BUCKLING STRESS FORMULAS FOR OVERHEAD SIGN BRIDGE SUPPORTS

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Contemporary freeway operations require large signs over traffic lanes. These signs are usually attached to trusses mounted on vertical supports. The supports are essentially restrained cantilever beams. The 1968 AASHO Specifications (1) require, among other things, that the critical buckling stress be limited by a formula based on theoretical and experimental investigations of a simple beam subjected to loads which produce pure bending (8, 9, 10). The purpose of this study is to examine this formula to determine its applicability to highway sign supports, and to develop a suitable alternate requirement. Rigorous adherence to the present AASHO requirement results in uneconomical open sections for overhead sign bridge supports which are fixed at the base, restrained from lateral movement at the top, and are subjected to wind and dead loads which produce lateral, transverse, axial, and moment forces. The same conditions exist on roadside signs with two or more supports, but the critical buckling stress requirement does not greatly affect the selection of economical flanged shapes for such supports. A formula is developed for critical buckling stress, and it is shown that this formula is in agreement with available test results. Comparisons of other theoretical buckling formulas with test data are also presented.

•THIS paper examines the critical buckling stress requirements for the design of supports for overhead sign bridges. The discussion which follows is intended to show that current AASHO Specification (1) requirement Section 6(a)(3) for critical buckling stress is uneconomical, because its application leads to heavier sections than are needed. It will be shown that compliance with the critical buckling stress requirement is not necessary, because it is based on theoretical and experimental investigations of a beam subjected to pure bending, whereas an overhead sign bridge support is a cantilever beam column in which the axial forces are small compared to other design loads. Economical design is always desirable, but, in addition, safety of the traveling public is also a prime consideration of contemporary design; and to meet the latter consideration, it becomes imperative to use the lightest supports required to meet stress requirements, maintain structural stability, and meet the other criteria of sound design procedure. Symbols and nomenclature used in this paper are shown in the appendix.

The AASHO Specifications for the Design and Construction of Structural Supports for Highway Signs (1), cites Formula 20 of the Design Manual for High Strength Steels (2) as its criterion for buckling stresses. This formula is:

$$f_{cr} = \frac{\Pi^2 E}{2\left(\frac{L}{d}\right)^2} \sqrt{\left(\frac{I_y}{2I_x}\right)^2 + \frac{KI_y}{2(1+\mu)\,I_x^2}\left(\frac{L}{\Pi d}\right)^2} \tag{1}$$

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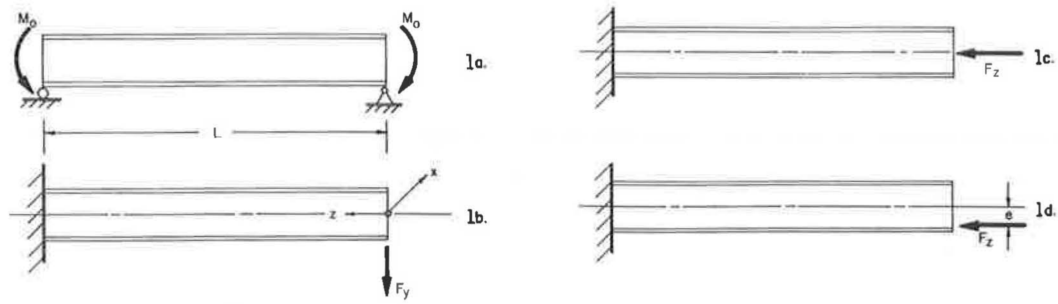


Figure 1. Support conditions and loadings for theoretical buckling cases.

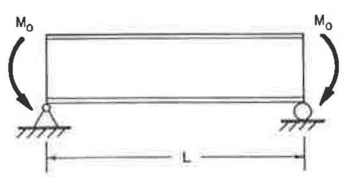
This formula was derived and investigated experimentally by Winter (8, 10); deVries (9) showed that when I_x is several times I_y, this relation reduces to:

$$f_{cr} = \frac{18.83 \times 10^6}{\frac{Ld}{bt}} \qquad (2)$$

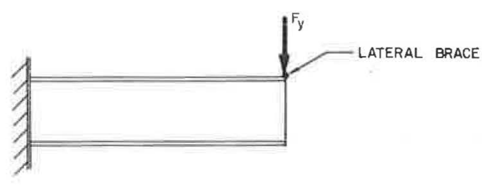
which is presented as Formula 21 in the U.S.S. Design Manual (2). Formulas 20 and 21 apply to lateral buckling of beams in pure bending (Fig. 1a). Timoshenko (5) gives a formula similar to Formula 20, and also presents an equation for lateral buckling of a cantilever beam under transverse end load (Fig. 1b), which will be discussed later, as Eq. 5.

The theoretical buckling stress for a perfectly straight slender member under axial compression (Fig. 1c) is given by the well known Euler equation

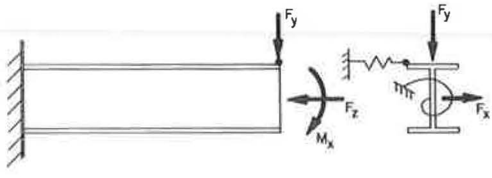
$$f_{cr} = \frac{\Pi^2 E}{\left(\frac{K_y L}{r_y}\right)^2} \qquad (3)$$



2a. THEORETICAL SUPPORT CONDITIONS—AASHO, AISC



2b. TEST MODEL SUPPORT CONDITIONS (4)



2c. OVERHEAD SIGN BRIDGE SUPPORT CONDITIONS

Figure 2. Support conditions and loadings.

Another expression for determining critical axial stress on columns with a small amount of eccentricity (Fig. 1d) is the Secant Formula

$$f_{cr} = \frac{P}{A}\left[1 + \frac{ec}{r^2}\sec\left(\frac{L}{2r}\sqrt{\frac{P}{AE}}\right)\right] \tag{4}$$

All of these expressions for critical buckling stress relate the geometrical properties of a member and its modulus of elasticity. The elastic modulus for steel is constant for all grades, but the yield strengths vary widely. Lighter members fabricated from higher strength steels are more susceptible to buckling than members fabricated from lower strength steels. Thus, the critical buckling stress governs more often as the strength of the steel increases. An overhead sign bridge support is subjected to a combination of loads (Fig. 2c). However, no formulas for calculating critical stress exist for general loading (6).

CANTILEVER BEAM COLUMNS WITH END RESTRAINT

Timoshenko (5) gives the following formula for critical buckling load of a cantilever beam under transverse end load (Fig. 1b).

$$P_{cr} = \frac{4.013\sqrt{EI_yGK}}{\left(1 - \sqrt{\dfrac{EI_yd^2}{4GKL^2}}\right)^2 L^2}\left(1 - \frac{a}{L}\sqrt{\frac{EI_y}{GK}}\right) \tag{5}$$

The term $\left(1 - \frac{a}{L}\sqrt{\frac{EI_y}{GK}}\right)$ is a correction factor to account for the load being applied at points other than the centroid. If the load is applied to the upper flange, a is $\frac{d}{2}$. The maximum bending stress would occur at the fixed end:

$$F_{cr} = \frac{Mc}{I} = \frac{P_{cr}\cdot L\cdot \frac{d}{2}}{I_x} = \frac{4.013}{I_x}\left(\frac{d}{2L}\right)\frac{\sqrt{EI_yGK}}{\left(1 - \dfrac{d}{2L}\sqrt{\dfrac{EI_y}{GK}}\right)} \tag{6}$$

If the effective lengths are taken into account, the formula becomes:

$$F_{cr} = 4.013\left(\frac{d}{2L}\right)\left(\frac{K_x^2}{I_x}\right)\frac{\sqrt{EG}\sqrt{\dfrac{I_y}{K_y^2}\cdot\dfrac{K}{K_z^2}}}{\left(1 - \dfrac{d}{2L}\sqrt{\dfrac{E}{G}\dfrac{I_y}{K_y^2}\cdot\dfrac{K_z^2}{K}}\right)} \tag{7}$$

where K_x, K_y, and K_z were each 2.0 in the original derivation for a cantilever beam.

This expression clearly shows that this type of buckling is a combination of weak axis bending and torsion. The effective length factor, K_z, refers to the effective length of the compression flange during buckling and is therefore the same as K_y, and thus, K_z will be replaced by K_y. Equation 7 can be written as

$$F_{cr} = 4.013 \left(\frac{d}{2L}\right)\left(\frac{K_x^2}{I_x K_y^2}\right) \frac{\sqrt{EGI_y K}}{\left(1 - \frac{d}{2L}\sqrt{\frac{E}{G}\frac{I_y}{K}}\right)} \quad (8)$$

The effective length, $K_y L$, can be obtained from Figure C1.8.3 of the AISC Manual of Steel Construction (3) using a G_A (flexibility coefficient) for the partially restrained end obtained in the following manner (Fig. 3).

The point "A" denotes the point of lateral support.

$$\delta_P = \frac{PL^3}{3E\left(\frac{I_y}{2}\right)} \quad (9)$$

$$\delta_T = \theta\left(\frac{d}{2} + e\right) = \frac{TL}{GK}\left(\frac{d}{2} + e\right) = \frac{PL\left(\frac{d}{2} + e\right)^2}{GK} \quad (10)$$

where δ_P is the lateral displacement of the compression flange due to a fictitious lateral load, P, and δ_T is the lateral displacement resulting in a resisting torque, T.

$$G_A = \frac{\delta_T}{\delta_P} = \frac{3}{2}\frac{EI_y}{GK}\left(\frac{d}{L}\right)^2\left(\frac{1}{2} + \frac{e}{d}\right)^2 \quad (11)$$

The deviation between test results and theoretical calculations increase as the members become lighter. It seems logical, therefore, to incorporate the term I_x/I_y into the factor of safety to be applied to the theoretical stress. Assume a factor of safety in the form,

$$F.S. = 2\left[1 + \left(\frac{1}{100}\frac{I_x}{I_y}\right)^2\right] \quad (12)$$

This factor of safety when applied to the theoretical stress is always greater than 2.0 and, when applied to the test results, the real margin of safety ranges from 1.67 for low ratios of I_x/I_y to 2.0 for very high values of I_x/I_y. The factor of safety is approximately 2.0 for most sections which are used for sign supports.

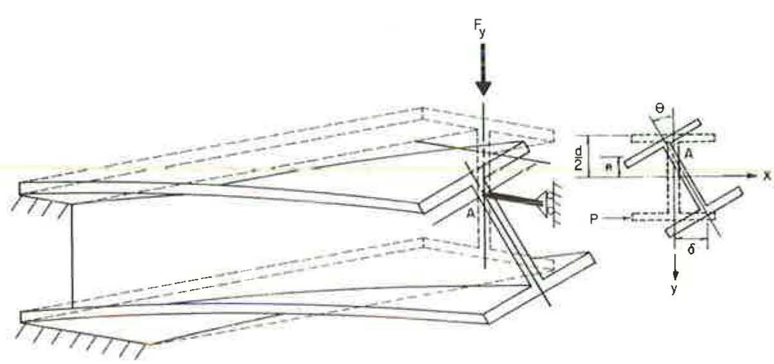


Figure 3. Model support conditions.

TAPERED BEAM COLUMNS

When the load on a cantilever beam is limited by yielding at the fixed end, it is apparent that some saving in material (and thus a saving in weight) can be made by tapering the web depth and flange width. A series of load tests were conducted by Krefeld (4) on cantilever beams with partial end restraint, subjected to end-point loads. A lateral end brace was attached to the tension flange which allowed rotation of the end, but did not permit the tension flange to move laterally (Fig. 2b), and the lateral brace restricted the motion of the compression flange also. These test conditions more closely approximate the loading and end conditions for an overhead sign bridge than those in any of the theoretical derivations; and, as shown in Figures 4 and 5, the experimental failure stresses are much larger than predicted by the formulas listed in Table 1.

Krefeld's test results led to the following empirical equations for untapered beams with end load and end brace:

$$f_{cr} = \frac{80,000,000}{\frac{Ld}{bt}} \text{ psi} \qquad \text{for } 5,000 > \frac{Ld}{bt} > 2,500 \qquad (13)$$

and

$$f_{cr} = \frac{110,000,000}{\frac{Ld}{bt}} - 7,000 \text{ psi} \qquad \text{for } 5,000 > \frac{Ld}{bt} > 1,000 \qquad (14)$$

where f_{cr} is the nominal stress at the support when elastic buckling occurs. Equation 13, although simpler, becomes increasingly conservative for high-yield-point materials below the limits of Ld/bt specified.

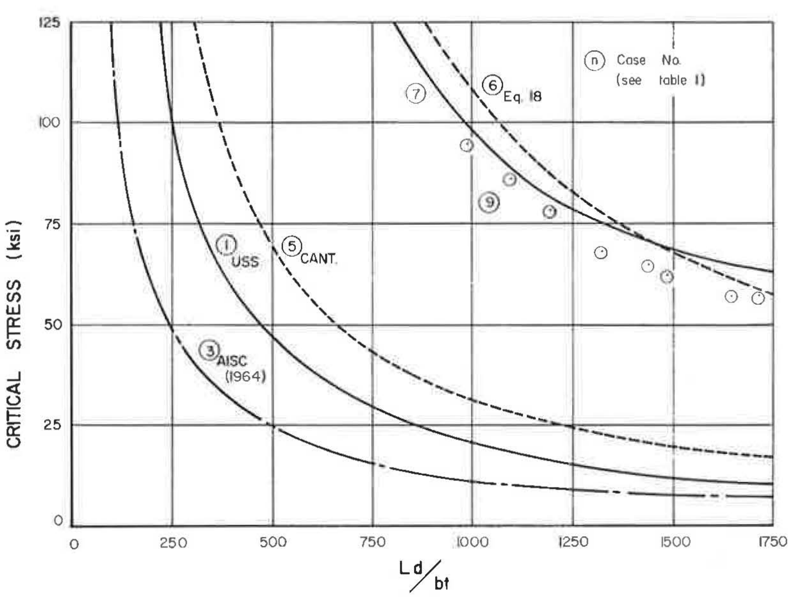


Figure 4. Comparison of critical stresses versus Ld/bt from various theoretical formulas and test results.

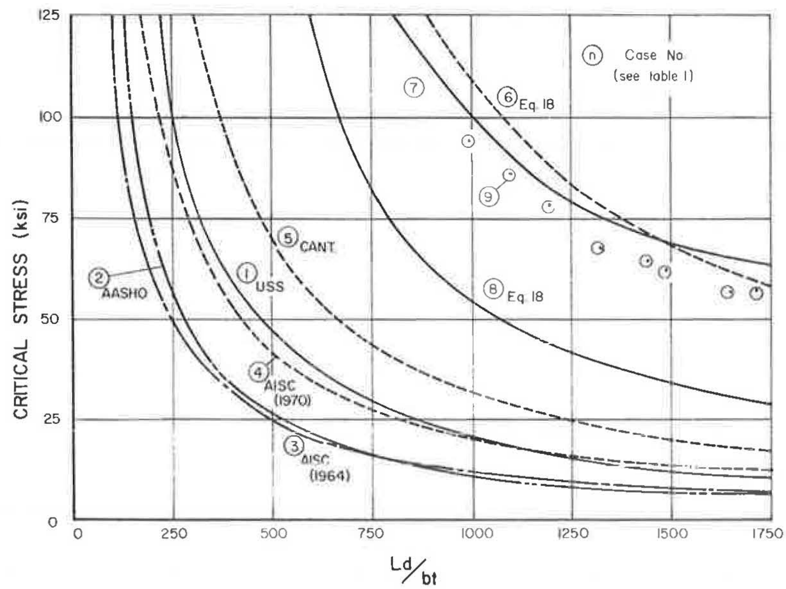


Figure 5. Comparison of critical stresses versus Ld/bt from various theoretical formulas with safety factors and test results.

Krefeld (4) noted

> . . . that the flange stress at points within the span increases with increasing taper and that for extreme tapers the flange stress may be greater at points near the end of the cantilever than at the support.

He introduced a stress-reduction factor for tapered beams, R, which is defined as

> . . . the ratio of the nominal stress at the support of a wedge beam to that of an untapered beam having the same section at the support when elastic buckling occurred.

TABLE 1

CRITICAL STRESS CASES

Case	Description	Source
1	Lateral buckling of simple beam in pure bending	USS Design Manual (2) Formula 20 (see also References 8, 9, and 10)
2	USS Formula 20 with a 1.8 safety factor	AASHO Specifications (1)
3	Allowable stress formula based on lateral buckling	AISC Steel Manual (3), Sixth Edition, 1964
4	Allowable stress formula based on lateral buckling modified to include effects of end moments	AISC Steel Manual (12), Seventh Edition, 1970
5	Lateral buckling of cantilever beams under transverse end load	Timoshenko (5), p. 258
6	Lateral buckling of restrained cantilever—includes effects of end conditions	Proposed Stress Formula, Equation 18
7	Lateral buckling of restrained cantilever—modified Equation 18	Equation 20
8	Proposed critical stress formula with 2.0 safety factor	Equation 18
9	Test data	Krefeld et al. (4)

The reduction factor is a function of the section moduli and flange dimensions at each end of the beam

$$R = F\left[\left(\frac{Z_0}{Z_1}\right), \left(\frac{b_0 d_1}{b_1 d_0}\right)\right] \tag{15}$$

The critical stress producing buckling of a tapered beam with a given span was found to vary with the parameter

$$\alpha = \frac{Z_0}{Z_1}\left(\frac{b_0 d_1}{b_1 d_0}\right)^{3/2} \tag{16}$$

and this equation agrees with test results. The reduction factor can be expressed in terms of α as follows; for end load, with end brace:

$$R = \frac{7 + \alpha}{5 + 3\alpha} \tag{17}$$

From this formula, it is possible to determine maximum tapers such that no reduction is necessary. Note that when R = 1.0, α = 1.0. Therefore, no reduction is necessary when α is 1.0. If there is an axial load in addition to the lateral load, then the taper must be less than that calculated by the above formula. A reasonable upper limit appears to be $Z_0/Z_1 = 4.0$.

PROPOSED CRITICAL STRESS FORMULA

The critical stress formula (Eq. 8) can now be rewritten to include the effects of taper on the column. The effects of eccentricities of loading, residual stresses, and imperfections are considered to be adequately compensated for in the factor of safety.

The proposed critical stress formula to be used for overhead sign bridge supports is

$$F_{cr} = \frac{1}{F.S.}\,\frac{R}{I_x}\left(\frac{2d}{L}\right)\left(\frac{K_x}{K_y}\right)^2 \frac{\sqrt{EI_y GK}}{\left(1 - \frac{d}{2L}\sqrt{\frac{EI_y}{GK}}\right)} \tag{18}$$

The terms in this equation are defined as:

F.S. = factor of safety (Eq. 12);
d/L = depth/length;
K_x = effective length factor based on end conditions for bending about the x-axis;
K_y = effective length factor based on end conditions for bending of the compression flange about the y-axis. This involves a special flexibility coefficent, G_A, to account for possible twisting of top of column (Eq. 11). After the G values are known, the K values are obtained from Figure C1.8.3 of the AISC Manual of Steel Construction (3);
E, G = modulus of elasticity, shearing modulus;
K = torsional rigidity of section, for I and wide flange sections (Eq. 19); and
R = a parameter to account for taper (Eq. 17).

If the web of the member contributes very little to I_x, I_y, and K, the following approximate expressions can be used (10).

$$I_x \doteq 2\, bt_f \left(\frac{d}{2}\right)^2, \quad I_y \doteq \frac{2t_f b^3}{12}, \quad K \doteq \frac{2}{3}\, bt_f^3 \tag{19}$$

Substitution of these terms into Eq. 18 yields:

$$F_{cr} \doteq \frac{4R}{3(F.S.)} \left(\frac{K_x}{K_y}\right)^2 \frac{\sqrt{EG}}{\left(\frac{Ld}{bt}\right)} \frac{1}{\left[1 - \frac{1}{4} \frac{\left(\frac{d}{t}\right)^2}{\left(\frac{Ld}{bt}\right)} \sqrt{\frac{E}{G}}\right]} \tag{20}$$

This equation yields allowable stress values which are usually a few percent higher than Eq. 18. This is comparable to Formula 5 of the AISC Code (3) derived by deVries (9). It incorporates a factor to correct for the position of the load since the load is usually applied to the flanges (Eq. 5). The last term in the denominator could be set equal to unity since Ld/bt is usually quite large. If typical values of the material constants for steel are substituted into the above equation ($E = 30 \times 10^6$ psi, $G = 11.5 \times 10^6$ psi),

$$F_{cr} \doteq \frac{24{,}800{,}000}{\left(\frac{Ld}{bt}\right)} \left(\frac{K_x}{K_y}\right)^2 \frac{R}{F.S.} \frac{1}{\left[1 - 0.4 \frac{\left(\frac{d}{t}\right)^2}{\left(\frac{Ld}{bt}\right)}\right]} \tag{21}$$

A conservative approximation for the critical stress for the case of a restrained cantilever, as shown in Figure 2b, can be obtained by using the following values:

$$K_x = 2.0, \quad K_y \doteq 1.0, \quad R = 1.0, \quad \left[1 - 0.4 \frac{\left(\frac{d}{t}\right)^2}{\left(\frac{Ld}{bt}\right)}\right] \doteq 1.0 \tag{22}$$

Equation 21 now reduces to

$$F_{cr} = \frac{99{,}000{,}000}{\frac{Ld}{bt}\,(F.S.)} \text{ psi} \tag{23}$$

This compares very closely with Krefeld's empirical critical stress formula for this case given in Eq. 14. This is similar in form to the critical stress formulas for simple beams subjected to end moments derived by deVries (9) and Winter (10) (i.e., the present AASHO formula).

This critical stress determined by Eq. 18 or 20 would be compared with the allowable bending stress, F_b. Equation 23 could be used for the specific case of the overhead sign bridge support. The lesser of the two values would be used for F_b in the AASHO interaction formula.

$$\frac{f_a}{F_a} + \frac{f_b}{F_b} + \left(\frac{f_v}{F_v}\right)^2 \leq 1.0 \tag{24}$$

The effects of lateral loading would be taken into account by using

$$\frac{f_a}{F_a} + \frac{f_{bx}}{F_{bx}} + \frac{f_{by}}{F_{by}} + \left(\frac{f_{vx}}{F_{vx}}\right)^2 + \left(\frac{f_{vy}}{F_{vy}}\right)^2 \leqq 1.0 \qquad (25)$$

COMPARISON OF CRITICAL STRESS FORMULAS

The graphs (Figs. 4 and 5) show various critical stress formulas, as described in Table 1. In each case, it is assumed that the stresses are below the yield point. The Krefeld test results (4) are for relatively light sections with high ratios of I_x/I_y. The tests were performed on tapered and untapered cantilever beams with lateral restraint of the tension flange at the point of application of the transverse load (Fig. 2b). This closely approximates the conditions existing on an overhead sign bridge support. More detailed graphs are given in a Texas Transportation Institute Technical Memorandum (11).

The proposed critical buckling stress formula, Eq. 18, closely predicts the buckling stresses determined by the tests because it has the ability to account for different end conditions through the effective-length factors K_x and K_y.

The 1970 AISC Specifications (12) modify the 1964 AISC allowable bending stress equations (3) to account for end moments and, in general, are less conservative (by a factor of 1 to 2.3). For the case of a restrained cantilever beam with no moment at the free end, the curve in Case 3 is shifted upward by a factor of 1.75 (Case 4).

Figure 4 has no safety factors applied to the U. S. S. Manual (2) formulas or the present formula. The AISC (3) formula has a built in factor of 1.67 or greater. Figure 5 applies safety factors to the curves in Figure 4. The AASHO Specifications (1) require a safety factor of 1.8 applied to the U. S. S. formula and closely matches the AISC (3) formula. A safety factor of 2.0 is applied to the proposed critical stress formula (Eq. 18).

With the factors of safety applied, the buckling stress predicted by the proposed formula is approximately 4.8 times the stress determined by the current AASHO Specifications. If a safety factor of 3.0 is applied to the proposed critical stress formula, the predicted stress would be 3.2 times the AASHO Specifications. If buckling stress is the limiting factor in the design, the use of the proposed formula could result in significant savings in material.

The proposed formula appears to be sufficiently conservative for overhead sign bridge supports and the AASHO requirement seems to be too conservative. This is of significant importance in the design of breakaway supports in which the mass of the support must be kept to a minimum in order to limit damage to vehicles and to prevent injury to passengers.

DISCUSSION

The AASHO Specifications specify two wind-load combinations: (a) full normal load (F_y) plus a 0.2 lateral component (F_x); and (b) a 0.6 normal load plus a 0.3 lateral component. The lateral component causes a lateral displacement of the top of the support and, of course, results in a lateral bending stress. Since this bending is taking place about the weak axis, this can have a significant contribution to the interaction formula.

There has been some discussion as to the effect of this lateral load on the critical stress formula. It definitely will have some effect. However, inclusion of this lateral force results in a nonlinear, large deflection problem, not an eigen-value buckling problem, and it thus cannot be incorporated into the critical stress formula. The critical stress formula was based on support conditions and loadings, as shown in Figure 2b. An actual sign bridge support is connected to the truss or to the sign at more than one point. Thus, it has more than one lateral support point. The support conditions at the upper end are as follows:

1. Unrestrained against normal translation (y-direction).
2. Unrestrained against rotation about the major axis (x-axis).
3. Elastically restrained against rotation about the minor axis (may be considered rigidly restrained in most cases due to the stiffness of the truss).

4. Elastically restrained against lateral translation.
5. Elastically restrained against torsional rotation (may be considered rigidly restrained if firmly fastened to truss).

The only exception to these conditions is a roadside sign with a single support, in which case there is no lateral or rotational support and both K_x and K_y are 2.0 (7).

The critical stress formula is based on these support conditions at the upper end:

1. Unrestrained against normal translation (y-direction).
2. Unrestrained against rotation about the major axis (x-axis).
3. Unrestrained against rotation about the minor axis.
4. Restrained against lateral translation of tension flange only.
5. Elastically restrained against torsional rotation due to the lateral brace.

By comparing the end conditions for overhead sign bridge supports and the model on which the critical stress formula is based, it is apparent that the model restraints are much less rigid than that provided by field conditions. Therefore, it appears that the formula, when applied to actual cases, will have a factor of safety larger than 2.0 in spite of the fact that the lateral load has some effect on decreasing the critical stress.

The lateral displacement will have some effect on the actual critical stress. However, the lateral displacement is restrained by the framework between the supports. It is the authors' considered opinion that the effects of lateral displacement are adequately accounted for in the interaction formula.

The effect which the lateral load has on the critical stress can be determined from a nonlinear computer program; such a procedure is discussed briefly in a Texas Transportation Institute Technical Memorandum (11). Use of this nonlinear program in conjunction with full-scale tests would form the basis for important theoretical work in this area.

CONCLUSIONS AND RECOMMENDATIONS

This report considers critical buckling stresses in open sections. The major conclusion is that the present AASHO critical buckling stress formula is overly conservative when applied to sign supports. A critical stress formula was developed for restrained cantilevers which closely approximates the boundary conditions existing on an overhead sign bridge. The buckling stress criterion may not govern for lower strength steels. However, it may be the limiting stress for higher strength steels which are being used increasingly to reduce the support mass.

The critical stress formula proposed in this report is recommended for consideration for adoption into future AASHO Specifications. Areas which might be explored in more detail include: (a) full-scale tests on large wide flange shapes to determine critical stresses under various loadings and end restraints [an extension of the Krefeld study (4)]; (b) development of a finite-element buckling program to include effects of lateral-torsional buckling; (c) correlation of proposed critical stress formula (Eq. 18) with (a) and (b) above; and (d) determination of the validity of the use of an interaction formula of the form of Eq. 25.

It is apparent that application of the AASHO Specifications (1) results in a grossly overdesigned, double-tapered column as tested by Krefeld (4), and as shown in Figures 4 and 5.

The buckling problem is dependent on the conditions of end restraint. Thus, requirements for critical stress should provide means for accounting for various end conditions. The proposed formula has this flexibility.

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Appendix

NOMENCLATURE

A, A_f, A_w	Areas of the cross section, flange, and web.
a	Distance from neutral axis to point of application of normal load.
b	Width of flange.
c	Distance from neutral axis to edge of beam (usually d/2).
d	Depth of beam.
e	Distance from neutral axis to lateral support point.
E	Modulus of elasticity.
F_a, F_b	Maximum allowable stresses in axial and bending respectively (AISC and AASHO)
F_y	Yield point stress.
F.S.	Factor of safety.
f	Calculated or actual stresses.
G	Shearing modulus.
G_A, G_B	End condition parameters used in determining effective lengths (Eq. 11).

I_x, I_y	Moments of inertia about the x and y axes.
K	Torsional rigidity (not equal to the polar moment of inertia for open sections, see Eq. 19).
K_x, K_y, K_z	Effective length factor for bending about the x, y, and z axes.
$\frac{K_y L}{r_y}$, $\frac{L}{r_{yf}}$, $\frac{Ld}{bt}$	Buckling parameters appearing in the critical stress formulas.
L	Unsupported length.
M_0	Applied end moment.
P	Applied point load.
R	Stress reduction factor for tapered beams.
r_x, r_y, r_z	Radii of gyration about the x, y, and z axes.
r_{yf}	Radius of gyration of one flange plus one-sixth the area of the web.
t_f, t_w	Thickness of flange and web.
x, y, z	Coordinate axes.
α	Taper parameter.
δ	Displacement.

Subscripts

a	Refers to axial stress.
b	Refers to bending stress.
cr	Refers to critical stress.
f, w	Denotes the flange and web.
x, y, z	Refers to the x, y, and z axes.
0, 1	Denotes support point and end point on a tapered beam.
$\doteq$	Approximately equal to.