ANALYSIS OF STRESSES AND DISPLACEMENTS IN THREE-LAYER VISCOELASTIC SYSTEMS

J. F. Elliott and F. Moavenzadeh, Department of Civil Engineering, Massachusetts Institute of Technology

A solution technique is presented for the analysis of the stresses and displacements induced in a viscoelastic three-layered body subject to boundary loads with different time configurations. The technique used is composed of two parts: (a) the use of a modified correspondence principle in the form of hereditary integrals to convert the elastic solution (to a boundary value problem with time-boundary conditions of the Heaviside type) to the viscoelastic solution, and (b) the use of the principles of the response of linear systems to imposed excitations to obtain the viscoelastic solutions to boundary value problems with time-boundary conditions other than the Heaviside type. The underlying principles of this method are discussed, and an example of the use of the method in the analysis of the stresses and displacements induced in a three-layer viscoelastic body subjected to uniformly distributed normal loads of the stationary, repeated, and moving types is presented.

•THIS PAPER presents a method of stress analysis for pavement structures by representing it as a three-layer mathematical model. The model is capable of predicting the response of the pavement to the following three sets of variables: (a) the mechanical properties of the materials in the layers, (b) the loading characteristics, and (c) the geometric parameters. Under heading (a) the quality of each layer and that of the combination of layers is considered. Under heading (b) the pertinent parameters used are the magnitude of the load, its duration (static, repeated, moving), and the frequency of the repetition. Under heading (c) the thickness of each layer, the offset distance of the load, and the location of the point of interest are considered.

STATEMENT OF PROBLEM AND METHOD OF SOLUTION

The geometrical model selected is a multilayered, semi-infinite half space consisting of three distinct layers as shown in Figure 1. It is assumed that the material properties of each layer can be characterized as linear elastic or linear viscoelastic. The load is considered to be uniform, normal to the surface, and acting over a circular area. The following loading conditions are considered:

1. A stationary load is applied and maintained at the surface (Fig. 1).
2. A repeated load is applied with a specified frequency to the surface of the pavement (Fig. 2).
3. A load travels at a constant velocity V along a straight path on the surface of the system (Fig. 3).

The variables of interest are the components of the stress tensor and the displacement vector at any point in the system.

The results for the normal deflection on the surface are presented in detail in this paper. [The other stress and deflection components can be determined in a similar manner (4).]

Sponsored by Committee on Mechanics of Earth Masses and Layered Systems.

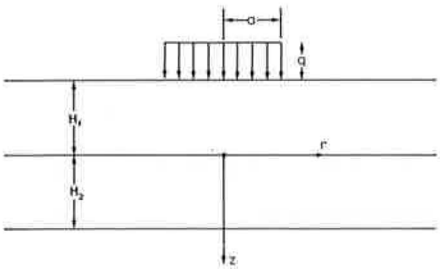

Figure 1. Cross section of three-layer system.

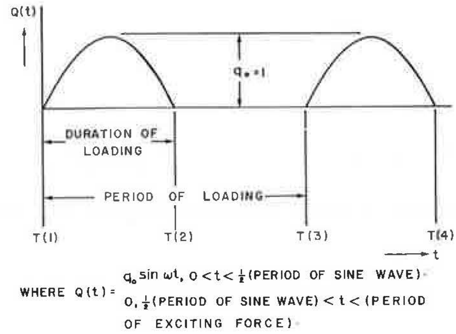

Figure 2. Time-varying repeated load configuration.

The correspondence principle is used to obtain the viscoelastic solution for the stationary load case. In addition, the principle of the response of initially relaxed linear systems to imposed excitations is used to obtain the solutions for the moving and repeated load boundary conditions. This method of analysis has been selected largely because the viscoelastic behavior of the system materials is assumed to be represented by stress-strain relations of the linear and nonaging type, using hereditary integrals.

The steps involved in this analysis consist of the following:

1. Obtaining the elastic solution for the surface deflection of the system due to a stationary applied load (2).
2. Applying the "correspondence principle" to the above solution, in the form of hereditary integrals for the stress-strain relations, to obtain the viscoelastic solutions (2).
3. Obtaining the viscoelastic solution for the surface deflection due to the repeated and moving loads, through the use of Duhamel's superposition integral for linear systems.

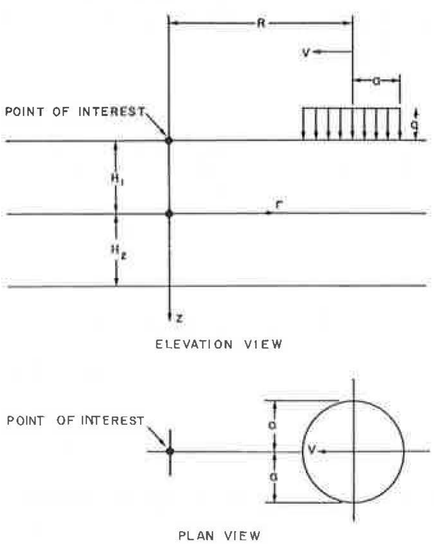

Figure 3. Moving load on viscoelastic half space.

The Stationary Load

The solution for the surface deflection of a three-layer linear elastic half space was first derived by Burmister (1) for an incremental boundary load $-mJ_0(mR)$. The present analysis uses, for mathematical convenience, an incremental boundary load $-J_0(mR)\,J_1(ma)$. The resulting expressions are then integrated from 0 to ∞ with respect to m, and multiplied by qa to yield the correct response of the system to the following boundary stress:

$$\sigma_z \Big|_{z = -H_1} = -qa \int_0^{\infty} J_0(mR)\,J_1(ma)\,dm \qquad (1)$$

where

q = intensity of the applied load, and
a = the radius of the loaded area.

$J_0(\,)$ and $J_1(\,)$ are Bessel functions of the first kind zero and first order respectively.

Using this approach, the stationary load solution for the surface deflection of a pavement system with elastic layers is given by the following equation:

$$W_s(r, -H_1) = qa \int_0^{\infty} \frac{H\,m}{m} \psi(m, -H_1)dm \tag{2}$$

where

$H\,m = J_0(mR)\, J_1(ma)$ product of Bessel functions,
$\psi(m, -H_1)$ = a rational function of elastic constants and their products,
m = a dummy variable of integration,
R = the offset distance at which a solution is desired, and
$-H_1$ = the value of the Z coordinate at the surface of the layered system.

To obtain the viscoelastic solution, the stress-strain relations of the viscoelastic layers are assumed to be of the following form:

$$\epsilon_i(R, Z, t) = D_{CRP_i}(0)\, \sigma_i(R, Z, t) - \int_0^t \sigma_i(R, Z, \tau) \frac{\partial}{\partial \tau} D_{CRP_i}(t - \tau)d\tau \tag{3}$$

where

$D_{CRP_i}(t)$ = creep function of each layer (i = 1, 2, 3),
$\epsilon_i(R, Z, t)$ = strain at location (R, Z) at time t, and
$\sigma(R, Z, t)$ = stress at location (R, Z) at time t.

The equivalent creep compliance for each layer in the viscoelastic domain will then be given in an integral operator form as

$$D_{CRP_i}(t) = D_{CRP_i}(0)\,(\,) - \int_0^t (\,) \frac{\partial}{\partial \tau} D_{CRP_i}(t - \tau)d\tau \tag{4}$$

When the correspondence principle (3) is applied to the elastic solution through the use of the creep integral operator in Eq. 4 for each layer, the viscoelastic solution for the surface deflection under a stationary load is obtained as

$$W_s(R, -H_1, t) = qa \int_0^{\infty} \frac{H\,m}{m} \psi(t, m, -H_1)\, dm \tag{5}$$

The surface deflection is time-dependent and the variable responsible for this time-dependence is $\psi(t, m, -H_1)$, which is the viscoelastic counterpart of $\psi(m, -H_1)$ in Eq. 2. If the mechanical properties of the system are changed, $\psi(t, m, -H_1)$ also changes.

The surface deflection $W_s(R, -H_1, t)$ due to a step load of intensity q applied to the surface of the system is a step response function of the system. It can be used when convolved with a time-varying load other than the step type applied to the boundary of the system to obtain the surface deflection for the new time-varying loading condition.

The Repeated Load Solution

Given $W_S(R, -H_1, t)$ as the response of the linear viscoelastic system to a step loading, the response due to a repeated load Q(t) can be determined from the following superposition integral:

$$W_R(R, -H_1, t) = \int_0^t \frac{\partial Q(t)}{\partial \tau} W_S(R, -H_1, t - \tau)d\tau \tag{6}$$

[The lower limit of the integral accounts for the discontinuities that may initially exist in $Q(t)$ or $W(R, -H_1, t)$.]

Figure 2 shows the repeated loading pattern used in this study where

$$Q(\tau) = \begin{cases} \sin W\tau \text{ for } [(N-1)\text{ period}] \leq \tau \leq [(N-1)\text{ period} + \text{duration}] \\ 0 \text{ for } [(N-1)\text{ period} + \text{duration}] \leq \tau \leq [(N)\text{ period}] \end{cases}$$

for any number of repetitions N of the applied load (N = 1, 2, 3...). A repetition of load is completed at the end of every period of application.

In this study, $W_S(R, -H_1, t)$ has been represented by a finite series of exponential terms of the form $\sum_{i=1}^{n} A_i \exp -(t - \tau)\delta_i$. The method of determining the coefficients of A_i and the exponents δ_i of such a series has been described in detail elsewhere (4, Appendix II).

The Moving Load Solution

When the load moves with a constant velocity V along a straight path on the surface (Fig. 3), the argument R in function $J_0(mR)$ of Eqs. 2 and 5 is $R - Vt$, where R is the offset distance of the load at zero time. The incremental load exciting the system will have the form $-J_0[m(R - V\tau)]J_1(ma)H(t - \tau)$ for loading and $+J_0[m(R - V\tau)]J_1(ma)H(t - \tau_1)$ for unloading ($>\tau_1>\tau$), where $H(t)$ is the Heaviside step function.

This form of loading thus corresponds to the summation of all such incremental terms over time. By integrating them over m and multiplying by qa the total boundary load is obtained in the following form:

$$\sigma_Z\Big|_{Z=-H_1} = -qa \int_0^{\infty} J_1(ma)\left\{\sum_{i=1}^{N-1} J_0[m(R - V\tau_i)]\,[H(t-\tau_i) - H(t-\tau_{i+1})] + J_0[m(R - V\tau_N)]\,H(t-\tau_N)\right\} dm \quad (7)$$

Then for appropriate values of $\Delta\tau = \tau_{i+1} - \tau_i$, the discrete load application given above corresponds to the continuous application of a moving load. The response for this case in terms of the surface deflection at location $(R, -H_1)$ due to a moving load is given by

$$W_M(R, -H_1, t) = qa \int_0^{\infty} \left\{\frac{J_1(ma)}{m} \sum_{i=1}^{N-1} J_0[m(R - V\tau_i)]\,[\psi(m, H_1, t-\tau_i) - \psi(m, H_1, t-\tau_{i+1})] + J_0[m(R - V\tau_N)]\,\psi(m, H_1, t-\tau_N)\right\} dm \quad (8)$$

from which the solution at any time t_N can be obtained.

In this arrangement again the $\psi(m, H_1, t - \tau_1)$ terms have been represented by finite exponential series of the form $\sum_{j=1}^{m} G_j \exp -t\alpha_j$ where G_j and α_j are determined using techniques previously described (4, Appendix II).

RESULTS

The following presentation is divided into two sections. A discussion is first presented on the dimensionless system parameters that describe the pavement system. Then the capabilities of the model are discussed with the aid of several typical pavement structures.

Dimensionless Pavement Parameters

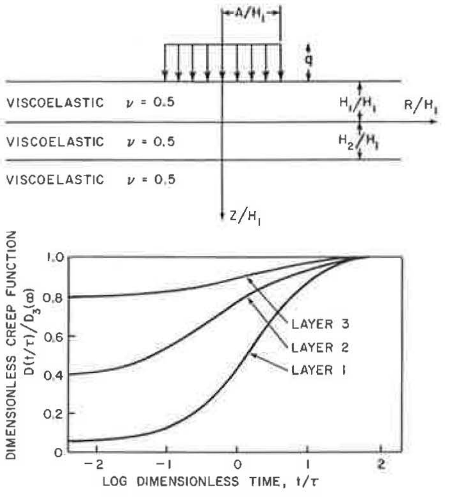

Figure 4. Dimensionless pavement system 2.

The pavement in this analysis is defined as a three-layer structure with a given set of dimensionless variables (Fig. 4). Each layer is incompressible, and the material in each layer is represented by a dimensionless creep compliance function. For ease of computation and use, the normalizing factor for the creep compliance of each layer is the value of the creep function of the third layer at infinity, $D_3(\infty)$.

All the components of the stress tensor at any point in the structure are expressed in terms of the intensity of the load. For the components of the displacement vector, the normalizing factor is the product of the intensity of the load q, the creep function of the third layer at infinity $D_3(\infty)$, and the height of the first layer H_1.

The following geometric variables are considered: the offset distance of the load R, the heights of layers one and two, H_1 and H_2 respectively, the depth of interest Z, the radius of the loaded area A—all of which are presented in dimensionless forms in terms of the height of the first layer. In addition, all times are dimensionless and in terms of an arbitrary time factor, τ.

The Model Response

The response of the model is discussed in three sections. First, the influence of the mechanical properties of the layers on the response of various pavement systems is discussed. Next, the influences of the loading variables on the response of two pavement systems are considered. Third, the influences of the geometric variables are discussed.

The Mechanical Properties of the Layer Materials—Four different pavement systems were selected to investigate the effect of the mechanical properties of the materials used in each layer on the response behavior (Fig. 5).

The material in each layer is assumed to be incompressible, linear, homogeneous, and isotropic. Due to the lack of availability of meaningful real data, the four systems used for the discussion are selected to demonstrate qualitatively the influence that the material properties may have on the pavement response. For this reason, no absolute values are given for the magnitude of the creep function of any layer in any of the systems used.

System 1 is composed of completely elastic materials. System 2 is completely viscoelastic. Layer one in system 3 is viscoelastic, and the second and third layers are considered elastic. System 4 is assumed to be partially viscoelastic by considering that only the third layer is viscoelastic.

System 2 (Fig. 4) is used as the basis for comparison. In this system each layer is viscoelastic and the creep compliance function of each layer has the same value at infinity. This provision serves as a check on the results of the static loading condition. At very large times, the creep functions of the three layers have the same value; the system acts as a homogeneous elastic half space and Boussinesq's elastic solution should thus be obtained.

In dimensionless form, systems 2 and 4 have the same initial values for the dimensionless creep functions $[D(t)/D_3(\infty)]$ in their respective layers; systems 1 and 3 also have the same initial values for the dimensionless creep functions in their respective layers.

Vertical Deflection—Figure 6 represents the vertical deflection under the center of the loaded area on the first interface for systems 1 through 4. It was mentioned before

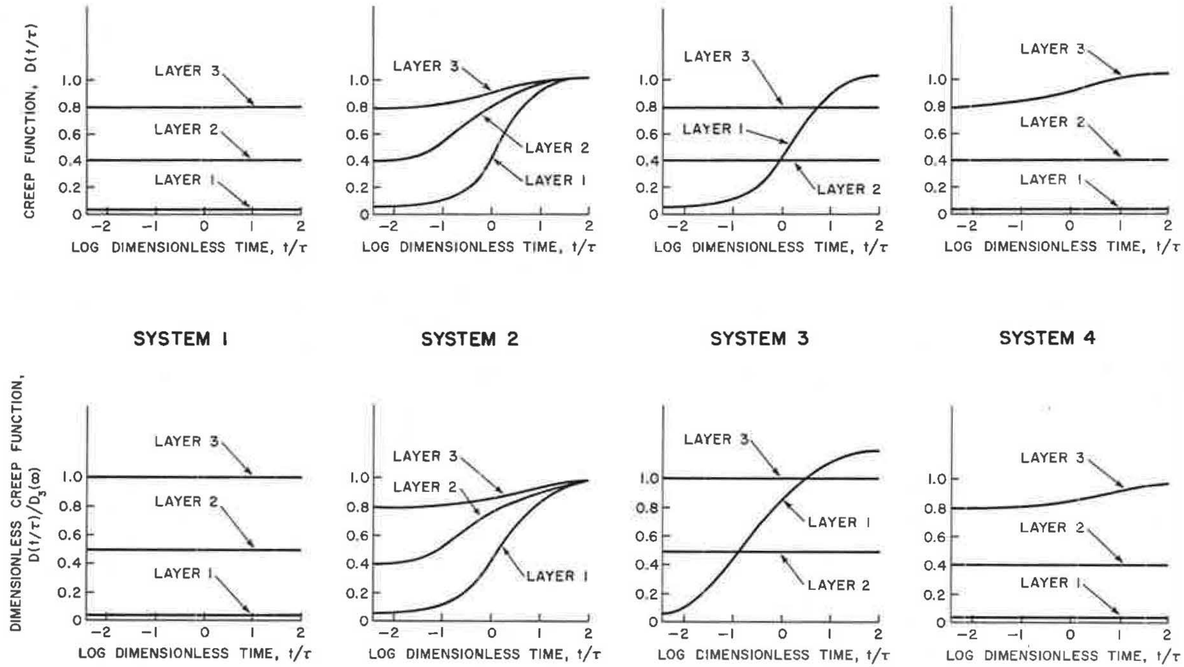

Figure 5. Creep function for systems 1 through 4.

that the zero time value of the creep functions for systems 1 and 3 are the same; therefore at zero time (not shown because of the logarithm scale) the two systems undergo the same magnitude of deflection. However, as the load is maintained, system 3 continues to deflect more because of the viscoelastic characteristics of its top layer. This shows the effect of the stiffness of the first layer on the deflection. The stiffer the surface material, the lesser is the deflection of the pavement.

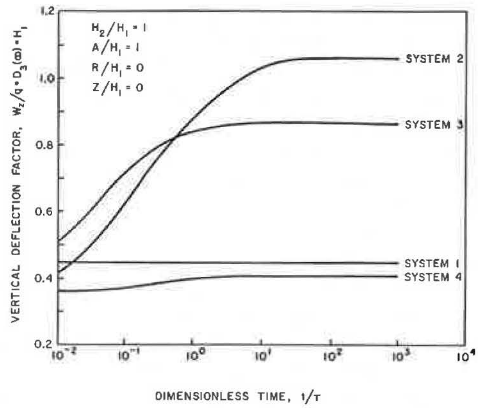

Figure 6. Influence of material variables on vertical deflection factor—4 systems.

Systems 2 and 4 have the same initial values of deflection. System 2, however, accumulates more deflection than system 4. Because the third layers of both systems possess the same mechanical properties, the discrepancy in results must be due to the uppermost layers; this supports Burmister's conclusion that when better quality materials are used in the base and surface course of the pavement, the pavement provides a blanket effect around the subgrade.

All the systems shown in Figure 5 are elastic at long times with system 4 displaying the least deflection factor and system 2 the greatest. In terms of stiffness, at long times system 2 is the weakest of all the systems and, therefore, deflects the most. Because systems 2 and 3 are later used to investigate the influence of the loading conditions and geometric variables on the mechanical response, it will be worthwhile to examine them more closely at the present time.

For very small values of time, system 2 is stiffer than system 3. Therefore, at short loading times the deflection factor for system 2 is less than that for system 3 when loads of the same magnitude are used. With the passage of time, however, system 2 becomes less stiff when compared to system 3. From the shape of the deflection curves, this occurs at dimensionless time of approximately one. When this event occurs, the deflection factor on the first interface of system 2 exceeds that of system 3 and continues to do so until it reaches a plateau.

For the stress (Fig. 7), however, system 3 consistently displays higher values of developed stresses than system 2 except at very short times where the stresses of system 2 are greater than those for system 3 (not shown in the figure). The reason for this behavior is that the two lower layers of system 3 are elastic and have stiffness values that do not change with time. The stresses developed on the first interface of system 3 increase considerably with time because of the rigidity of the bottom two layers. The lower layers of system 2 offer no such resistance; therefore, the stresses developed are not as great as those for system 3.

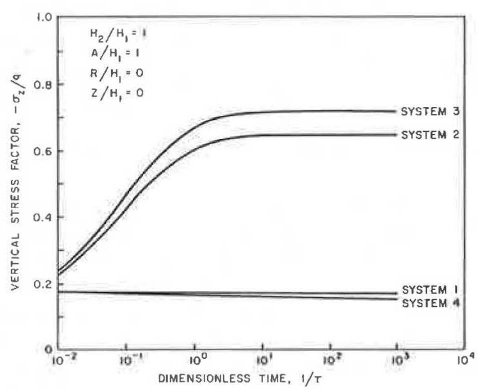

Figure 7. Influence of material variables on vertical stress factor—4 systems.

Vertical Stresses—Figure 7 shows the curves obtained for the vertical stress factor on the first interface for systems 1, 2, 3, and 4 (Fig. 5). The stresses similar to vertical deflection start with the same initial values for systems 2 and 4 and for systems 1 and 3.

However, while the stress on the first interface of system 2 increases with time, that on system 4 decreases. This may be because the stiffness of the upper two layers of system 4 remains fixed while that of the lower layer decreases with time. The overall effect is to cause a decrease in the stresses on the first interface. For elastic system 1, the stresses remain constant with time as expected.

In Figure 7, the stresses developed on the first interface of system 3 exceed those of system 2. This increase is due to the differences between the two systems, which resulted in a difference in their vertical deflections shown in Figure 6.

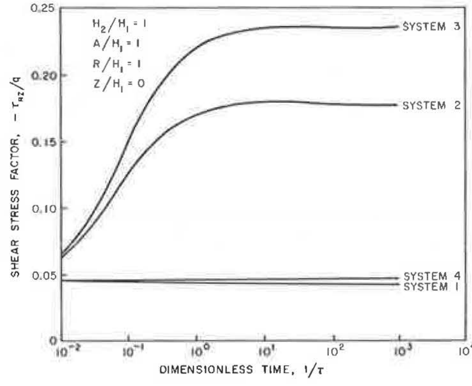

Figure 8. Influence of material variables on shear stress factor—4 systems.

Shear Stress—Figure 8 shows the effect of the mechanical properties of the layer materials on the shear stress developed on the first interface directly under the edge of the loaded area. The effect is almost similar to that for vertical stress except for system 4, where the shear stress slightly increases with time while the normal stress showed a slight decrease. This may be because the deflection on the interface under the axis of the load is increasing, causing an increase in the curvature of the interface. This would cause an increase in the shear stress because such stresses, as Burmister shows, are deflection dependent (6).

The results indicate that the nature of the system response depends on the mechanical characteristics of the materials in the layers. The interaction between the material properties of each layer produces what may be called a system function that is not only a function of the location of the point of interest but also of the kind of the response function being investigated (i.e., a stress or a deformation output).

Loading Conditions—The vertical stress, the vertical deflection, and occasionally the shear stress are presented to investigate the effect of the loading conditions and geometric variables.

The Stationary Loading Condition—For the stationary loading condition, the magnitude of the components of the stress and displacement factors generally increases with the increase in dimensionless time factor for all systems, as shown in Figures 6, 7, and 8 for vertical deflection, vertical stress, and shear stress respectively.

It is interesting to note that for both systems 2 and 3 the magnitude of all the three responses tends toward an asymptotic value at a value of dimensionless time corresponding to that when the dimensionless creep compliance functions become asymptotic. The system response therefore depends on the response characteristics of the layer materials. The extensive variation in stress and deformation with time under the constant load for both systems is a marked contrast to the constant distribution (of these quantities) exhibited by a structure with elastic properties. This phenomenon may have an important influence in the design of such structures. The rational design of the structural components (especially those exhibiting viscoelastic response) should, therefore, use the complete history of the stress and displacement distribution, rather than a single value of these components.

At values of dimensionless time greater than 1,000, the magnitudes of the stress and displacement components are equal to those obtained when the system behavior is elastic; i.e., the mechanical properties of the materials in the layers are equivalent to those of the given creep functions at infinity.

This capability serves as a check on the validity of the viscoelastic representations because the results thus obtained are comparable to those acquired by other authors for elastic systems having the appropriate properties. The model can therefore account for the manner of variation of all the pertinent stress and deflection factors at any point in three-layer pavement structure for this boundary condition.

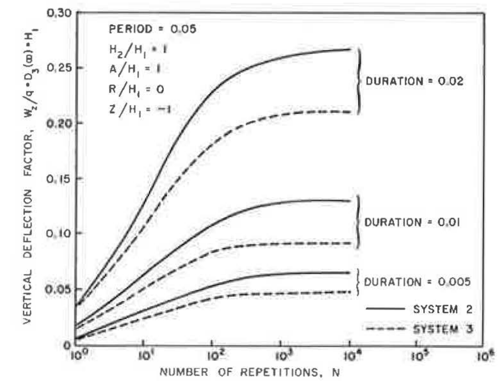

Figure 9. Influence of duration of vertical deflection factor.

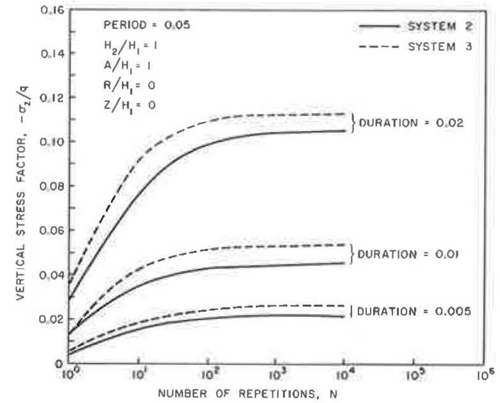

Figure 10. Influence of duration on vertical stress factor.

The Repeated Loading Condition—For this loading condition, the influence of the duration and period of the loading on the response of systems 2 and 3 was investigated using the vertical deflection on the surface directly under the center of the loaded area and the vertical stress on the first interface.

A load repetition is considered to be completed at the end of the period of the loading (see Fig. 2), and the magnitude of the stress or deflection factor is measured at this time. The results that are presented for a dimensionless period of 0.05 and durations of the loading equal to 0.005, 0.01, and 0.02 can be noted in Figures 9 and 10; the stress and deflection factors of the systems increase with increasing number of load repetitions.

Figure 9 shows the results obtained for the vertical deflection on the surface. The magnitudes of the deflection factor for system 3 are consistently lower than those for system 2. This indicates that system 2 behaves more viscously, a result that is evident from the dimensionless creep compliance functions for this system (Fig. 4).

Figure 10 shows the curves obtained for the vertical stress factors on the first interface. The stresses developed in system 3 are again consistently higher than those for system 2. This is because, for the same load, the system that is stiffer will develop the greater stresses. An examination of the creep functions for systems 2 and 3 in Figure 5 shows system 3 to be stiffer than system 2.

For each system, the greater the duration of the loading the greater is the developed stress or deflection factor. This indicates that the severity of the structural response is directly related to the duration of the loading. The longer the load remains on the system, the greater is the damaging effect.

The Moving-Load Condition—Figures 11 through 13 show the manner in which systems 2 and 3 respond to the application of a moving load traveling with a constant velocity along a straight line on the surface (see Fig. 3). The curves in Figure 11 are for the vertical deflection on the surface. The vertical stress

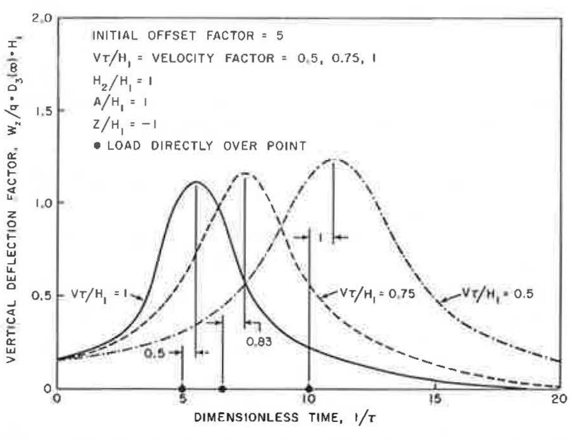

Figure 11. Moving load—vertical deflection factor, system 2.

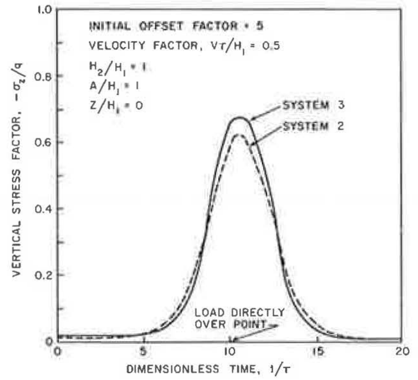

Figure 12. Moving load comparison—vertical stress factor.

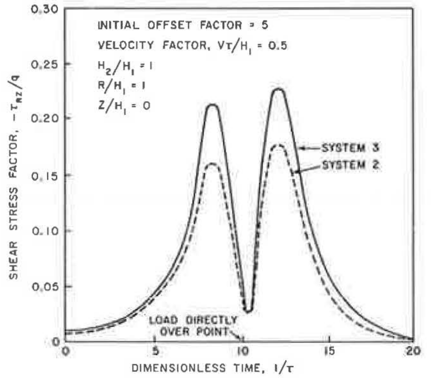

Figure 13. Moving load comparison—shear stress factor, systems 2 and 3.

and the shear stress on the first interface are shown in Figures 12 and 13 respectively.

The curves indicate that the system response in terms of stress and deflection factors is not symmetrical with respect to time. The viscoelastic behavior of the systems is such that there is a time lag between the observance of a response and the time of application of the agent causing it. Therefore, at the time when the load is directly over the point of interest, the magnitudes of the response in terms of stress and deflection factors are not at the maximum. In addition, it must be noted that the maximum value of either stress or deflection factor is obtained after the load has passed over the point.

The observed time lag in the viscoelastic responses is also velocity dependent. The greater the velocity of the moving load, the smaller is the time lag. This velocity dependence is discussed using the solutions obtained for the vertical deflection of systems 2 and 3. Figure 11 shows the curves obtained for the vertical deflection of system 2. It is seen that the peak deflection of the system increases with decreasing velocity factors, indicating that the longer the load remains within the region of influence the greater is the damaging effect that it has on a point of interest within the structure. It is also interesting to note that the lag in response time increases with decreasing velocity factors, for these same reasons. The same behavior is displayed by system 3.

The results obtained for the vertical stress and shear stress factors on the first interface of systems 2 and 3 also show a consistent behavior as indicated in Figures 12 and 13 respectively. In the case of the shear stress factor, however, there are two peaks in the curve. This indicates that as the load approaches the point the shear stress builds up to some limiting value and starts to decrease. At the time when the load is directly over the point, the shear stress is zero. However, the superposition of effects from previous loads actually prevents the total shear stress from going to zero. The effect of the decrease is to cause a marked reduction in the shear stress. As this load moves away from the point, the shear stress builds up again to a maximum peak and decreases.

The Geometric Variables—The height factor, depth factor, radius factor, and offset distance factor are discussed in the following paragraphs.

The Height Factor of the Second Layer—The influence of the height factor on the system response is investigated using the vertical deflection factor on the surface of systems 2 and 3 when they are subjected to repeated loading.

Figure 14 shows that for both systems the deflection increases with the number of repetitions for a fixed value of the height factor. The deflection factors for a given number of repetitions, however, decrease with increasing H_2/H_1, indicating that the thicker the system, the lower are the deflections. An alternative method of lowering surface deflection is to have stiffer materials in the layers, as was discussed earlier.

The dependence of the surface vertical deflection factor on the height factor of the second layer is more marked for system 3 than for system 2. For system 3, this marked dependence is displayed at every load repetition. At lower numbers of load repetition the dependence is pronounced. At higher levels it is absent. This is not surprising when the mechanical properties of system 2 are taken into account (Fig. 5). Because all the creep functions of system 2 tend toward the same value at long times, one would expect the system to become homogeneous eventually. When this occurs, the deflection at the surface is independent of H_2/H_1 because the system is a semi-infinite homogeneous mass.

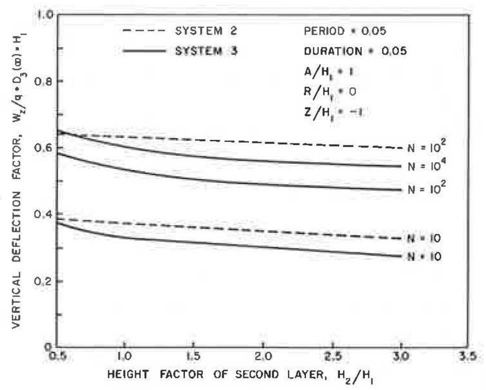

Figure 14. Influence of height factor on vertical deflection factor.

The Depth Factor—The influence of the depth factor is investigated using the vertical deflection under the center of the loaded area. The curves so obtained for both systems 2 and 3 are shown in Figure 15. There is a marked reduction in vertical deflection through the layers as shown in this figure. The reduction is greater as the number of load repetitions increases. For points within the third layer, an increase in the number of repetitions does not cause significant increase in the deflection. However, this effect is more severe for points that are nearer to the point of load application.

The Radius Factor—The effect of the radius factor was investigated using the vertical deflection factor on the first interface directly underneath the load (Fig. 16). The curve generally indicates that, as the radius of the loaded area is increased, the deflection is accordingly increased. Again system 2 deflects more than system 3 for reasons previously discussed.

The damaging effect on the first interface increases considerably with the increase in the size of the loaded area confirming the rather obvious result that for a given period and duration of loading heavier loads do more damage than lighter ones.

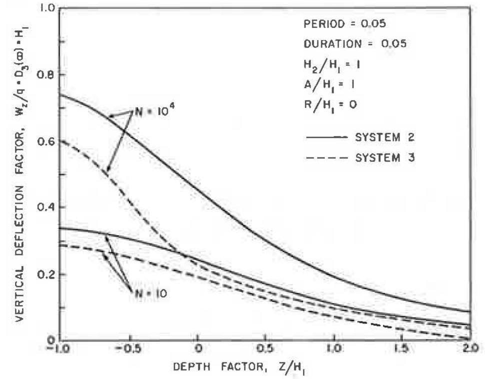

Figure 15. Influence of depth factor on vertical deflection factor.

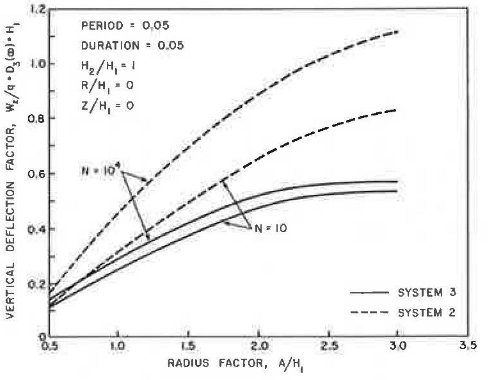

Figure 16. Influence of radius factor on vertical deflection factor.

The Offset Distance Factor—Figure 17 shows that the surface deflection factor decreases with increasing offset distance for both systems 2 and 3. This result is what is expected: the effect of the applied load is more severe near the point of interest.

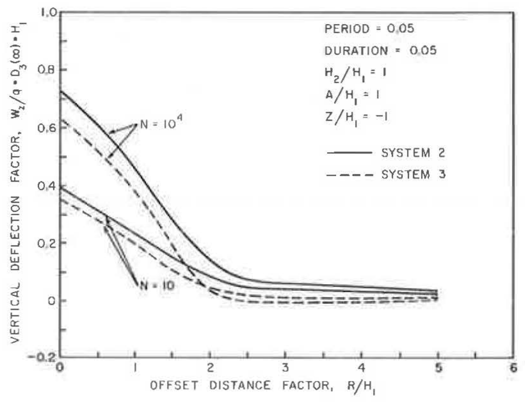

Figure 17. Influence of offset distance factor on vertical deflection factor.

SUMMARY AND CONCLUSIONS

The preceding discussion has served to emphasize the fact that the physical characteristics of the pavement system depend, among other things, on geometric measurements such as thickness, arrangement of the component layers, and the basic properties that characterize material behavior. The system response consequently involves the behavior of the physical structure when it is subjected to load and climatic inputs. When these act on the system, a condition that describes the mechanical state results. Measurable quantities such as the deformation and stress are then acquired. These quantities are calculated in this study based on the assumption that the material in each layer is linear viscoelastic.

The model presented accounts for the response behavior of a three-layer linear viscoelastic system. It is capable of predicting the response of pavement structures in terms of the developed stresses and displacements at any location.

In the development of the model, however, several assumptions were made. For instance, the effects of inertia and the fact that the real structure has finite geometrical boundaries are neglected. The materials in layers are assumed to be linear, homogeneous, and isotropic. The validity of such assumptions must therefore be further investigated.

In addition, the determination of realistic inputs other than material properties into a suitable model for the stress and deformation analysis should be investigated. A proper and adequate identification of the characteristics of the applied load and the determination of appropriate failure criteria for existing pavement structures would aid in this area.

The present viscoelastic analysis is believed to be a step in the right direction primarily because, unlike the linear elastic analysis, the rate and accumulation effects, the influence of parameters such as the duration of loading, the system geometry, the system properties, and the load configuration on the mechanical response of the system can all be accounted for.

ACKNOWLEDGMENT

The work reported here was sponsored by the U.S. Department of Transportation, Federal Highway Administration, under a project entitled "Moving Load on a Viscoelastic Layered System Phase II."

The opinions, findings, and conclusions expressed in the publication are those of the authors and not necessarily those of the Federal Highway Administration.

REFERENCES

1. Burmister, S.M. The General Theory of Stresses and Displacements in Layered Soil System, I, II, III. Jour. Appl. Phys., Vol. 16, No. 2, pp. 80-96; No. 3, pp. 126-127; No. 5, pp. 296-302, 1945.
2. Ashton, J. E., and Moavenzadeh, F. Linear Viscoelastic Boundary Value Problems. Jour. Eng. Mech. Div., Proc. ASCE, Feb. 1968.

3. Bland, D. R. Linear Viscoelasticity. Pergamon Press, London, 1960.
4. Elliott, J. F., and Moavenzadeh, F. Moving Load on Viscoelastic Layered Systems, Phase II. Dept. of Civil Eng., M.I.T., Cambridge, Research Rept. R69-64, Sept. 1969.
5. Tobolsky, A. V. Stress Relaxation Studies of the Viscoelastic Properties of Relaxation Studies of the Viscoelastic Properties of Polymers. Jour. Appl. Phys., Vol. 27, No. 7, July 1956.
6. Burmister, Donald M. Application of Layered System Concepts and Principles to Interpretations and Evaluations of Asphalt Pavement Performances and to Design and Construction. Internat. Conf. on Structural Design of Asphalt Pavements, Univ. of Michigan, 1962.