	0.0181	0.84	7.70	1.22	4.01	0.3206	0.95	0.0171	0.83	7.64	10.31	4.02
		20	0.1565	0.98	0.0026	0.99	24.21	0.93	4.01	0.1259	0.98	0.0021	0.70	24.06	18.85	4.06
			h_2/a = 1.250							h_2/a = 5.000						
	2	2	0.9178	0.68	0.2365	0.54	1.57	2.83	4.82	0.7552	0.64	0.2186	0.51	1.40	22.72	4.41
		20	0.3354	0.90	0.0304	0.64	4.47	1.64	6.94	0.1603	0.83	0.0219	0.52	2.97	23.68	4.70
	20	2	0.6193	0.86	0.0856	0.74	2.93	1.80	4.12	0.5029	0.84	0.0773	0.72	2.63	16.75	4.05
		20	0.2522	0.95	0.0116	0.80	8.78	1.11	4.38	0.1230	0.93	0.0080	0.72	6.24	21.03	4.21
	200	2	0.3376	0.95	0.0177	0.84	7.71	1.17	4.01	0.2914	0.95	0.0163	0.83	7.23	15.29	4.01
		20	0.1486	0.98	0.0025	0.61	24.54	0.78	4.04	0.0907	0.97	0.0018	0.82	20.53	23.20	4.08
1.2500			h_2/a = 0.625							h_2/a = 2.500						
	2	2	0.7913	0.71	0.1782	0.51	1.80	2.48	4.57	0.6888	0.69	0.1661	0.48	1.68	10.63	4.68
		20	0.3108	0.91	0.0237	0.61	5.32	1.49	4.64	0.1929	0.88	0.0183	0.47	4.28	14.38	5.37
	20	2	0.3840	0.91	0.0315	0.68	4.94	1.43	3.94	0.3523	0.90	0.0302	0.64	4.72	10.48	3.15
		20	0.1719	0.97	0.0041	0.77	16.86	0.96	2.79	0.1310	0.97	0.0034	0.69	15.48	18.56	3.09
	200	2	0.1796	0.97	0.0044	0.77	16.50	1.06	2.65	0.1751	0.97	0.0044	0.75	16.23	17.55	2.70
		20	0.0807	0.99	---	--	59.42	0.80	2.49	0.0752	0.99	---	--	58.28	37.24	2.57
			h_2/a = 1.250							h_2/a = 5.000						
	2	2	0.7441	0.71	0.1711	0.50	1.76	3.48	4.73	0.6387	0.67	0.1639	0.48	1.58	24.20	4.60
		20	0.2551	0.90	0.0206	0.53	5.02	1.96	5.33	0.1400	0.84	0.0173	0.44	3.28	26.12	5.08
	20	2	0.3733	0.91	0.0311	0.65	4.86	1.65	3.12	0.3220	0.90	0.0295	0.63	4.43	17.03	3.17
		20	0.1585	0.97	0.0039	0.70	16.68	0.99	2.92	0.0954	0.96	0.0031	0.64	12.50	25.03	3.20
	200	2	0.1785	0.97	0.0044	0.74	16.36	1.05	2.69	0.1665	0.97	0.0043	0.74	15.75	20.66	2.71
		20	0.0798	0.98	---	--	59.32	0.70	2.53	0.0644	0.99	---	--	54.18	37.63	2.64
2.5000			h_2/a = 0.625							h_2/a = 2.500						
	2	2	0.6118	0.68	0.1451	0.43	1.71	4.86	9.18	0.5687	0.65	0.1428	0.34	1.61	16.40	9.90
		20	0.1966	0.89	0.0156	0.47	5.10	2.57	6.84	0.1463	0.87	0.0136	0.38	4.36	20.55	9.07
	20	2	0.2176	0.90	0.0168	0.48	5.24	2.38	4.76	0.2079	0.90	0.0153	0.44	5.51	16.65	4.53
		20	0.0913	0.97	0.0018	0.55	20.21	1.50	3.77	0.0816	0.97	0.0016	0.52	20.36	32.25	3.89
	200	2	0.0926	0.97	0.0018	0.57	20.33	1.64	3.60	0.0917	0.98	0.0017	0.52	21.82	33.58	3.36
		20	0.0411	0.99	---	--	84.08	1.16	3.04	0.0403	0.99	---	--	89.25	82.33	2.88
			h_2/a = 1.250							h_2/a = 5.000						
	2	2	0.5939	0.68	0.1439	0.49	1.67	6.42	9.68	0.5400	0.63	0.1421	0.33	1.54	31.27	9.91
		20	0.1769	0.89	0.0158	0.49	4.52	3.43	8.78	0.1147	0.84	0.0133	0.37	3.50	32.68	9.50
	20	2	0.2153	0.90	0.0171	0.53	5.09	2.74	4.95	0.1972	0.90	0.0152	0.44	5.27	22.61	4.62
		20	0.0886	0.97	0.0018	0.61	19.51	1.57	4.01	0.0677	0.97	0.0016	0.48	17.48	37.41	4.28
	200	2	0.0924	0.98	0.0019	0.59	20.01	1.67	3.61	0.0900	0.97	0.0017	0.53	21.56	35.41	3.38
		20	0.0411	0.99	---	--	83.58	1.11	3.12	0.0387	0.98	---	--	87.61	75.78	2.97

* C.F. is the correction factor for dual tires.

Discussion

N. K. VASWANI, Virginia Highway Research Council—Huang's paper is a great step toward the application of theory to practice.

In Virginia we have been evaluating our pavements on the basis of their maximum deflection and spreadability. We define spreadability as a nondimensional quantity as follows:

$$\text{Spreadability} = \frac{d_{max} + d_1 + d_2 + d_3 + d_4}{5\, d_{max}} \times 100 \text{ percent}$$

where

d_{max} = maximum deflection of the pavement, and
d_1, d_2, d_3 and d_4 = the deflections at 1, 2, 3 and 4 ft from the point of the maximum deflection in the deflected basin.

Thus, spreadability as defined here and curvature as defined by the author are interrelated.

Theoretical analyses based on the elastic layered system were carried out in Virginia, and we have determined that if spreadability and maximum deflections were to be considered, the multilayer system could be reduced to a two-layer system consisting of the semi-infinite subgrade and the overlying pavement. The modulus of elasticity of the overlying pavement, for the purpose of practical application, would be the average modulus of elasticity obtained by

$$E_{av} = \frac{h_1E_1 + h_2E_2 + \ldots}{h_1 + h_2 + \ldots}$$

where

E_{av} = the average modulus of elasticity of the overlying pavement, and
E_1, E_2, ... = the moduli of elasticity of the materials in the pavement having thicknesses equal to h_1, h_2, ... respectively.

The author has theoretically determined that, when the pavements are of different types and with a wide variety of surface thicknesses and modulus ratios, the deflection-curvature ratio is no longer a constant. I have found this to be true in my evaluation of pavements in Virginia on the basis of the deflection-spreadability ratio, and also in my evaluation of the deflection data reported by Scrivner et al. (22). Some examples of the evaluations made in Virginia have been reported (23), and there appears to be a good relationship between the curvature as defined by the author and spreadability as adopted by Virginia.

In this discussion, I present the case of a pavement in southeastern Minnesota. The pavement consisted of a 3-in. layer of asphaltic concrete with 3 in. of crushed rock and 9 in. of sand gravel underneath it. The subgrade soil was a silty clay loam. Figure 5 shows the magnitude of the variation in the ratio of spreadability vs. maximum deflection from December 1966 to August 1967. The temperatures recorded at the top of the pavement during this period varied as follows: December 16, 1966—36 F; February 22, 1967—23 F; March 27, 1967—60 F; April 11, 1967—68 F; and August 8, 1967—93 F. These temperature data present a picture of the seasonal variations and the hydrothermal effects on the pavement and the subgrade, and, hence, the variations in the moduli of elasticity of the pavement and the subgrade.

For an understanding of Figure 5, the following points need to be recognized.

1. The modulus of elasticity of the subgrade for any point on the curve could be obtained by extrapolating a coordinate parallel to the coordinate inclined at $\tan^{-1} 0.47$ degrees (Fig. 5). The point at which this extrapolated coordinate hits the base line is the modulus of elasticity of the subgrade.

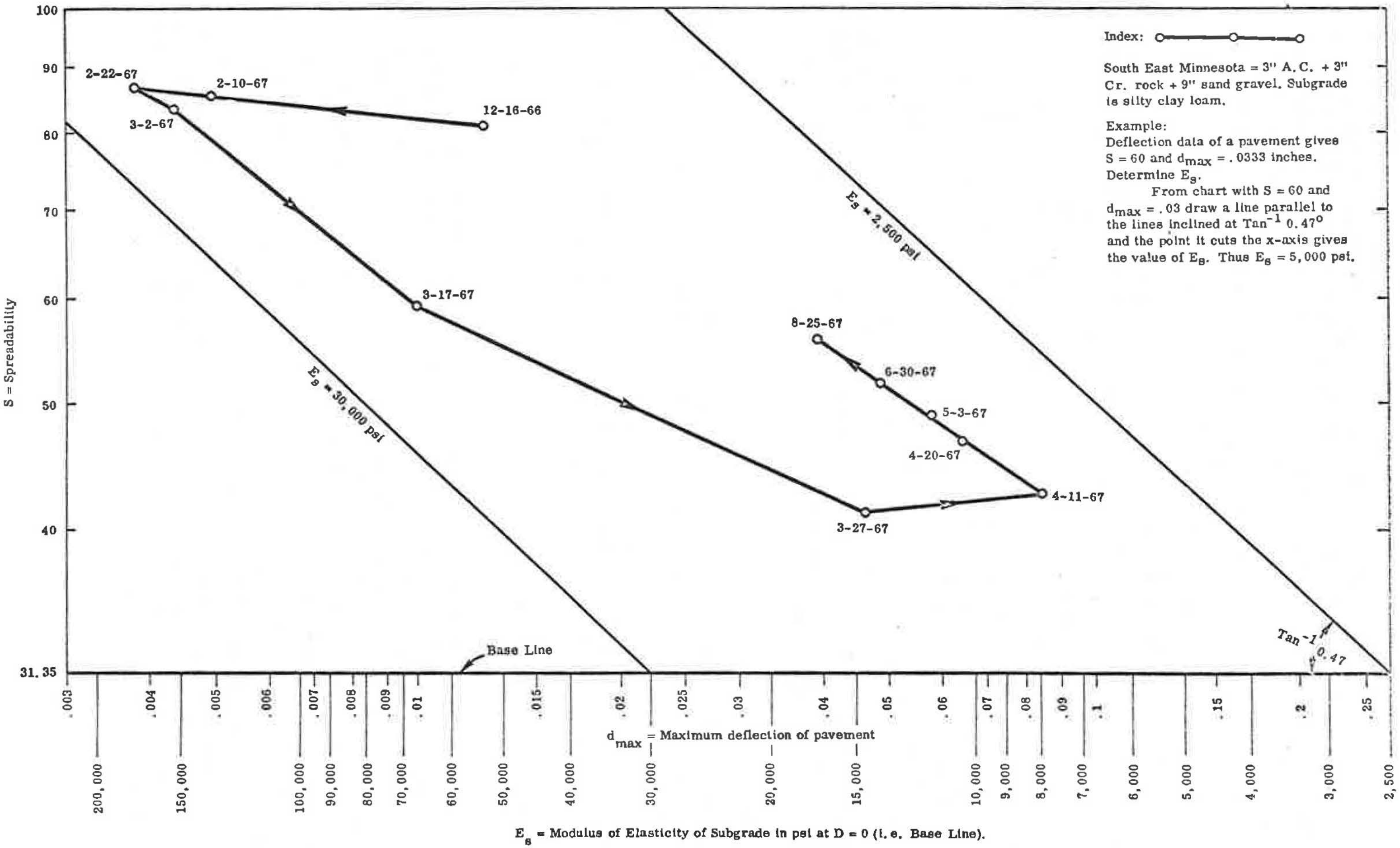

Figure 5. An example of pavement evaluation.

2. For the same pavement thickness, the spreadability value would increase (and thus d_{max} would decrease) as the average modulus of elasticity of the pavement increases, and vice versa, although the subgrade modulus remains the same.

3. For the same average modulus of elasticity of the pavement, the spreadability value would increase (and thus d_{max} would decrease) as the thickness of the pavement increases, and vice versa, although the subgrade modulus would remain the same.

Thus, Figure 5 shows that the modulus of elasticity of the subgrade increased from about 6,000 psi on December 16, 1966, to about 18,000 psi on February 22, 1967, with very little increase in the modulus of elasticity of the pavement. This was probably caused by the frozen condition of the subgrade.

From February 22 to March 17, 1967, the modulus of elasticity of the pavement decreased as indicated by the reduction in spreadability from 88 to 60. This might be due to the increase in the temperature of the asphaltic concrete and a thawing of the moisture in the crushed rock and sand gravel. During this period the modulus of elasticity of the subgrade did not change much, which indicates that the subgrade continued to remain frozen and thus retained the increased modulus caused by freezing.

The period between March 17 to March 27, 1967, shows an increased thawing of the pavement and the thawing of the subgrade, which resulted in the reduced modulus of elasticity of the pavement and the subgrade.

The period March 27 to April 11, 1967, indicates no change in the modulus of elasticity of the pavement but an increased thawing of and more moisture in the subgrade.

The period April 11 to August 25, 1967, shows an increased spreadability and thus an increased modulus of elasticity of the pavement and a slight increase in the subgrade modulus. These changes are probably the results of water draining away from the pavement and the subgrade.

References

22. Scrivner, F. H., Poehl, R., Moore, W. M., and Phillips, M. B. Detecting Seasonal Changes in Load Carrying Capabilities of Flexible Pavements. NCHRP Rept. 76, 1969.
23. Vaswani, N. K. A Method for Evaluating the Structural Performance of Subgrades and/or the Overlying Flexible Pavements. Presented at HRB 50th Annual Meeting and to be published in Highway Research Record series.

Y. H. HUANG, Closure—Vaswani's interest in the discussion of my paper is greatly appreciated. The spreadability used by him is a ratio of the off-center deflections to the center deflection and is, therefore, quite similar to the radius of curvature used in my paper to indicate the difference between off-center and center deflections, keeping in mind that curvature increases as spreadability decreases. It is gratifying that his field study in Virginia as well as other data also shows that the deflection-spreadability ratio changes with the change in modulus ratios, and that the effect of pavement thickness and modulus ratios on spreadability bears the same trend as that on curvature. Because the main purpose of my paper is to relate the curvature to the tensile strain at the bottom of the asphalt layer, which is maximum directly under the load, I have to determine the curvature near to the load by using a small chord length. If I had used the spreadability defined by Vaswani by measuring the off-center deflection as far as 4 ft away from the center, the correlation between the curvature and the tensile strain could not possibly have been established.