

DYNAMIC DETERMINATION OF PILE LOAD-BEARING CAPACITY

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Consideration has been given to the development of an accurate method for obtaining the load-bearing capacity of a pile based on dynamic pile driving information. Based on the neglect of inertial forces and the assumption that the pile is a rigid body, the bearing capacity is approximately equal to the centroid (along the force axis) of the interaction force-time curve. The interaction force is obtained by the use of a load cell situated between the hammer and pile top. A more accurate determination of load-bearing capacity may be obtained by including the inertial effect in the analysis. The agreement between the dynamic predictions and the static load-test values for 8 full-scale piles is quite good. On the basis of force-displacement curves derived from force-time and displacement-time traces, approximately 95 percent of the rated energy output of the hammer is transferred to the soil-pile system. Finally, initial consideration is given to the model simulation of a hammer-pile-soil system on an analog computer.

•THE PRIMARY purpose of this study is to provide a predictive estimate of the load-bearing capacity of a pile based on dynamic information obtained during the driving process. The "static" bearing capacity of a pile generally changes with time subsequent to driving; i.e., the initial capacity of a pile immediately after driving may be more or less than that measured at some future date. The analyses presented here utilize information obtained during the driving process; hence, the calculated values of load-bearing capacities are of an initial nature.

The theoretical treatment of the dynamic features of hammer-pile-soil interaction as given here is patterned after the analytical approach and subsequent results obtained by Scanlan and Tomko (1). By the use of one-dimensional elastic wave theory, they demonstrate that the elastic contribution to the motion of the pile is small compared to the rigid-body contribution. Their investigation disclosed that the predicted load-bearing capacities of full-scale piles are not appreciably altered when the elastic portion of the pile motion is neglected. The elastic-rigid dynamic theory requires that the total dynamic displacement of a pile due to a hammer blow be the sum of the elastic and the rigid-body displacements. The rigid-body dynamic theory disregards the elastic motion of the pile.

By neglecting inertial forces and assuming that a rigid-body condition exists during the driving, we relate here the bearing capacity of a pile to the maximum interaction force developed between the hammer and the pile-soil system. Subsequently, the analysis is improved by inclusion of the inertial effects during pile motion. Comparisons are made between measured and predicted values of load-bearing capacities of full-scale piles. The rated kinetic energy outputs of the hammers are compared with the corresponding values of energy transfer during driving. Finally, a model of the hammer-pile-soil system is simulated on an analog computer.

FIELD EXPERIMENTATION

During the course of the investigation, 21 full-scale piles were driven and subsequently load-tested at 5 different soil locations. Dynamic measurements of driving force and pile displacement were obtained for 7 of the 21 piles. The piles were fabricated from Douglas fir, precast concrete, steel H-beam, and steel pipe.

A diagram of the dynamic force-displacement measurement system and apparatus configuration is shown in Figure 1. The interaction force between the pile-soil system and the hammer was measured as a function of time by means of a calibrated load cell. The pile displacement as a function of time was measured with a linear variable differential transformer (LVDT). The same time base was used in recording the hammer force and pile displacement simultaneously on 2 channels of a magnetic tape recorder. Examples of the force and displacement-time traces are shown in Figures 2 through 5.

The force-time curves shown in Figures 2, 3, and 4 were recorded at a high sensitivity level on the magnetic tape recorder. Because of the hammer spike after the pulse maximum, the high-sensitivity FM amplifier on the recorder was overloaded, resulting in a distortion of the remainder of the force-time trace. In particular, the "tensile" portions of the traces are exaggerated. The force-time curve shown in Figure 5 was recorded on dc to 10 kHz regular FM and does not exhibit a negative pulse feature. However, because of internal reflections within the hammer-pile-soil system, the existence of a tensile "tail" on the interaction force-time record would not be physically unreasonable.

Static load tests were conducted by use of a hydraulic jack and a dial indicator in conjunction with a reaction beam and a configuration of anchor piles around the test pile. All piles were statically tested within 1 to 2 weeks after being driven and during a period of 3 to 12 months thereafter. During a test, the load was applied in 15-ton increments until the ultimate or failure load was attained. The minimum elapse time between load increments was 0.5 hour. Typical load-displacement curves are shown in Figure 6.

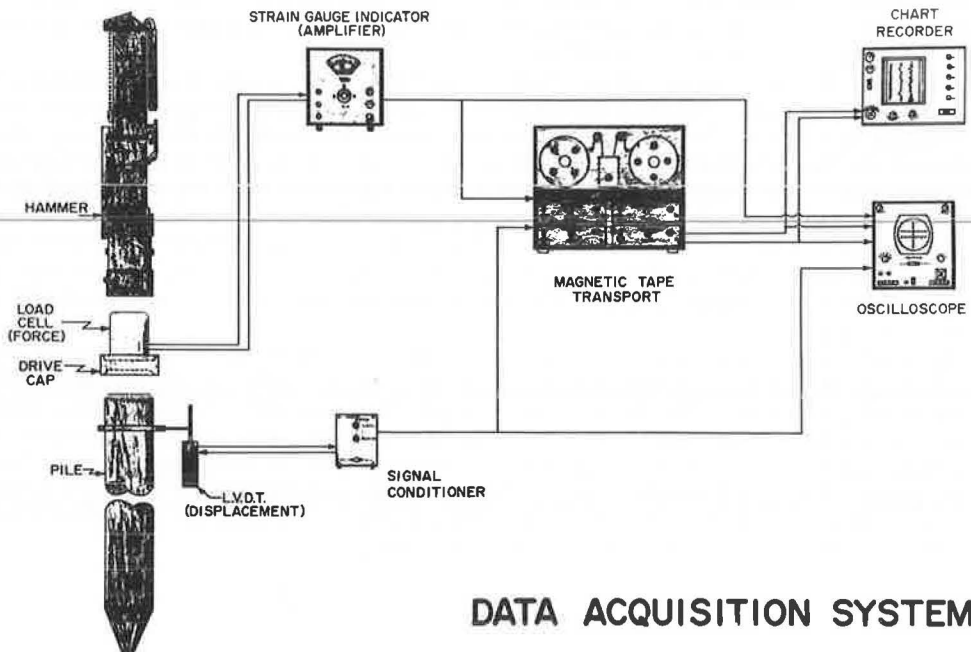


Figure 1. Schematic diagram of force-displacement measurement system and apparatus configuration for dynamic pile testing.

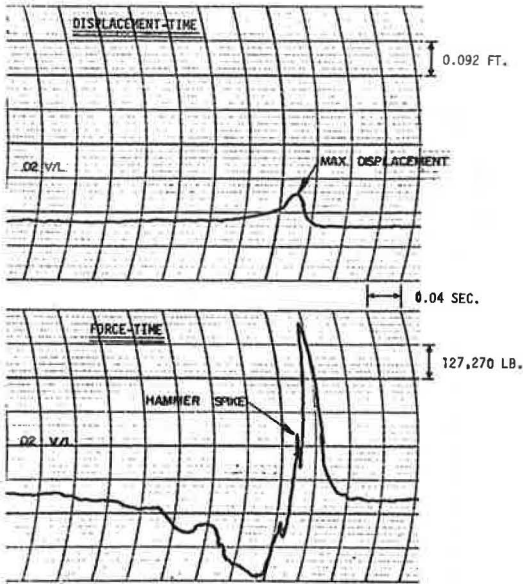


Figure 2. Displacement-time and force-time records for a 59-ft steel-pipe pile driven by a Link Belt 520 at the Watertown field site.

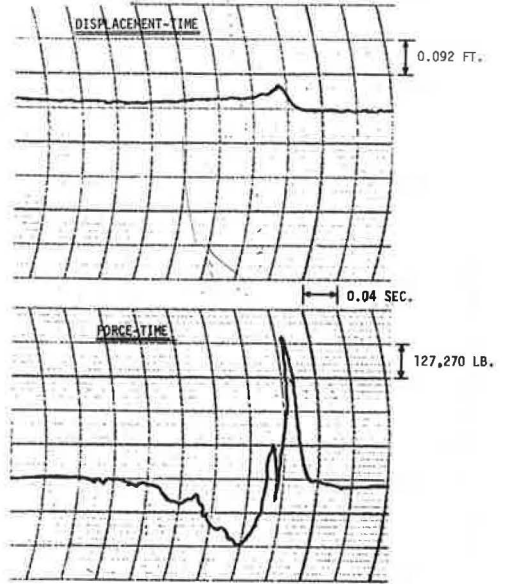


Figure 3. Displacement-time and force-time records for a 30-ft Douglas fir pile driven by a Link Belt 520 at the Madison field site.

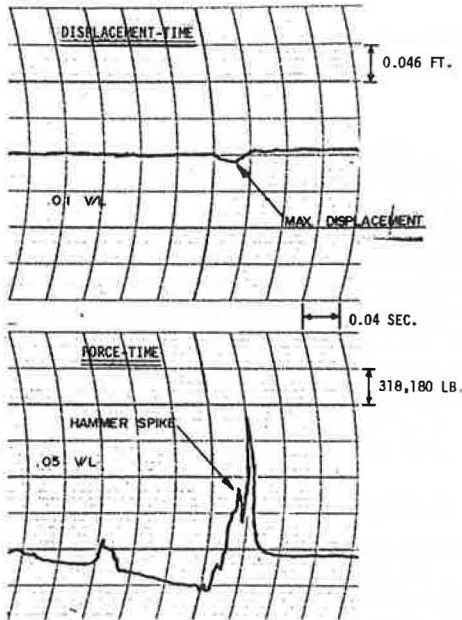


Figure 4. Displacement-time and force-time records for a 59-ft concrete octagonal precast concrete pile driven by a Link Belt 520 at the Watertown field site.

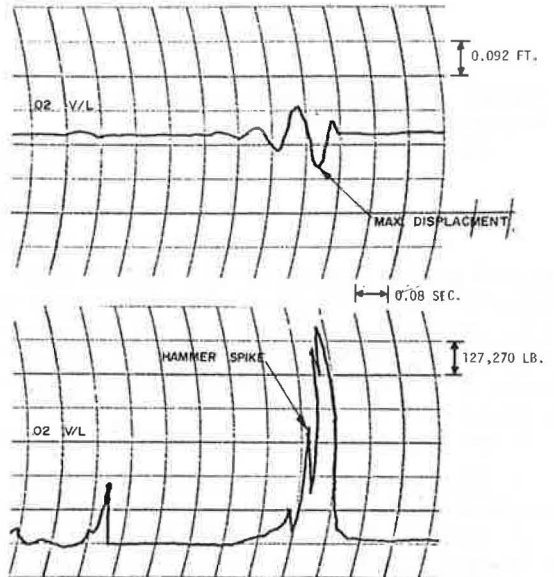


Figure 5. Displacement-time force-time records for a 123-ft steel H-beam pile driven by a Delmag D-22 at the Chamberlain field site.

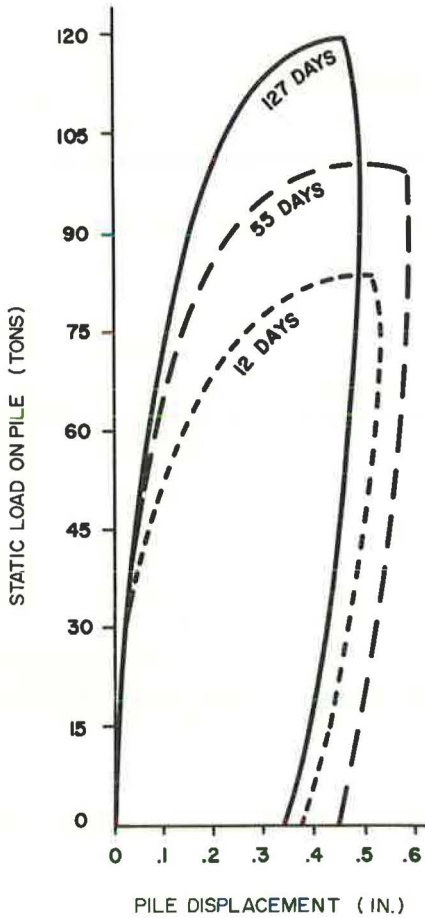


Figure 6. Load-displacement curves obtained from static load tests on a 35-ft tapered concrete pile at the Madison field site.

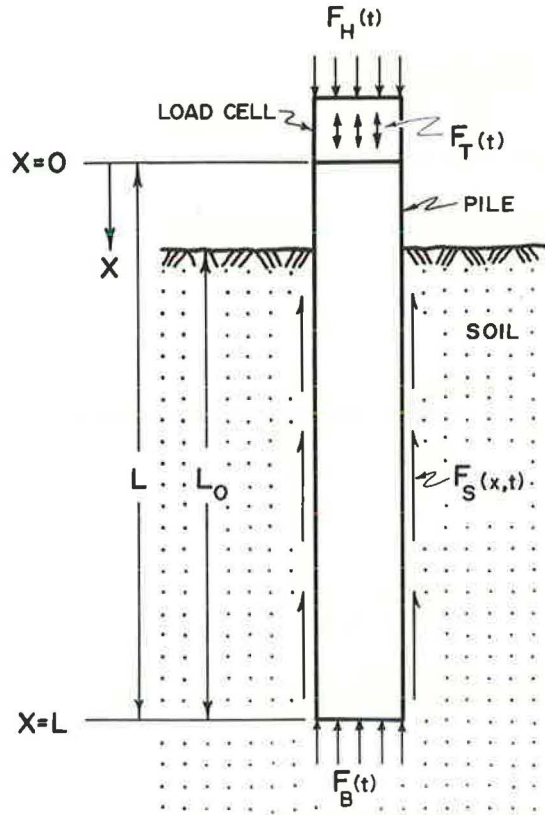


Figure 7. Schematic diagram of a pile subjected to an idealized system of driving and resistive soil forces.

ANALYSIS OF PILE LOAD-BEARING CAPACITY

Simplified Pile Model

Consider a pile of total length L and weight W_p situated in a soil mass, as shown in Figure 7; the length of pile actually in the soil is designated as L_0 , where $L_0 \leq L$. The top of the pile, i.e., at $x = 0$, is subjected to a time-dependent hammer force $F_H(t)$. The weight of the hammer is designated as W_H . The soil exerts a normal force $F_B(t)$ on the base of the pile, and a distributed "frictional" force $F_S(x, t)$ on the sides of the pile, i.e., for $L - L_0 \leq x \leq L$. If the mechanical behavior of the soil is rate dependent, then the base and side forces must be considered as being time dependent.

In the analyses to follow, we use the following assumptions:

1. That the elastic wave motion in the pile is immediately dissipated by the soil and the predominant pile motion is effectively that of a rigid body; and
2. That the forces exerted by the soil on the pile at the base and along the sides can be lumped together into a composite resistive force of the form

$$F_B(t) + F_S(x, t) = F_R + cv(t) \quad (1)$$

where F_R is the "static" bearing resistance of the soil, c is a "viscous" damping constant for the soil, and $v(t)$ is the instantaneous rigid-body velocity of the pile.

If $u(t)$ is the instantaneous rigid-body displacement of the pile and $v(t)$ is the pile velocity, then the following energy terms are pertinent to the composite hammer-pile-soil system:

$$\begin{aligned} KE &= \frac{1}{2}[Mv(t)^2] \\ PE &= \frac{1}{2}[ku(t)^2] \\ DE &= \frac{1}{2}[cv(t)^2] \end{aligned} \quad (2)$$

where

$$\begin{aligned} KE &= \text{kinetic energy, ft-lbf;} \\ PE &= \text{potential energy, ft-lbf;} \\ DE &= \text{dissipation energy by viscous damping, ft-lbf;} \\ M &= (W_p + W_H)/g = \text{combined mass of hammer and pile, lb}_f\text{-sec}^2/\text{ft;} \\ W_p &= \text{pile weight, lb}_m; \\ W_H &= \text{hammer weight, lb}_m; \\ g &= 32.174 \text{ lb}_m\text{-ft/lb}_f\text{-sec}^2; \text{ and} \\ k &= \text{elastic constant for the composite hammer-pile-soil system,} \\ &\quad \text{lb}_f/\text{ft.} \end{aligned}$$

The energy dissipated by Coulomb damping, i.e., by the resistive force F_R , is

$$DE = \int F_R du \quad (3)$$

The time-dependent force $F_T(t)$, as measured by a load cell situated between the hammer and the top of the pile, is not simply the hammer force $F_H(t)$; rather, $F_T(t)$ is the interaction force between the hammer and the pile-soil system. In view of data shown in Figures 2 through 5, $F_T(t)$ can be approximated by a sine function over the first half cycle.

$$F_T(t) = F_0 \sin(\omega t) \quad \left(0 \leq t \leq \frac{\pi}{\omega}\right) \quad (4)$$

where F_0 is the maximum force and ω is the frequency. The corresponding work or energy expended during the displacement of the pile is

$$W = \int F(u)du = \int F(t)v(t)dt \quad (5)$$

where $du = (du/dt)dt = v(t)dt$.

Load-Bearing Capacity Based on Energy-Impulse Considerations

In order to obtain a reasonable estimate of the initial load-bearing capacity of a pile, let us consider an energy balance over the hammer-pile-soil system. The work done by the forces of the hammer and the soil on the rigid pile during a displacement is equal to the change in kinetic energy of the pile during the same displacement. The change in kinetic energy of the pile over a time period of $t = \pi/\omega$, where ω is the frequency of the interaction force $F_T(t)$, is

$$E_H = \int_0^{\pi/\omega} F_T(t)v(t)dt \quad (6)$$

where E_H is the rated kinetic energy output of the hammer. The work done by the dissipative resistive force of the soil on the rigid pile over the same time interval is

$$E_R = \int_0^{\pi/\omega} [F_R + cv(t)]v(t)dt \quad (7)$$

The elastic strain energy stored in the entire system, including that of the hammer, pile, and soil, over the same time interval is

$$E_O = \int_0^{\pi/\omega} ku(t)v(t)dt \quad (8)$$

Let us suppose that the energy stored in the system over the time interval $0 \leq t \leq \pi/\omega$ is entirely dissipated in work done by the soil on the pile as the pile displaces into soil.

$$E_R = E_O \quad (9)$$

Basically, this implies that the entire hammer-pile-soil system is first compressed in a springlike manner with a potential energy E_O , and then the stored energy is completely dissipated by Coulomb and viscous damping as the pile is permanently displaced into the soil. Thus, an energy balance over the time interval $0 \leq t \leq \pi/\omega$, $E_O + E_R = E_H$ or $2E_R = E_H$, by Eq. 9 is written as

$$\int_0^{\pi/\omega} [F_T(t) - 2F_R]v(t)dt = 0 \quad (10)$$

where use is made of Eqs. 6 and 7, and the viscous damping coefficient c is taken as zero for simplification. As a good approximation, we may suppose

$$v(t) \cong BF_T(t) \quad (11)$$

where $B = \text{constant}$. This supposition implies that, if the function $F_T(t)$ is known, then $v(t)$ is approximately a function of the same form, differing only by a constant; experimental field results indicate that $F_T(t)$, $u(t)$, and $v(t)$ are all of a sinusoidal nature, thus lending credence to the assumption. Substitution of Eq. 11 into Eq. 10 gives

$$F_R = \frac{\int_0^{\pi/\omega} F_T(t)^2 dt}{2 \int_0^{\pi/\omega} F_T(t) dt} \quad (12)$$

According to this result, the bearing capacity of a pile is equal to the centroid of the interaction force-time curve. In particular, if $F_T(t) = F_O \sin(\omega t)$, then

$$F_R = (\pi/8)F_O \quad (13)$$

where F_O is the maximum interaction force.

From another viewpoint, this result implies that the impulse of the hammer force $F_H(t)$ must equal the impulse of the resistive bearing force F_R of the soil, and the sum of the 2 impulses must equal the impulse of the interaction force $F_T(t)$ as measured by the load cell. This result is intuitively reasonable on physical grounds. The primary deficiency in the analysis is the disregard of the effect of the inertial force of the hammer-pile system. The effect is additive with regard to the bearing force F_R . As the pile is displaced with relative ease into the soil, the inertia term is small as compared with F_R . However, at refusal, the inertial contribution is significant. In summary, this analysis yields a lower bound or minimum value on the bearing capacity of a pile; inclusion of the inertia term in the formulation will improve the estimate of the load-bearing capacity of the pile.

Load-Bearing Capacity Based on Energy-Displacement Considerations

The pile-soil system, when dynamically acted on by the hammer, behaves in a manner analogous to that of a damped simple harmonic oscillator. This premise is made with the assumptions that the pile acts as a rigid body and the pile-soil interaction acts as a damped elastic medium. The premise is substantiated by the fact that during a hammer blow the pile first exhibits a maximum displacement and then rebounds to a permanent displacement.

Under the assumption that the pile-soil system behaves as a simple harmonic oscillator, the sum of the kinetic energy and potential energy corresponding to the unrestricted movement of the oscillator will always be a constant. At the point of maximum displacement u_0 , corresponding to zero velocity, the kinetic energy is zero and the potential energy is

$$PE = \frac{1}{2}(K u_0^2) \quad (14)$$

where K is the elastic constant of the oscillator. At any instant when the oscillator is in motion, the total potential and kinetic energy is equal to the potential energy at the point of maximum displacement, or zero velocity. The natural frequency of the displacement is

$$\omega_n = \sqrt{K/M} = 2\pi/T \quad (15)$$

where M is the total mass of the system, i.e., hammer plus pile, and T is the period of 1 cycle of the pile displacement. Thus, at any instant the energy required for the pile to exhibit simple harmonic motion is

$$E_0 = \frac{1}{2}[M(\omega_n u_0)^2] \quad (16)$$

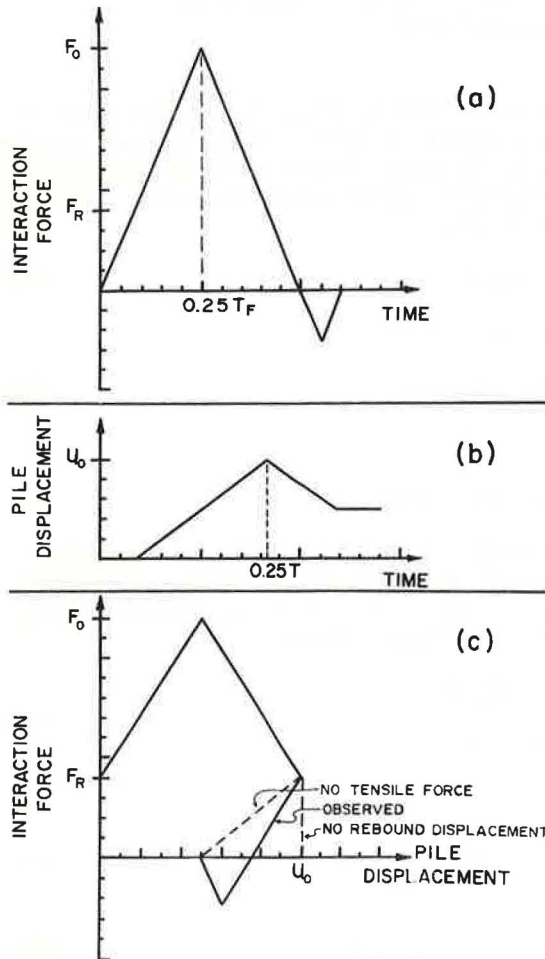


Figure 8. Simplified force-displacement-time curves for approximate simulation of dynamic pile behavior.

$$E_H - E_0 = F_R u_0 + \frac{1}{2}(F_0 - F_R)u_0 \quad (17)$$

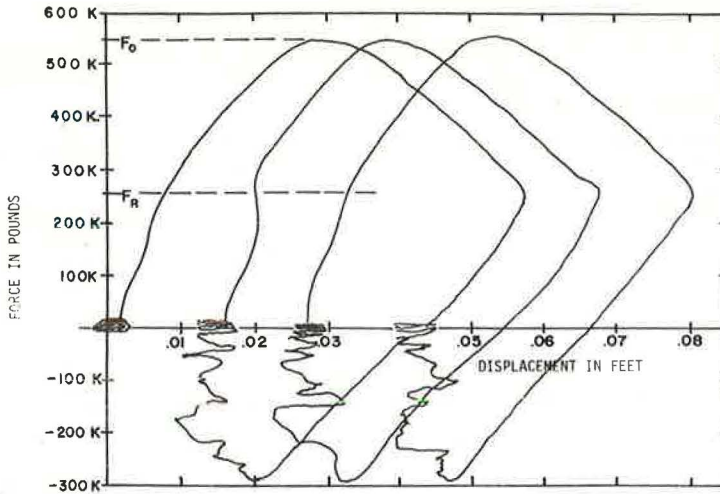


Figure 9. Dynamic force-displacement curves obtained from force-time and displacement-time records for a 59-ft steel-pipe pile driven by a Link Belt 520 at the Watertown field site.

According to the theory developed in the previous section, where the inertial force is neglected, the bearing capacity F_R of the pile is equal to the maximum interaction force F_0 multiplied by the centroid of the force-time function. In this case, the forcing function is of a triangular shape, as shown in Figure 8a, and the centroid is equal to one-third of the maximum value of the interaction force; i.e.,

$$F_R = F_0/3 \quad (18)$$

By substitution of Eq. 18 into Eq. 17, we obtain

$$F_0 = \frac{3}{2}[(E_H - E_0)/u_0] \quad (19)$$

A force balance on the pile yields

$$M\{d[v(t)]/dt\} + F_R = F_T(t) \quad (20)$$

where $v(t)$ is the instantaneous rigid-body velocity of the pile, and M is the combined mass of the hammer and the pile. Consider the instant of time during the pile motion at which the pile displacement is maximum, i.e., $u(t_0) = u_0$. At $t = t_0$, the pile velocity is zero and the pile acceleration is a negative maximum. Hence,

$$F_T(t_0) = F_0/3 \quad (21)$$

$$d[v(t_0)]/dt = -u_0\omega_n^2$$

Substitution of Eq. 21 into Eq. 20 gives

$$F_R = (F_0/3) + Mu_0\omega_n^2 \quad (22)$$

or

$$F_R = \frac{1}{2}(E_H/u_0) + \frac{3}{4}(Mu_0\omega_n^2) \quad (23)$$

where use is made of Eqs. 16 and 19. Thus, the bearing capacity F_R of the pile can be related to the hammer output energy E_H and the maximum pile displacement u_0 ; this analysis includes the effect of the hammer-pile inertia.

TABLE 1

COMPARISON OF CALCULATED AND MEASURED LOAD-BEARING CAPACITIES OF FULL-SCALE PILES

Pile	Total Length (ft)	In-Place Length (ft)	u_0 Period (sec)	Maximum u_0 (ft)	Total Weight of Hammer-Pile System, $W_H + W_p$ (lbm)	Rated Out-Put Energy of Hammer, E_H (ft-lbf)	Maximum Force, F_0 (lbf)	Load-Bearing Capacity (tons)		
								Equation 13	Equation 23	Static Field Test
Madison, tapered concrete	35	26	0.104	0.66	13,360	18,700	432,730	85	105	82 to 120
Madison, precast concrete	35	26.5	0.108	0.0734	13,640	20,700	483,640	95	110	80 to 115
Madison, wood	30	26	0.100	0.0734	8,855	17,700	560,000	110	90	90 to 140
Watertown west, wood	58.5	28.5	0.088	0.0624	9,945	19,500	636,360	125	115	135
Watertown east, wood	59.5	29.5	0.160	0.0550	9,985	13,700	432,730	85	72	80
Watertown, octagonal concrete	59	29	0.080	0.0174	18,200	21,200	1,209,090	237	326	330+
Watertown, steel pipe	59.3	29.5	0.100	0.041	9,080	17,700	661,820	130	125	140
Spearfish, steel H-beam	30	30	0.160	0.0587	8,175	12,500	560,000	110	61	50 to 75
Chamberlain, steel H-beam	123	123	0.112	0.0918	18,995	29,100	789,090	155	143	None

Comparison of Calculated and Measured Load-Bearing Capacity

On the basis of force-time and displacement-time data obtained during the actual pile-driving process, the load-bearing capacity of a pile can be calculated from either Eq. 13 or Eq. 23. The effect of the inertial force is included in the second expression, but not in the first. A comparison of the calculated load-bearing capacities with those measured statically for 8 full-scale piles is given in Table 1. In general, the quantitative correlation between the dynamically predicted and statically measured load-bearing capacities is quite good. A dynamic force-displacement curve, as obtained from a cross plot of the force-time and displacement-time records, can be utilized for an additional check on the general accuracy of the predictive techniques. Typical force-displacement curves for a steel-pipe pile driven with a Link Belt 520 are shown in Figure 9. The load-bearing capacity of the pile corresponds to the first break, or "knee," of the individual curves. From the curves for the steel-pipe pile, F_R is approximately 130 tons as compared with calculated values of 125 and 130 tons and a field-test value of 140 tons.

The area under a force-displacement curve represents the work done by the interaction force on the hammer-pile-soil system. In order to evaluate the transfer of energy between the hammer and the pile-soil system, the work done by the interaction force must be compared with the rated energy output of the hammer. Table 2 gives a tabulation of the percentage of energy transferred for a variety of piles driven by a Link Belt

TABLE 2

ENERGY TRANSFER FOR PILES DRIVEN BY LINK BELT 520

Pile	Rated Hammer-Energy Output (ft-lbf)	Work Obtained By Integration of Force-Displacement Curve ^a (ft-lbf)	Approximate Energy Transfer (percent)
Watertown west, wood	19,500	19,400	99
Watertown east, wood	13,700	12,850	94
Watertown, octagonal concrete	21,200	20,295	96
Watertown, steel pipe	17,700	17,575	99
Madison, wood	17,700	17,570	99

^aIntegration performed on compressive force portion of force-displacement curve.

520. Within the accuracy of the dynamic measurements of force and displacement as functions of time, the transfer of energy from the hammer to the pile-soil system appears to be in the order of 94 to 99 percent.

Analog-Computer Simulation of a Hammer-Pile-Soil System

Employing the premise that the linear impulse is equal to the change in linear momentum of a mass over a time interval, we may write

$$\frac{d(KE)}{dv} = \int_{t_1}^{t_2} \left[F_0 \sin(\omega t) - F_R - \frac{d(PE)}{du} - \frac{d(DE)}{dv} \right] dt \quad (24)$$

where $u(t)$ and $v(t)$ are the instantaneous displacement and velocity respectively of the "rigid" pile. The quantity $[F_0 \sin(\omega t) - F_R]$ may be regarded as the generalized external force acting on the pile. By substitution of Eq. 2 into Eq. 24, we have after some simplification

$$M(d^2u/dt^2) + c(du/dt) + ku + F_R = F_0 \sin(\omega t) \quad (25)$$

k may be regarded as a constant that represents the total elastic features of the hammer-pile-soil system, and c is a viscous damping constant for the soil. If the load-bearing capacity term F_R is considered to be analogous to friction at the soil-pile interface, then no pile motion can occur until the forcing function $F_0 \sin(\omega t)$ is equal to F_R ; i.e., for $F_R = F_0 \sin(\omega t)$, we must have $u(t) = v(t) = 0$. If we assume that F_R and F_0 are related by Eq. 13, then the initial conditions on Eq. 25 are

$$u(t) = v(t) = 0 \quad \text{for } 0 \leq t \leq \frac{1}{\omega} \sin^{-1}(\pi/8) \quad (26)$$

Inspection of Figures 2 through 5 reveals that this initial condition is indeed quite realistic; i.e., pile displacement is not initiated until the value of the forcing function exceeds $\pi F_0/8$.

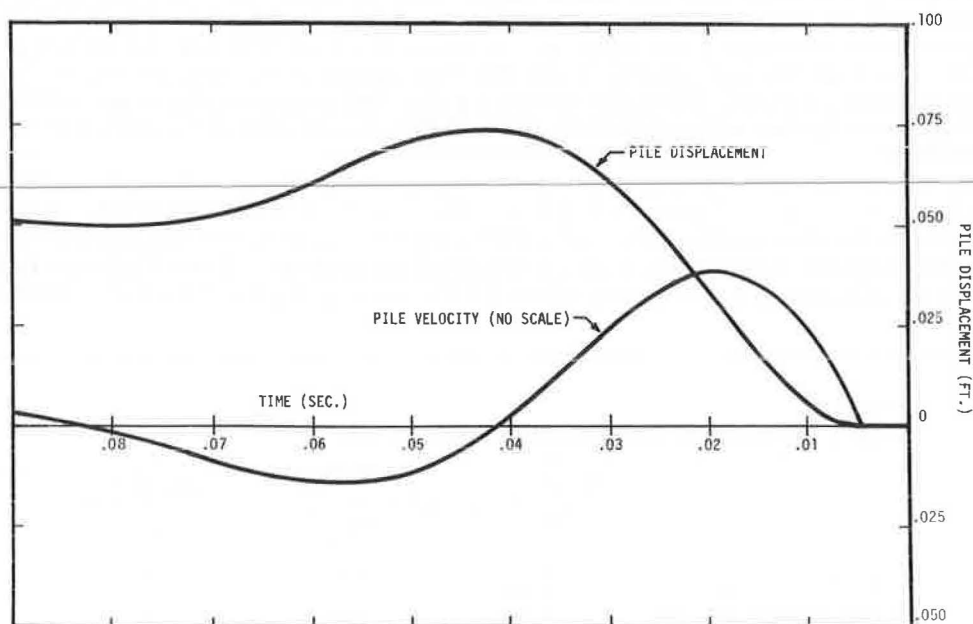


Figure 10. Displacement-time curve obtained by an analog simulation of a Douglas fir pile driven by a Link Belt 520 at the Madison field site.

The second-order, ordinary linear differential Eq. 25 governing the pile displacement can be adapted for solution to an analog computer, wherein the initial conditions (Eq. 26) are utilized. As an example, consider the 30-ft Douglas fir pile at the Madison site. From Figure 5, we obtain $F_0 = 560,000$ lbf and $\omega = 63$ cps. In addition, we take $W_D + W_H = 8,855$ lb and $F_R = (\pi/8) F_0 = 219,910$ lb. According to data shown in Figure 5, a maximum displacement of $u_0 = 0.0734$ ft occurs at time $t_2 = 0.04$ sec. The values of the elastic and viscous constants, i.e., k and c , can be varied until the proper values of u_0 and t_2 are obtained. In this manner, a displacement-time curve for the pile can be determined. A comparison of the "theoretical" curve shown in Figure 10 with the experimental curve shown in Figure 5 reveals quantitative as well as qualitative similarities; the simulation appears to be relatively realistic, especially with regard to the permanent pile displacement. For this particular field example, the elastic and viscous constants were found to be $k = 1,127,500$ lbf/ft and $c = 192,500$ lbf-sec/ft.

The simulation assumes in effect that the pile behaves as a rigid body as it is displaced into the soil. This assumption is reasonably valid if Young's modulus of elasticity between the soil and the pile is approximately a factor of ten, with the pile having the larger numerical value. The elastic constant k is essentially representative of the elasticity of the soil.

CONCLUDING REMARKS

The primary purpose of this study has been to develop a predictive estimate of the "static" load-bearing capacity of a pile on the basis of "dynamic" information obtained during the driving process. By assuming that a pile behaves as a rigid body during the driving process and by neglecting the inertial force, we found the load-bearing capacity to be equal to the centroid (along the force axis) of the hammer-pile-soil interaction force-time curve; this force-time relationship is determined by means of a load cell situated between the hammer and the top of the pile. A subsequent analysis, involving the energy output of the hammer and the maximum pile displacement, includes the inertial effect and yields a more accurate estimate of the load-bearing capacity. The values calculated from both analyses for full-scale piles are found to be in good agreement with static-load test results. In view of the relative simplicity of the theoretical considerations, the quantitative comparison between the calculated and measured values of load-bearing capacity is indeed quite satisfying. Additional verification of the calculated capacities is obtained from dynamic force-displacement curves as constructed from force-time and displacement-time records.

Because the area under a force-displacement curve represents the work done on the entire system by the interaction force, an evaluation of the energy transfer from the hammer is possible. On this basis, approximately 95 percent or more of the rated energy output of the hammer is transferred to the pile-soil system. This energy-transfer percentage is considerably higher than previously anticipated.

Through the use of an analog computer, the simulation of a hammer-pile-soil system is possible. The particular model considered in this study includes the viscous behavior of the soil and the gross elastic features of the entire system. Employing a matching procedure, we can obtain pile displacement-time curves with reasonable quantitative accuracy with regard to comparison with full-scale pile data.

In summary, the primary accomplishments of this investigation have been (a) to develop relatively simple but accurate methods of predicting the static load-bearing capacity of a pile on the basis of dynamic information obtained during the driving process; (b) to demonstrate that the energy transfer from the hammer to the pile-soil system is of the order of 95 percent or better; and (c) to provide preliminary insight into the model simulation of a hammer-pile-soil system on an analog computer.

ACKNOWLEDGMENTS

This investigation was undertaken in cooperation with the Federal Highway Administration. A special acknowledgment is made to W. W. Grimes, formerly of the South Dakota Department of Highways, for his efforts in initiating the pile-bearing capacity study and interest in continuation of the project to a successful conclusion. The

opinions, findings, and conclusions expressed in this paper are those of the authors and not necessarily those of the Federal Highway Administration.

REFERENCE

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DISCUSSION

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The authors are to be complimented for the work presented and particularly for the dynamic measurements made on load test piles. So far as the writers are aware, this work represents the first application of analog magnetic tape for recording pile dynamic measurements.

Questions do arise regarding the accuracy of the force measurements due to the presence of tension forces at the end of the record. The authors indicate that this portion of the record should be disregarded. It is possible to hypothesize on the reasons for this tension force. However, any possible explanation will imply as a consequence that earlier portions of the record are also in error. It is also observed that the rise time on the force records is very slow particularly when compared with the records given in a Michigan report (2). It would be expected that the force records obtained by the South Dakota researchers would resemble those obtained in Michigan (2) because of the similarity of the force transducer and driving assembly. Furthermore, because the pile force is proportional to the velocity, this implies that the peak force is dependent on the hammer velocity and not on the pile resistance. This is counter to the resistance prediction given by Eq. 22.

It is noted that the results given in Table 2 show a variety of values for the rated energy for a single hammer, the Link Belt 520. It is assumed that these values are obtained from measurements of the bounce chamber pressure. When they are compared with the manufacturer's rating, they are similar to those reported by others.

Reference

2. A Performance Investigation of Pile Driving Hammers and Piles. Michigan State Highway Commission, Lansing, March 1965.

AUTHORS' CLOSURE

The authors would like to thank Goble and Rausche for their comments concerning the paper. With regard to the accuracy of the force-time measurements, attention is directed to the records shown in Figures 4 and 5. The force-time trace shown in Figure 4 was recorded at a high sensitivity level on the magnetic tape unit. As a consequence of the hammer spike after the pulse maximum, the FM amplifier on the recorder was overloaded, whereupon the remainder of the trace was distorted. The force-time trace shown in Figure 5 was recorded at the correct sensitivity level. By comparison, the only significant difference in form of the 2 pulses occurs after the hammer spike. In particular, the tensile forces shown in Figure 5 are noticeably absent. As a consequence, the authors do not feel that the portions of the force-time records shown in Figures 2, 3, and 4 prior to the hammer spikes are in error. It is not unreasonable to expect a tensile force in the later portion of the records, because of the physics of

the composite hammer-pile-soil system; however, it is felt that the tensile forces should be negligible as compared to the compressive forces, as indeed is shown in Figure 5. In addition, the rise times of the force records shown in Figures 2, 3, and 4 are comparable with the rise times of the force-time traces obtained with an oscilloscope by the investigators in Michigan (2). However, the majority of the force-time traces they illustrate were graphically constructed from hand-digitized data obtained from oscillograph records. By comparison with these traces, the rise times of our force-time records are indeed slow. Consideration must also be given to the improvement in the recording system used to obtain our data as compared to the system employed by the investigators in Michigan. Because of the nature of the instrumentation system, the authors are of the opinion that careful consideration should be given to the use of the dynamic data given in the Michigan report as a standard or a reference frame for the evaluation of the results of subsequent investigations.

On physical grounds, it is certainly plausible to expect that soil behaves in a viscous fashion during loading. As a consequence, the resistive force of the soil on the pile is dependent on the velocity of the pile as indicated in Eq. 1. In the derivation of Eq. 22, the assumption is made that the behavior of the pile-soil system during dynamic loading is analogous to that of a damped simple harmonic oscillator. At the instant of maximum pile displacement, the pile velocity is zero and the pile acceleration is a negative maximum. Because the calculation of the pile-bearing resistance is made at the instant of maximum displacement, the viscous contribution in the force balance on the system is zero. With reference to this condition, the energy required to permanently displace the pile into the soil is the difference between the rated output energy of the hammer and the energy required for simple harmonic motion of the pile. This energy difference is related to the work done by the hammer-pile-soil system interaction force and the pile resistive force. The interaction force is measured by means of a load cell situated between the hammer and the top of the pile. Because the interaction force is effectively measured internally within the hammer-pile-soil system, it must reflect the elastic characteristics of the entire system, as well as the viscous nature of the soil resistance to pile motion.