# CONSISTENCY IN TRANSPORTATION MODAL-SPLIT AND EVALUATION MODELS 

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#### Abstract

This paper is addressed to the evaluation of traveler benefits associated with transportation system alternatives. It is asserted that the different steps in the transportation planning process, which are carried out by models that represent travel behavior, are frequently not consistent with one another and that evaluation of traveler benefits, which is the last step in the process, may provide further inconsistency. These inconsistencies may lead to erroneous conclusions regarding the relative desirability of one system over another. A unified approach to modal split and evaluation calculation is presented and demonstrated in the context of three specific modal-split models-linear, logit, and impedance fraction. Two specific methods of calculation of benefits are presented, one based on probability theory and one based on a modified consumer's surplus formulation. An overall procedure for demand model development and evaluation calculations is outlined.


- IT HAS BECOME increasingly common in recent years for transportation system planners to include a formal evaluation procedure as the final step in the transportation planning process. Although such evaluations have included the effects of the transportation system not only on the traveler but also on the system operator and the community, this paper is addressed only to the computation of traveler benefits. Presumably, better decision-making results from the application of a formal evaluation method than is possible without the use of such a method.

In the planning process, a number of models are used to represent choices of travelers. Usually, separate models are used for purposes of estimating trip generation, trip distribution, modal split, and traffic assignment. These models each purport to represent various aspects of travel decisions made by travelers in the area under study or, in other words, to represent the value system of travelers. To this quartet of models is added another, that is, evaluation. It is clear that the evaluation model (the traveler portion of the model) should reflect the same values of travelers as those used to derive the other models.

In practice, however, the models are usually developed separately. Different demographic, economic, and transportation system variables are included in the different models. In addition, even in cases in which a particular variable is included in more than one model, the specific formulation of that variable may differ between models. For example, travel time, t, may appear as follows in the trip-distribution, modalsplit and evaluation models:

| Model | Representation of Value Associated With Travel Time |
| :---: | :---: |
| Trip distribution | $\left(t_{\text {automobile }}\right)^{\mathbf{k}_{1}}$ |
| Modal split | $\mathrm{k}_{2}\left(\mathrm{t}_{\text {automobile }} /\right.$ ttransit $)$ |
| Evaluation | $\mathrm{k}_{3}\left(\mathrm{t}_{\text {automobile }}-\mathrm{t}_{\text {transit }}\right)$ |

[^0]where $k_{1}, k_{2}$, and $k_{3}$ are derived parameters that weigh the importance of travel time in relation to other variables. It is clear that each model presents different representations of travelers' perceptions and values associated with travel times.

Without considering in detail a number of theoretical and statistical problems, it is clear that the models are not tied together well. This fact suggests that misleading evaluations of alternatives may result. For example, a modal-split model might forecast increasing use of transit under the assumption of an improved transit system. Persons who are forecast to change from automobile travel to transit travel must do so because they perceive a benefit; yet, a benefit calculation resulting from an evaluation model that is independently developed may show a disbenefit. What is needed is a consistent approach to demand modeling and traveler-evaluation modeling. It is toward such an objective that this paper is addressed, with particular attention to the interaction between modal-split and evaluation models.

First, an example of the previously mentioned inconsistency is presented. Then, methods of achieving consistent evaluations are developed for three types of modal-split models. These models are a linear model, a logit model, and an impedance fraction model. The methods entail use of a probability distribution of the willingness to pay and the use of the consumer's surplus theory. (Consideration of producer's surplus as it affects the evaluation process is beyond the scope of the present paper. This is tantamount to the assumption that supply curves in the normal representation are horizontal.) A general procedure is then recommended to achieve consistency. The exposition is mainly in terms of transit improvements. Highway improvements can also be handled with a similar procedure, as suggested in the final section.

## EXAMPLE OF AN INCONSISTENT BENEFIT CALCULATION

In a certain location, assume that transportation system planners are considering an improved transit system. For a particular zone pair, travelers at present must choose between automobile travel and transit travel under the following situation, called alternative $0, \mathrm{~A} 0$ : automobile cost $=50$ cents per trip, automobile time $=30 \mathrm{~min}$ per trip, transit cost $=30$ cents per trip, and transit time $=40 \mathrm{~min}$ per trip. Under the improved transit system, called alternative 1, A1, both cost and time improve. The costs and times are as follows: automobile cost $=50$ cents per trip, automobile time $=$ 30 min per trip, transit cost $=25$ cents per trip, and transit time $=37 \mathrm{~min}$ per trip.

Because both cost and time by transit improve, additional travelers are drawn to transit. The modal-split values estimated from a certain model are as follows: A0 transit patronage $=5$ percent, A1 transit patronage $=30$ percent.

The evaluation is conducted by comparing traveler benefits with system costs. Traveler benefits are defined as the difference between total travel costs for the two alternatives. Total travel costs are the sum of dollar costs plus the value of travel time multiplied by the amount of travel time. For a value of travel time of 5 cents per minute, the total travel costs per traveler are as follows:

| Alternative | Automobile | Transit |
| :---: | :---: | :---: |
| 0 | $50+5 \times 30=200$ | $30+5 \times 40=230$ |
| 1 | $50+5 \times 30=200$ | $25+5 \times 37=210$ |

The benefits, assuming that there are 100 travelers between the two zones, are
B = costs for A0 - costs for A1
$=$ (number of automobile users $\times$ unit automobile cost + number of transit users $x$ unit transit cost) A0
(number of automobile users $x$ unit automobile cost + number of transit users $x$ unit transit cost) A1
$=[(0.95)(100)(200)+(0.05)(100)(230)]$
$-[(0.70)(100)(200)+(0.30)(100)(210)]$
Surprisingly, the calculation shows a disbenefit.

This result can be appraised by considering that the travelers are divided into three groups:

1. Transit travelers who continue to use transit;
2. Automobile travelers who change to transit; and
3. Automobile travelers who continue to use automobiles.

The first group must benefit because both transit cost and time are less than before. The second group must benefit because they would not have changed modes without perceiving a benefit. The third group finds its situation unchanged (except perhaps for reduced congestion, which can be handled in a separate calculation). Because two groups benefit and the third perceives no change, a net benefit must result.

In comparison, the benefit calculation shows a disbenefit; the inconsistency is apparent. However, it can be shown that another value of travel time factor ( $\mathrm{V}<4.06$ in this case) will produce a benefit. Thus, the comparison of alternatives depends crucially on the value of time chosen. More will be said on this subject.

## EVALUATION IN CONJUNCTION WITH A LINEAR MODAL-SPLIT MODEL

Consider a situation in which a modal-split model has been calibrated for actual observed travel choices made in a particular metropolitan area. Suppose the model is

$$
P_{t}=0.6+0.005 \Delta C+0.025 \Delta T
$$

where
$\mathrm{P}_{\mathrm{t}}=$ proportion of travelers using transit;
$\Delta \mathbf{C}=$ dollar cost of automobile travel minus dollar cost of transit travel, in cents; and
$\Delta T=$ time for automobile travel minus time for transit travel, in minutes.
The graph of this relationship is shown in Figure 1. The value $z^{*}$ is defined as follows:

$$
\mathrm{z}^{*}=0.6+0.005 \Delta \mathrm{C}+0.025 \Delta \mathrm{~T}
$$

and $\mathrm{z}^{*}$ is interpreted as the total perceived cost difference between the two modes. This model would forecast 100 percent transit travel for a situation in which $\mathrm{Z}^{*}=1.0,0$ percent for a situation in which $\mathrm{Z}^{*}=0$, and so on.

Another way of viewing the modal-split relationship and the graph is that of a cumulative frequency distribution of the willingness to pay for the difference between transit and automobile travel. (This point of view states that, as the $Z^{*}$ values increase, the cumulative fraction of transit users increases.) For any particular value of $z^{*}$, a tiny


Figure 1. Linear modal-split model.
fraction of the travelers are at the margin such that small shifts in $Z^{*}$ will cause them to change modes. Because they are at the margin, it can be asserted that neither transit nor automobile travel produces a benefit over that of the other (1). The balance of the transit travelers must perceive a benefit by riding transit. In other words, they would be willing to pay more, in terms of cost and travel time, than they are paying.

Now, let there be a particular zone pair for which the following values exist: $\Delta \mathrm{C}=$ $25, \Delta T=-19$. For this zone pair, the particular cost and time values produce a $Z^{*}$ value of 0.25 , and the transit travel is forecast as 25 percent. Next, consider a tiny fraction of travelers who would be at the margin if the $\mathrm{Z}^{*}$ - value were 0.15 . Application of the modal-split model will show that they lie at the fifteenth percentile point on the distribution. Regardless of the actual situation $\left(Z^{*}\right)$, their willingness to pay is 0.15 . The term $Z$ will be used to indicate willingness to pay. The difference between what they are willing to pay (at $\mathrm{Z}=0.15$ ) and the cost (at $\mathrm{Z}^{*}=0.25$ ) must be a measure of their perceived benefit of traveling by transit. The situation for the tiny fraction at the fifteenth percentile point is shown in Figure 2.

One set of costs and times that would piace this tiny fraction at the margin is $\Delta C=$ 5 and $\Delta T=-19$. In other words, the travelers are faced with a situation in which the automobile travel costs 25 cents more than travel by transit, yet they would be on the margin if the automobile only cost 5 cents more than transit. Under the assumption that the modal-split model is a good representation of the expected value system of travelers, the benefit that this tiny fraction perceives must be 20 cents. Similarly, if they were presented a situation wherein $\Delta \mathrm{C}=25$ and $\Delta \mathrm{T}=-23$, they would still be on the margin with a willingness to pay, $Z$, of 0.15 . However, being actually faced with the $\Delta C=25$ and $\Delta T=-19$ situation, they perceive a benefit that is traceable to the difference between the 19- and $23-\mathrm{min}$ travel times.

The benefit, $B$, is defined as the difference between the cost and their willingness to pay

$$
\mathrm{B}=\mathrm{Z}^{*}-\mathrm{Z}
$$

where
$Z^{*}=$ cost for the actual situation; and
$Z=$ willingness-to-pay value of $Z$, as determined by the point $P_{t}$ on the probability distribution at which the tiny fraction of travelers lie.


Figure 2. Willingness to pay versus cost.

However, inspection of the equation for Z will reveal that this difference is stated in unknown (undefined) units. To reduce the expression to a more desirable form, in which benefits are measured in dollar units, it is necessary to divide by the coefficient of $\Delta C$. Therefore, the perceived benefit is given by

$$
\mathrm{B}=\frac{0.6+0.005 \Delta \mathrm{C}+0.025 \Delta \mathrm{~T}-\mathrm{Z}}{0.005}
$$

For the tiny fraction lying at the fifteenth percentile, the benefit is

$$
B=\frac{0.6+0.005(25)+0.025(-19)-0.15}{0.005}=20
$$

In general, for a modal-split model of the form

$$
\mathrm{P}_{\mathrm{t}}=\alpha+\beta \Delta \mathrm{C}+\gamma \Delta \mathrm{T}
$$

the dollar-measured benefit of transit travel over automobile travel accruing to a tiny fraction of travelers at a given point, $\mathrm{P}_{\mathrm{t}}$, on the cumulative distribution is

$$
\mathrm{B}=(\alpha / \beta)+\Delta \mathrm{C}+(\gamma / \beta) \Delta \mathrm{T}-(\mathrm{Z} / \beta)
$$

Now, consider the total benefit that accrues to the population of transit users. To simplify the exposition, we will divide the population into groups of 5 percent intervals. Group 1 consists of those lying between the 0 and 5 percentile points on the distribution. The value that will stand for this group is 2.5 percent, the median of the group interval. If the linear model is used for 100 travelers facing a $\Delta C=25$ and $\Delta T=-19$ travel situation, 25 percent will choose transit. The benefits for each of the five groups are derived from the preceding formula for the individual traveler lying at the group median and multiplied by the number of persons in the group.

| Percentage Range <br> of Each Group | Median <br> Percent | Benefit per <br> Traveler (cents) | Total Benefit <br> (cents) |  |
| :---: | :---: | :---: | :---: | :---: |
|  |  | 2.5 |  | 45 |
| 5 to 5 | 7.5 |  | 35 |  |
| 10 to 10 |  | 25 |  | 175 |
| 15 to 20 |  | 12.5 |  | 25 |
| 20 to 25 | 17.5 |  | 15 |  |

The total benefit for all transit travelers is 625 cents.
Next, assume that transit planners are considering an improved transit system that would offer 2 min less travel. The two alternatives are

| Alternative 0 |  |
| :---: | :---: |
| $\Delta \mathrm{C}=25$ <br> $\Delta \mathrm{~T}=-19$ |  |$\quad$| Alternative 1 |
| :---: |
| $\Delta \mathrm{C}=25$ |
| $\Delta \mathrm{~T}=-17$ |

Now, 5 percent more travelers will be drawn to transit, as indicated by

$$
\begin{aligned}
\mathrm{P}_{\mathrm{t}} & =0.6+0.005 \Delta \mathrm{C}+0.025 \Delta \mathrm{~T} \\
& =0.6+0.005(25)+0.025(-17) \\
& =0.30
\end{aligned}
$$

The total benefits are calculated as follows:

| Percentage Range of Each Group | Median <br> Percent | Benefit per Traveler (cents) | Total Benefit (cents) |
| :---: | :---: | :---: | :---: |
| 0 to 5 | 2.5 | 55 | 275 |
| 5 to 10 | 7.5 | 45 | 225 |
| 10 to 15 | 12.5 | 35 | 175 |
| 15 to 20 | 17.5 | 25 | 125 |
| 20 to 25 | 22.5 | 15 | 75 |
| 25 to 30 | 27.5 | 5 | 25 |

and the total benefit is 900 cents.
Between these two alternatives, the net benefit is the difference between the two benefits.

$$
\begin{aligned}
\mathrm{NB} & =\mathrm{B}_{1}-\mathrm{B}_{0} \\
& =900-625 \\
& =275
\end{aligned}
$$

The algebraic calculation for each alternative is performed as follows:

$$
B_{i}=N \sum_{j=1}^{m} B_{i j} \Delta g_{j}
$$

where
$\mathrm{B}_{\mathrm{i}}=$ benefits accruing to the ith alternative,
$\mathrm{N}=$ total number of travelers,
$B_{i j}=$ benefits accruing to the $j$ th group under the $i$ th alternative,
$\Delta g_{j}=$ fraction of travelers in the $j$ th group, and
$\mathrm{m}=$ last group on the cumulative distribution choosing transit.
This form of calculation, in general, will produce only approximate results. A more precise method will result from use of the calculus. Expanding the previous expression results in

$$
\begin{aligned}
\mathrm{B}_{\mathrm{i}} & =(\mathrm{N} / \beta) \sum\left(\mathrm{Z}^{*}-\mathrm{Z}\right) \Delta \mathrm{g}_{\mathrm{j}} \\
& =(\mathrm{N} / \beta) \sum \mathrm{z}^{*} \Delta \mathrm{~g}_{\mathrm{j}}-(\mathrm{N} / \beta) \sum \mathrm{Z} \Delta \mathrm{~g}_{\mathrm{j}}
\end{aligned}
$$

Carrying $\Delta \mathrm{g}_{\mathrm{j}}$ to the infinitesimal results in

$$
\mathrm{B}_{\mathrm{i}}=(\mathrm{NZ} / \beta) \int_{-\infty}^{\mathrm{Z}^{*}} \mathrm{f}(\mathrm{Z}) \mathrm{dZ}-(\mathrm{N} / \beta) \int_{-\infty}^{\mathrm{Z}^{*}} \mathrm{Z} \mathrm{f}(\mathrm{Z}) \mathrm{dZ}
$$

where


Figure 3. Summation of benefits.

$$
\begin{aligned}
& f(Z)=\text { probability density function } \\
& \mathrm{Z}^{*}=\text { of } \mathrm{Z}, \text { and } \\
& \mathrm{Z} \text { at } \mathrm{P}_{\mathrm{t}} .
\end{aligned}
$$

Graphically, the process of integration estimates the area under the modalsplit curve and to the left of the $\mathrm{Z}^{*}$ value, as shown in Figure 3. The use of the calculus to solve the example results in the following:

For a uniform distribution, which is correct for the linear model, $\mathrm{P}_{\mathrm{t}}=$ $\mathrm{Z}^{*}$ or, if the standard probability notation is used, $F(Z)=Z$ and $f(Z)=$
$\mathrm{dF}(\mathrm{Z}) / \mathrm{dZ}=\mathrm{dZ} / \mathrm{dZ}=1$. The relationship, therefore, reduces to

$$
\begin{aligned}
\mathrm{B}_{0} & =(\mathrm{NZ} / \beta) \int_{-\infty}^{\mathrm{Z}^{*}} \mathrm{f}(\mathrm{Z}) \mathrm{dZ}-(\mathrm{N} / \beta) \int_{-\infty}^{\mathrm{Z}^{*}} \mathrm{ZdZ} \\
& =(\mathrm{NZ} / \beta)\left[\mathrm{F}\left(\mathrm{Z}^{*}\right)\right]-(\mathrm{N} / \beta)\left[\left(\mathrm{Z}^{2} / 2\right)\right]_{0}^{\mathrm{Z}^{*}} \\
& =\{[(100)(0.25)] / 0.005\}(0.25)-(100 / 0.005)\left\{\left[(0.25)^{2} / 2\right]-\left(0^{2} / 2\right)\right\} \\
& =625
\end{aligned}
$$

A similar calculation for alternative 1 will produce a value of 900 , producing a net benefit of 275 as before.

It turns out that this argument, based on an analysis of the probability distribution for modal split, is equivalent to a consumer's surplus formulation of the benefit calculation. As stated earlier, the cost used here is the cost difference between modes; normally, the consumer's surplus is formulated in terms of individual costs rather than differences in cost ( $\mathbf{2}, \underline{3}, 4)$. The expression for approximating the consumer's surplus in terms of a cost difference between alternative 1 and alternative 0 is

$$
N B=(1 / 2)\left(\Delta C_{1 p}-\Delta C_{0 p}\right)\left(N_{1}+N_{0}\right)
$$

where
$\Delta C_{i p}=$ perceived cost difference for the $i$ th alternative, and
$\mathrm{N}_{\mathrm{i}}=$ number of transit travelers for the $i$ th alternative.
The equivalency of the two approaches is obtained if the price difference is defined as

$$
\Delta \mathrm{C}_{\mathrm{ip}}=(\alpha / \beta)+\Delta \mathrm{C}+(\gamma / \beta) \Delta \mathrm{T}
$$

In the example, the net benefits of alternative 1 over alternative 0 are calculated to be

$$
\mathrm{NB}=(1 / 2)\{[120+25+(5)(-17)]-[120+25+(5)(-19)]\}(25+30)=275
$$

Thus, the two theoretical approaches to benefit estimation produce the same results because the linear approximation is exact for a linear model.

The value of 275 is considered to be the correct evaluation of benefits. Such calculations will produce consistent modal-split and evaluation results and, therefore, solve the problem of potential inconsistencies that was demonstrated in an earlier section.

Beyond the problem of inconsistency is the problem of inappropriate calculation methods. As cited earlier, the choice of a value of time is an important factor in benefit calculations. However, it turns out that, even if a "good" value of time factor is chosen, calculations of benefits may still be made inappropriately. This methodology question has been debated extensively in the literature on the subject of consumer's surplus. The alternative calculation is performed erroneously, summing the total travel costs in a manner such as was demonstrated in the previous section. The net benefits are calculated algebraically as follows:

$$
\begin{aligned}
\mathrm{NB}_{1-0}= & \left(\mathrm{P}_{\mathrm{a} 0}\right)(\mathrm{N})\left(\mathrm{C}_{\mathrm{a} 0}+\mathrm{VT}_{\mathrm{a} 0}\right)+\left(\mathrm{P}_{\mathrm{t} 0}\right)(\mathrm{N})\left(\mathrm{C}_{\mathrm{t} 0}+\mathrm{VT}_{\mathrm{t} 0}\right) \\
& -\left(\mathrm{P}_{\mathrm{a} 1}\right)(\mathrm{N})\left(\mathrm{C}_{\mathrm{a} 1}+\mathrm{VT}_{\mathrm{a} 1}\right)-\left(\mathrm{P}_{\mathrm{t} 1}\right)(\mathrm{N})\left(\mathrm{C}_{\mathrm{t} 1}+\mathrm{VT}_{\mathrm{t} 1}\right)
\end{aligned}
$$

where
$P_{\mathrm{ai}}=$ proportion of travel by automobile for the ith alternative,
$P_{t i}=$ proportion of travel by transit,
$\mathrm{C}_{\mathrm{ai}}=$ automobile cost,
$\mathrm{C}_{\text {ti }}=$ transit cost,
$\mathrm{T}_{\mathrm{ai}}=$ automobile time,
$\mathrm{T}_{\mathrm{ti}}=$ transit time,
$\mathrm{V}=$ value of travel time, and
$\mathrm{N}=$ total number of travelers.
In the example problem analyzed earlier, assume that the $\Delta C$ and $\Delta T$ values result from

| $\mathrm{C}_{\mathrm{a} 0}=50$ | $\mathrm{C}_{\mathrm{a} 1}=50$ |
| :--- | :--- |
| $\mathrm{C}_{\mathrm{t} 0}=25$ | $\mathrm{C}_{\mathrm{t} 1}=25$ |
| $\mathrm{~T}_{\mathrm{a} 0}=30$ | $\mathrm{~T}_{\mathrm{a} 1}=30$ |
| $\mathrm{~T}_{\mathrm{t} 0}=49$ | $\mathrm{~T}_{\mathrm{t} 1}=47$ |

The net benefits could be calculated by using the previous formula, with the choice of a "good" value of time derived from the modal-split model. The value is the ratio $\gamma / \beta$ or 5 cents per minute.

$$
\begin{aligned}
\mathrm{NB}= & (0.75)(100)(50+5 \times 30)+(0.25)(100)(25+5 \times 49) \\
& -(0.70)(100)(50+5 \times 30)-(0.30)(100)(25+5 \times 47)=-50
\end{aligned}
$$

which is not the same as the correct 275 benefit measure calculated earlier. Therefore, the inappropriateness of this method can be asserted on the basis of the theoretical arguments presented earlier and is demonstrated by this numerical example: A disbenefit result cannot logically result from an improvement (in travel time) to the transit system. Unfortunately, the inappropriate calculation has been used in most of the recent transit benefit-cost studies.

## EVALUATION IN CONJUNCTION WITH A LOGIT MODAL-SPLIT MODEL

More sophisticated approaches to modal-split model development frequently use nonlinear forms of a modal-split equation. One such form is that of the logistic distribution or logit function ( 5,6 ). Lisco (7) presents a similar approach using probit analysis. The model takes the form

$$
\left.P_{t}=e^{Z^{*} /\left(1+e^{*}\right.}\right)
$$

An advantage of this formulation is that probability values are limited to the range between 0 and 1, whereas in the linear model described in the foregoing either a discontinuous function must be specified, or probability values of less than 0 and greater than 1 could be computed from an indiscriminate use of the formula.

Suppose, for a certain situation, a logit model has been calibrated with a $\mathrm{Z}^{*}$ as follows:

$$
\mathrm{Z}^{*}=-0.60+0.06 \Delta \mathrm{C}+0.30 \Delta \mathrm{~T}
$$



Figure 4. Logit modal-split model.

The graph of this relationship is shown in Figure 4.

In this example, let there be a zone pair for which the following values exist: $\Delta C=25$ and $\Delta T=-6.7$. Here, the transit travel would be forecast as 25 percent:

$$
\begin{aligned}
\mathrm{z}^{*}= & -0.60+0.06(25) \\
& +0.30(-6.7)=-1.11 \\
\mathrm{P}_{\mathrm{t}}= & \mathrm{e}^{-1.11} /\left(1+\mathrm{e}^{-1.11}\right)=0.25
\end{aligned}
$$

The same arguments as the foregoing can be made with respect to the will-
ingness to pay for transit service. For example, one small fraction of the travelers would be at the margin if their Z-value were $\mathbf{- 1 . 7 4}$. They lie at the fifteenth percentile point on the distribution.

$$
P_{t}=e^{-1.74} /\left(1+e^{-1.74}\right)=0.15
$$

One set of costs and times that would place this fraction at the margin is $\Delta C=14.5$ and $\Delta \mathrm{T}=-6.7$. In other words, they are faced with a situation in which automobile travel costs 25 cents more than transit, and they would be on the margin if the automobile only cost them 14.5 cents more than transit. The benefit that this small fraction perceives must be $25-14.5=10.5$ cents.

The perceived benefit for any fraction-identified by the point, $P_{t}$, on the cumulative distribution-is given by

$$
B=\frac{-0.60+0.06 \Delta C+0.30 \Delta T-\ln \left(P_{t} /\left[1-P_{t}\right)\right]}{0.06}
$$

In general, for the logit modal-split model

$$
P_{t}=e^{Z^{*} /\left(1+e^{Z^{*}}\right)}
$$

in which $\mathrm{Z}^{*}=\alpha+\beta \Delta \mathrm{C}+\gamma \Delta \mathrm{T}$. The benefit accruing to a small fraction of travelers lying at a given point $P_{t}$ on the cumulative distribution is

$$
\mathrm{B}=(\alpha / \beta)+\Delta \mathrm{C}+(\gamma / \beta) \Delta \mathrm{T}-\left\{\left[\ln \left(\mathrm{P}_{\mathrm{t}} / 1-\mathrm{P}_{\mathrm{t}}\right)\right] / \beta\right\}
$$

Over the entire proportion of the travelers who choose transit, the benefits are computed by the expression given previously:

$$
\mathrm{B}_{\mathrm{i}}=(\mathrm{NZ} / \beta) \int_{-\infty}^{\mathrm{Z}^{*}} \mathrm{f}(\mathrm{Z}) \mathrm{dZ}-(\mathrm{N} / \beta) \int_{-\infty}^{\mathrm{Z}^{*}} \mathrm{Zf}(\mathrm{Z}) \mathrm{dZ}
$$

In this case,

$$
\begin{aligned}
\mathrm{B}_{\mathrm{i}} & =(\mathrm{NZ} / \beta)[\mathrm{F}(\mathrm{Z})]_{-\infty}^{\mathrm{Z}^{*}}-(\mathrm{N} / \beta) \int_{-\infty}^{\mathrm{Z}^{*}}\left\{\mathrm{Ze} \mathrm{Z} /\left[\left(1+\mathrm{e}^{\left.\left.\mathrm{Z})^{2}\right]\right\} d \mathrm{Z}}\right.\right.\right. \\
& =\left(\mathrm{NZ} \mathrm{Z}^{*} / \beta\right)\left[\mathrm{F}\left(\mathrm{Z}^{*}\right)\right]-(\mathrm{N} / \beta)\left[\left\{\left[\mathrm{Ze} \mathrm{Z} /\left(1+\mathrm{e}^{\mathrm{Z}}\right)\right]-1 \mathrm{n}\left(1+\mathrm{e}^{\mathrm{Z}}\right)\right\}\right]_{\mathrm{Z}_{0}^{*}}^{\mathrm{Z}^{*}}
\end{aligned}
$$

Because the lower limits of integration are the same, the net benefits between alternative 1 and alternative 0 are

$$
\mathrm{NB}_{1-0}=\left[\left(\mathrm{NZ}^{*} / \beta\right) \mathrm{F}\left(\mathrm{Z}^{*}\right)\right]_{\mathrm{Z}_{0}^{*}}^{\mathrm{Z}_{1}^{*}}-(\mathrm{N} / \beta)\left[\left\{\left[\mathrm{ZeZ} /\left(1+\mathrm{e}^{\mathrm{Z}}\right)\right]-\ln \left(1+\mathrm{e}^{\mathrm{Z}}\right)\right\}\right]_{-\infty}^{\mathrm{Z}^{*}}
$$

Consider now a situation in which two alternatives are to be evaluated:


The modal-split calculation for alternative 0 is shown in the foregoing, resulting in $P_{t}=0.25$. For alternative 1

$$
z=-0.60+0.06(30)+0.30(-5.0)=-0.30
$$

$$
P_{t}=e^{-0.30} /\left(1+e^{-0.30}\right)=0.425
$$

Solving for net benefits using the formula derived in the preceding results in

$$
\begin{aligned}
\mathrm{NB}= & {[(100)(-0.30) / 0.06](0.425)-[(100)(-1.11) / 0.06](0.25) } \\
& -100 / 0.06\{[(-0.30)(0.742) / 1.742]-0.555\} \\
& +(100 / 0.06)\{[(-1.11)(0.33) / 1.33]-0.285\}=453
\end{aligned}
$$

The net benefits can also be approximated by the consumer's surplus method,

$$
N B=(1 / 2)\left(\Delta C_{1 p}-\Delta C_{0 p}\right)\left(N_{1}+N_{0}\right)
$$

if the cost is defined as

$$
\Delta C_{i p}=\frac{\alpha}{\beta}+\Delta C+\frac{\gamma}{\beta} \Delta T
$$

The net benefit is, therefore,

$$
\mathrm{NB}=\frac{1}{2}\{[30+5.0 \times(-5)]-[25+5.0 \times(-6.7)]\}[25+42.5]=455
$$

Again, the two theoretical approaches produce approximately the same results. The probability approach is the more theoretically correct.

## EVALUATION IN CONJUNCTION WITH AN IMPEDANCE FRACTION MODAL-SPLIT MODEL

Consider a situation in which a modal-split model has been formulated in terms of a fractional relationship of impedance measures, such as the following:

$$
\mathrm{P}_{\mathrm{t}}=\frac{0.005+\mathrm{C}_{\mathrm{t}}^{-1.4}+\mathrm{T}_{\mathrm{t}}^{-1.2}}{0.005+\mathrm{C}_{\mathrm{t}}^{-1.4}+\mathrm{T}_{\mathrm{t}}^{-1.2}+0.008+\mathrm{C}_{\mathrm{a}}^{-1.4}+\mathrm{T}_{\mathrm{a}}^{-1.2}}
$$

Let there be two transportation system alternatives as follows:

| Alternative 0 |  | Alternative 1 |
| :---: | :---: | :---: |
|  | $\mathrm{C}_{\mathrm{t}}=25$ $\mathrm{C}_{\mathrm{t}}=20$ <br> $\mathrm{C}_{\mathrm{a}}=50$  <br> $\mathrm{~T}_{\mathrm{t}}=40$  <br> $\mathrm{~T}_{\mathrm{a}}=25$  <br> $\mathrm{~T}_{\mathrm{t}}=35$  <br>   <br> $\mathrm{~T}_{\mathrm{a}}=25$  |  |

Under alternative 0

$$
P_{t}=\frac{0.005+0.0113+0.0120}{0.005+0.0113+0.0120+0.0080+0.0042+0.0210}=0.44
$$

Under alternative 1

$$
P_{t}=\frac{0.005+0.0152+0.0141}{0.005+0.0152+0.0141+0.0080+0.0042+0.0210}=0.51
$$

To apply the consumer's surplus argument requires that the prices of the two alternatives be derived. In this case, the most direct way to perform the computation is to
find the difference in equivalent price, $\mathrm{C}^{\mathrm{e}}$, between the two alternatives. If only a difference in dollars were relevant, the $\mathrm{C}^{\mathrm{e}}$ difference would be simply the difference between the dollar costs of the two alternatives, or 5 cents. However, both a time and a cost difference are presented.

The equivalent price $\mathrm{C}^{\mathrm{e}}$ is obtained by finding the equivalent cost difference that would produce the same modal split as the split that results from the combination of cost and time differences presented in the actual case. The calculation can be made for alternative 1 by using the time value of alternative 0 .

$$
0.51=\frac{0.0050+\mathrm{C}_{\mathrm{t}}^{-1.4}+0.0120}{0.0050+\mathrm{C}_{\mathrm{t}}^{-1.4}+0.0120+0.0080+0.0042+0.0210}
$$

where $\mathrm{C}_{\hat{E}}^{\mathrm{e}}=18$. Thus, the combination of $\mathrm{C}_{t}=20$ and $\mathrm{T}_{\mathrm{t}}=35$ presented in alternative 1 will produce the same modal split as would the combination $C_{t}=18$ and $T_{t}=40$. The equivalent price between the two alternatives is $(50-18)-(50-25)=7$ cents. If there are 100 travelers, the net benefits are

$$
\begin{aligned}
\mathrm{NB} & =(1 / 2)\left(\mathrm{C}_{1}^{\mathrm{e}}-\mathrm{C}_{0}^{\mathrm{e}}\right)\left(\mathrm{N}_{1}+\mathrm{N}_{0}\right) \\
& =(1 / 2)(7)(44+51)=332
\end{aligned}
$$

## RECOMMENDED PROCEDURE FOR MAKING CONSISTENT MODAL-SPLIT AND EVALUATION CALCULATIONS

The procedure that is recommended for resolving potential inconsistencies between modal-split model results and evaluation calculations is as follows:

1. Postulate a number of modal-split models by using various formulations of the overall model and of the formulation of individual independent variables. Overall model formulations include linear models, logit models, probit models, and impedance fraction models. Individual independent variable formulations include differences between modes and ratios between modes.
2. Calibrate the models on observed travel choices in the area.
3. Apply theoretical, statistical, and empirical tests to select the most appropriate model. The resulting model will be used to make both modal-split and evaluation calculations.
4. Estimate the modal split for each transportation system alternative and for the existing situation for each zone pair.
5. Compute the equivalent perceived price for the existing situation and each improvement alternative. Symbolically, if the modal split for the existing situation is $\mathrm{P}_{\mathrm{o}}=\mathrm{h}\left(\mathrm{C}_{0 \mathrm{a}}, \mathrm{C}_{0 \mathrm{t}}, \mathrm{T}_{0 \mathrm{a}}, \mathrm{T}_{0 t}\right)$ and for improvement alternative 1 is $\mathrm{P}_{1}=\mathrm{h}\left(\mathrm{C}_{1 \mathrm{a}}, \mathrm{C}_{1 \mathrm{t}}, \mathrm{T}_{1 \mathrm{a}}\right.$, $\left.\mathrm{T}_{1 t}\right)$, then find the equivalent price $\mathrm{C}_{t}^{*}$ by solving the expression $\mathrm{P}_{1}=\mathrm{h}\left(\mathrm{C}_{1 \mathrm{a}}, \mathrm{C}_{\mathrm{t}}^{\mathrm{e}}, \mathrm{T}_{1 \mathrm{a}}\right.$, $T_{0 t}$ ), for $C_{t}^{e}$. The equivalent price difference is $\mathrm{C}_{0 t}-\mathrm{C}_{t}^{e}$.
6. Use the consumer's surplus formula to approximate the net benefits.

$$
\mathrm{NB}=(1 / 2)\left(\mathrm{C}_{0 \mathrm{t}}-\mathrm{C}_{\mathrm{t}}^{\mathrm{e}}\right)\left(\mathrm{P}_{0}+\mathrm{P}_{1}\right)(\mathrm{N})
$$

## HIGHWAY IMPROVEMENTS

The examples presented in this paper considered improvements to a transit system. The groups that benefit are previous transit users who benefit through better transit service, and previous automobile users who switch from automobiles to transit and thereby benefit. The group that does not benefit consists of previous automobile users who do not switch. These persons may actually benefit because of reduced congestion. Such benefits can be computed separately and added to the benefits of the other two groups.

A similar procedure can easily be derived for an improvement to the highway system. The groups that benefit are previous automobile users who benefit through better
highway travel, and previous transit users who switch from transit to automobiles and thereby benefit. The group that does not benefit consists of previous transit users who do not switch. The procedure would compute benefits for automobile travelers by integration, finding the area above the modal-split curve and to the right of the Z -value as shown in Figure 5 for each alternative and then subtracting one benefit from the base case to obtain the net benefit.

CONCLUSIONS


Figure 5. Benefits for highway improvements.

This paper demonstrates that traveler benefit evaluations may be computed in an inconsistent manner compared with associated computations that are made to develop demand estimates by mode of travel. Based on arguments regarding the willingness to pay for transportation versus the cost of transportation, a theoretical benefit model is developed from probability theory. This model provides essentially equivalent results to those that are developed from this application of a modified consumer's surplus model. A general procedure is presented that will provide traveler evaluations that are consistent with demand models.

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[^0]:    Sponsored by Committee on Transportation Systems Design and presented at the 50th Annual Meeting.

