

# THE EFFECT OF DISCRETE TIME MEASUREMENT ON SPEED DATA

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Time series measurements of vehicle speeds are often performed by measuring the travel time of each vehicle between 2 given points in units of pulse counts from a pulse generator of known frequency. The travel time is then measured in discrete equidistant values, but 2 discrete values are possible for each actual travel time because of end effects in the counting technique. Such a measurement technique effects a transformation of the continuous spectrum of actual vehicle speeds into a discrete spectrum of observed speed values. Because speed is inversely proportional to the measured time, the separation of the discrete speed values increases hyperbolically with increasing speeds. A computer program was developed to simulate such a measurement technique. The data simulated by this program were used for a statistical analysis to quantify the errors and special effects of such a measurement technique on selected distributions of speeds, distributions of relative speeds, and space parameters computed from the time series data. Among the results of this analysis are that the effects of the measurement technique can generally be ignored for speed distributions, if a certain set of intervals is used for classifying the speed values, but they cannot be made negligible for relative speed distributions.

•**TIME SERIES** measurements of vehicle speeds at a given highway location are often performed by measuring the travel time of each vehicle between 2 points that are a small, accurately known distance apart. The travel time is usually measured by electronically counting the number of pulses from a continuously running pulse generator of known frequency. The time is thus measured in discrete, equidistant values, but 2 such discrete values are possible for each actual travel time because of end effects in the pulse-counting technique. Because speed is inversely proportional to the measured time, the separation of the resulting discrete speed values increases hyperbolically for increasing speed values. Such a measurement technique effects a transformation of the continuous spectrum of actual vehicle speeds into a discrete spectrum of observed speeds with certain probabilities for the assignment of discrete values to each actual value.

Because the observed speed values are used in many inferences about traffic flow characteristics, it is important to know the effects of such measurement techniques on speed data. To quantify these effects, one would ideally like to compare known actual speed distributions for a highway with the corresponding speed distributions that are measured on the highway by this technique. In reality, however, only measured speed distributions are available. Unfortunately, it is not possible to reconstruct accurately the actual from the observed speed distributions; therefore, the use of real observed data is insufficient for such a study. To circumvent this problem, refuge was taken in the use of a simulation technique.

A computer program was developed to simulate the measurement technique, and a statistical analysis was performed to quantify the errors and special effects introduced by this measurement technique into speed data such as the distribution of speeds, the

distribution of relative speeds, and space parameters computed from the time series data. Because not all the various implementations in practice of such a measurement technique could be anticipated, this study was performed by using the traffic analyzer of the Federal Highway Administration as a typical example. Even though the results from studying this example may not apply quantitatively to some other implementations of such a measurement technique, their qualitative features apply in any case.

DESCRIPTION OF THE MEASUREMENT TECHNIQUE

A typical setup for the measurement of vehicle speeds is the so-called speed trap that consists of 2 vehicle detectors positioned a distance L apart in the same lane. As a vehicle passes the first detector, it activates a counter that then starts to record the number of pulses from a continuously running generator of square wave pulses. When the vehicle passes the second detector, the counter is deactivated, and the accumulated number of pulses is recorded as the so-called speed code. The speed code is thus a measure of the travel time of a vehicle through the speed trap of length L. Any fraction of a square wave pulse that may be created at the time of the counter activation or deactivation is counted as a full pulse. As a consequence, the speed code observed for a given vehicle travel time varies as a function of the relative timing between the moment of counter activation and the sequence of square wave pulses.

This will be demonstrated for the case of the traffic analyzer, which generates square wave pulses with a time period of 0.01 sec and a duty cycle of 50 percent, i.e., the time length of each pulse equals one-half of the time period. If, for example, the travel time of a vehicle through the speed trap happens to equal exactly 2 times the time period of the square wave pulse, speed codes 2 and 3 are possible (Fig. 1). Assuming that the activation points occur at random, codes 2 and 3 are observed with probability of 0.5 each. If the travel time equals exactly 2.5 times the period of square wave pulses, the probability of observing a speed code 2 is 0, but it is 1 for observing a speed code of 3.

For any value of actual travel time, Figure 2 shows the probability of obtaining a particular speed code. Figure 2 applies, however, only to cases in which the arrival times of successive vehicles are independent of each other. For the case of dependent arrival times of successive vehicles, a dependency would be introduced into the speed codes for each vehicle, because the first pulse from each successive vehicle would exhibit a certain predictable timing with respect to the series of generated square wave pulses.

In real traffic, of course, time headways are dependent because of the interaction between vehicles that are traveling relatively close to each other in the same lane. Such a dependency is of the order of seconds (2).

In order to obtain a dependency of observed speed codes for successive vehicles, however, the dependency between successive arrival times has to be of the order of 0.01 sec. A dependency in the order of seconds does not introduce any dependency

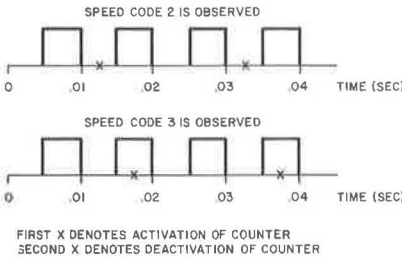


Figure 1. Demonstration example. (For a given car travel time of 0.20 sec through the speed trap, 2 different speed codes are possible depending on the relative timing between the square wave pulses and the pulses from the speed trap.)

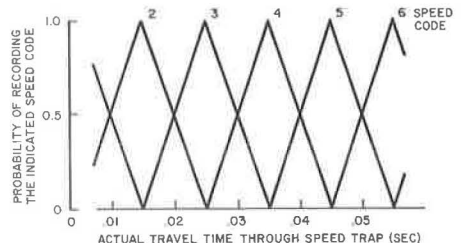


Figure 2. Probability function of observing the indicated speed codes.

in the digits of the order of 0.01 sec in the time headway. With respect to the square wave pulses generated in the traffic analyzer, arrival times can, therefore, be considered completely independent of each other. Then, Figure 2 shows the probability of observing a certain speed code for each vehicle on the highway.

Figure 2 shows that, for a recorded speed code  $U$ , the actual travel time of the vehicle through the speed trap must have been in the range from  $[U - (\frac{1}{2})] 0.01$  sec to  $[U + (\frac{1}{2})] 0.01$  sec. The midpoint of this interval is  $[U - (\frac{1}{2})] 0.01$  sec, and the corresponding vehicle speed is

$$v = \frac{L}{[U - (\frac{1}{2})] 0.01 \text{ sec}} \quad (1)$$

The speeds computed from consecutive speed codes by Eq. 1 form a series of discrete values. The separation of these speed values increases hyperbolically with increasing values of speed.

To permit a study of the effects of this measurement technique on speed data, a computer program was developed that simulates this measurement technique. This program samples actual speeds from a specified speed probability density. For each actual speed value, the actual travel time through the speed trap is computed, and a speed code is found with the probabilities shown in Figure 1. Equation 1 is then used to compute the value of the observed speed. A detailed description of this computer program is presented in another report (1).

#### THE EFFECT OF THE MEASUREMENT TECHNIQUE ON SPEED DISTRIBUTIONS

In the following, the effect of the measurement technique on realistic distributions of vehicle speeds on a highway is studied. It has been shown for a highway with 2 lanes one-way (2) that the measured speed distributions can be approximated quite well by normal distributions and that the standard deviations of these distributions are small relative to the mean. The effect of the speed transformation was, therefore, studied for such normal speed distributions and subsequently for relative speed distributions and space parameters derived from these normal speed distributions.

##### Comparison of Actual and Observed Distributions

From the measured speed distributions (2) the distribution  $N(52.3; 22.0)$  for the fast lane and the case of light traffic has been selected for this study, where  $N(x;y)$  is a normal probability density of mean  $x$  and variance  $y$ . In order to study the effect of the measurement technique as a function of different variances, the distribution  $N(50.0; 49.0)$  was also selected for this study. This distribution approximates the speed distribution measured for vehicles in lanes 1 and 2.

From each of these normal distributions, 100,000 statistically independent speed values were sampled by the first part of the computer program developed for this analysis. These values were designated as actual speeds. The second part of this computer program then transformed these speeds into the observed speeds by simulating the effect of the traffic analyzer.

For each of these normal distributions, the actual speeds thus obtained were classified into speed intervals, and the actual speed distributions

$$s_i = m_i / MI_i \quad i = 1, 2, 3, \dots \quad (2)$$

were formed, where  $m_i$  is the number of actual speed values in the  $i$ th speed interval of length  $I_i$ , and  $M$  is the total number of generated speeds, which is 100,000 in all cases. Division by the interval length  $I_i$  normalizes the histogram area to 1. The observed speed distributions,  $s_i$ ,  $i = 1, 2, 3, \dots$ , were formed in analogy to Eq. 2.

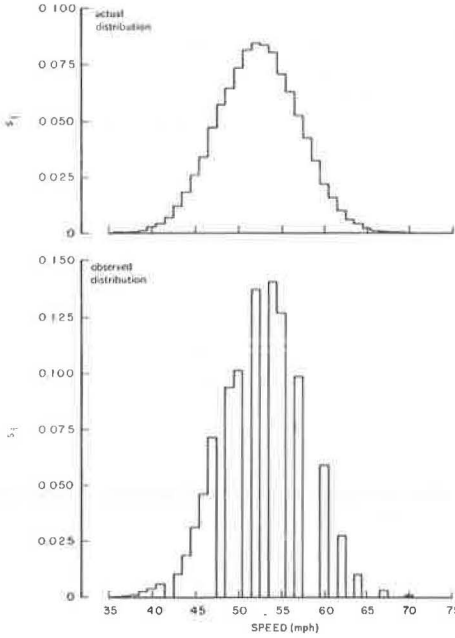


Figure 3. Speed distribution with speed intervals of constant width,  $N(52.3; 22.0)$ , and speed trap length of 24 ft.

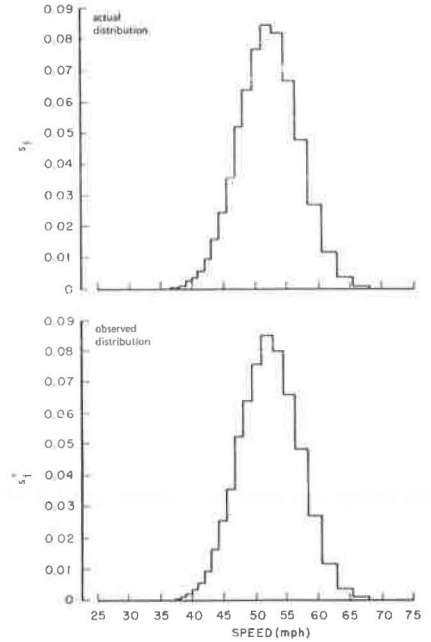


Figure 4. Speed distribution with speed intervals of varying width,  $N(52.3; 22.0)$ , and speed trap length of 24 ft.

Figure 3 shows the resulting distributions for  $N(52.3; 22.0)$ , for the case of  $I_i = 1$  mph for all  $i$ , and a speed trap length of 24 ft. This figure shows that many speed intervals that contain actual speed values do not contain any observed speed values, whereas others contain much more observed than actual speed values, and the dissimilarity between the actual and observed speed distributions grows with increasing speeds. This is due to the fact that the observed speed values are discrete and their separation increases hyperbolically with increasing speeds.

In order to account for the hyperbolically increasing separation between the discrete observed speed values, the same distribution is therefore presented classified into the variable intervals

$$\left[ \frac{L}{U(0.01 \text{ sec})} \times \frac{L}{(U-1)(0.01 \text{ sec})} \right] \quad (3)$$

around the discrete speed values given by Eq. 1. The result is shown in Figure 4. In this case, the observed distribution simulates the actual distribution so much more accurately that it was decided to present all succeeding speed distributions with this variable interval size.

Figure 5 shows actual and observed distributions for  $N(50.0; 49.0)$  for the case of a speed trap length of 24 ft. The influence of the speed trap length on an observed speed distribution was studied by obtaining the observed speed distribution for a speed trap length of 30 ft for  $N(52.3; 22.0)$ . The result is shown in Figure 6.

Visual comparison of the actual and observed distributions in Figure 4, 5, and 6 shows a good agreement. This was quantified by performing a chi-square test for each of these cases. The chi-square statistics are given by

$$\chi^2 = \sum_i \frac{(m_i - m'_i)^2}{m_i} \quad (4)$$

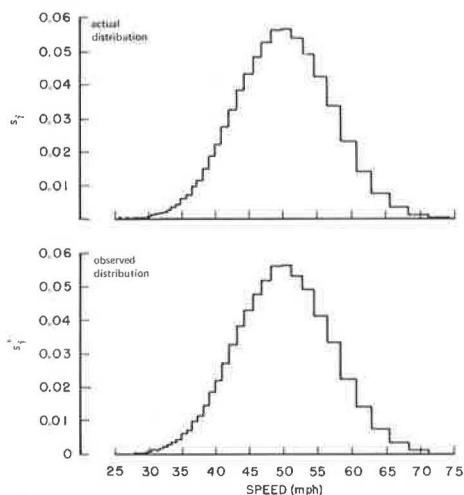


Figure 5. Speed distribution with speed intervals of varying width,  $N(50.0; 49.0)$ , and speed trap length of 24 ft.

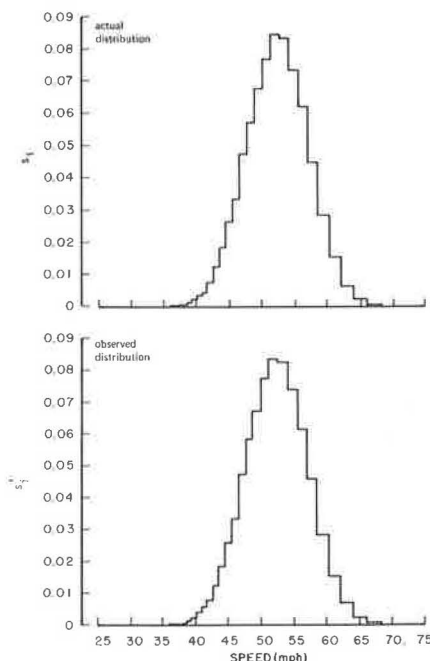


Figure 6. Speed distribution with speed intervals of varying width,  $N(52.3; 22.0)$ , and speed trap length of 30 ft.

where  $m_i$  and  $m_j$  are the numbers of actual and observed speed values in the  $i$ th speed interval, and the summation is to be made over all speed intervals considered. In each speed interval used for this test, the number of vehicle pairs was well above six, so that the chi-square statistics apply. The number of degrees of freedom has been assumed as one less than the number of speed intervals used for this test. The results are given in Table 1.

With these results, the hypothesis that there is no effect of the measurement technique on the speed distributions in Figures 4, 5, and 6 cannot be rejected by chi-square tests taken at the 10 percent significance level (and thus cannot be rejected, of course, at the more usual 5 percent and 1 percent significance levels).

#### Mean and Variance of Actual and Observed Distributions

After this comparison of the shapes of actual and observed speed distributions, this section discusses the effect of the measurement technique on the mean and variance of the speed distributions. In order to include a speed distribution with a significantly different mean from the distributions studied in the foregoing, the speed distribution  $N(39.6; 21.6)$  for the fast lane in the case of fairly heavy flow (2) was added to the subsequent study. The influence of both different mean with equal variance and different variance with equal mean could be evaluated.

TABLE 1  
CHI-SQUARE TEST FOR DIFFERENCE BETWEEN  
ACTUAL AND OBSERVED SPEED DISTRIBUTIONS

Distribution	Trap Length (ft)	Speed Interval	Computed $\chi^2$	Degrees of Freedom	Tabulated $\chi^2_{0.10}$
$N(52.3; 22.0)$	24	Variable	13.1	19	27.2
$N(50.0; 49.0)$	24	Variable	31.5	42	54.1
$N(52.3; 22.0)$	30	Variable	20.0	26	35.6

TABLE 2  
MEAN AND VARIANCE OF SPEED DISTRIBUTIONS

Distribution	Actual or Observed	Mean	Standard Deviation of Mean	Variance	Standard Deviation of Variance
N(52.3;22.0)	Actual	52.3590	0.216	21.94	1.35
	Observed	52.3686	0.220	22.45	1.40
N(39.6;21.6)	Actual	39.5788	0.214	21.57	1.33
	Observed	39.5819	0.215	21.73	1.35
N(50.0;49.0)	Actual	50.0434	0.322	48.88	3.02
	Observed	50.0517	0.327	49.31	3.08

The 100,000 speed values that were sampled from each assumed normal distribution using a 24-ft trap length were subdivided into 200 subsamples of 500 speed values each. The mean and the variance were computed for each subsample. These obtained values were then used to compute the mean of the entire sample, the variance of this mean, the mean variance, and the variance of this variance.

The values that resulted for each of the 3 normal distributions considered in this report are given in Table 2. To these results, the Wilcoxon Signed-Rank Test was applied to test the hypothesis that there is no difference between the means of the 3 actual and observed speed distributions. The alternative hypothesis is that the observed values are larger than the actual values. For the 3 assumed speed distributions, the actual values were subtracted from the observed values. The 3 resulting differences were ranked according to their absolute values, and the sign of the difference was attached to the ranks. The sum of the positive ranks,  $T_3$ , is computed. The probability  $\Pr [T_3 \leq a]$ , where  $a$  is the computed value of  $T_3$ , is then taken from the table (3). The hypothesis is rejected if  $\Pr [T_3 \leq a] \geq 1 - \alpha$ , where  $\alpha$  is the significance level for the test.

This test was then applied also to the variances of the distributions. The results of this test are given in Table 3. They indicate that, for any chosen significance level, both means and variances of the observed speed distributions are larger than the corresponding values for the actual speed distributions. However, the differences in the means appear to be so small that they can generally be ignored for all practical purposes.

#### Space Parameters Computed From Time Series Data

The data from the traffic analyzer are time data; i.e., they are taken at a fixed location on the highway over a certain period of time. In contrast to such data are space data; i.e., data that are taken at a fixed moment of time over a certain stretch of highway. (Aerial photography is one method that yields space data.) Sometimes the mean and the variance of the speed distribution in space are of interest, but only time data are available. Then, use is often made of the relationship (4)

$$f_s(v) = (\bar{v}_s/v) f_t(v) \quad (5)$$

where  $f_t(v)$  is the speed probability density in time,  $f_s(v)$  is the speed probability density in space, and  $\bar{v}_s$  is the space mean speed.

Integrating Eq. 5 over all speeds and solving for  $\bar{v}_s$  yield the equation

$$\bar{v}_s = \frac{1}{\int_0^{\infty} \{ [f_t(v)]/v \} dv} \quad (6)$$

By the use of this equation, the space mean speed can be computed as the inverse

TABLE 3  
WILCOXON SIGNED-RANK TEST OF RESULTS  
IN TABLE 2

Observed Minus Actual Mean	Signed Rank	Observed Minus Actual Variance	Signed Rank
+ 0.0096	+ 3	+ 0.51	+ 3
+ 0.0031	+ 1	+ 0.16	+ 1
+ 0.0083	+ 2	+ 0.43	+ 2
a = sum + = 6 Pr { $T_3 \leq 6$ } = 1.000		a = sum + = 6 Pr { $T_3 \leq 6$ } = 1.000	

TABLE 4  
MEAN AND VARIANCE OF SPEED DISTRIBUTIONS IN SPACE

Distribution	Actual or Observed	Space Mean Speed	Standard Deviation of Space Mean Speed	Variance	Standard Deviation of Variance
N(52.3;22.0)	Actual	51.9349	0.218	22.03	1.38
	Observed	51.9353	0.222	22.50	1.42
N(39.6;21.6)	Actual	39.0207	0.219	21.77	1.38
	Observed	39.0202	0.220	21.91	1.39
N(50.0;49.0)	Actual	49.0309	0.334	49.64	3.18
	Observed	49.0316	0.337	50.01	3.21

of the harmonic mean of the speed probability density in time. Multiplying Eq. 5 by  $v^2$  and integrating over all speeds yield the variance of the speed probability density in space. If the square of the space mean speed is subtracted from this variance, the variance about the mean

$$\sigma_s^2 = \bar{v}_s \bar{v}_t - \bar{v}_s^2 \quad (7)$$

is obtained, where  $\bar{v}_t$  is the time mean speed. The use of this equation together with Eq. 6 then makes it possible to compute the variance of the speed probability density in space from time data.

For each of the speed distributions in time studied in the previous sections, the means and variances of the corresponding speed distribution in space were also computed by using Eqs. 6 and 7. Again, the data sample was subdivided into subsamples of 500 speed values each, and space mean speed and variance were computed for each subsample. These values were then used to compute the space mean speed for the entire sample, the standard deviation of this mean, the mean variance, and the standard deviation of this variance. The resulting values are given in Table 4.

The Wilcoxon Signed-Rank Test, described earlier, was applied to the differences between the actual and observed space mean speeds, as well as to the differences between the actual and observed variances. The test results are given in Table 5. These results indicate that the measurement technique does not effect a statistically significant change in the space mean speed but that it effects a statistically significant increase of the variance.

#### THE EFFECT OF THE MEASUREMENT TECHNIQUE ON RELATIVE SPEED DISTRIBUTIONS

In the following, the effect of the measurement technique on distributions of relative speeds is studied for the 3 selected underlying speed distributions of N(52.3; 22.0), N(39.6; 21.6), and N(50.0; 49.0) and for the 2 speed trap lengths of 24 and 30 ft.

#### Comparison of Actual and Observed Distributions

Relative speeds were computed as the differences between successive values of the sampled speeds,

$$(\Delta v)_j = v_j - v_{j-1}$$

$$j = 2, 3, \dots, 100,000$$

From the computed relative speeds, the actual relative speed distribution

TABLE 5  
WILCOXON SIGNED-RANK TEST OF RESULTS  
IN TABLE 4

Observed Minus Actual Space Mean Speed	Signed Rank	Observed Minus Actual Variance	Signed Rank
+ 0.0014	+ 2	+ 0.47	+ 3
- 0.0005	- 1	+ 0.14	+ 1
+ 0.0017	+ 3	+ 0.37	+ 2
a = sum + = 5 Pr{ $T_3 \leq 5$ } = 0.875		a = sum + = 6 Pr{ $T_3 \leq 6$ } = 1.000	

$$r_i = n_i / NI_i \quad i = 1, 2, 3, \dots \quad (8)$$

was formed, where  $n_i$  is the number of actual relative speeds in the  $i$ th relative speed interval of length  $I_i$ , and  $N$  is the total number of such values, which is 99,999 in this case. This procedure was then repeated for the observed speed values, and the observed relative speed distribution,  $r'_i$ , was formed in analogy to Eq. 8.

Because the actual speed distributions are normal distributions,  $N(\mu_V; \sigma_V^2)$ , with mean  $\mu_V$  and variance  $\sigma_V^2$ , the actual relative speed distribution is the normal distribution  $N(0; 2\sigma_V^2)$ .

Figure 7 shows the distributions for the relative speed intervals  $(-20, -19], \dots, (-1, 0], (0, 1], \dots, (19, 20]$ , where a parenthesis indicates an open end of the interval and a bracket a closed end. In this case, the resulting observed distribution is not symmetrical about zero because those observed relative speed values that are exactly equal to zero (this occurs when 2 adjacent observed speed values are equal) are classified into the interval  $(-1, 0]$ . This classification was discarded because the actual distribution is symmetrical about zero. In the following, only those classifications are used where the value zero is the midpoint of a relative speed interval.

For the 2 classifications  $(-20.5, -19.5], \dots, (-0.5, 0.5], \dots, (19.5, 20.5]$  and  $(-19, -17], \dots, (-1, 1], \dots, (17, 19]$ . Figures 8 through 13 show the relative speed distributions for  $N(52.3; 22.0)$ ,  $N(50.0; 49.0)$ , and  $N(39.6; 21.6)$  and for a speed trap length of 24 ft. Figures 14 and 15 show the relative speed distributions for  $N(52.3; 22.0)$  resulting from the same classifications but for a speed trap length of 30 ft.

For each of these cases, a chi-square test, analogous to Eq. 4, was performed for the relative speed distributions. The results of this test are given in Table 6. These results indicate that actual and observed relative speed distributions are significantly different statistically in each case. It is apparent that a classification in 2-mph is always relatively preferable. The disagreement between actual and observed relative

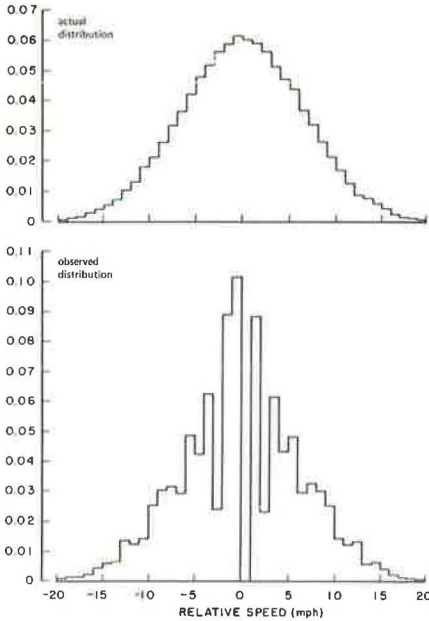


Figure 7. Relative speed distribution with 1-mph intervals (zero being the end point of one of them),  $N(52.3; 22.0)$ , and speed trap length of 24 ft.

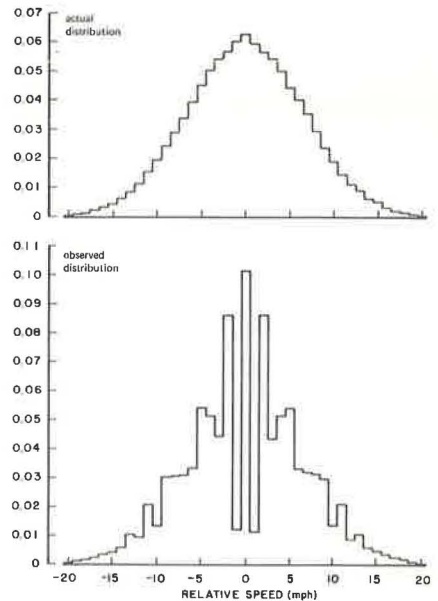


Figure 8. Relative speed distribution with 1-mph intervals (zero being the midpoint of one of them),  $N(52.3; 22.0)$ , and speed trap length of 24 ft.



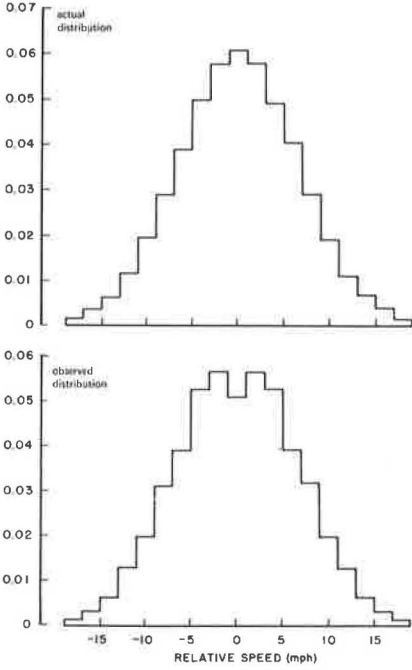


Figure 9. Relative speed distribution with 2-mph intervals,  $N(52.3; 22.0)$ , and speed trap length of 24 ft.

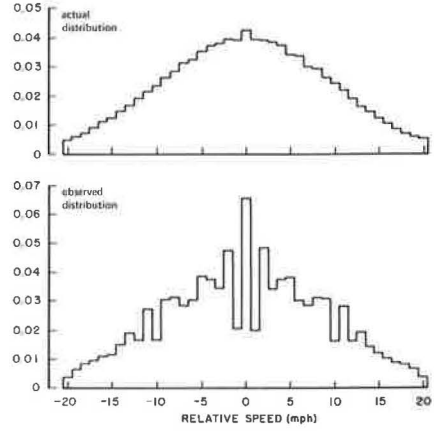


Figure 10. Relative speed distribution with 1-mph intervals,  $N(50.0; 49.0)$ , and speed trap length of 24 ft.

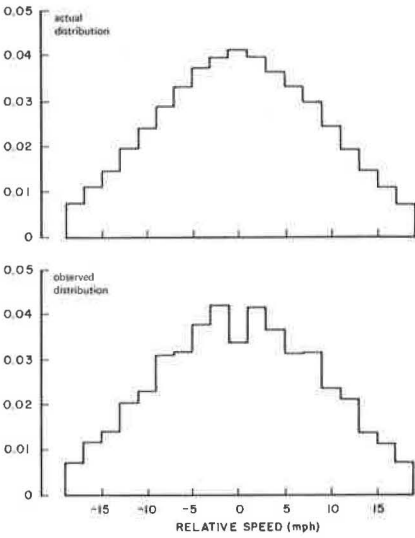


Figure 11. Relative speed distribution with 2-mph intervals,  $N(50.0; 49.0)$ , and speed trap length of 24 ft.

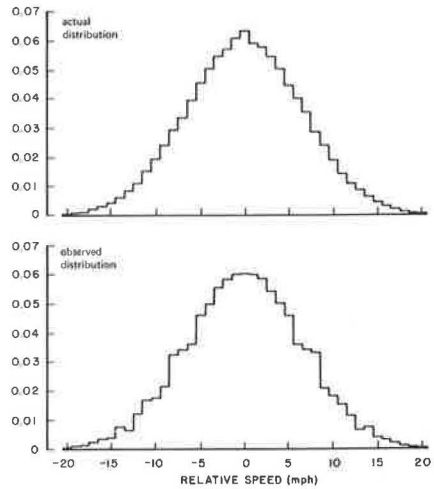


Figure 12. Relative speed distribution with 1-mph intervals,  $N(39.6; 21.6)$ , and speed trap length of 24 ft.

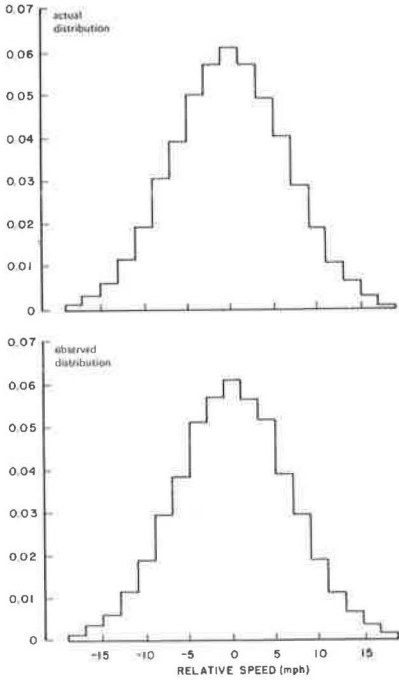


Figure 13. Relative speed distribution with 2-mph intervals,  $N(39.6; 21.6)$ , and speed trap length of 24 ft.

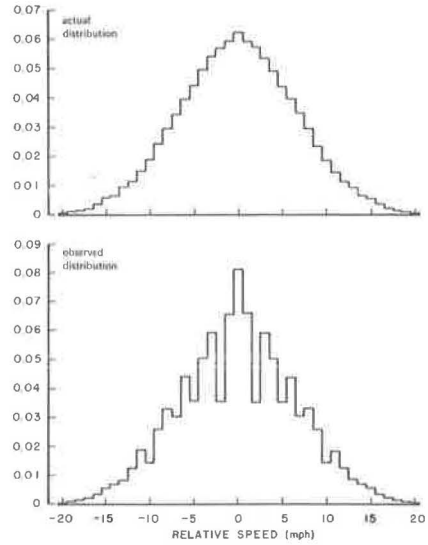


Figure 14. Relative speed distribution with 1-mph intervals,  $N(52.3; 22.0)$ , and speed trap length of 30 ft.

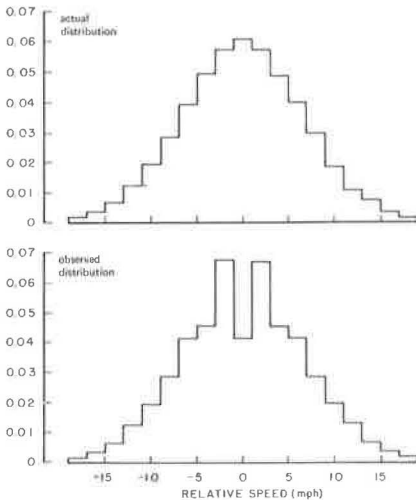


Figure 15. Relative speed distribution with 2-mph intervals,  $N(52.3; 22.0)$ , and speed trap length of 30 ft.

speed distributions is worst in the case corresponding to Figure 7; this is one more reason to discard that kind of classification.

The change from a 24-ft to a 30-ft speed trap definitely reduces the disagreement between actual and observed distributions in the case of the 1-mph classification. Here the improvement by a 2-mph classification is relatively small.

Tentative results for a 3-mph classification have not yielded any substantial reduction in the disagreement between actual and observed distribution, and this was, therefore, not pursued further. Thus, it can be concluded that in the cases considered, no statistically significant similarity between actual and observed relative speed distributions can be achieved.

#### Mean and Variance of Actual and Observed Distributions

This section discusses the effect of the measurement technique on the mean and variance of these relative speed distributions. For this purpose, the same sample of 99,999 relative speeds utilized earlier was subdivided into 199 subsamples

TABLE 6

CHI-SQUARE TEST FOR DIFFERENCE BETWEEN ACTUAL AND OBSERVED RELATIVE SPEED DISTRIBUTIONS

Distribution	Corresponding Figure	Trap Length (ft)	Relative Speed Interval (mph)	Computed $\chi^2$	Degrees of Freedom	Tabulated $\chi_{0.01}^2$
N(52.3;22.0)	7	24	1 <sup>a</sup>	16,934	39	62.4
N(52.3;22.0)	8	24	1	15,354	40	63.7
N(52.3;22.0)	9	24	2	590	18	34.8
N(50.0;49.0)	10	24	1	4,743	40	63.7
N(50.0;49.0)	11	24	2	502	18	34.8
N(39.6;21.6)	12	24	1	958	40	63.7
N(39.6;21.6)	13	24	2	212	18	34.8
N(52.3;22.0)	14	30	1	1,682	40	63.7
N(52.3;22.0)	15	30	2	1,187	18	34.8

<sup>a</sup>Only classification in which central interval is not centered about zero.

TABLE 7

MEAN AND VARIANCE OF RELATIVE SPEED DISTRIBUTIONS

Generating Distribution	Actual or Observed	Mean	Standard Deviation of Mean	Variance	Standard Deviation of Variance
N(52.3;22.0)	Actual	-0.001128	0.0129	43.88	3.32
	Observed	-0.001160	0.0130	44.97	3.36
N(39.6;21.6)	Actual	-0.001118	0.0128	43.13	3.27
	Observed	-0.001141	0.0128	43.49	3.29
N(50.0;49.0)	Actual	-0.001684	0.0193	97.75	7.40
	Observed	-0.001643	0.0191	98.68	7.55

TABLE 8

WILCOXON SIGNED RANK TEST OF RESULTS IN TABLE 7

Observed Minus Actual Mean	Signed Rank	Observed Minus Actual Variance	Signed Rank
+0.000032	+ 2	+ 1.09	+ 3
+0.000023	+ 1	+ 0.36	+ 1
-0.000041	- 3	+ 0.93	+ 2
$a = \text{sum} = 3$ $\Pr\{T_3 \leq 3\} = 0.625$		$a = \text{sum} = 6$ $\Pr\{T_3 \leq 6\} = 1.000$	

of 500 relative speeds each and 1 sub-sample of 499 relative speeds. In analogy to the earlier computations, mean and variance were computed for each of these 200 subsamples, and then the mean of the entire sample, the variance of this mean, the mean variance, and the variance of this variance were computed. The results are given in Table 7.

Again, the Wilcoxon Signed-Rank Test was applied to the differences between the actual and observed means, as well as to the differences between the actual

and observed variances. The results of this test are given in Table 8. They indicate that there is no statistically significant difference between the actual and observed means of the relative speed distributions, whereas, the variance of the observed relative speed distribution appears to be statistically significantly larger than the actual variance.

## CONCLUSIONS

The effect of the measurement technique caused by discrete time measurements on speed data was studied by the use of a computer program. This program sampled statistically independent speeds from normal speed distributions that approximated actually measured speed distributions quite well. Relative speed distributions were derived by computing differences between successive values of the sampled speeds. This way, the effect of car interaction that introduces a certain speed dependency has been ignored in the analysis of relative speed distributions. In reality, therefore, the effect

of the measurement technique on relative speed distributions may deviate quantitatively from the results of this study, especially for heavy traffic flow. However, a speed dependency would not alter the qualitative features of the results obtained for the relative speed distributions in this analysis.

Differences in the actual and observed speed distributions due to the effect of the measurement technique can be made statistically insignificant, if the interval selected for classifying the speed values increases hyperbolically with increasing speeds. However, the measurement technique causes a statistically significant increase of the mean and variance of the speed distributions, whereby the change in mean is so small that it can generally be ignored for practical purposes. The space mean speed computed from the time series data appeared to remain unchanged in contrast to the corresponding variance, which increases significantly.

The specific way chosen for classifying relative speeds into intervals appears to be a prime factor in the quality of the measured relative speed distributions. Even though it was concluded that any classification makes actual and observed distribution significantly different statistically, a 2-mph classification appears to be preferable over a 1-mph classification. The value zero should be the midpoint, not the end point, of one of the relative speed intervals. The mean of the relative speed distributions is not significantly changed, whereas the variance is increased significantly.

In any study that uses speed data from time series measurements, special consideration should be given to the effect of discrete time measurements on the characteristics of the derived speed data. Results from such a consideration not only should influence the data analysis but also should be especially used to optimize the experimental setup prior to the acquisition of the data.

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#### REFERENCES

1. Pahl, J. The Effect of Discrete Time Measurement on Speed Data. Univ. of California, Los Angeles, Eng. Rept. 70-69, Aug. 1969.
2. Pahl, J., and Sands, T. Vehicle Interaction Criteria From Time Series Measurements. Univ. of California, Los Angeles, Eng. Rept. 70-65, July 1969.
3. Owen, D. B. Handbook of Statistical Tables. Addison-Wesley Publishing Co., Reading, Mass., 1962, pp. 325-326.
4. Breiman, L. Point and Trajectory Processes in One-Way Traffic Flow. System Development Corp., TM-3858/001/00, July 15, 1968.