FLANGE PROPORTIONS FOR CURVED PLATE GIRDERS

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The results of analytical studies dealing with local flange buckling of horizontally curved plate girders are discussed. The relationship between the factor of safety against local buckling and the ratio of warping torsional normal stress to bending stress is plotted for several values of the flange width-thickness ratio. Two possible allowable stress design criteria are used to determine the factor of safety. In the first, the total stress at the flange tip, warping plus bending, is limited to $0.55F_y$. In the second, the total stress at the quarterpoint of the flange width is limited to $0.55F_y$. The influence of residual stresses is included in the results for girders fabricated by heat-curving or flame-cutting curved plates and then welded. Both A-36 and A-441 steel girders are considered. The results indicate that the factor of safety against local buckling using the first criterion for curved girders fabricated by welding or heat-curving and having width-thickness ratios equal to or less than those currently given by the AASHO specification remains substantially the same as for straight girders. For the second criterion, the factor of safety decreases as the ratio of warping to bending normal stress increases. Critical width-thickness requirements for girders fabricated by cold-bending are shown to be smaller than the present AASHO limit if local buckling is to be prevented during the fabrication process.

•DESIGN specifications for highway bridges (1) contain provisions limiting the width-to-thickness ratio for the compression flanges of beams and plate girders. These provisions based on local buckling considerations are currently limited to straight girders. Similar provisions do not exist for horizontally curved girders. The purpose of this paper is to summarize existing analytical work (2, 3, 4) on local buckling of curved plate girders. The results may be used to establish design requirements for curved girder flanges. Provisions for spacing of lateral bracing (usually expressed in terms of the flange width) based on overall lateral torsional buckling will not be considered here. Comparison of the analytical results with recent tests on curved plate girders is presented elsewhere (5).

BACKGROUND

The following 3 factors affect the local buckling behavior of curved girders: (a) flange curvature (geometrical effect) expressed in terms of the width of half of the flange, b, and the radius of the girder, R_W, i.e., b/R_W; (b) prebuckling stress condition (stresses due to applied loads); and (c) fabrication process, which affects the residual stresses. Values of flange curvature up to $b/R_W = 0.01$ were considered in the analytical studies. This would correspond to a girder with a 2-ft flange on a 100-ft radius. A survey of 28 curved bridges built in the United States indicated that an average value of this parameter was $b/R_W = 0.0025$. Numerical results indicated that, for flange curvatures less than or equal to 0.01, curvature was not important. Differences between the buckling stress for a straight girder, $b/R_W = 0$, and that for a curved girder, $b/R_W = 0.01$, were only 2 to 3 percent. The results presented in the next sec-

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tion for $b/R_W = 0.01$ are, therefore, applicable within the entire range of practical flange curvatures.

The prebuckling stress condition in a curved plate girder is different from that in a straight girder. Because of the curvature, the girder twists under load and warping normal stresses develop in addition to the bending stresses (3). The magnitude of these torsional stresses, which vary linearly across the flange width, is influenced by the torsional rigidity of the bridge cross section. The efficient use of lateral diaphragms can significantly reduce the magnitude of these stresses. Results presented here will be given for a wide range of warping to bending stress ratios, σ_W/σ_B (Fig. 1a).

Residual stresses developed in the fabrication process significantly affect the flange buckling behavior. Girders fabricated as follows will be considered: (a) flame-cutting the flanges to the desired curvature and then welding the flanges and web; (b) cold-bending a straight girder or rolled beam; and (c) heat-curving. Residual stress patterns produced by these 3 fabrication processes are presented elsewhere (2, 6).

NUMERICAL RESULTS

Before the requirements for curved girders are considered, it is of interest to evaluate the "factor of safety" inherent in the AASHO specification for straight girders. For an A-36 steel girder, the width-thickness requirement in this specification is $b/t = 11.5$. Using the residual stress pattern for a welded plate girder with $b/t = 11.5$ and solving the buckling problem for a girder subjected to pure bending gave a critical moment (moment at which local buckling occurs) of 99 percent of the yield moment, $M_y = \sigma_y S$ (2). For an allowable stress of $0.55\sigma_y$, this gives a factor of safety of 1.80. Similar computations for a girder with A-441 flanges and $b/t = 10$ gave a factor of safety of 1.69. Dividing the critical moment for a curved girder by the allowable design moment to determine the factor of safety provides a direct means of comparison between the straight and curved girder.

To determine the design moment requires that the allowable stress criteria first be established. One possibility involves limiting the total stress at the flange tip, warp-

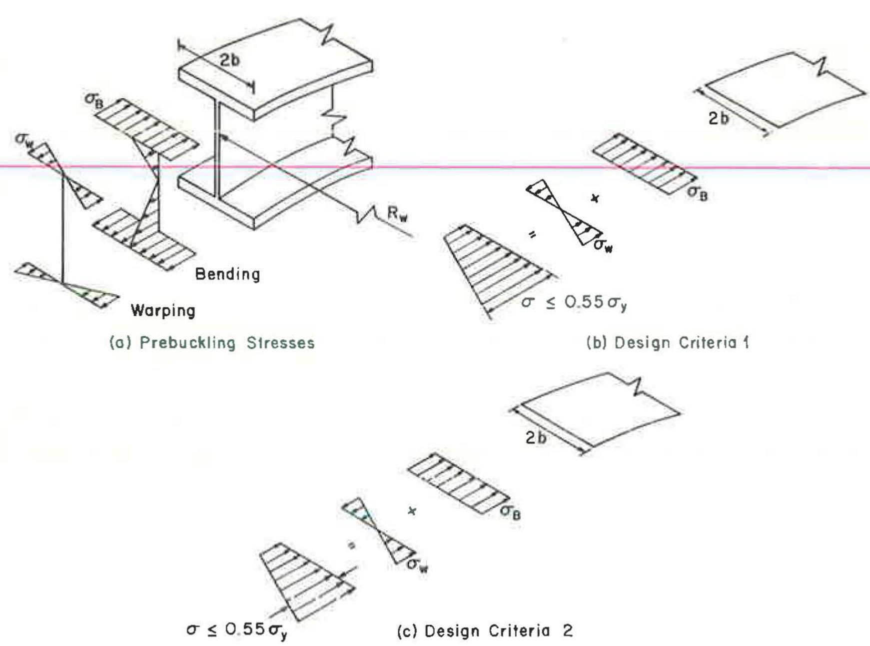

Figure 1. Possible allowable stress design criteria.

ing plus bending, to $0.55\sigma_y$ as shown in Figure 1b. In view of the fact that the warping stresses vary over the flange width, some designers choose to limit the average stress, warping plus bending at the middle of half the flange or at the quarterpoint of the flange, to $0.55\sigma_y$ (Fig. 1c). Obviously the total stress at the flange tip for this case is greater than that currently allowed for straight girders. Both criteria will be considered here.

Graphs of the factor of safety as a function of the warping to bending stress ratio for A-36 and A-441 welded and heat-curved girders are shown in Figures 2 through 5 for both design criteria. Curves for specific values of b/t are given. The points plotted represent the values of σ_W/σ_B for which the critical moments were calculated. No attempt was made to consider all the possible combinations of this ratio. The factor of safety was determined by dividing the critical bending moment for the particular ratio of σ_W/σ_B by the bending moment for each of the 2 design criteria using the same ratio of σ_W/σ_B. The influence of residual stresses was taken into account in determining the critical moment.

The residual stress pattern used for the welded girder was obtained from measured values used in previous studies (2). For the heat-curved girder, calculated residual stresses for a type 3 heat and a temperature of 1,150 F were used (6). Because the residual stress pattern for a welded girder is the same for both halves of the flange, the results shown in Figures 2 and 3 do not depend on the direction of the warping stress gradient (compression on inside or outside flange tip). For the heat-curved girder, however, the residual stress pattern is not symmetric about the web. The results were obtained for compressive warping stresses on the inside flange tip (toward the center of curvature). Because for this heating condition (type 3) the tensile residual stresses produced by heat-curving are theoretically somewhat lower on the inner flange tip than on the outer flange tip and are beneficial from the standpoint of local buckling (2, 4),

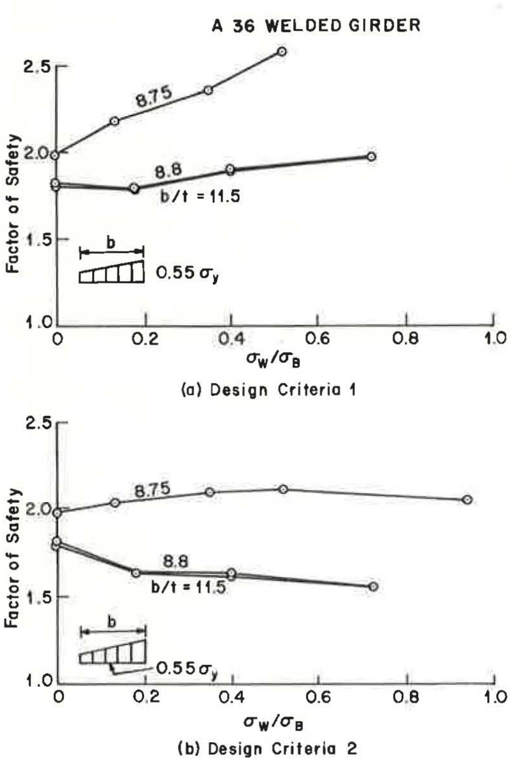

Figure 2. Factor of safety for A-36 welded girder.

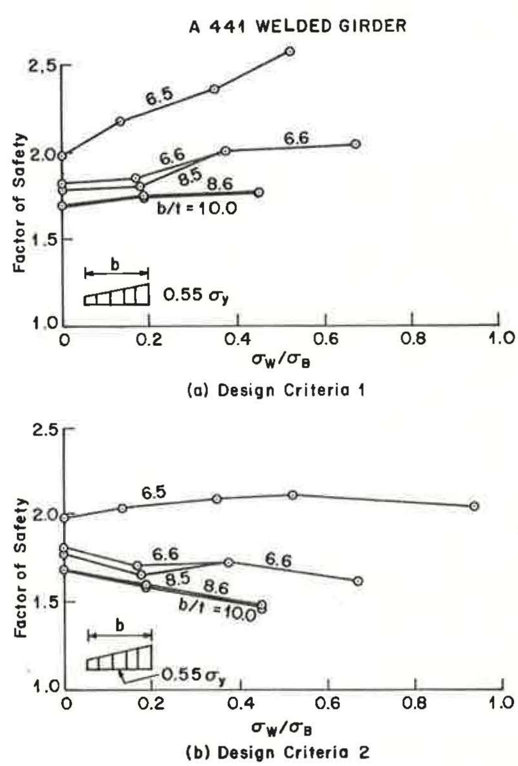

Figure 3. Factor of safety for A-441 welded girder.

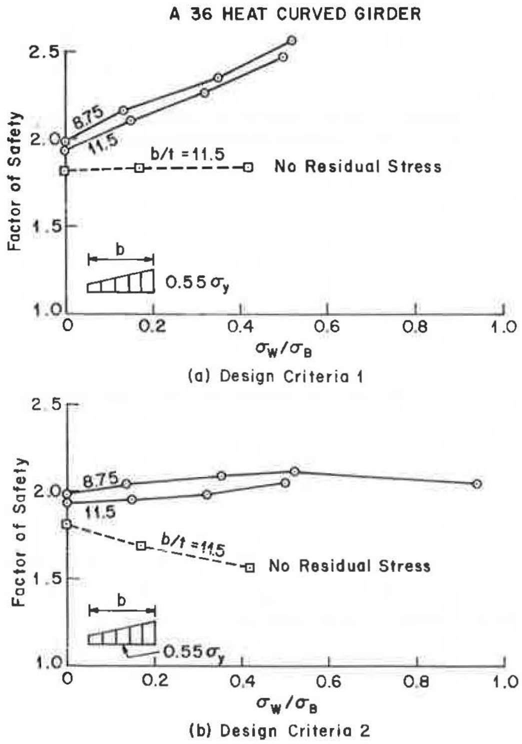

Figure 4. Factor of safety for A-36 heat-curved girder.

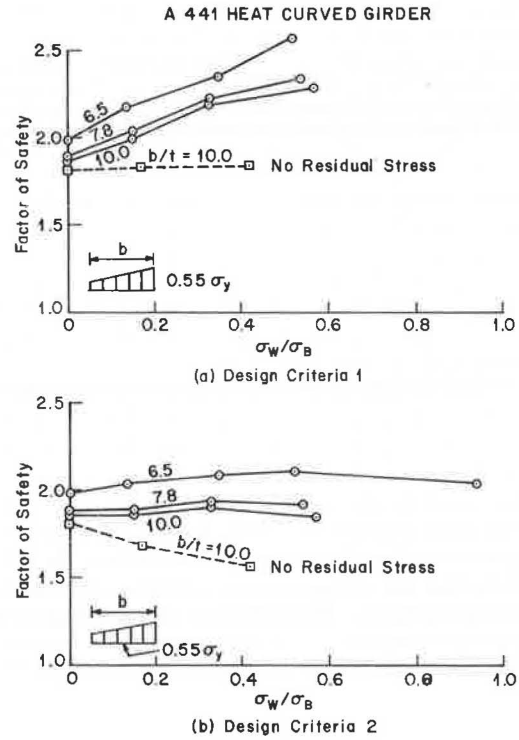

Figure 5. Factor of safety for A-441 heat-curved girder.

the results presented are conservative. The tensile residual stresses produced by lower heat-curving temperatures are less than those produced by the type 3 heat used in the analytical study (2). In Figures 4 and 5 curves are, therefore, shown for the case of zero residual stress. These curves represent lower bounds for heat-curved girders because the beneficial effect of the tensile residual stress at the flange tip produced by the heat-curving has been neglected.

Figure 2a shows that the factor of safety for b/t = 11.5 remains almost constant regardless of the value of σ_W/σ_B. Decreasing b/t to 8.8 hardly affects the results. This minor influence of width-thickness ratio on the factor of safety is due to the behavior of the plate buckling curves in the elastic-plastic region. The transition curves in this region differ from the overall column buckling slenderness ratio curves in that yielding at the outer flange tip produces a rapid reduction in buckling strength. For b/t = 8.75, however, the factor of safety is substantially increased. For this value of b/t, it is possible to fully yield half of the flange before local buckling occurs. For $\sigma_W/\sigma_B = 0$ and b/t = 8.75, the factor of safety is higher than that provided by the AASHO b/t value of 11.5. Because the girder can reach the fully plastic moment, the factor of safety reflects the shape factor, Z/S, of the cross section [$FS = M_p/0.55M_y = (M_y/0.55M_y)(Z/S) = (1.80)\,1.1 = 1.98$]. As σ_W/σ_B increases, the factor of safety with b/t = 8.75 increases. This is due to the fact that the rate of reduction in the plastic moment due to combined bending and torsion is less than the rate of reduction in allowable bending moment for combined bending and warping normal stress.

For the second criterion, Figure 2b shows a reduction in the factor of safety as σ_W/σ_B increases. Because the critical moment is not affected by the design criteria, this decrease is due to the higher allowable moment permitted by criterion 2 that limits the average flange stress rather than the maximum stress. It should be pointed out that the state of stress at the flange tip has a significant effect on the buckling strength. For example, yielding of only a small portion of the flange near the tip drastically decreases the buckling strength (2). Note that, for $\sigma_W/\sigma_B \simeq 0.7$, the factor of safety is still 1.55. For b/t = 8.75, no reduction in factor of safety occurs.

The results shown in Figure 3 for an A-441 flange are similar to those shown in Figure 2 for A-36 steel.

The factors of safety shown in Figures 4 and 5 for a heat-curved girder designed by using criterion 1 actually increase as σ_W/σ_B increases. This is due to the beneficial effect of the tensile residual stresses produced at the flange tip by the heat-curving process. As noted previously for type 1 and type 2 heat-curving, this increase would be less. Also, because subsequent loading and unloading of the girder tends to reduce the residual stresses (7), this increase would be somewhat less after the first few loadings of the bridge. When criterion 2 is used, the tensile residual stresses offset the reduction because of the use of an average stress or a higher allowable design moment, and the factor of safety remains relatively constant as σ_W/σ_B increases.

Results for a cold-bent girder are shown in Figure 6. Instead of the factor of safety being determined, the flange proportions required to prevent local buckling during the cold-bending process were determined. Figure 6a shows the percentage of half flange width, ηb, which must be yielded during cold-bending to produce a particular curvature, b/R_W. This stress distribution was used to determine the corresponding values of b/t required to prevent local buckling during fabrication (Fig. 6b). Figure 6b shows that to cold-bend a girder to a curvature of b/R_W = 0.001 requires that the value of b/t be less than or equal to 9.1. These results indicate that the width-thickness ratios necessary to cold-bend girders to practical curvatures, b/R_W > 0.0005, are less than currently allowed in the AASHO specification.

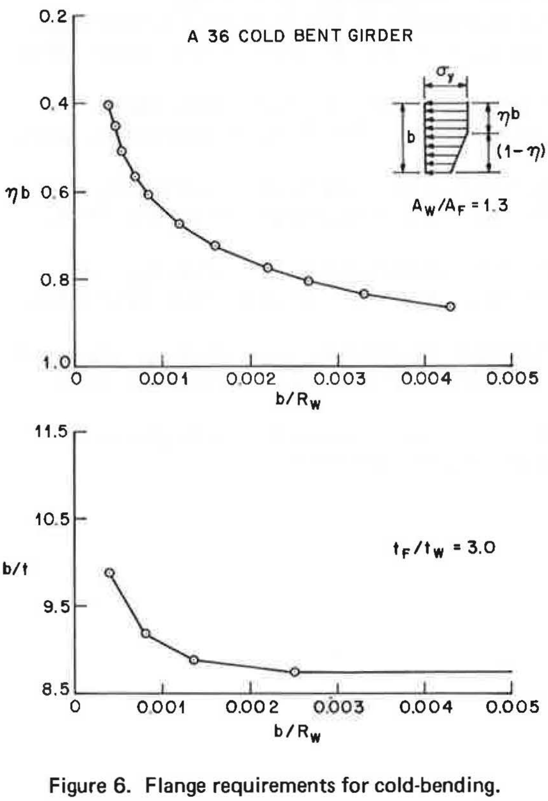

Figure 6. Flange requirements for cold-bending.

SUMMARY AND CONCLUSIONS

The factor of safety against local buckling for various flange width-thickness ratios for horizontally curved girders was evaluated. Two possible design criteria were considered. For the case in which the total stress at the flange tip, warping stress plus bending stress, is limited to $0.55\sigma_y$, the factor of safety for curved girders using present straight girder flange proportions, b/t, remains substantially the same regardless of the ratio of σ_W/σ_B. If the average stress in the most critically stressed half of the flange is limited to $0.55\sigma_y$, thus allowing the stress at the flange tip to exceed $0.55\sigma_y$, the factor of safety decreases if straight girder flange proportions are used. The maximum decrease is 14 percent for a welded girder with σ_W/σ_B = 0.7 (σ_W/σ_B = 0, FS = 1.80; σ_W/σ_B = 0.7, FS = 1.55).

The results presented here were concerned with allowable stress design. If a "load factor" design philosophy (8) is adopted for curved girder bridges, more stringent flange width-thickness requirements would be necessary (2). These requirements are similar to those proposed for straight beams and girders (8).

NOTATION

The following symbols are used in this paper:

A_F = area of 1 flange;
A_W = area of web;
b = width of half of flange;
M = bending moment;
M_p = plastic moment;
M_y = yield moment;
R_w = centerline radius of girder;
S = section modulus;
t = flange thickness;
t_w = web thickness;
Z = plastic section modulus;
η = percentage of half flange that is yielded;
σ = normal stress;
σ_B = bending stress;
σ_w = warping normal stress; and
σ_y = yield stress.

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