

# NEW METHOD FOR DETERMINATION OF TENSILE STRENGTH OF SOILS

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This paper describes a new and simple test technique for determining the tensile strength of soils. A cylindrical soil specimen is used by applying two steel punches at the center on both top and bottom surfaces of the specimen. Based on the perfect plasticity theory, a simple formula for computing the tensile strength of soils is developed. The fundamental relationship between tensile strength and environmental variables is examined. The comparisons of tensile strength determined from double punch tests and split tensile tests for various materials including concrete, mortar, and bituminous concrete are presented. It is concluded that the double punch test could be used easily for both laboratory and field to determine the tensile characteristic of soils.

•TENSILE STRENGTH of soil is one of the important strength parameters in the field of soil mechanics. However, engineers often consider that the tensile strength of soil is assumed to be zero because it is a relatively small value compared with compression strength and because of the lack of a satisfactory measuring technique.

The importance of cracking failure related to the tensile strength of materials in many highway pavements and earthfill dams has been given considerable attention in recent years. Leonards and Narain (14) developed a laboratory measuring technique to measure the tensile-bending stress of soil by use of clay-beam and to predict the cracking behavior of earth dams. George (10) has applied the theory of brittle fracture to evaluate the cracking growth and the effects on stabilized soil-cement.

For measuring the tensile strength of material, the split tensile test has been widely used for concrete (1, 5, 20) and has been extended to measure the tensile strength of bituminous concrete (4, 15), lime-stabilized soil (16), and soil-cement (12, 13). Tschebotarioff et al. (18) and Winterkorn (19) have used a modified Briquet Gang Model type to measure the tensile strength of various clay minerals. Recently, Chen (7, 8) proposed a double punch test that has been suggested as an alternative test method for determination of tensile strength of concrete.

The purpose of this paper is to develop both theoretically and experimentally the application of a double punch test to cohesive soils, which includes (a) development of an equation based on the perfect-plasticity theory that the tensile strength of soil can be computed; (b) development of the fundamental relations between tensile strength and environmental variables; and (c) comparisons of tensile strength results determined from double punch tests and split tensile test for various materials including concrete, mortar, and bituminous concrete.

## DESCRIPTION OF DOUBLE PUNCH TEST

Using two steel discs centered on both top and bottom surfaces of a cylindrical soil specimen, the vertical load is applied slowly on the discs until the specimen reaches failure. The tensile strength of the specimen can be calculated from the maximum load by the theory of perfect plasticity. Schematic diagrams and photographs of the double punch test are shown in Figures 1 and 2.

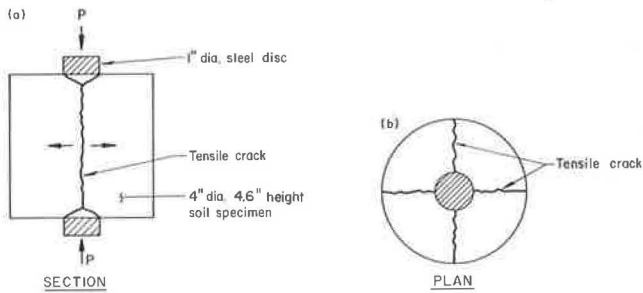
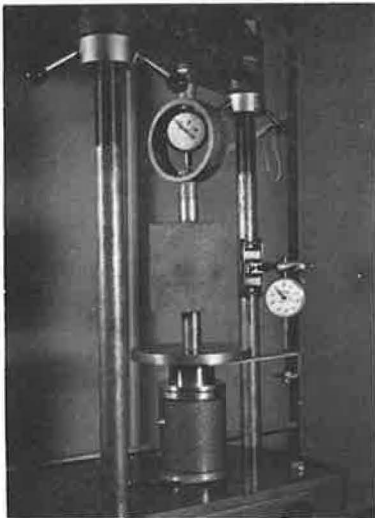


Figure 1. Schematic diagram of a double punch test.

The effect of the sample size and the dimensions of the disc have been studied by Hyland and Chen (11). Based on the test of concrete and mortar, they have found that the effect of height-to-diameter ratio and disc size on the tensile strength is approximately a linear relation. Fang (9) has found that a height-to-diameter ratio of the specimen varying from 0.8 to 1.2 and a ratio of diameter of the specimen to the diameter of the disc varying from 0.2 to 0.3 are suitable for this test. For convenience, the Proctor mold (4 by 4.6 in.) and CBR mold (6 by 6 in.) with 1 in. and 1.33 in. (CBR piston) disc respectively are recommended (9). For this study the Proctor mold was used for preparation of the soil specimen with a 1-in. diameter disc. The disc should be rigid so that no bending occurs during the loading test.



(a)



(b)

Figure 2. A double punch test: (a) test setup; and (b) modes of failure.

#### THEORETICAL ANALYSIS

The theoretical basis of the formula for computing the tensile strength of a split tensile test has been derived from the theory of linear elasticity (17). It has the simple form

$$\sigma_t = \frac{2P}{\pi Ld} \quad (1)$$

where

- $\sigma_t$  = simple tensile strength, psi;
- $P$  = applied load, lb;
- $L$  = length of specimen, in.; and
- $d$  = diameter of specimen, in.

It has been shown recently by limit analysis (7) that an identical formula of the problem can also be derived from the theory of perfect plasticity. A plasticity treatment of the double punch test for the concrete has been developed by Chen (8), and results for predicting the bearing capacity of concrete and rock are available (6, 7, 11). It would appear that the same

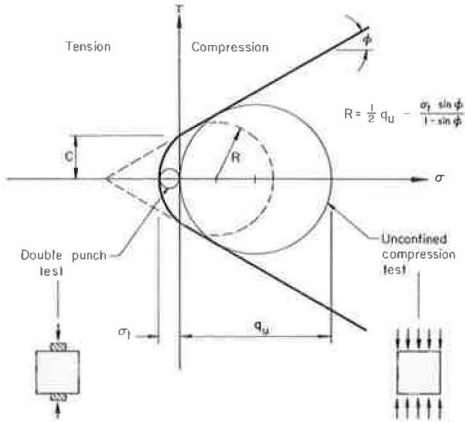


Figure 3. Modified Mohr-Coulomb criterion.

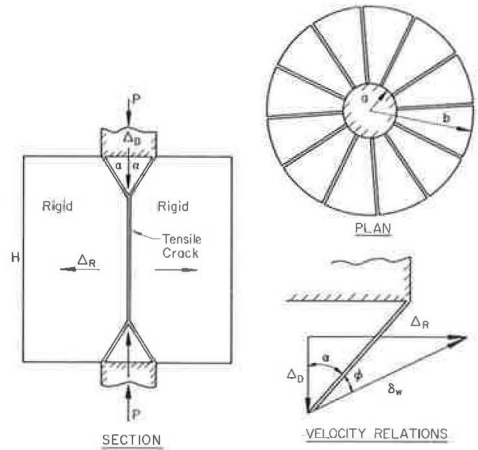


Figure 4. Failure mechanism of a double punch test.

theory should be applicable to the soil double punch test because the bearing capacity behavior for soils can be closely related to the bearing capacity behavior of concrete blocks or mortar.

The theory cited by Chen and Drucker (6) is based on two assumptions. The first assumption is that sufficient local deformability of soils in tension and in compression does exist to permit the application of the generalized theorems of limit analysis to soils idealized as a perfectly plastic material. The second assumption is that a modified Mohr-Coulomb failure surface in compression and a small but non-zero tension cutoff is postulated as a yield surface for soils (Fig. 3). In Figure 3,  $q_u$ ,  $\sigma_t$ ,  $c$ , and  $\phi$  denote the unconfined compression, simple tension strength, cohesion, and the internal friction angle of the soil respectively.

Figure 4 shows an ideal failure mechanism for a double punch test on a cylinder specimen. It consists of many simple tension cracks along the radial direction and two cone-shaped rupture surfaces directly beneath the punches. The cone shapes move toward each other as a rigid body and displace the surrounding material sideways. The relative velocity vector  $\delta_w$  at each point along the cone surface is inclined at an angle  $\phi$  to the surface (6). The compatible velocity relation is also shown in Figure 4. It is a simple matter to calculate the areas of the surfaces of discontinuity. The rate of dissipation of energy is found by multiplying the area of each discontinuity surface by  $\sigma_t$  times the separation velocity  $2\Delta_r$  across the surface for a simple "tensile" crack or  $q_u(1 - \sin \phi)/2$  times the relative velocity  $\delta_w$  across the cone-shaped rupture surface for simple "shearing" (6). Equating the external rate of work to the total rate of internal dissipation yields the value of the upper bound on the applied load  $P$ ,

$$\frac{P}{\pi a^2} = \frac{1 - \sin \phi}{\sin \alpha \cos (\alpha + \phi)} \frac{q_u}{2} + \tan (\alpha + \phi) \left( \frac{bH}{a^2} - \cot \alpha \right) \sigma_t \quad (2)$$

in which  $\alpha$  is the as yet unknown angle of the cone,  $a$  is the radius of the punch, and  $b$  and  $H$  are the specimen dimensions (Fig. 4).

The upper bound has a minimum value when  $\alpha$  satisfies the condition  $\partial P^u / \partial \alpha = 0$ , which is

$$\cot \alpha = \tan \phi + \sec \phi \left[ 1 + \frac{\frac{bH}{a^2} \cos \phi}{\frac{q_u}{\sigma_t} \left( \frac{1 - \sin \phi}{2} \right) - \sin \phi} \right]^{1/2} \quad (3)$$

valid for

$$\alpha \geq \tan^{-1} \left( \frac{2a}{H} \right)$$

and Eq. 2 can be reduced to

$$\frac{P}{\pi a^2} = \sigma_t \left[ \frac{bH}{a^2} \tan(2\alpha + \phi) - 1 \right] \quad (4)$$

Using typical values of  $q_u = 10 \sigma_t$  and  $\phi = 20$  deg, and assuming  $2a = 1$  in.,  $2b = 4$  in. and  $H = 4.6$  in., the upper bound has a minimum value at the point where  $\alpha = 14.2$  deg, and Eq. 4 gives

$$P \leq P^u = \pi (1.12 bH - a^2) \sigma_t \quad (5)$$

It is found that the value of the coefficient 1.12, which appeared in Eq. 5, is not too sensitive to the internal friction angle  $\phi$ . For example,  $\phi$  varies from 0 to 30 deg and the value of the coefficient varies from 0.84 to 1.32 respectively. The average value of the coefficient is 1.08.

As concluded by Chen and Drucker (6), the upper bound solution so obtained is in fact close to the correct values. It seems, therefore, reasonable to take

$$\sigma_t = \frac{P}{\pi (1.0 bH - a^2)} \quad (6)$$

as a working formula for computing the tensile strength in a double punch test for all soils.

## LABORATORY EXPERIMENTS

### Specimen

Medium plasticity soil (liquid limit = 31, plasticity index = 10) was selected for the study. Soil samples passed No. 10 sieves and were air-dried. A 4- by 4.6-in. Proctor mold was used for preparation of the soil specimen. Specimens were compacted in three layers with a 5.5-lb hammer and 12-in. drop; 15, 25, and 55 blows per layer were applied. For the double punch test the procedures were followed as suggested by Fang (9). One-inch diameter steel discs were used as shown in Figures 1(a) and 2(a). The rate of load application was 2 in. per min. Simultaneously, duplicated specimens were made for the split tensile test (2) and unconfined compression test (2, 3).

### Test Results

The load-deflection data and maximum load were recorded for all tests, which include double punch, split tensile and unconfined compression tests. The test results are shown in Figures 5 through 10. The double punch tensile strength was computed from Eq. 6 where  $b = 2$  in.,

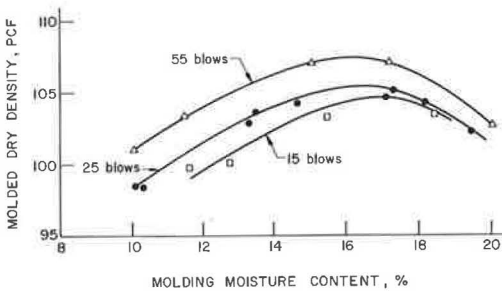


Figure 5. Molded dry density versus molding moisture content.

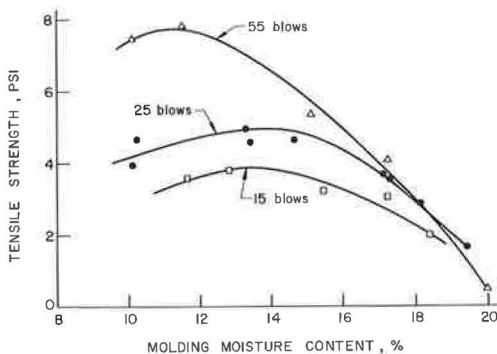


Figure 6. Tensile strength versus molding moisture content.

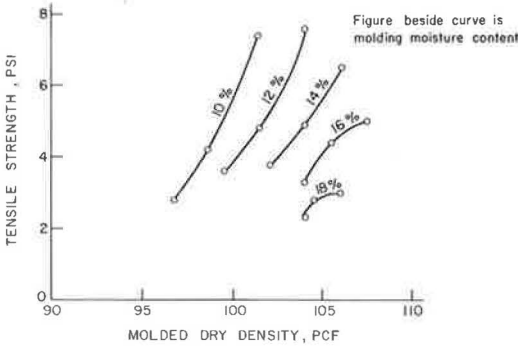


Figure 7. Tensile strength versus molded dry density.

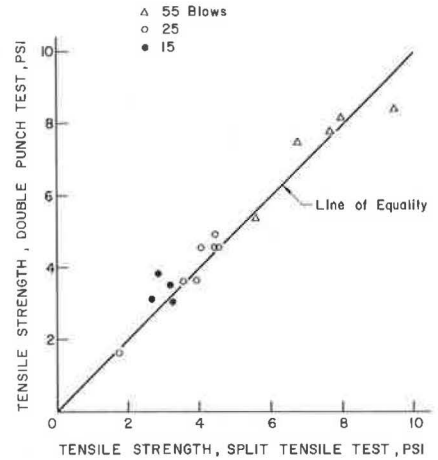


Figure 8. Comparisons of tensile strength of soil determined by double punch and split tensile tests.

H = 4.6 in., and a = 0.5 in. The split tensile strength was calculated from Eq. 1 where L = 4.6 in. and d = 4 in. For both equations, P is the maximum load for the specimen. The cracking pattern for the double punch test is shown in Figures 1(b) and 2(b). The cone-shaped formation with 2- or 3-piece cracks is generally observed for the soils.

Figure 5 shows the density-moisture content relationships with three compactive efforts. Figure 6 shows the tensile strength versus molding moisture content with various compactive efforts and indicates that maximum tensile strength exists on the dry

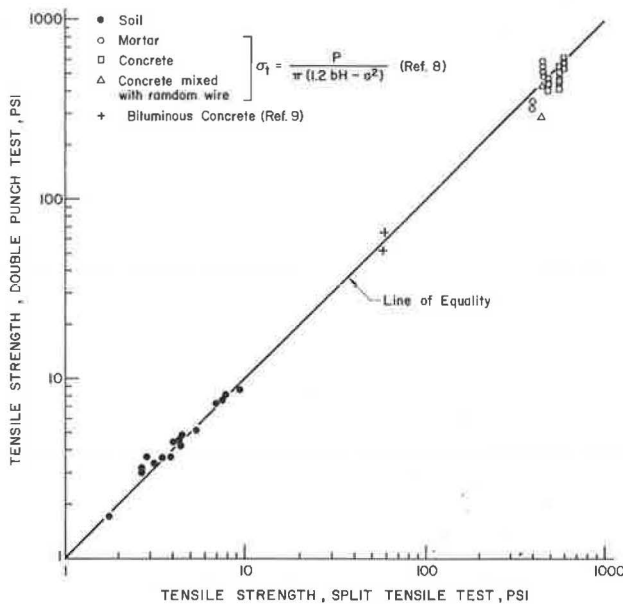


Figure 9. Comparison of tensile strength of various materials determined by double punch and split tensile tests.

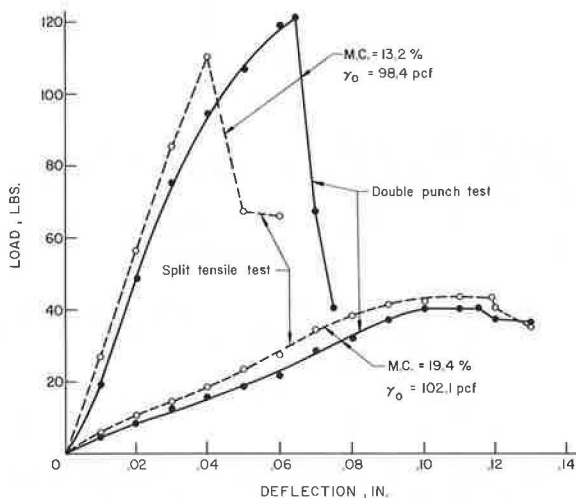


Figure 10. Load-deflection curves.

side of the optimum moisture content. Figure 7 was interpreted from Figures 5 and 6 and indicates that, at higher moisture content, as density increases the tensile strength increases slightly; however, at lower moisture content, as density increases, the tensile strength increases sharply.

Figures 8 and 9 show the comparisons of the tensile strength determined by double punch and split tensile tests. Figure 8 shows only one type of soil with various molding moisture contents and compactive efforts. However, Figure 9 shows the tensile strength of soil comparisons with other materials such as concrete, mortar, and bituminous concrete. Good agreement between two tensile strength test results is indicated. Figure 10 shows the typical load-deflection curves for both double punch and split tensile tests. For all the cases, the similar load-deflection patterns were found for both double punch and split tensile tests.

#### SUMMARY AND CONCLUSIONS

1. The double punch test is a simple test and easy to perform. No additional equipment is needed for the test, which could be tied in with routine CBR or compaction tests.
2. Based on the plasticity theory, a simple equation (Eq. 6) has been developed for computing the tensile strength of soils. This equation agrees both theoretically and experimentally with the equation used for the split tensile test.
3. Higher tensile strength existed on the dry side of the optimum moisture condition.
4. When the cracking failure is significant, it is necessary to examine the tensile strength of the material. The double punch test can be used for both laboratory and field construction control.

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## REFERENCES

1. Akazawa, Tsuneo. Tension Test Method for Concrete. Union of Testing and Research Laboratories for Materials and Structures, Bull. 16, Nov. 1953, pp. 11-23.
2. ASTM. 1970 Annual Book of ASTM Standards, Part 11.
3. Bowles, J. E. Engineering Properties of Soils and Their Measurement. McGraw-Hill, New York, 1970, 187 pp.
4. Breen, J. J., and Stephens, J. E. Split Cylinder Test Applied to Bituminous Mixtures at Low Temperatures. ASTM, Jour. of Materials, Vol. 1, No. 1, March 1966.
5. Carneiro, Fernando L. L. B., and Aguinaldo, Barcellos. Tensile Strength of Concrete. Union of Testing and Research Laboratories for Materials and Structures, Bull. 13, March 1952, pp. 97-127.
6. Chen, W. F., and Drucker, D. C. Bearing Capacity of Concrete Blocks or Rock. Jour. of Eng. Mech. Div., Proc. ASCE, Vol. 95, No. EM4, Aug. 1969, pp. 955-978.
7. Chen, W. F. Extensibility of Concrete and Theorems of Limit Analysis. Jour. of Eng. Mech. Div., Proc. ASCE, Vol. 96, No. EM3, June 1970, pp. 341-352.
8. Chen, W. F. Double Punch Test for Tensile Strength of Concrete. ACI, Vol. 67, Dec. 1970, pp. 993-995.
9. Fang, H. Y. Method of Test for Tensile Strength of Soil, Rock, and Stabilized Materials by Double Punch Test. Lehigh Univ., Fritz Eng. Lab. Rept. 348.4, July 1970.
10. George, K. P. Theory of Brittle Fracture Applied to Soil Cement. Jour. of Soil Mech. and Found. Div., Proc. ASCE, Vol. 96, No. SM3, May 1970, pp. 991-1010.
11. Hyland, M. W., and Chen, W. F. Bearing Capacity of Concrete Blocks. ACI, Vol. 67, March 1970, pp. 228-236.
12. Kennedy, T. W., and Hudson, W. R. Application of the Indirect Tensile Test to Stabilized Materials. Highway Research Record 235, 1968, pp. 36-48.
13. Kezdi, A. Erddrucktheorien. Springer-Verlag, Berlin, 1962.
14. Leonards, G. A., and Narain, J. Flexibility of Clay and Cracking of Earth Dams. Jour. of Soil Mech. and Found. Div., Proc. ASCE, Vol. 89, No. SM2, March 1963, pp. 47-98.
15. Livneh, M., and Shklarsky, E. The Splitting Test for Determination of Bituminous Concrete Strength. Proc. AAPT, Vol. 31, 1962, pp. 457-476.
16. Thompson, M. R. The Split-Tensile Strength of Lime-Stabilized Soils. Highway Research Record 92, 1965, pp. 11-23.
17. Timoshonko, S. Theory of Elasticity. McGraw-Hill, New York, 1934, pp. 104-108.
18. Tschebotarioff, G. P., Ward, E., and DePhilippe, A. A. The Tensile Strength of Disturbed and Recompacteds Soils. Proc. Third Internat. Conf. on Soil Mech. and Found. Eng., Vol. 3, 1953, pp. 207-210.
19. Winterkorn, Hans F. The Science of Soil Stabilization. HRB Bull. 108, 1955, pp. 1-24.
20. Wright, P. J. F. Comments on an Indirect Tensile Test on Concrete Cylinders. Magazine of Concrete Research, Vol. 7, No. 20, July 1955, pp. 87-96.