

OPTIMIZATION OF URBAN TRANSPORTATION TERMINAL NETWORK

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The objective of this study was to develop a method of optimizing a terminal network to provide transportation service to urban areas. The optimum terminal network was considered to be one that minimized a total cost function. Two criteria provided the bases of the analysis: first, rectilinear travel routes for access to terminals prevail in the urban area; and, second, a multiterminal structure for transportation vehicles is required to provide an adequate level of service to the major area of a city. Many variables associated with the urban terminal-destination environment were reviewed and were subjectively evaluated to obtain a practical optimum solution. Characteristics required of a terminal distribution system were also specified. A curtailed enumeration technique, the Golden Section Search procedure, and a computer simulation program combined with an economic trade-off analysis were used to determine the optimum terminal density and the location and capacity for all terminals in the network under the stated conditions. The computer simulation program was used to help evaluate the worth of each state specified by the optimization technique. Experiments conducted by using the program were reviewed by application of the solution method to a parking terminal network in an urban area. The program successfully resolved the terminal network for a random demand distribution, and it should provide direction in analyzing any actual system.

•WITH THE INCREASE in travel demand as well as in the cost of transportation, it is becoming more and more important to consider the total systems approach in the design of not only the transportation media but also the terminal network. Terminal availability, to a great extent, determines the efficiency with which a transportation mode serves an area, because the access portion of travel not only takes time but also imposes costs on the user. A total-trip-route concept is, therefore, mandatory for the design of a truly optimum transportation system.

The optimum terminal network, which is to provide service within a specific locality, must balance several factors. The network must consider the geographical distribution of travel demand, the characteristics of demand, the relative level of demand in the area, and the cost of providing and using the terminal facility. The requirements for a terminal network will vary with the type of mode and its applications. For systems operating along major radial routes, the terminal should provide for connections with feeder modes. For systems operating in the denser urban center, the terminal should provide convenient walking access to the user. A mode of transportation is typically concerned with providing travel service to a large portion of city area, and, in most cases, to an entire city. In making this provision the public is best served and customer goodwill is best developed because the mode attains the flexibility of servicing any possible destination point in the city. Because each terminal unit can service only a limited area in close proximity to itself, a network of terminals is required to provide city-wide service.

A number of diverse technologies can be applied to determine the optimum terminal network. However, previous work has not really provided adequate system concepts to determine the optimum transportation terminal network with the objective of minimizing total cost and, at the same time, maximizing transportation service.

Hakimi's study (11) concerning the absolute median of a graph and the subsequent extensions to transportation terminals by Geodeon (10) and Frank (9), who considered probabilistic demand, provide valuable insight for the current study, but many factors associated with transportation terminals are not considered in any of these studies. Some of these factors not considered are (a) the economics both of providing a terminal and of traveling to use it, (b) competitive interaction among modes and the propensity of people to travel various distances to avail themselves of other conveniences offered by another mode, and (c) number and location of terminals in a network.

Vuchic (18) considered rectilinear access routes to terminals in a network, but his model reduced access routes to unidimensional paths. His work gives no direction in cases where two-dimensional paths between the users' destinations and terminal locations prevail. His objective of minimizing total travel time did not consider the different value that various customers may assign to travel time. Cramer (6) and O'Doherty (15) suggested methods of locating terminals in two dimensions, but neither considered the general rectilinear case. Both methods lack the sensitivity to locate a large number of terminals, which is the basic purpose of the current study. Neither Cramer's nor O'Doherty's study considered the economic cost of travel or the proclivity of customers to use a particular mode of travel. Each of the studies mentioned in the foregoing makes an important contribution under its own set of assumptions. However, each is found to be somewhat limited in the context of the current study, which attempts to treat a wide variety of identifiable variables associated with an urban transportation terminal system. Classical theories of optimization, facility location, simulation, economics, decision-making, and central places, combined with psychological and traffic engineering concepts, were all applied in this study.

STUDY OBJECTIVES

The problem examined by this study was the determination of an optimum terminal network to provide the best possible transportation service to an urban area. The economic criterion of minimizing the total cost (i.e., the sum of the user cost and the cost of the terminal facilities) was used as the objective defining the optimum system. The terminal access impedance was measured by the total access time and, therefore, by the costs involved in traveling from origin points to the nearest terminal (or from the terminal to destination points). The capacity of terminal facilities provided needed only to correspond to the periods of peak accumulation of parked vehicles because this demand represents the maximum load that the facility must service. While this criterion of minimizing total cost may not provide the results desired by the profit-seeking entrepreneur, it is particularly applicable to modes of transportation under public ownership. The term "optimum" used in this study is more accurately defined as "a near optimum solution under the stated conditions".

The current study represents a departure from previous work in two important ways: rectilinear travel was assumed to prevail, and the system involving a large number of terminals was optimized from the viewpoint of minimizing total cost, as defined by a unique formula. The criteria used in the study not only considered many of the variables that affect the urban terminal environment but also attempted to model the system in a way realistic enough to provide valid assistance for an actual application.

It is hoped that the solution technique will be applicable to the transportation system that has definite locations of users' access in urban areas, i.e., terminal networks such as automobile parking facilities, bus stops, taxi stands, minicar transit stations, and rail and rapid transit stations. In addition, this study will provide particularly valuable insight into the transportation system planning for new towns as well as the planning of new transportation systems for existing cities.

URBAN TERMINAL ENVIRONMENT

The terminal network must be analyzed in terms of the competitive environment within which it operates. It is, therefore, important to make explicit the environmental factors considered by this study so that the analysis will yield a solution method that would be meaningful in an actual application.

Travel demand is a random variable that can be adequately quantified only after intensive study in any city. The level of demand must be specified as a function of the origin; destination; time of day, week, month, and year; and corresponding attributes of the traveler such as his trip purpose, sex, income, and age. Accurate estimates of the level of demand available to any given mode are critical to the optimization of the terminal network. Planned changes within an urban area must be included to adequately describe the demand to be served over the life cycle of the system. Because demand varies stochastically by time periods (reaching some peak daily), the statistical distributions of demand must be described by developing a mean and standard deviation of available demand at any point of destination.

The portion of available demand that a mode captures, called the attracted demand, is a function of the distance from the terminal to the point of destination, because fewer people are willing to walk longer distances. The effectiveness of the terminal is defined as the percentage of available demand attracted. The distance-reduction function must be a composite of the distances people are currently willing to travel under the prevailing conditions and an ideal function derived for the optimum transportation condition. Because the grid pattern of street networks is predominant in American cities (16), access to terminals must be by travel along rectilinear routes. Based on the assumption of the economic and psychological makeup of people, the traveler is inclined to use the terminal nearest his destination. The duration of a trip at the destination is a function of trip purpose. Parkers, for example, usually are willing to walk long distances only if they intend to remain at their destination for a long time.

The user's terminal-access cost is the subjective value placed on the user's access time, discomfort, and inconvenience. Some of the available demand may be lost, because the poor positioning of the terminal increases the user's total walking distance. A user is always expected to return to the terminal to obtain transportation to his trip origin point; thus, the access cost is for both ways between the terminal and destination. Terminal facility cost includes the cost of land, the cost of construction, and the cost of maintaining and operating the terminal. The total cost objective function that is to be minimized should, therefore, include a penalty charge to the system for each demand unit lost because of the specified position of the terminal. This penalty rate must be determined subjectively; this study uses a penalty that approximates the fee a user might pay for using the terminal facility. This definition may be most appropriate for automobile parking facilities, but it could be modified in the context of other modes. If this penalty is not incorporated, the optimum terminal system would have no terminals, because total cost would be minimized by providing no transportation. The penalty, in a sense, is an actual cost, because a different terminal location might attract more demand.

CONCEPTS OF OPTIMIZATION

Service Area Considerations

If only a single terminal is provided, it will serve an area bounded simply by the distance people are willing to walk. If terminal service is required for a larger area, a multiterminal structure is necessary to provide service for all points in the area. Because this study was ultimately concerned with the optimization of a terminal network and not the optimization of a single terminal, it became important to distinguish the bounded optimum geometric configuration of the area to be served by each terminal. By definition, the optimum service area pattern in terms of the area, shape, and size must provide minimum average access distance in comparison to other feasible configurations.

A previous study (22) by the authors was conducted to examine a number of identifiable variables associated with the urban terminal environment to determine the optimum spatial pattern of transportation terminal locations in urban areas. Study results indicated that the optimum pattern is one having the following characteristics: The

terminal location pattern uses diamond-shaped service areas that are centered at intersections (with the major axis parallel to the urban streets) with the boundary lines of the service areas parallel to street-block diagonals. The service area diagonal must be an integer multiple of block sides, with the distance from the center of the service area to the vertexes being only one block, two blocks, three blocks, and so forth. Larger penalties are assessed in the larger service-area size increments, because less demand is attracted from greater distances. The penalty rate may, therefore, force a particular size of service area to be optimum in terms of minimum total cost.

The optimum set of service areas can be determined by three different ways: demand approach, uniform-size approach, and sectioning approach. Each approach has certain advantages and disadvantages.

The demand approach is based only on the specific demand in the immediate vicinity of each terminal. Each service area would be of a different size, but each terminal would be optimum by the criterion within a given area. This has several limitations: (a) Because of the geometry of service areas, it may be impossible to provide service for the entire city; (b) the total network may be suboptimized by starting at the individual terminal level; (c) service area boundaries will not be easily discernible to the user; and (d) the terminal may be difficult for users to find because of the nonuniformity involved.

The uniform-size approach assumes that all service areas in the system must be the same size. This approach has several advantages: (a) Service would be provided to the entire city; (b) the total network would not be suboptimized because the system approach would dictate the size of each service area; (c) service area boundaries would be easily discernible; and (d) terminals would be easily found by users. However, this approach presents another problem. That is, the concentrated demand in certain areas may force a particular, nonoptimum service area size in other areas of the city.

The sectioning approach is based on the assumption that a city is composed of a number of sections in which demand may be considered relatively homogeneous. The condition of uniform demand distribution within a section of a city is fairly realistic, derived by assuming that points of similar demand tend to aggregate. If this is indeed not true, it may prove uneconomical or impossible to conduct studies in the depth required to describe the actual demand at each destination point.

Because the demand is considered to be essentially uniform within specific sections in the city, the uniform network approach should be applied to each of these sections. Thus, the best solution in each section could be easily found. If the service areas of two bordering localities are of varying sizes, then the bordering service areas must be analyzed in detail in order to determine the best geometry of transition between the two service area sizes. In this context, the sectioning approach retains all of the advantages of the first two approaches and eliminates the disadvantages.

Terminal Facility Considerations

The optimum terminal network is defined by the specific terminal locations within the service areas and by the required terminal capacities. Specification of a terminal at the center of its service area, although optimum mathematically, is not practical because this point is at the center of an intersection. For the uniform demand case, the terminal location can be displaced small distances from the service area center and still retain optimum characteristics in terms of terminal accessibility. This fact provides great flexibility in determining the actual location. For practical purposes, the site where land is available and most compatible with the traffic flow in the area should be selected.

By a more sophisticated optimization theory, the economically desirable location of a terminal within its service area can directly follow the well-known Golden Section Search technique (20) with the objective of minimizing total cost. The following economic system may be optimized by the Golden Section Search technique. Expected values of each of the random variables are used.

$$E(TS) = E(TC) + E(WC)$$

$$E(TC) = \sum_i \left[(D_p)_{ij} (CT)_j + PR(D_q)_{ij} \right]$$

$$E(WC) = 2 \sum_i \frac{(CW)_i (D_t)_i (RWD)_{ij} (R_d)_i}{(WR)}$$

where

- $E(TS)$ = expected total cost;
- $E(TC)$ = expected terminal cost;
- $E(WC)$ = expected user access cost;
- $(D_p)_{ij}$ = expected daily attracted demand for point i at the time the terminal at location j experiences peak demand;
- $(CT)_j$ = daily cost of providing one terminal space;
- PR = penalty rate per available customer not attracted to mode;
- $(D_q)_{ij}$ = expected daily total available demand at point i that is not attracted (i.e., lost) to a terminal at location j ;
- $(CW)_i$ = expected access cost for users at destination i , dollar per hour;
- $(D_t)_i$ = expected total daily demand available at point i ;
- $(RWD)_{ij}$ = rectilinear access distance between point i and terminal location j ;
- $(R_d)_i$ = expected distance reduction factor for users at point i or percentage of users to use the terminal at $(RWD)_{ij}$; and
- (WR) = average access rate of users, feet per hour.

These economic functions also consider the reduction of demand, related directly to increased distance from the terminal, as a function of trip purpose at point i . The greatest personal mobility that can be economically afforded is provided.

After the service area is specified and the best terminal location is found, the optimum terminal capacity remains to be determined. The capacity problem is essentially that of predicting the duration of terminal users as well as the cyclical nature of demand. The optimum capacity must minimize the total cost—the cost for users having to wait to enter the facility when it is filled to capacity, the cost of providing extra terminal spaces, and the cost incurred by the facility for the lost demand. To prevent the waiting time from becoming intolerable requires that the terminal capacity be great enough to accommodate the peak demand. However, from the economic point of view, the terminal could probably permit some waiting and thereby reduce investment requirements. These facts are important in actual applications.

The terminal capacity may be initially determined on the basis of maximizing service to the peak-demand accumulation. The terminal cost is then traded off with the cost of waiting. Because the distributions of available peak demand and the distance-reduction factor for each destination point are known, the distribution of the attracted peak demand for the terminal can be found either by statistical methods or by simulation.

COMPUTER ANALYSIS

Based on the general optimization concept, a computer program was developed with the objective that it be easily applied to the optimization of an actual terminal network. To promote adaptation to any particular case, the program was written in modular form so that each subroutine performs only one function. An example of the flow of operation within the computer program is shown in Figure 1. A microscopic presentation of the logic involved is prohibited by the length of the program.

The first data set consists of the means and standard deviation of the distributions of the characteristics of demand for each point of destination. The second data set includes the X and Y coordinates (in feet) of the street-block corners. Square and rectangular block configurations with parallel streets are considered, although other configurations could be used with only minor modification to the program. The last data set consists of the X and Y coordinates of the prospective terminal locations to be analyzed.

One block is processed side at a time to determine the S and Y coordinates of points of destination. The number of points per block side are randomly assigned, and a number is assigned to each point that relates to the set of demand characteristics at that point. For simplicity, only one demand point at the center of each block side was considered to represent the expected location of all points on the block side. In a realistic application, actual values of building locations may replace these estimates. Exact locations and demand characteristics of all points in the city could be resolved by the program, but it may be uneconomical to gather data in this detail. This block-side composite has the advantage, however, of smoothing variances in the demand characteristics along a block side.

For a given terminal location within a defined service area, a subroutine determines the service area increment for which each point of destination falls. An increment number is assigned to each point. This identifier, combined with the X and Y coordinates and the demand characteristic identifier, completely defines each point of destination.

The average and total rectilinear access distances, the access cost, the terminal cost to accommodate the demand attracted, the penalty charge for lost demand, and the total cost are calculated for each service area increment. The appropriate distance-reduction factor is used to determine the demand attracted for the actual distance to each point of destination. Another subroutine was set up to calculate the cumulative value of each of these variables through each service area size. At any terminal location, the optimum size of corresponding service area is determined.

Within any defined set of service area boundaries, a program section uses Golden Section Searches in two city-street directions to determine the optimum terminal location required to service the area demand at minimum total cost.

For any given terminal location, a simulation program is applied to find the optimum terminal capacity. A subroutine is included to select random variables from the (assumed) normal distribution of each of the demand characteristics for each simulation trial. After a large number of trials, the distribution function of peak demand can be developed. This function indicates the necessary capacity of the optimum terminal unit.

OPTIMIZATION BY COMPUTER APPLICATION

The optimization steps carried out by the developed computer program were used to analyze an urban parking terminal network as an example application. For study purposes, several important assumptions were made in an attempt to represent the actual urban parking situation. It is important to first review these assumptions to gain a more thorough understanding of the resultant network. Final decision can then be made with a full understanding of the network so that the objectives of the parking facility system are fully realized. The assumptions on which this example analysis was based are as follows:

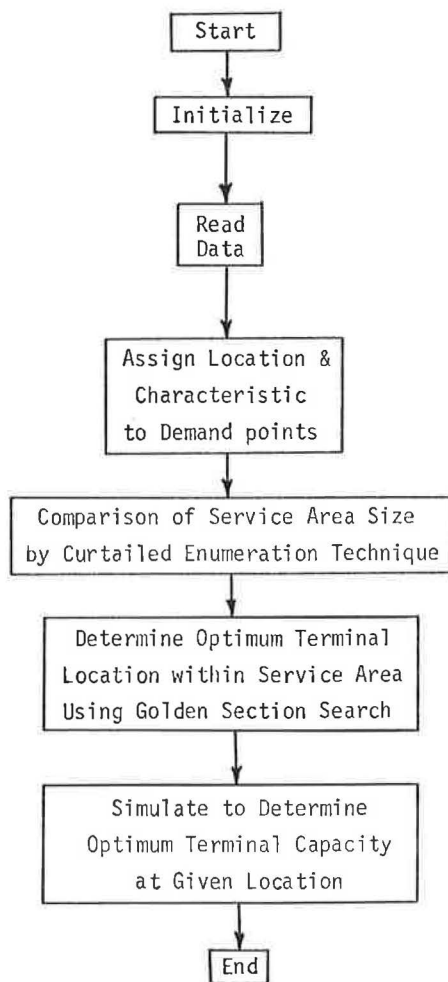


Figure 1. Computer flow diagram.

1. As previously indicated, the rectilinear walking paths are considered because the grid street pattern is predominant, with very few exceptions, in American cities. Also, it was assumed that a terminal user always used the terminal nearest his origin or destination.

2. It was assumed that total daily demand at any origin and destination point followed a normal distribution. Also, points of similar demand (level and characteristics) tend to aggregate. The term "uniform demand distribution" used implies equivalence among those normal demand distributions.

3. The trip purpose included work, shop, business, and others. Because demand for transportation service varies stochastically as a function of trip purpose, the study considered this differentiation in demand by treating demand characteristics of trip purpose. A short-duration trip category representing part of each of the four trip purposes was used because the short-duration parker, who is willing to walk only small distances, has a volatile effect on system optimization.

4. A walking rate of 15,000 ft per hour or 4 fps (ft per sec) was considered to be representative and was used in this analysis (3). The cost of walking time is a subjective value depending on the trip purpose of parkers.

5. It was assumed that all available demand was attracted within a walking distance of 200 ft and that, beyond this distance, the demand reduction was linear to zero at some distance depending on trip purpose (7, 13).

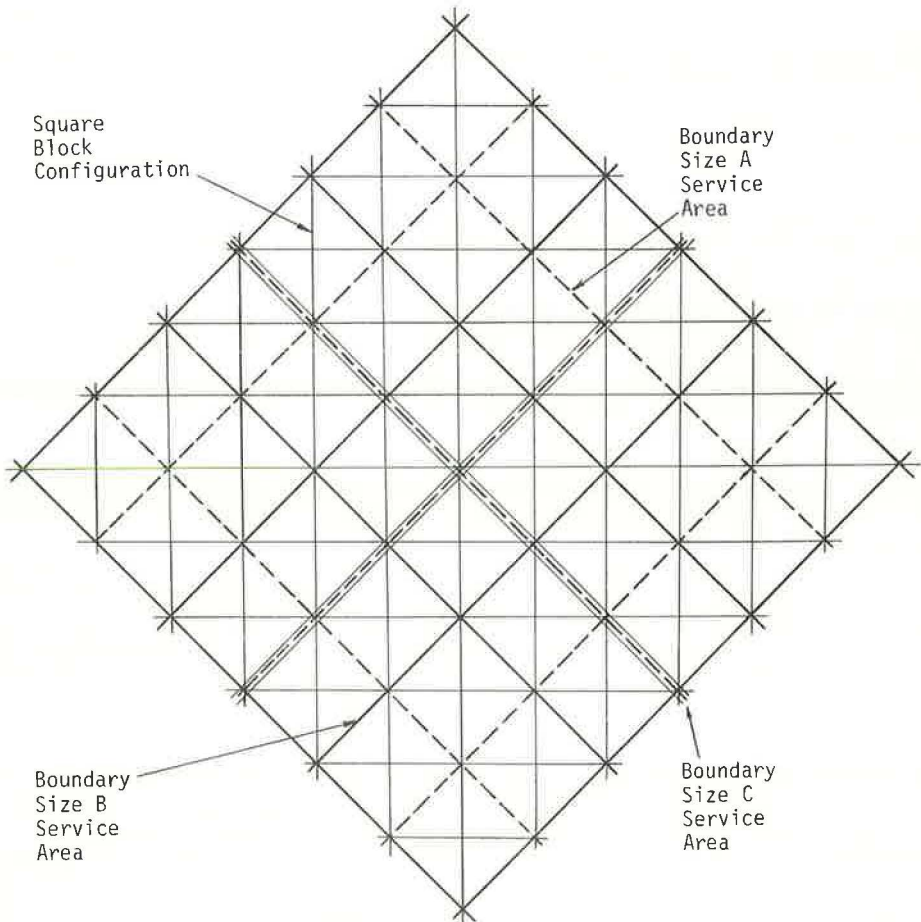


Figure 2. Comparison of equivalent service areas.

6. Four-hundred foot square street blocks were used with a parallel 50-ft wide street, because these dimensions were found to be typical in American cities (23). The size of an individual service area larger than three blocks in the distance from the service area center to the vertex was assumed to be nonfeasible, because the maximum desirable walking distance of 1,250 ft was used as the upper limit. The configuration of three possible sizes of the service area is shown in Figure 2. This represents the smallest equivalent area that can be covered by all feasible service area sizes.

Optimization of Terminal Density

As indicated previously, determining the optimum terminal density is essentially a resolution of the most economic size of service area. All three service area sizes ought to be compared on an equivalent-area basis in an attempt to minimize the total cost. As shown in Figure 2, 36 size A areas are equivalent to 9 size B areas and 4 size C areas. In fact, these configurations provide the only consistent basis for comparing possible service area sizes, because the area served and the available demand served are the same for all service area sizes. If this basis of comparison is not possible in an actual case, comparison becomes somewhat more abstract and could include a per-area or a per-demand attracted ratio of the total costs involved. Total costs associated with locating a terminal at the center of a service area and servicing areas of size A, B, and C were calculated by the computer program. Several runs were then made, changing the terminal location to other possible points along the block side and also shifting the service area pattern so that the terminal remained at the center of the pattern. Each of these "sensitivity" runs produced total costs greater than the cost originally calculated for each respective service area size. This confirmed the prediction that service areas should be centered at intersections. The effect of the reduction of demand as a function of distance is shown in Figure 3. This figure shows the average walking distance of attracted users for each service area size.

The network analysis needs to consider only sets of intersection points (the centers of the service areas) that have the potential of providing service to the entire section. For example, only two sets of size A service areas are possible. Total cost and other criteria may be calculated for one set of intersection points (size A service areas) that provides service to the entire locality. Then, the network pattern is shifted by one intersection distance in any direction and the complement of the first pattern is obtained. A comparison of the total costs of these two cases (collectively exhaustive for the size A service area) would indicate the best set of points for the centers of size A service areas. Similarly, only a few sets of size B and size C intersection points exist, and the best size B and best size C sets could be easily found. Because of the area encompassed by these larger sizes, the physical geometry of the section may serve to curtail some of the potentially possible sets.

Experiments made during this study, by using hypothetical data, indicated that comparison of complete sets may not be necessary. For instance, only a minor improvement can be obtained by shifting to the second possible set of size A service areas. Therefore, it may be possible to distinguish the optimum service area size within a section by comparing only one size A area with one size B area and one size C area. After selecting the best size, the best set of service areas of that size would be found by enumeration of only a few possible sets.

Table 1 gives the results of the computer solution to the network analysis. On the basis of total cost, average walking distance, user walking cost, and percentage of available demand attracted, the size A configuration was found superior in comparison to both the size B and the size C configurations. On the basis of terminal cost, the size C configuration was found superior but to the detriment of these other criteria.

The total cost for a given configuration is a function of the subjectively assigned penalty rate. This total cost function for the three configurations is shown in Figure 4. For penalty rates less than \$0.19, the size C configuration has the lowest total cost, while the size A configuration has the lowest total cost for penalty rates greater than \$0.19. The penalty rate should be selected to force the desired level of effectiveness. Figure 5 shows functional relationships of all of these factors versus terminal density.

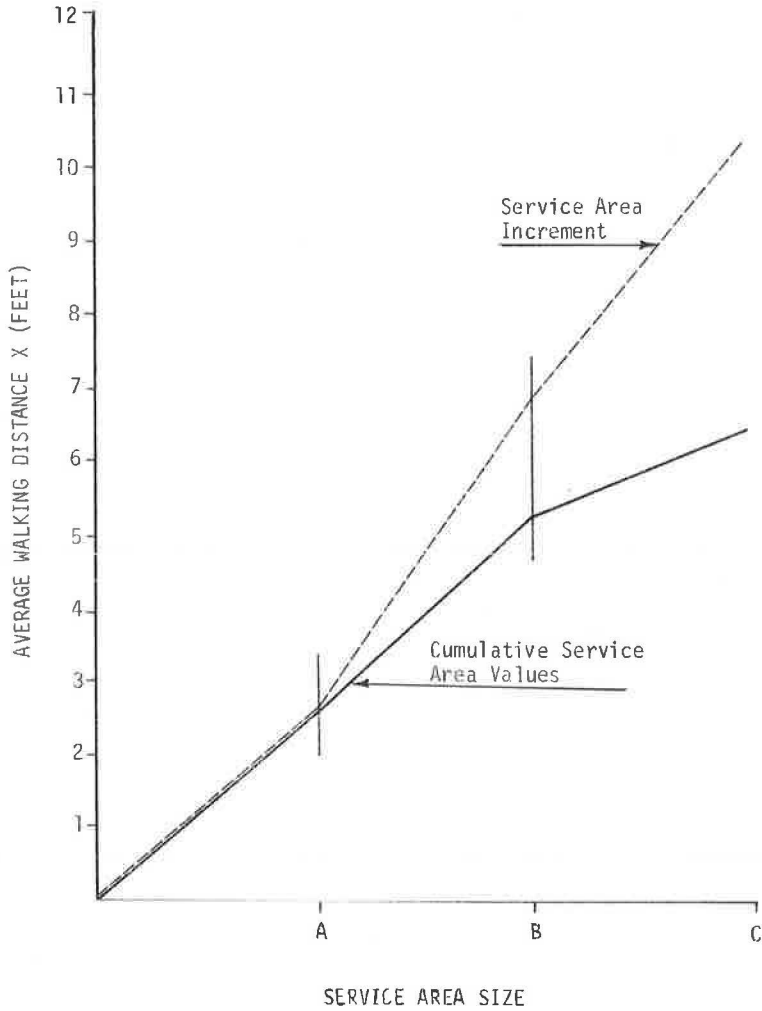


Figure 3. Average walking distance by service area.

TABLE 1
ECONOMICAL COMPARISON OF SERVICE AREA SIZE EQUIVALENCY

Service Area Equivalency		Average Walking Distance (ft)	Available Demand Attracted (percent)	Terminal Cost (dollars)	Walking Cost (dollars)	Penalty Cost (dollars)	Total Cost (dollars)
Number	Size						
36	A	250	79	1,458	720	561	2,729 ^a
9	B	524	54	999	1,053	1,159	3,211
4	C	647	30	557	724	1,763	3,044

^aOptimum.

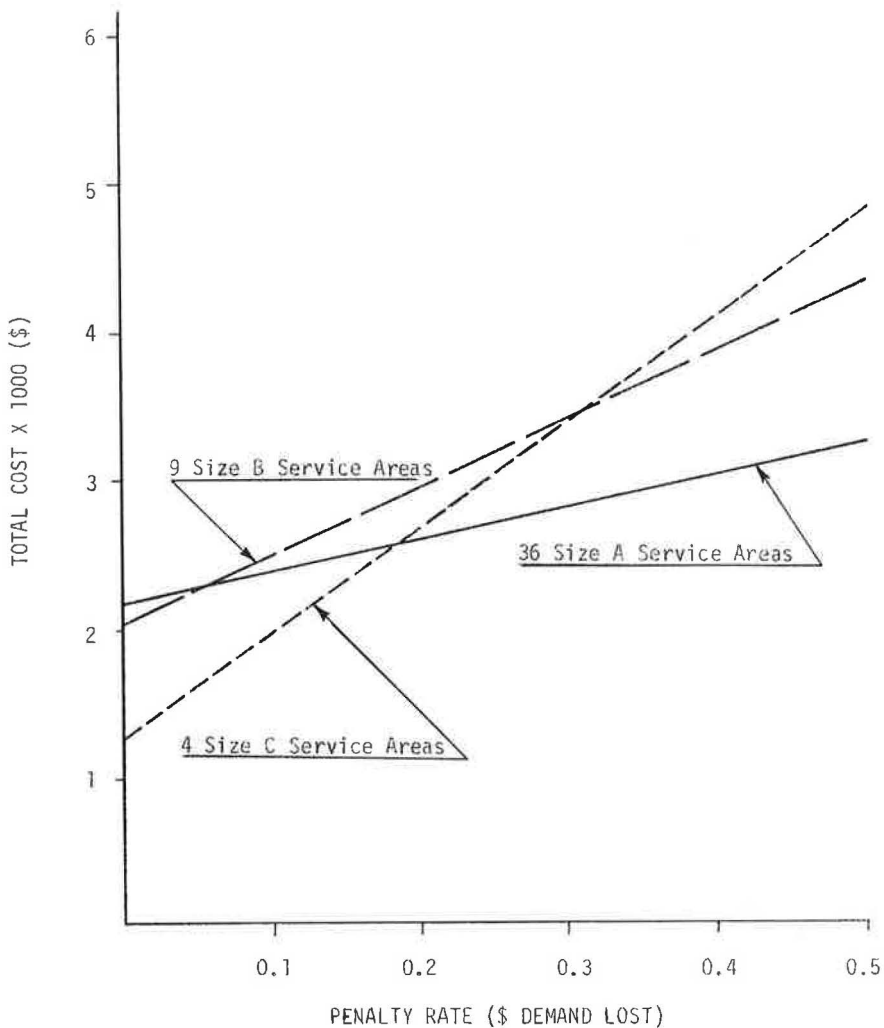


Figure 4. Effect of various penalty rates.

Optimization of Terminal Location

First, the uniform demand case was analyzed by the computer program. Input data consisted of the following demand characteristics:

<u>Characteristics</u>	<u>Mean</u>	<u>Standard Deviation</u>
Total average available, daily demand	35.00	2.70
Peak available, daily demand	14.00	0.86
Distance-reduction factor	1.30	0.05
Cost of walking (dollars per hour)	2.80	0.15

The mean and standard deviation of each of these parameters were used to describe the respective normal distributions. The distance-reduction factor reflects the fact that people are less willing to walk longer distances to use a mode, because other modes exert more competitive influence by providing shorter distances. Because the demand

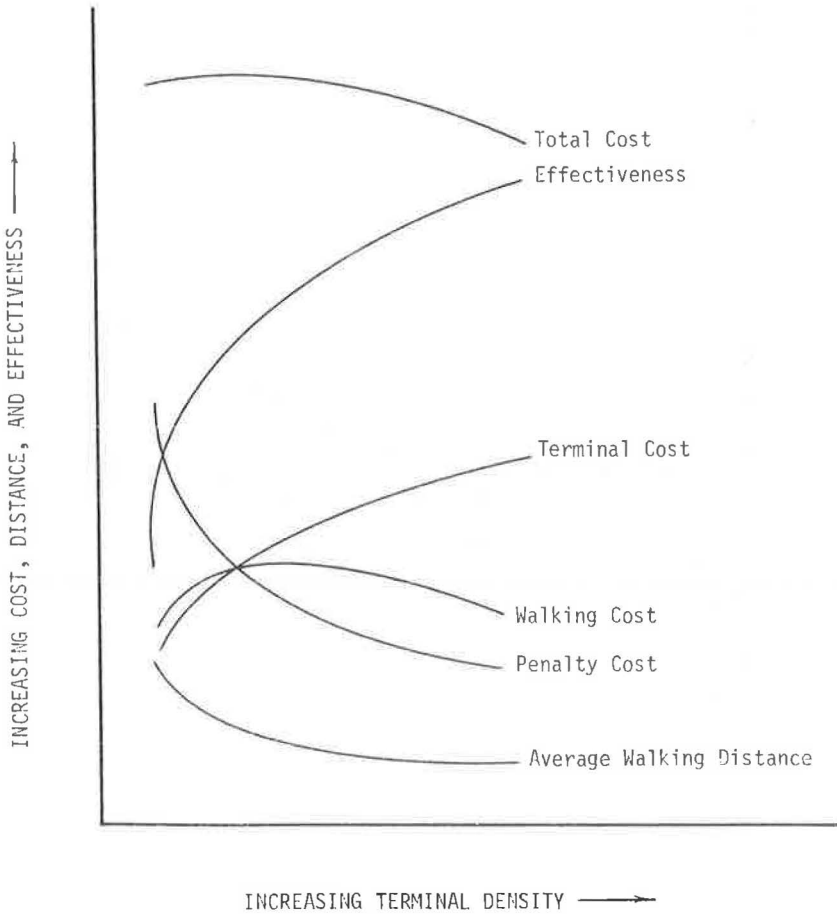


Figure 5. Criteria as a function of terminal density.

within the service area is normally distributed, the terminal must be located at the center of the service area to minimize the average and total walking distances for all users. However, if the traffic congestion or land availability limit a central location, a small displacement of the terminal from the center is possible without changing the average and total walking distances (22).

If the demand within a defined service area is nonuniform (nonhomogeneous), then the optimum terminal location is not at the service area center. The case reflecting practical conditions—the mean values of the distributions of the demand characteristics vary from point to point—was treated next. It was assumed that each demand point represented a certain trip purpose, rather than a composite of purposes, and acted at the middle point of block sides. The trip purpose assigned to any destination point was assigned in a random fashion, and the percentages of each purpose are given in Table 2 along with the demand characteristics assumed.

Because the terminal was constrained to lie along the outer periphery of blocks, determination of the optimum terminal location was accomplished by two unidimensional Golden Section Searches. This search was limited in each direction to within 200 ft of the intersection, and the total cost objective function was minimized. Next, the best costs found in each direction were compared, and the better of the two was selected. This

TABLE 2
DEMAND CHARACTERISTICS

Trip Purpose	Percent of Total Demand Points	Total Demand (daily)		Peak Demand (daily)		Distance-Reduction Factor		Cost of Walking (dollars/hour)	
		μ	σ	μ	σ	μ	σ	μ	σ
Shop	40	60	5.0	15	1.5	0.8	0.04	2.0	0.15
Business	25	25	1.0	5	0.5	1.0	0.05	4.0	0.10
Work	20	20	2.0	15	1.5	1.6	0.15	3.0	0.25
Miscellaneous	10	35	2.7	14	0.9	1.3	0.05	2.8	0.15
Short duration, all purposes	5	65	1.0	4	0.5	0.4	0.02	3.5	0.05

Note: Input data for a nonuniform stochastic demand case. μ = mean and σ = standard deviation.

location was at the center of the street, and the best side of the street was taken as the side favored by the search along the other axis direction.

Results of this nonuniform demand case are given in Table 3. In addition, it was indicated that the optimum service area should be centered at an intersection, but that the unique optimum location of the terminal must be found in accordance with the characteristics of demand and location of destination points within the service area. The optimum terminal tends to be displaced toward points of greater demand.

Optimization of Terminal Capacity

The optimum capacity of the terminal under the known terminal location and density can be determined through computer simulation and economic analysis. The attracted demand from each destination point may reach a peak at different times; dependence between the component peak demand distributions can be treated with simple algebraic functions.

After the peak attracted-demand density function was defined, it was then necessary to specify the corresponding terminal capacity. The terminal capacity that would serve virtually all daily peak demands under the condition of no waiting to enter the terminal would be the $\mu + 3\sigma$ value of the density function. However, this capacity would result in inefficient use of some spaces because they would be used very infrequently. If the facility operates at some utilization below 100 percent, capacity must be increased to consider this inefficiency. Automobile parking places become more difficult to find when facilities are nearly full, so lots and garages operate at 85 or 90 percent utilization (23).

For the uniform demand case given in Table 4, the standard deviation of the total peak attracted-demand distribution for independent component demand distributions was about 2.0, while a case assuming complete dependency among the component distributions yielded a standard deviation of 6.24. Adequate description of the correlation between

TABLE 3
OPTIMUM LOCATION OF TERMINAL

Service Area Size	Number of Demand Points	Total Demand Attracted	Available Demand Attracted (percent)	Average Walking Distance (ft)	Total Walking Distance \times 1,000 (ft)	Expected Terminal Capacity (spaces)	Penalty Cost (dollars)	Terminal Cost (dollars)	Walking Cost (dollars)	Total Cost (dollars)
A	8	168	76	250	42.1	86	13.00	38.50	16.10	68.60
B	32	389	36	495	192.9	177	175.10	81.50	74.87	331.47
C	72	435	16	561	243.9	208	567.00	95.70	95.91	758.61
A ^a	8	173	79	249	43.1	84	11.73	38.71	17.55	67.89

Note: Nonuniform stochastic demand case.

^aOptimum terminal location for size A service area: X coordinate = 2,700, Y coordinate = 2,482.

TABLE 4
OPTIMUM CAPACITY OF TERMINAL

Service Area Size	Number of Demand Points	Total Demand Attracted	Available Demand Attracted (percent)	Average Walking Distance (ft)	Total Walking Distance × 1,000 (ft)	Expected Terminal Capacity (spaces)	Penalty Cost (dollars)	Terminal Cost (dollars)	Walking Cost (dollars)	Total Cost (dollars)
A	8	221	73	250	55.1	88	14.76	40.00	21.05	75.81
B	32	602	54	524	315.6	241	129.80	111.10	115.91	356.81
C	72	757	30	647	490.2	303	441.00	139.20	180.85	761.05
A ^a	8	221	79	250	55.1	88	14.76	40.00	21.05	75.81

Note: Uniform stochastic demand case.

^aOptimum terminal capacity for size A service area: mean spaces = 88, standard deviation = 1.99.

component demand distributions is seen to be critical in determining the optimum terminal capacity.

SUMMARY AND CONCLUSIONS

The objective function advocated by this study was to provide an urban terminal network with minimum total cost. The specific contribution of this study is an approach that yields a solution technique meaningful in actual applications. Total cost was defined as the sum of walking cost for the total attracted daily demand plus the cost to provide terminal space to efficiently serve most attracted daily demand plus a theoretical penalty charge for available demand that is not attracted because of the relative length of the terminal-to-destination distance.

To specify the optimum terminal density, a curtailed enumeration method was presented to define the optimum set of service areas. The approach was based on the assumption that various sections within an urban area may be characterized by nearly uniform demand. The assumption of uniform, or equal, means of the normal distributions of the respective demand characteristics at all destination points within an urban section appeared to adequately represent the demand pattern and demonstrated the assumption that points of similar demand tend to aggregate. Also, if these expected values are indeed not uniform, it may be uneconomical or impossible to characterize the actual demand at each destination point by in-depth studies. All efforts should be made to gather the data on which the network is to be based; but a practical and economical limitation is reached somewhere, and the true idiosyncrasies of the actual demand may never be known.

It is quite evident that minimizing the terminal access distance tends to favor smaller service areas for all terminals in the network. The cost of providing terminal facilities, however, would increase greatly under such a criterion. Conversely, on the basis of minimizing capital investment, no terminals would be specified. The capital investment concept could be used in an incremental analysis in which the incremental investment required for a smaller service area is compared with the incremental improvement in effectiveness or cost of walking. An incremental analysis that subjectively compares the utility of the effectiveness and total cost for various, feasible service area increments is possible. The desired level of effectiveness must be predetermined and a subjective method of comparing cost to effectiveness must be determined to use this cost-effectiveness approach.

Within each of these specified service areas, further improvements in total cost were obtained by finding the best terminal location to serve the demand peculiar to the service area. Two Golden Section Searches were used to find this optimum terminal location within each service area. In an actual case, land availability may prove to be a limiting factor. City zoning and other restrictions may hamper location of the terminals at the locations and in the numbers specified. If land is not available at the locations specified by the solution techniques, use of nearby land must be analyzed to determine the effect on the mathematically optimized system. Use of nearby land available at lower cost than at the specified location must also be analyzed in this respect. The

truly optimum solution must consider these practical limitations so that the "optimum" as defined mathematically may be adjusted to a practical optimum, the best solution possible under the prevailing conditions.

The terminal capacity required to most efficiently serve demand is important to applications such as automobile parking facilities. Although the terminal must serve the total daily demand, the capacity of the terminal must be just large enough to efficiently accommodate the peak attracted demand. Because the terminal capacity must be sufficiently large to serve the defined service area for a number of years, planned changes in the demand structure should be included in the analysis. Analysis of the probability of certain changes and the probable effects on the terminal network should be necessary to determine the long-run adequacy of the terminal capacity. In this study, an economic trade-off analysis by computer simulation combined with the statistical theory was used to determine the optimum terminal capacity.

An application of the developed solution technique to a type of transportation terminal including automobile parking was described to indicate the ability of the method to resolve an actual case. Although the technique shows promise for use in the optimization of the transportation terminal network, the technique is dependent on the accuracy of the data that describe the urban terminal environment. It is, therefore, recommended that a complete set of actual data be carefully gathered to further test the suggested solution method.

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