

HIGHWAY RESEARCH RECORD

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| 7 Reports

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FOREWORD

A wide variety of subjects of interest to designers, construction engineers, specification writers, and those responsible for materials and testing is contained in this RECORD. Three of the papers are related in that they are concerned with acceptance sampling plans and specifications having a rational basis in their recognition of product variability, i. e., "statistical specifications." The others are related also, not in the subject matter being discussed but rather in the criticality of the problems under consideration. What highway organization is not concerned with new and better information on vibration, pavement friction, coatings for bridge decks, and pavement seals? There is much to be learned in all these areas.

Bower and Gerhardt on the subject of the effect of vibration on pavement durability conclude, "A new construction specification for pavement consolidation is needed." They elaborate further on good versus poor practices and their effects on the finished product.

The papers by Jorgenson (one published as an Abridgment) and the one by Anday should be read carefully by all who deal with or are affected by the development and implementation of acceptance sampling plans and statistical specifications. These papers include proposed systems for acceptance of base and subbase materials, compacted embankments, and asphalt construction. The text of the recommended specifications will be of particular interest to many readers.

Rose and Ledbetter provide a summary of the contributions of surface finish, fine aggregate, mix proportions, and construction practices to the resulting friction properties of portland cement concrete pavements. These factors, they state, contribute most to friction properties. This is an excellent report of what is known on the subject. It distills for the reader the findings and conclusions of 39 previously published reports on various aspects of frictional properties of concrete pavements and is a valuable addition to the literature.

Spellman and Stratfull describe experience with a nondestructive test method that has promise of providing information that may be useful in predicting the performance of sealants. They describe the development of the instrument and its use in the field.

The final paper by Vyas is concerned primarily with the development of analytical tools for predicting the load-deflection characteristics of neoprene seals having a particular geometry. According to Vyas, the analysis is "basically a variation of the well-known problem of elastica, first investigated by Euler." The results of the analysis are used to predict deformations for a seal, and these are compared to results derived experimentally.

—J. F. McLaughlin

CONTENTS

THE EFFECT OF GOOD VIBRATION ON THE DURABILITY OF CONCRETE PAVEMENT

Laurence C. Bower and Burrell B. Gerhardt 1

DEVELOPMENT AND TRIAL USE OF ACCEPTANCE SAMPLING PLANS FOR COMPACTED EMBANKMENTS

James L. Jorgenson 24

DEVELOPMENT AND TRIAL USE OF ACCEPTANCE SAMPLING PLANS FOR ASPHALT CONSTRUCTION

James L. Jorgenson 35

STATISTICAL SPECIFICATION FOR THE ACCEPTANCE OF PUG MILL-MIXED BASE AND SUBBASE MATERIALS

M. C. Anday 39

SUMMARY OF SURFACE FACTORS INFLUENCING THE FRICTION PROPERTIES OF CONCRETE PAVEMENTS

J. G. Rose and W. B. Ledbetter 53

AN ELECTRICAL METHOD FOR EVALUATING BRIDGE DECK COATINGS

Donald L. Spellman and Richard F. Stratfull 64

ANALYSIS OF ELASTOMERIC PAVEMENT SEALS

Ravindra K. Vyas 72

THE EFFECT OF GOOD VIBRATION ON THE DURABILITY OF CONCRETE PAVEMENT

Laurence C. Bower and Burrell B. Gerhardt, Colorado Department of Highways

Because a noticeable difference in durability of concrete pavements in Colorado seems to be associated with differences in consolidation and void content of the concrete, an effort has been made to determine the relationship. Test sections were laid out on 2 pavements; one was composed of a fine-aggregate mix and one was composed of a coarse-aggregate mix. The paver used with the coarse-aggregate mix was equipped with a surface pan vibrator and 2 internal vibrators that could be spaced for any desired setting. The other test sections had internal vibration by vibrators having eccentrics with 4 different diameters and 3 different vibration speeds. In addition, the angle of the vibrators was changed, as was the height of the vibrator from the base of the slab. Paver speeds were also varied from 10 to 19 ft/min. After having been in place for only 4 months, little can be said about durability of the concrete, but much has been determined about consolidation and segregation of the particles. To acquire good consolidation, 1 $\frac{1}{4}$ -in. slump concrete required considerable vibration. Proper amplitude, frequency, and spacing of vibrators will provide up to 100 per cent consolidation of concrete pavements without segregation of the aggregate. The nuclear density device is a reliable indicator of the consolidation of freshly poured concrete pavement and, when used by a capable operator, may be used to check and control densities during construction.

•A COMPARISON of the performance of concrete pavements in Colorado to the effort during construction to vibrate the concrete will reveal a consistent relationship. Where the effort and cost were relatively high, the wearing surface has usually provided good, maintenance-free service. Where very little attention was given to consolidation of the fresh concrete, the ensuing years of heavy loads, studded-tire use, and freezing weather have left the surface badly abraded.

Figure 1 shows the results of a cursory investigation of concrete pavements in Colorado over which there has been considerable traffic in the past 10 years. The percentage of densification shown in the figure is based on core weights and rodded cylinder tests. No doubt factors other than vibration such as age, air entrainment, and water-cement ratio affect durability of concrete pavements, but densification appears to be one factor that we might improve by means of better specifications. At the present time, many specifications for the placement of concrete are vague in regard to consolidation.

PRESENT SPECIFICATIONS

The following wording of the standard Colorado specification for consolidation of pavements is typical of that used by many other construction agencies:

Vibrators for full width vibration of concrete paving slabs may be either the surface pan type or the internal type with either immersed tube or multiple spuds. They may be attached to the spreader or the finishing machine, or may be mounted on a separate carriage. The frequency of the surface vibrators shall not be less than 3,500 impulses per minute and the frequency of the

internal type shall not be less than 5,000 impulses per minute for tube vibrators and not less than 7,000 impulses per minute for spud vibrators.

Some doubt is created by other standard specifications such as those of the U. S. Corps of Engineers that state, "Vibrating equipment shall be of the internal type, and the number and power of each unit shall be adequate to properly consolidate all of the concrete. The amplitude of vibration shall be sufficient to produce satisfactory consolidation of the concrete with the vibrator spacing used." The main questions are, Will an effort to consolidate concrete pavements during construction consistently increase pavement durability? What realistic densification effort should be specified to acquire durable concrete pavement?

This project was started in an attempt to help answer these 2 questions. Interest from other agencies such as the Highway Research Board, the Corps of Engineers, The American Concrete Paving Association, and other state highway departments has been high, indicating that the problem is universal. The Federal Highway Administration approved our request for this study in April 1970 and made some very helpful suggestions involving procedure and items for investigation. Contractors and suppliers in the area readily agreed to provide equipment for the study.

LABORATORY FINDINGS

Two classes of concrete are commonly used for concrete pavements in Colorado. The coarse-aggregate mix (Class A) is made up of about 65 percent plus No. 4 particles, 35 percent sand, and 6 sacks/cu yd of cement. The fine-aggregate mix (Class AX) is made up of about 35 percent plus No. 4 particles, 65 percent sand, and 6½ to 7 sacks/cu yd of cement. The consolidation characteristics of the coarse-aggregate mix were investigated north of Denver on the 2-mile, 4-lane project I270-6(7). Consolidation studies on the fine-aggregate mix were made in northeastern Colorado on the 15-mile, 4-lane project I80S-2(20) and on the 23-mile, 2-lane project I70-5(21). The designed mixes for two of these projects were investigated in the laboratory prior to and during construction. Table 1 gives typical values determined in the laboratory with each mix.

An attempt was made to determine both the static pressure and the vibratory effort to consolidate both classes of concrete. The results shown in Figure 2 indicate that

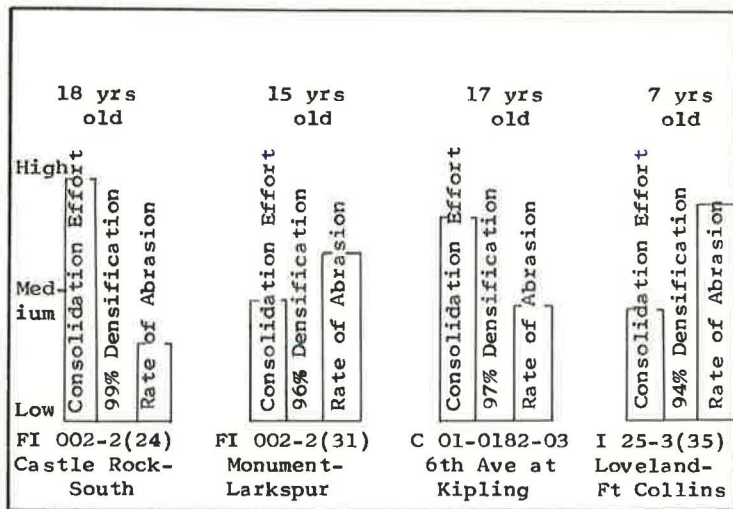


Figure 1. Consolidation effort indicated by extra work orders and close vibrator control.

TABLE 1
CHARACTERISTICS OF CONCRETE MIXES

| Characteristic | Fine-Aggregate Mix on Project I80S | Coarse-Aggregate Mix on Project I270 |
|--|------------------------------------|--------------------------------------|
| Theoretical density air free, pcf | 147.56 | 152.24 |
| Theoretical density 4 percent air, pcf | 141.66 | 146.15 |
| Average rodded density, pcf | 140 | 146 |
| Average density of cylinders from field, pcf | 138 | 145 |
| Average density of beams from field, pcf | 137 | 145 |
| Swiss hammer readings, psi | 2,800 to 3,300 | 3,400 to 4,100 |
| Sonic modulus of elasticity, psi | 1,185,700 | 1,356,700 |
| Average 14-day beam strength, psi | 450 | 450 |
| Average 28-day cylinder strength, psi | 4,470 | 4,480 |
| Water-cement ratio | 0.45 | 0.44 |

there is considerably more effort needed to consolidate 1-in. slump concrete than 2-in. slump concrete. Because both construction paving projects were scheduled to use 1- to 1½-in. slump concrete, it is easy to imagine that, unless there was good vibration on both projects, the number of voids (rat holes) would be large.

From unvibrated samples of 1¼-in. slump concrete prepared in the laboratory, it appeared that 127 pcf might be the lower limit of density that we could ever expect in the field for the fine-aggregate mix. The oscillating screeds provide some consolidation, however, so a value of 130 to 132 pcf appeared to be a more realistic value to expect as a lower limit for density values of the fine-aggregate mix. The paver for the coarse-aggregate mix was a "form" type equipped with a vibrating pan. Internal vibrators were added to provide the needed information for this research. The relationship between density and void space is shown in Figure 3.

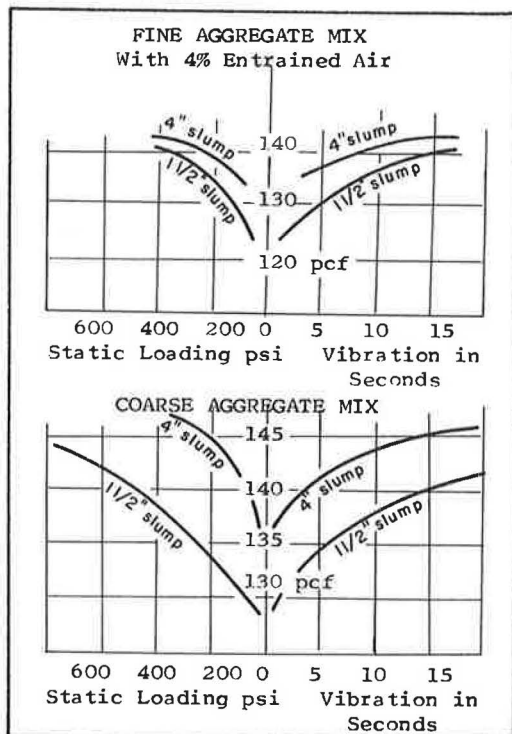


Figure 2. Effort necessary to consolidate stiff and plastic mixes as indicated by vibration in the laboratory and static loading.

FIELD TEST LAYOUTS

With some knowledge derived from laboratory tests, it was possible to lay out a series of test sections on the paving projects that would provide information on consolidation by 4 different sizes of vibrators. These sections would also serve as durability test sections from 1970 until 1975 or longer, because both roadways would be serving about 8,000 vehicles per day. Data on the layout of the test sections and the variations in vibrators are given in Tables 2 and 3.

On project I80S where the fine-aggregate mix was used, the unaltered or "control" sections were usually in the passing lane next to the experimental driving lane.

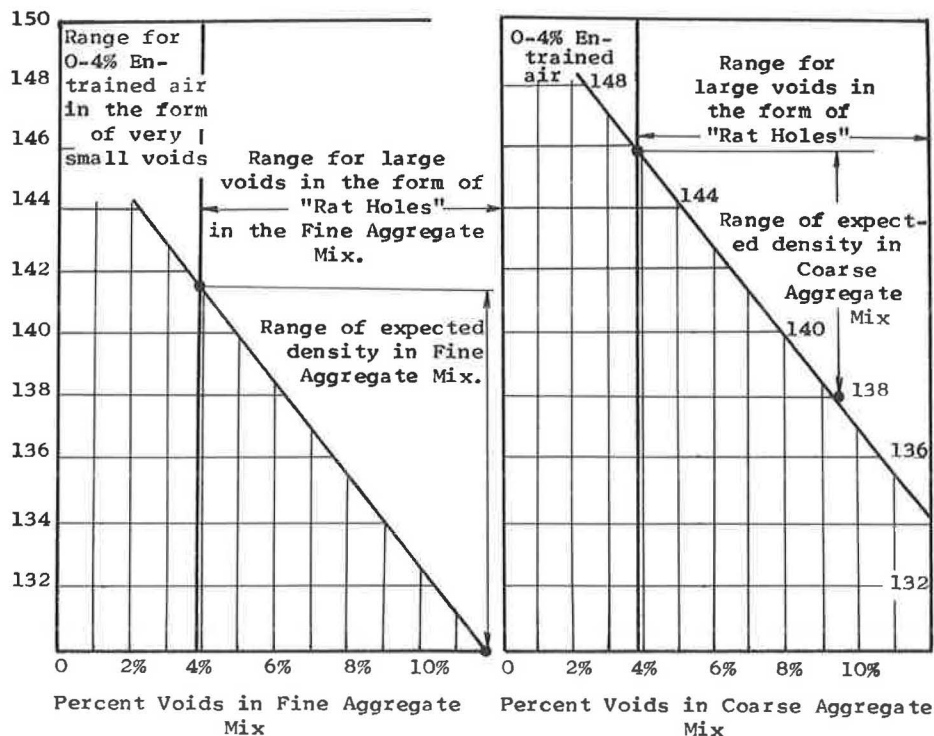


Figure 3. Relationship between density and percentage of voids for the fine-aggregate and the coarse-aggregate mixes.

TABLE 2
LAYOUT OF TEST SECTIONS FOR FINE-AGGREGATE MIX ON PROJECT 180S

| Paver Speed (ft/min) | Amplitude of Vibration (eccentric diameter in in.) | Approximate Spacing Between Vibrators (in.) | Angle of Vibrator (deg) | Height of Vibrator From Base (in.) | rpm of Vibrator | Section | Station | Typical Density (pcf) | |
|----------------------|--|---|-------------------------|------------------------------------|-----------------|-----------------|-----------------|-----------------------|-------------------|
| | | | | | | | | Over Vibrators | Between Vibrators |
| 13 | 1 $\frac{1}{8}$ | 24 | 0 | - ^a | 9,000 | 12 ^b | 4994+35-4997+65 | 137 | 134 |
| | | | 0 | 5 | 9,000 | 13 ^b | 4997+65-5000+50 | 137 | 133 |
| | | | 30 | - ^a | 9,000 | 14 ^b | 5000+50-5003+50 | 137 | 133 |
| | | | 30 | - ^a | 10,800 | 5 | 4940+00-4913+50 | 139 | 136 |
| | | | 0 | - ^a | 10,800 | 9 | 4970+50-4978+02 | 138 | 136 |
| | 1 $\frac{5}{16}$ | 15 | 0 | - ^a | 9,000 | 8 | 4961+50-4968+75 | 135 | 134 |
| | | | 0 | - ^a | 10,800 | 9 | 4970+50-4978+02 | 138 | 136 |
| | | | 0 | 5 | 9,000 | 7 | 4949+25-4958+40 | 135 | 135 |
| | | | 0 | 5 | 10,800 | 6 | 4940+00-4948+50 | 137 | 136 |
| | | | 30 | - ^a | 9,000 | 11 | 4987+50-4994+35 | 134 | 133 |
| | 1 $\frac{7}{16}$ | 12 | 0 | - ^a | 10,800 | 10 | 4978+75-4986+50 | 138 | 136 |
| | | | 0 | - ^a | 9,000 | 16 | 5013+50-5018+50 | 135 | 133 |
| | | | 0 | - ^a | 10,800 | 15 | 5008+50-5013+50 | 137 | 135 |
| | | | 0 | 5 ^c | 9,000 | 17 | 5024+50-5032+00 | 135 | 133 |
| | | | 0 | 5 ^c | 10,800 | 18 | 5032+00-5038+50 | 137 | 135 |
| | 1 $\frac{1}{4}$ | 9 | 0 | 5 ^c | 10,800 | 19 | 5040+50-5042+50 | 136 | 134 |
| | | | 30 | - ^a | 9,000 | 21 | 5049+50-5057+00 | 135 | 134 |
| | | | 30 | - ^a | 10,800 | 20 | 5042+50-5049+50 | 138 | 136 |
| | | | 0 | - ^a | 9,000 | 23 | 5088+50-5095+50 | 134 | 133 |
| | | | 0 | - ^a | 10,800 | 22 | 5081+50-5088+50 | 136 | 135 |
| 10 | 1 $\frac{7}{16}$ | 12 | 0 | 5 | 9,000 | 24 | 5095+50-5100+50 | 134 | 133 |
| | | | 0 | 5 | 10,800 | 25 | 5100+50-5107+50 | 136 | 135 |
| | | | 30 | - ^a | 9,000 | 27 | 5114+50-5122+00 | 134 | 133 |
| | | | 30 | - ^a | 10,800 | 26 | 5107+50-5114+50 | 135 | 133 |
| 19 | 1 $\frac{7}{16}$ | 12 | 30 | 5 | 10,800 | 2 | 4874+54-4880+40 | 140 | 136 |
| 13 | 1 $\frac{7}{16}$ | 12 | 30 | 5 | 10,800 | 2 | 4880+60-4887+68 | 139 | 135 |
| 13 | 1 $\frac{7}{16}$ | 12 | 30 | 5 | 10,800 | 3 | 4888+00-4894+42 | 140 | 135 |
| 16 | 1 $\frac{7}{16}$ | 12 | 30 | 5 | 10,800 | 4 | 4894+60-4901+50 | 139 | 135 |

^aMidway between base and top of surcharge.

^bPassing lane; all other test sections in driving lane.

^cFast power speed.

TABLE 3
LAYOUT OF TEST SECTIONS FOR COURSE-AGGREGATE MIX ON PROJECT I270

| Amplitude of Vibration (eccentric diameter in in.) | Spacing Between Vibrators (in.) | rpm of Vibrator (in air) | Section | Station | Typical Density (pcf) | | | |
|--|--|--------------------------------|---------|---------------|-----------------------|----------------------|-----|-----|
| | | | | | Over Vibrators | Between Vibrators | | |
| 1 $\frac{1}{6}$ | 12 | 7,000 | 16 | 95+50-97+00 | 144 | 144 | | |
| | | 9,000 | 17 | 100+50-102+00 | 145 | 145 | | |
| | | 11,000 | 18 | 104+00-105+50 | 146 | 146 | | |
| | 18 | 7,000 | 15 | 94+50-95+50 | 144 | 143 | | |
| | | 9,000 | 14 | 93+50-94+50 | 145 | 144 | | |
| | | 11,000 | 13 | 91+27-93+50 | 146 | 144 | | |
| | 24 | 7,000 | 10 | 85+10-85+96 | 143 | 140 | | |
| | | 9,000 | 11 | 85+96-87+43 | 144 | 141 | | |
| | | 11,000 | 12 | 90+27-92+27 | 145 | 143 | | |
| | 1 $\frac{5}{6}$ | 12 | 7,000 | 3 | 72+00-73+50 | 143 | 141 | |
| | | | 9,000 | 2 | 70+60-72+00 | 144 | 143 | |
| | | | 11,000 | 1 | 68+20-70+60 | 145 | 144 | |
| 18 | | 7,000 | 4 | 73+60-75+50 | 143 | 141 | | |
| | | 9,000 | 5 | 75+50-77+50 | 144 | 141 | | |
| | | 11,000 | 6 | 77+50-78+40 | 144 | 141 | | |
| 24 | | 7,000 | 9 | 81+50-83+50 | 143 | 138 | | |
| | | 9,000 | 8 | 80+50-81+50 | 144 | 139 | | |
| | | 11,000 | 7 | 78+40-80+50 | 144 | 140 | | |
| 1 $\frac{7}{16}$ | | 30 | 11,000 | X | 83+63-84+60 | 144 | 139 | |
| | | | 12 | 7,000 | 22 | 109+75-111+00 | 142 | 140 |
| | | | | 9,000 | 23 | 111+00-112+25 | 143 | 140 |
| | 11,000 | 19 | | 106+00-107+30 | 144 | 143 | | |
| | 18 | 7,000 | 21 | 108+50-109+75 | 143 | 140 | | |
| | | 9,000 | 24 | 114+50-116+00 | 144 | 141 | | |
| | | 11,000 | 20 | 107+30-108+50 | 145 | 142 | | |
| | 24 | 7,000 | 27 | 118+00-119+00 | 143 | 140 | | |
| | | 9,000 | 25 | 116+00-117+00 | 144 | 140 | | |
| | | 11,000 | 26 | 117+00-118+00 | 145 | 141 | | |
| | 1 $\frac{1}{4}$ | 12 | 7,000 | 28 | 119+70-121+00 | 142 | 141 | |
| | | | 9,000 | 29 | 121+00-122+00 | 143 | 142 | |
| 11,000 | | | 33 | 133+00-134+00 | 144 | 143 | | |
| 18 | | 7,000 | 36 | 136+50-137+50 | 141 | 140 | | |
| | | 9,000 | 35 | 135+25-136+50 | 142 | 141 | | |
| | | 11,000 | 34 | 134+20-135+25 | 143 | 142 | | |
| 24 | | 7,000 | 31 | 124+00-125+50 | 141 | 139 | | |
| | | 9,000 | 30 | 122+85-124+00 | 142 | 140 | | |
| | | 11,000 | 32 | 125+50-127+00 | 143 | 141 | | |

Occasionally the passing lane became the experimental lane and the driving lane was designated as the control section. The base course for this project was a 4-in., 3 percent SS-K emulsified asphalt-treated sand. The paver and vibrator mounting are shown in Figures 4 and 5.

On project I270 where the paver was equipped with a surface pan vibrator operating at 4,100 rpm, only 2 internal vibrators were used for each test section because the large auxiliary 189-cycle, 15-kw generator for the vibrator used on project I80S was not available. A smaller auxiliary 5-kw generator was used for the 2 vibrators. One internal vibrator was mounted 32 in. from the edge of the driving lane shoulder, and the other was positioned 12, 18, or 24 in. from the first vibrator for different test sections. The remainder of the 24-ft roadway became the control section, although there were many control sections placed between test sections as well. The base course was a 4-in. layer of crushed gravel and sand. The form



Figure 4. Paver used on project I80S with 15-kw, 180-cycle generator on right front leg of paver.

paver and vibrator are shown in Figures 6 and 7.

The vibrators on both pavers were changed at the end of the day so that the changeover to different eccentrics would not interfere with the contractor's operation. The rpm of the vibrator (frequency) and the angle could be changed while the pavers were in motion. Inspectors measured speed of the paver every 15 min or whenever there appeared to be a change. There were occasional "stops" due to the paver or haul truck slowdown. All such irregularities were documented as to location so that future coring and testing would not be affected by them.

In addition to the regular tests for control of the construction, special tests recommended by the Federal Highway Administration officials were performed for segregation of particles, density, air content, and pressure around the vibrators. Selection of the sample for the particle size analyses was made in the finished pavement by pushing a 21-in. diameter cylinder (bottomless garbage bucket) into the mix and scooping out the top third, middle third, and bottom third of the concrete into separate containers for separate tests.

Probably the most valuable information was obtained with the nuclear device that was used to measure density of the concrete immediately after placing. A means was devised for opening a small hole in the pavement and inserting the probe in a very exact manner. Within 3 min after the pavement was placed, a wet-density value could be determined. Figure 8 shows the use of the nuclear tester on project I80S. Figure 9 shows the use of the open-end cylinder for sampling fresh concrete for the particle size tests to determine segregation of the aggregate.

FIELD TEST RESULTS

Test results from this study will be in the process of development for several years. This is particularly true for durability tests that will require thousands and even millions of vehicle traverses to tell the complete story. However, much information about consolidation has already come to light. Average densities determined for the test sections are given in Tables 2 and 3. From the available data on field consolidation, it



Figure 6. Form paver used on project I270 with machine used to insert plastic transverse weakened-plane joint and with bridge for burlap drag.



Figure 5. Internal vibrator mounted on paver.

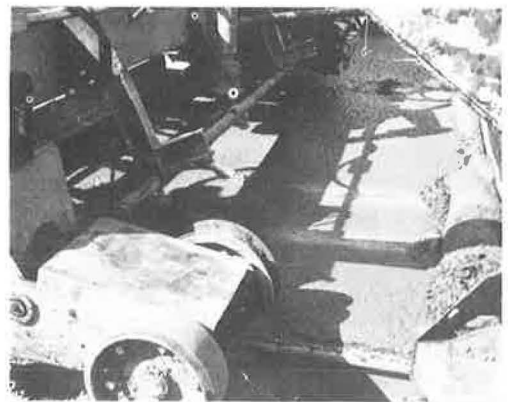


Figure 7. Internal vibrator mounted on paver and "wake" developed as vibrator passes through 1½-in. slump concrete.

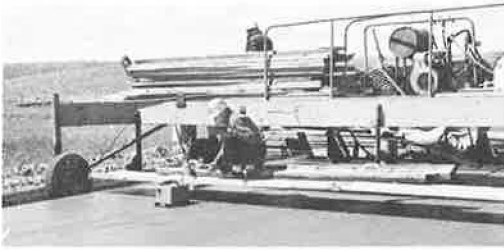


Figure 8. Nuclear test being taken in fresh concrete to determine density immediately after laydown.

appears the the density limits predicted from laboratory tests are quite applicable. There were densities as low as 127 pcf and as high as 140 pcf from cores and nuclear readings in the fine-aggregate mix.

On project I270 where the coarse-aggregate mix was used, the typical density pattern from the cores and nuclear readings was supplemented by density values from beams sawed out of the pavement. The coarse-aggregate mix varied in density from 138 to 146 pcf. Low densities were accompanied by a high percentage of rat holes. Soon after coring operations began, testers found the need for a procedure to measure the bulk specific gravity rather than the specific gravity obtained by the usual procedure of weighing cores in air and in water and basing the density on the difference between these 2 values. Bulk specific gravity was finally determined by sawing the ends off the cores as squarely as possible. The density was then based on the weight in air and a volume calculated from the height and the diameter. Some of the samples had 5 or 6 percent air voids in addition to the entrained air and this procedure became necessary even though it required more time to perform.

Even after everyone was satisfied that the procedure for determining density from the cores was giving accurate results, it was apparent that the density data were not going to present a perfect pattern. Computer outputs showed rather poor correlation of density with vibrator effort, and standard deviation values were high. The trouble appeared to be associated with erratic slump values during construction. Apparently slump was one of the most important variables in the study, and no special significance had been attached to it in tests on project I80S or project I270. Data had been recorded for slump values throughout the tests, but neither of the 2 test sites had test sections with variables and controlled slump values.

By chance, paving on project I70-5(21) in eastern Colorado between Seibert and Bethune was about to start when the need was realized for more data on slump. The aggregate for this project was similar to that for project I80S and so was the designed mix. A conference with representatives of the Federal Highway Administration resulted in approval for the establishment of the additional experimental sections just south of Vona (Table 4). A discussion of this project follows.



Figure 9. Sampling procedure for particle size analysis.

TABLE 4

FIELD TESTS ON PROJECT I70 FOR THE EFFECT OF SLUMP ON CONSOLIDATION

| Station | Slump (in.) | Density (pcf) at Distance From Vibrator | | | |
|-----------------|-------------|---|-------|--------|--------|
| | | 0 in. | 6 in. | 12 in. | 18 in. |
| 1958+50-1957+75 | 2¼ | 140 | 138.5 | 137.5 | 135.5 |
| 1956+85-1956+38 | 1 | 139 | 138 | 136 | 134 |
| 1954+45-1954+00 | 3 | 139 | 138 | 136 | 133 |
| 1953+85-1953+80 | ½ | 138 | 136.5 | 134.5 | 132 |
| 1950+18-1950+00 | 1½ | 140 | 138.3 | 137 | 135.2 |
| 1949+82-1949+52 | 4 | 133.5 | 132 | 131 | 130 |
| 1948+05-1947+93 | ½ | 138 | 136.5 | 134.5 | 132 |

ANALYSIS OF DATA

Effect of Slump on Density

Laboratory tests on the coarse-aggregate mix and on the fine-aggregate mix showed that, for a particular vibrator effort, the final density depended very much on the slump or consistency of the mix. The tests were made in molds 6 in. wide, 5 in. deep, and 48 in. long in order that a small vibrator with a $\frac{3}{4}$ -in. eccentric and 5,500 rpm frequency might be pulled through the mix at approximately 10 fps. The results are shown in Figure 10.

The density appears to increase approximately 2 pcf as the slump increases from $\frac{1}{2}$ to 2 in., but it shows a gradual decrease thereafter because the specific gravity of water is about a third of the specific gravity of cement and aggregate. The flexure strength on the beams made from these mixes showed that, although the $\frac{1}{2}$ - and 1-in. slump concrete contained large air voids, it was stronger than the concrete made with 3- and 4-in. slump concrete.

Field tests to determine the effect of slump on consolidation were performed on project I70 by using a paver equipped with 11 internal vibrators spaced as follows from the north edge to the south edge of the pavement: 0.5, 2.5, 2.7, 2.4, 2.6, 2.5, 2.0, 0.9, 2.2, 2.5, and 0.6 ft.

The densities given in Table 4 for the different mixes indicate the increase in consolidation that may be expected with 2-in. slump concrete as compared with less than 1-in. slump concrete. The laboratory and field tests agree quite well that slump values in the 2-in. range allow the vibrators to develop densities approximately 2 pcf higher than would be developed with the low slump of $\frac{1}{2}$ in. or with the high slump of 4 in.

Figure 11 shows a summation of the findings from field tests with the $1\frac{7}{8}$ -in diameter eccentric on projects I80S and I270 after appropriate allowance is made for slump. The estimated level of compaction achieved by each type of vibration is also shown.

Effect of Vibrator Angle and Height

Vibrators were arranged on all the pavers so that their angles could be changed from 0 deg, or horizontal, to 30 deg during paving operations. No difference in consolidation of the concrete could be attributed to this change of angle so long as the entire vibrator was within the plastic concrete. When the height was varied far enough to withdraw the vibrator from the concrete mix, there was a noticeable loss of consolidation effort imparted to the concrete. There was also a noticeable gain in frequency, as the vibrator

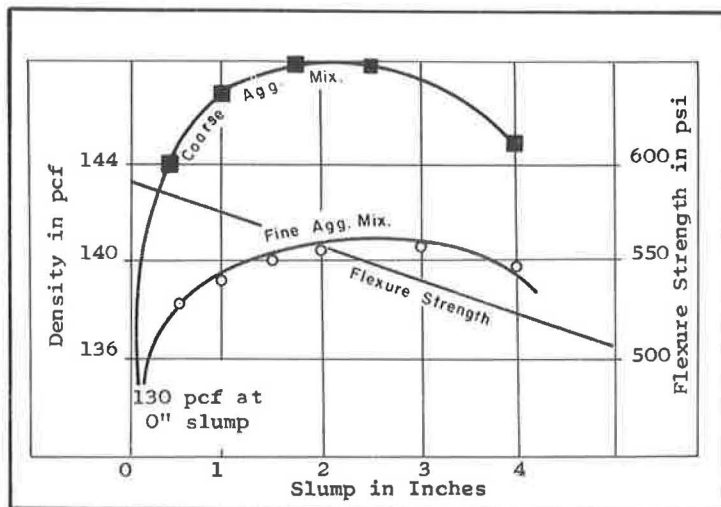


Figure 10. Results of laboratory tests.

was pulled out of the mix, indicating that the vibrator was not working against so much back pressure and, hence, was doing less work.

Some paver operators insist that a slight angle of the vibrator assists in "packing" the mortar in. No special tests were performed to affirm or refute this contention because the regular series of tests showed no advantage in placing the vibrators at an angle.

The tests on this project seem to indicate that a horizontal vibrator (or one only slightly tipped), placed at the mid-point of the slab height, is in optimum position for consolidation of the plastic concrete.

Effect of Paver Speed on Density

The variation of density was checked with paver speed by operating the paver on project I80S as near as possible at constant speeds of 10, 13, 16, and 19 ft/min with the $1\frac{7}{8}$ -in. diameter vibrators operating at 9,600 rpm in the mix and 10,800 rpm in air. Uncontrollable variations caused by changes in slump and base course preparation resulted in considerable scatter of points. There was only the slight indication of loss in density with increased speed shown by test sections 1, 2, 3, and 4 (Table 2).

With a difference of only 1 pcf (and this directly over the vibrator path) between the 19-ft/min speed and the 10-ft/min speed, it would have to be agreed that paver speed between these ranges resulted in very little variation in consolidation of the concrete with the big paver and the large high frequency vibrators.

Effect of Amplitude and Frequency on Density

According to the theory and research, the amplitude and the rpm of vibration determine the force or pressure that the vibrator is able to impart to the concrete. For the vibrators used, the relationship is shown in Figure 12. At 10,800 rpm (the normal

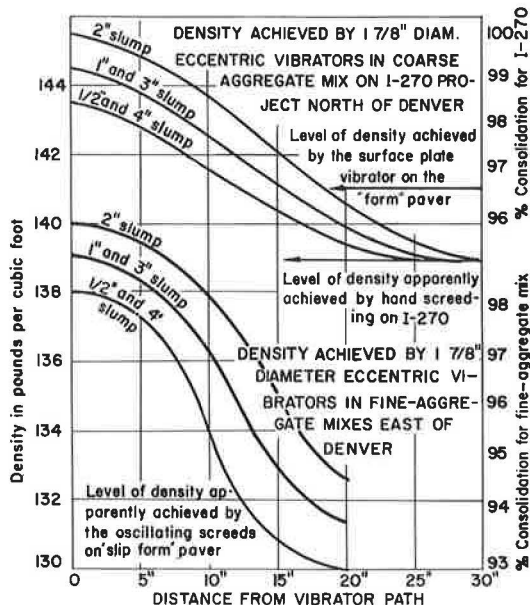


Figure 11. Relationship between density and distance from vibrator path for fine-aggregate and coarse-aggregate mixes.

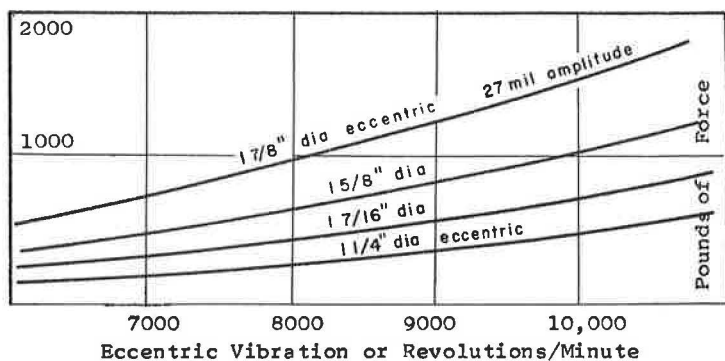


Figure 12. Force imparted by vibrators used on test sections of projects I80S and I270.

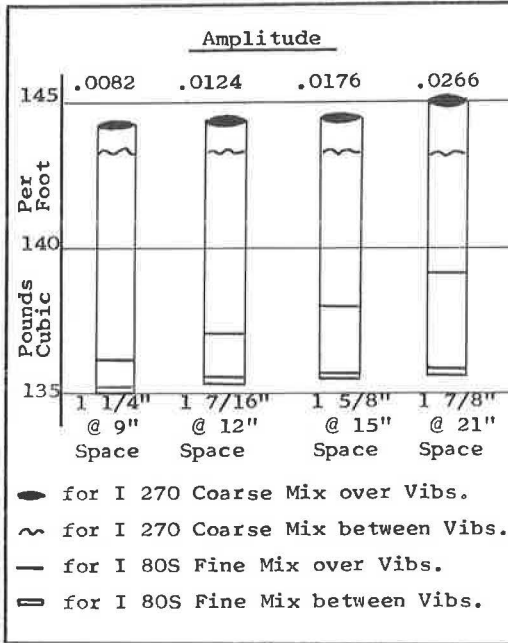


Figure 13. Variation of density with amplitude of vibration for different eccentrics.

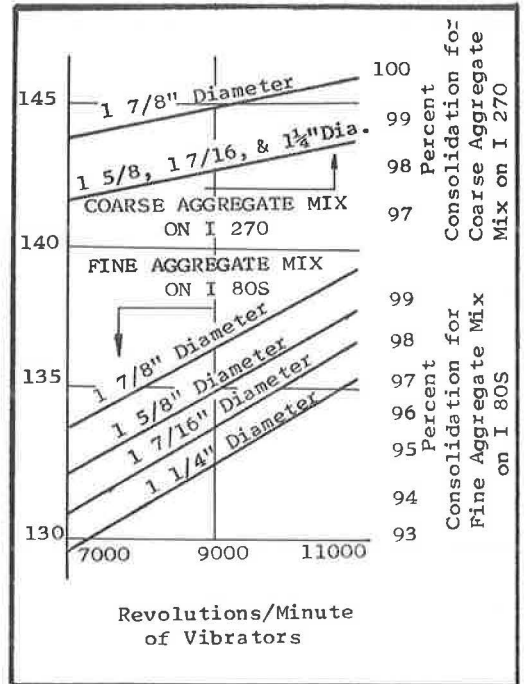


Figure 14. Variation of density with frequency of vibration.

vibration in air for the internal vibrators used on this research project), the densities developed over the vibrator path and between vibrators were as shown in Figure 13. The same vibrators run at different frequencies showed a relationship of vibration rate with density as shown in Figure 14.

From these relationships, it might be concluded that there is a gradual increase in density with an increase in either amplitude, frequency, or both. Not indicated by the data but possibly true is the fact that low slump grouts react most effectively to frequencies between 7,000 and 15,000 per minute. It is easy to imagine that, if the frequency and amplitude were too high, there would be a loss of vibrator-grout contact. What is desired is the proper number of "differential pushes" in rapid succession to effectively liquidize a stiff mix that has large entrapped air voids. Under liquid conditions, the air bubbles rise to the surface and are expelled, leaving a strong (because it has a low water-cement ratio) well-consolidated mass that should be durable.

The graphs show that the 1 7/8-in. diameter eccentric vibrated at 10,800 vibrations per minute in air and 9,600 rpm and in the mix will consolidate both types of concrete close to 100 percent of the laboratory rodded density value. The other eccentrics vibrated at somewhat lower speeds are not quite so effective, even when spaced closer together.

Effect of Vibration on Strength

Figures 15 and 16 show the typical appearance of 700 cores taken and photographed for this study. In general, the fine-aggregate mix from project I80S showed the greater percentage of large air voids. However, in the case of both the fine-aggregate mix and the coarse-aggregate mix from project I270, there were fewer voids in the cores taken in the path of the vibrator than between the vibrator paths. In fact, many of the 2-in. diameter cores taken between vibrator paths on project I80S broke during the drilling operation. Others must have been badly fractured because some of them showed

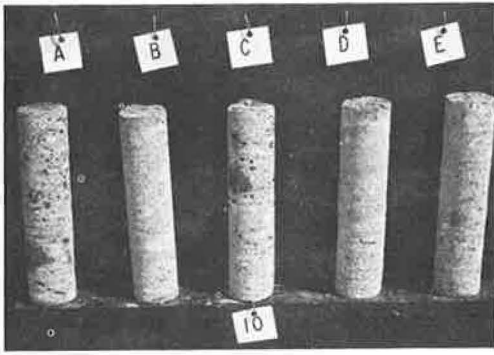


Figure 15. Cores from section 10 on project 180S where fine-aggregate mix was used (cores B and D were taken over path of vibrator, and cores A, C, and E were taken between paths of 1⁵/₈-in. diameter eccentric vibrator).

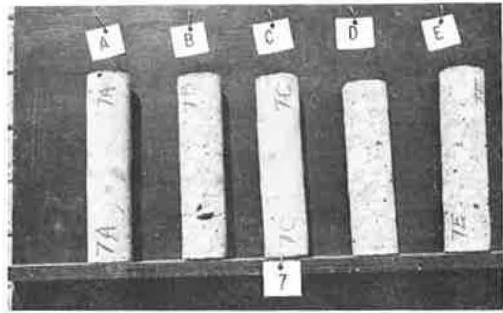


Figure 16. Cores from section 7 on project 1270 where coarse-aggregate mix was used (cores A and C were taken over path of vibrator, and cores B, D, and E were taken between paths of 1⁵/₈-in. diameter eccentric vibrator).

exceptionally low strength values when tested in the laboratory. In most cases, these low values were included in the data analysis because they generally reflect the overall strength of the pavement.

Compression test data are shown in Figures 17, 18, 19, and 20. Average values of 2-in. diameter cores 4 in. long show that the concrete in the path of the vibrators is on the average 10 percent stronger than concrete found between the paths of the vibrators on project 1270 where the paver

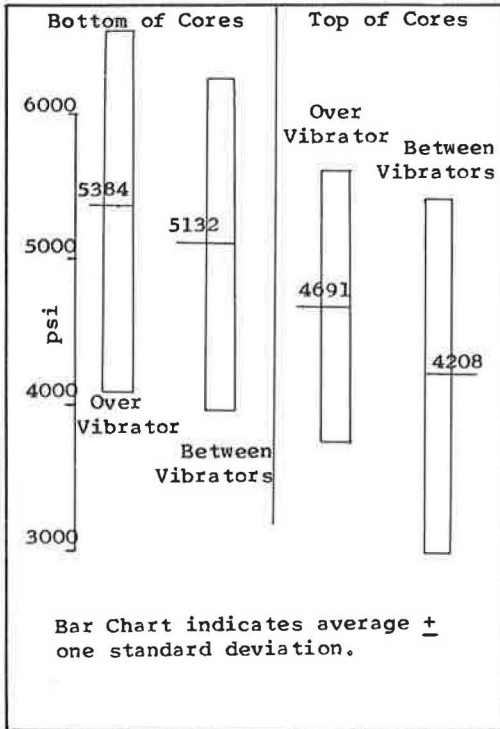


Figure 17. Comparison of compressive strength for top and bottom of concrete cores over and between vibrator paths on 1270.

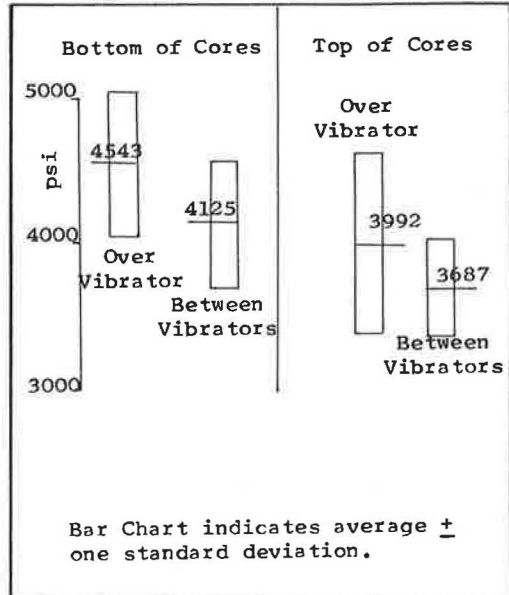


Figure 18. Comparison of compressive strength for top and bottom of concrete cores over and between vibrator paths on project 180S.

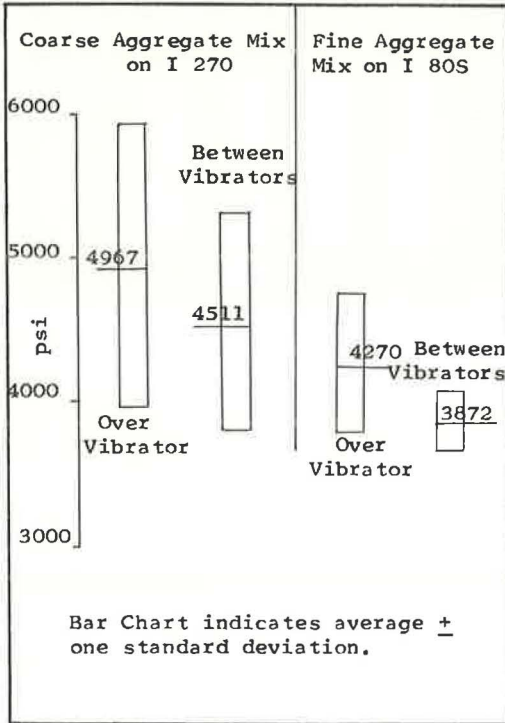


Figure 19. Comparison of compressive strength for concrete cores over and between vibrator on projects I270 and I80S.

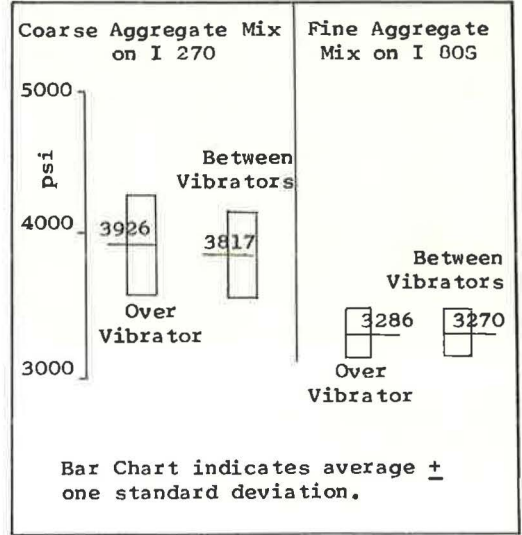


Figure 20. Comparison of compressive strength as indicated by Swiss hammer.

was equipped with a surface pan vibrator or on project I80S where only the internal vibrators provided the consolidation. The bottom 4 in. of the 8-in. slab appears to be slightly stronger than the top 4 in. (Figs. 17 and 18).

The Swiss hammer tests show lower strength values than the regular compressive tests for possibly 2 reasons: (a) They were taken when the concrete was only 28 days old, whereas the cores were broken when the concrete was 5 months old, and (b) the correlation between Swiss hammer readings and this particular concrete mix was not necessarily perfect. The general correlation supplied by the manufacturer was used to convert Swiss hammer readings to psi readings, and experience has shown this correlation to be a general one that does not apply to all mixes. However, the Swiss hammer readings do show that the concrete in the path of the vibrators is slightly stronger than the concrete between the vibrator paths. The standard deviation for Swiss hammer readings is certainly lower, indicating that the range of variance in readings is less on this nondestructive test than on the regular compression test.

Swiss hammer readings have shown very good correlation with performance of concrete pavements in Colorado. Figure 21 shows typical correlation and average core strength values from 3 projects that have been observed closely for cracking and abrasion by studded tires. It is interesting to note that Swiss hammer readings from 3 cores taken over paths of "dead" or nonoperating vibrators on project I80S showed values of 20, 22, and 24, which would correspond to psi values of 1,800, 2,000, and 2,400.

Data on the strength of beams cut from project I270 are given in Table 5 and indicate that the coarse-aggregate mix developed almost the same flexure strength with surface vibration as it did with a combination of surface vibration and internal vibration. The average strength of the beams tested with the top surface in tension is 10 percent higher than the average strength found when the beams were tested with the bottom side in tension. This fact may indicate a gain in strength due to additional surface vibration.

Figure 22 shows the gain in flexure strength as the density is increased from 144 to 146 pcf for beams taken from project I270. Based on limited data (24 flexure tests), flexure tests on project I270 indicate that there is a small increase in strength with an

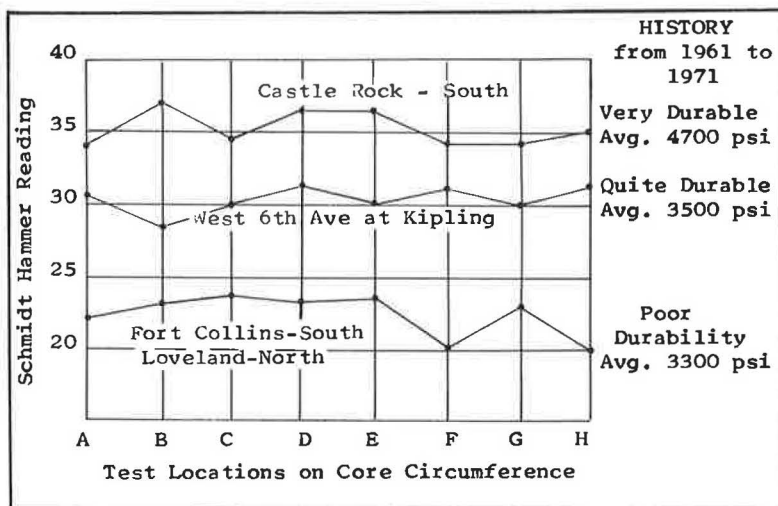


Figure 21. Swiss hammer test results.

TABLE 5
FLEXURE STRENGTH OF COURSE-AGGREGATE MIX ON PROJECT I270

| Diameter of Eccentric (in.) | Distance From Internal Vibrator (in.) | Flexure Strength (psi) | | | Density (pcf) |
|-----------------------------|---------------------------------------|------------------------|----------------|--------------|---------------|
| | | Surface Side up | Bottom Side up | Per ASTM C78 | |
| 1¼ | 0 to 8 | 605 | 760 | 567 | 145 |
| | 8 to 16 | 707 | 787 | 553 | 145 |
| | 16 to 24 | 619 | 683 | 498 | 144 |
| | 24 to 32 | 673 | 713 | 639 | 144 |
| 1⅞ | 0 to 8 | 713 | 702 | 582 | 146 |
| | 8 to 16 | 623 | 743 | 633 | 146 |
| | 16 to 24 | 679 | 702 | 668 | 145 |
| | 24 to 32 | 598 | 638 | 538 | 144 |

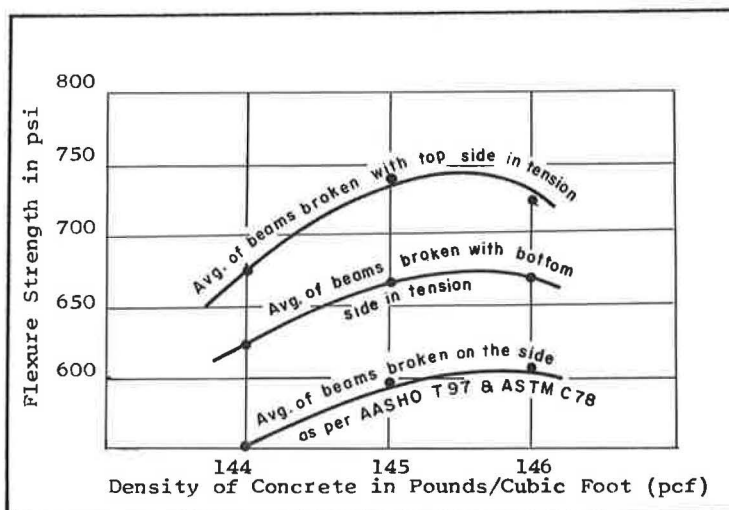


Figure 22. Flexure strength of coarse-aggregate mix from project I270.

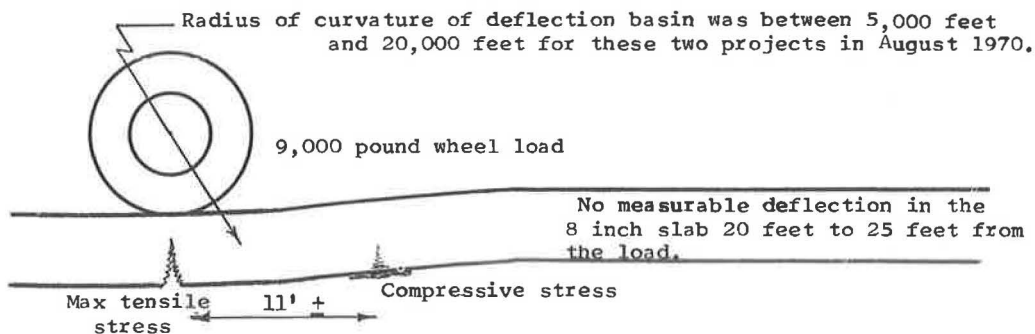


Figure 23. Texas basin beam tests on projects 180S and 1270.

increase in density if all other factors remain steady. However, when surface vibration is provided as well as internal vibration, there is very little indication that the internal vibration has contributed much to an increase in strength.

A nondestructive field test, which was hoped would provide some information about the benefits of vibration, was the Texas basin beam evaluation for deflection of the test sections. The test is performed by the use of a 9,000-lb wheel load and a special beam with a deflection dial reading to $\frac{1}{10,000}$ in. (Fig. 23). Limited use of this deflection measuring system in Colorado had shown that it has reasonably good correlation with concrete pavement performance. The summary of findings is given in Table 6. When the test sections were only a month old, they did not show much effect of the different types of vibration. The radius of curvature of the deflection basin under the 9,000-lb wheel load is generally large (9,000 to 20,000 ft), and the induced stresses are quite low at the bottom of the slab. This information may be of more help in future evaluations than it appears to be at this time.

In summarizing the results of the strength tests, the point should be made that there seemed to be a greater indication of effect of vibration in the appearance and density of the cores than in the strength values. In other words, the overall average of the strength tests indicate only a small advantage in good vibration. From visual observation, there would seem to be a significant advantage in good vibration. This fact is not hard for testers and field engineers to understand after years of observing breaks on "ratty" cylinders made of low-slump concrete. It is very common for a porous cylinder made from a dry mix to show greater compressive strength than for a cylinder made from higher slump concrete that was vibrated down into a well-consolidated mass. There was some variation in factors such as slump, entrained air content, and subbase support on these 2 research projects during construction. The question that follows is whether durability will finally be associated more with strength, density, entrained air content, appearance, or with some other variable that is not apparent at this time.

Effect of Vibration on Pulse Velocity Readings

Field tests of the speed of an acoustic wave through concrete has been a method of evaluating the quality of concrete for many years. There are several reliable ways of producing a "pulse" and measuring the velocity, but one of the most recently developed instruments is the microseismic timer. This instrument was rented for an evaluation of test sections prepared for this study.

The instrument consists of a sliding drop hammer that produces a start pulse for the timer. When the elastic wave front in the concrete reaches the microseismic transducer (phonograph needle pickup), a stop signal is produced. The elapsed time is then indicated to the nearest millionth of a second by lamps on the timer panel.

Readings were made near the locations from which the cores were taken. There was a large scatter of points in the correlation between wave velocity density and distance

TABLE 6

AVERAGE VALUES FROM TEXAS BASIN BEAM TESTS MADE MIDWAY BETWEEN TRANSVERSE JOINTS ON PROJECTS I80S AND I270

| Project | Section | Eccentric Diameter (in.) | rpm of Vibrator | Minimum Radius of Curve (ft) | Maximum Stress (psi) at Bottom of Slab | | Project | Section | Eccentric Diameter (in.) | rpm of Vibrator | Minimum Radius of Curve (ft) | Maximum Stress (psi) at Bottom of Slab | |
|---------|---------|--------------------------------|-----------------|------------------------------|--|-------------|---------|--------------------------------|--------------------------|-----------------|------------------------------|--|-------------|
| | | | | | Ten-sile | Compressive | | | | | | Ten-sile | Compressive |
| I80S | 1 | 1 ⁵ / ₈ | 10,800 | 15,154 | 84 | 15 | 6 | 1 ⁵ / ₈ | 10,800 | 8,774 | 145 | 31 | |
| | 2 | 1 ⁵ / ₈ | 10,800 | 10,856 | 118 | 22 | 7 | 1 ⁵ / ₈ | 10,800 | 9,806 | 130 | 61 | |
| | 3 | 1 ⁵ / ₈ | 10,800 | 13,892 | 91 | 15 | 8 | 1 ⁵ / ₈ | 9,000 | 8,335 | 152 | 84 | |
| | 4 | 1 ⁵ / ₈ | 10,800 | 14,000 | 95 | 15 | 9 | 1 ⁵ / ₈ | 7,000 | 8,335 | 152 | 84 | |
| | 5 | 1 ⁵ / ₈ | 10,800 | 13,988 | 94 | 15 | X | 1 ⁵ / ₈ | 10,800 | 9,261 | 137 | 46 | |
| | 6 | 1 ⁵ / ₈ | 10,800 | 14,746 | 90 | 22 | 10 | 1 ⁵ / ₈ | 7,000 | 10,419 | 122 | 31 | |
| | 7 | 1 ⁵ / ₈ | 9,000 | 12,365 | 103 | 15 | 11 | 1 ⁵ / ₈ | 9,000 | 10,419 | 122 | 31 | |
| | 8 | 1 ⁵ / ₈ | 9,000 | 15,912 | 80 | 20 | 12 | 1 ⁵ / ₈ | 10,800 | 15,154 | 84 | 15 | |
| | 9 | 1 ⁵ / ₈ | 10,800 | 14,522 | 88 | 38 | 13 | 1 ⁵ / ₈ | 10,800 | 12,823 | 99 | 15 | |
| | 10 | 1 ⁵ / ₈ | 10,800 | 10,856 | 118 | 27 | 14 | 1 ⁵ / ₈ | 9,000 | 13,892 | 91 | 22 | |
| | 11 | 1 ⁵ / ₈ | 9,000 | 16,670 | 76 | 50 | 15 | 1 ⁵ / ₈ | 7,000 | 5,557 | 228 | 23 | |
| | 12 | 1 ⁵ / ₈ | 9,000 | 18,754 | 68 | 22 | 16 | 1 ⁵ / ₈ | 7,000 | 7,577 | 168 | 15 | |
| | 13 | 1 ⁵ / ₈ | 9,000 | 14,746 | 87 | 19 | 17 | 1 ⁵ / ₈ | 9,000 | 10,419 | 122 | 15 | |
| | 14 | 1 ⁵ / ₈ | 9,000 | 17,596 | 72 | 20 | 18 | 1 ⁵ / ₈ | 10,800 | 12,823 | 99 | 23 | |
| | 15 | 1 ⁷ / ₁₆ | 10,800 | 15,912 | 80 | 19 | 19 | 1 ⁷ / ₁₆ | 10,800 | 11,907 | 107 | 15 | |
| | 16 | 1 ⁷ / ₁₆ | 9,000 | 15,281 | 84 | 15 | 20 | 1 ⁷ / ₁₆ | 10,800 | 13,892 | 91 | 23 | |
| | 17 | 1 ⁷ / ₁₆ | 9,000 | 15,154 | 84 | 22 | 21 | 1 ⁷ / ₁₆ | 7,000 | 11,907 | 107 | 15 | |
| | 18 | 1 ⁷ / ₁₆ | 10,800 | 12,823 | 99 | 22 | 22 | 1 ⁷ / ₁₆ | 7,000 | 18,522 | 69 | 23 | |
| | 19 | 1 ⁷ / ₁₆ | 10,800 | 13,352 | 95 | 22 | 23 | 1 ⁷ / ₁₆ | 9,000 | 13,892 | 91 | 31 | |
| | 20 | 1 ⁷ / ₁₆ | 10,800 | 17,596 | 72 | 15 | 24 | 1 ⁷ / ₁₆ | 9,000 | 11,907 | 107 | 23 | |
| | 21 | 1 ⁷ / ₁₆ | 9,000 | 15,281 | 84 | 23 | 25 | 1 ⁷ / ₁₆ | 9,000 | 18,522 | 69 | 23 | |
| | 22 | 1 ⁷ / ₁₆ | 10,800 | 13,530 | 96 | 30 | 26 | 1 ⁷ / ₁₆ | 10,800 | 16,670 | 76 | 15 | |
| | 23 | 1 ⁷ / ₁₆ | 9,000 | 15,281 | 84 | 22 | 27 | 1 ⁷ / ₁₆ | 7,000 | 15,154 | 84 | 15 | |
| | 24 | 1 ⁷ / ₁₆ | 9,000 | 16,670 | 76 | 23 | 28 | 1 ⁷ / ₁₆ | 7,000 | 16,670 | 76 | 15 | |
| | 25 | 1 ⁷ / ₁₆ | 10,800 | 23,814 | 53 | 15 | 29 | 1 ⁷ / ₁₆ | 9,000 | 18,522 | 69 | 15 | |
| | 26 | 1 ⁷ / ₁₆ | 10,800 | 16,207 | 81 | 15 | 30 | 1 ⁷ / ₁₆ | 9,000 | 18,522 | 69 | 38 | |
| | 27 | 1 ⁷ / ₁₆ | 9,000 | 18,546 | 76 | 15 | 31 | 1 ⁷ / ₁₆ | 7,000 | 13,892 | 91 | 15 | |
| I270 | 1 | 1 ⁵ / ₈ | 10,800 | 12,823 | 99 | 30 | 32 | 1 ⁷ / ₁₆ | 10,800 | 10,419 | 122 | 23 | |
| | 2 | 1 ⁵ / ₈ | 9,000 | 11,907 | 106 | 23 | 33 | 1 ⁷ / ₁₆ | 10,800 | 13,892 | 91 | 15 | |
| | 3 | 1 ⁵ / ₈ | 7,000 | 11,907 | 106 | 8 | 34 | 1 ⁷ / ₁₆ | 10,800 | 15,154 | 84 | 15 | |
| | 4 | 1 ⁵ / ₈ | 7,000 | 9,261 | 137 | 31 | 35 | 1 ⁷ / ₁₆ | 9,000 | 16,670 | 76 | 15 | |
| | 5 | 1 ⁵ / ₈ | 9,000 | 9,806 | 130 | 76 | 36 | 1 ⁷ / ₁₆ | 7,000 | 20,838 | 61 | 15 | |

from the vibrator. This was probably due to variation in factors such as air content, slump, and surface finish. Figure 24 shows the correlation between wave velocity and vibrator location. Wave velocities may be slightly higher for the fine-aggregate mix on project I80S than for the coarse-aggregate mix on project I270 because project I80S concrete was 3 months older when tested and contained a half sack of cement more than the mix for project I270.

Figure 25 shows the correlation between modulus of elasticity within the range of a vibrator and completely beyond the range of vibrator. Young's modulus of elasticity was computed from the following:

$$E = \frac{\text{density} (1 + 0.3)(1 - 0.6) \text{vel}^2}{32.2(1 - 0.3)}$$

where 0.3 was used for the value of Poisson's ratio.

Although individual microseismic tests seem to indicate very little about the value of consolidation, overall averages of microseismic data seem to indicate some advantage in good vibration. It is very doubtful that the microseismic instrument could be used as a means of control for consolidation of concrete pavements in the field.

Effect of Vibration on the Modulus of Elasticity

ASTM Test C 215 was used to determine Young's modulus of elasticity. The testing consists of the use of an amplified variable frequency audio oscillator connected to a

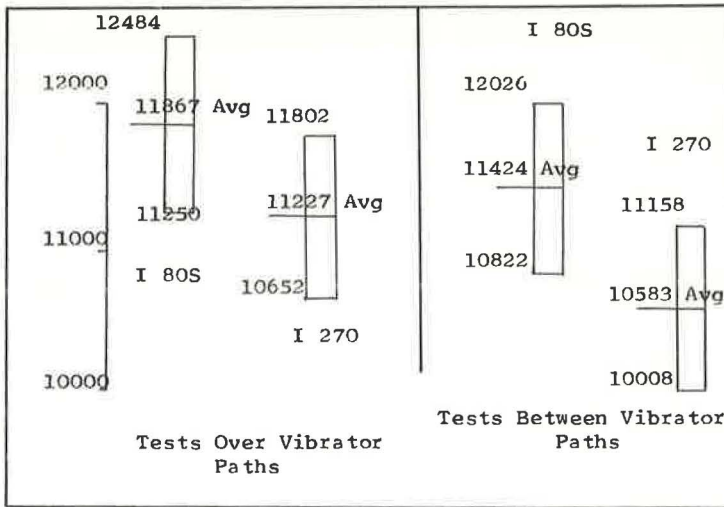


Figure 24. Impact elastic wave velocities for projects I270 and I80S.

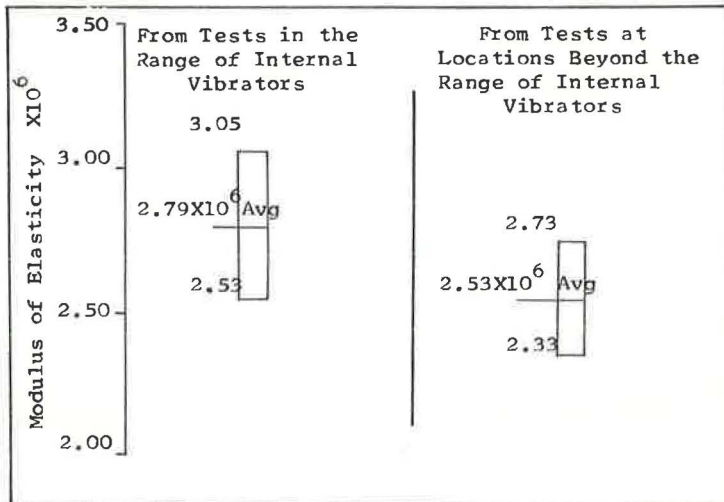


Figure 25. Modulus of elasticity values from project I270 (computed from pulse velocity device).

driving unit (modified audio speaker) that is fashioned to vibrate a concrete specimen. By adjusting the frequency of the oscillator and observing the reaction of the concrete specimen, as the vibrations are picked up with a phonograph cartridge and oscillograph, a frequency may be found that brings the specimen into resonance. A formula relating the frequency, weight, length, and diameter is then used to determine Young's dynamic modulus of elasticity. The presence of unsound material or invisible cracks is quickly detectable with this apparatus, and all cores taken for this study were tested to help classify and identify cores that were typical of certain areas. Figures 26 and 27 show the findings in regard to vibrator operations.

For both the fine-aggregate mix and the coarse-aggregate mix, modulus of elasticity values were greater for the concrete over the paths of the vibrator than for the concrete between the vibrators. Figure 27 shows that, for pavement sections on project I270

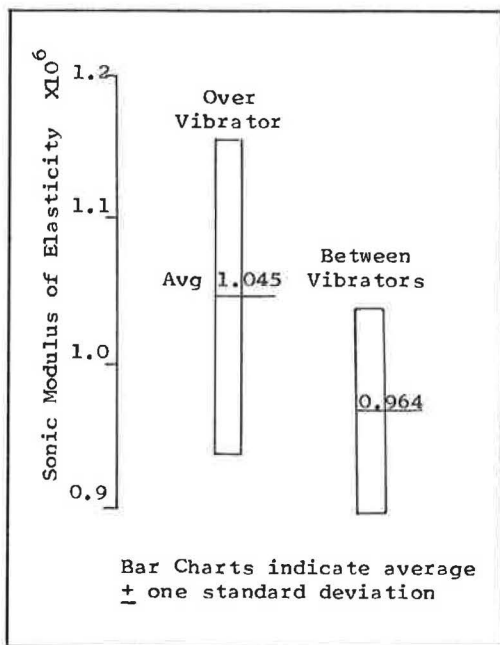


Figure 26. Sonic modulus of elasticity on cores from the fine-aggregate mix on project 1805.

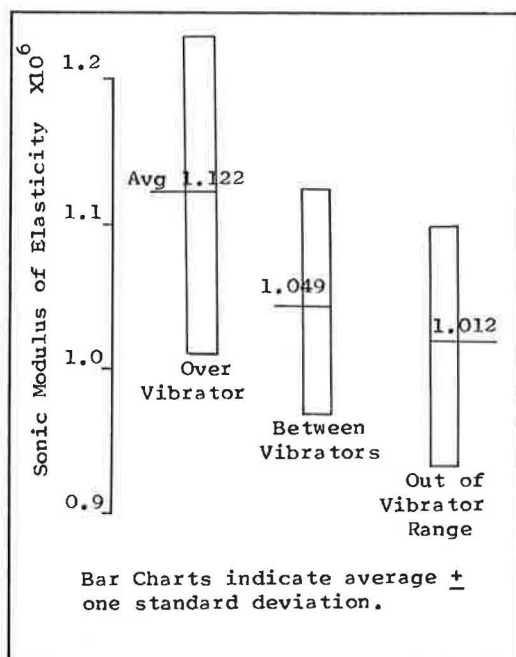


Figure 27. Sonic modulus of elasticity on cores from the coarse-aggregate mix on project 1270.

(coarse-aggregate mix) where only surface pan vibration had been provided, the average modulus of elasticity was even less than it was between internal vibrators.

Effect of Vibration on Segregation

One of the main concerns of this study was to either confirm or invalidate the theory that heavy vibration causes segregation of the aggregate particles. This study has shown that it takes a great deal of vibration to cause segregation of particles in an 8-in. layer of concrete having only $1\frac{1}{4}$ -in. slump. There was no visual indication of segregation of particles around any of the vibrated concrete that was placed on these 2 projects.

Sieve analyses of plus No. 4 aggregate on samples tested in the laboratory are given in Table 7. The data are too limited for the application of statistical analysis, but there is only a slight evidence of segregation on the coarse-aggregate mix. There is really none on the fine-aggregate mix.

Effect of Vibration on Entrained Air Content

It is generally agreed that heavy vibration will tend to expel all sizes of air voids in a concrete mix including the very small ones that are purposely implanted by means of additives to improve the freeze-thaw durability. Although a thorough study of the loss of this entrained air by vibration could be, and has been, the object of extensive research in that field alone, there is a possibility that some information along this line could be gathered from this study because of the control in the variation of amplitudes, frequencies, and paver speeds on the test sections.

The facilities for the measurement of pore size and number are available at the laboratories of a cement company in Colorado. Officials of that company readily agreed to perform tests necessary to determine entrained air content on cores from various test sections as well as cores from pavements with durable and nondurable service records in Colorado.

The results of a preliminary group of these tests will be available during the winter of 1970-1971. If it appears at that time that future efforts will be rewarding, a request

TABLE 7
SIEVE ANALYSIS OF NO. 4 AGGREGATE CORES FROM PROJECTS I270 AND I80S

| Project | Portion of Core | Station | Sieve (percent passing) | | | | Project | Portion of Core | Station | Sieve (percent passing) | | | |
|---------|-----------------|---------|-------------------------|-------------------|-------------------|-------------------|---------|-----------------|---------|-------------------------|-------------------|-------------------|-------------------|
| | | | 1 In. | $\frac{3}{4}$ In. | $\frac{1}{2}$ In. | $\frac{3}{8}$ In. | | | | 1 In. | $\frac{3}{4}$ In. | $\frac{1}{2}$ In. | $\frac{3}{8}$ In. |
| I270 | Top 2.5 in. | 76 | 76 | 51 | 25 | 14 | | | 5085 | 100 | 83 | 55 | 41 |
| | | 105 | 75 | 48 | 25 | 14 | | | 5118 | 100 | 78 | 53 | 39 |
| | | 115 | 72 | 49 | 24 | 12 | | | 4911 | 100 | 82 | 50 | 35 |
| | Middle 2.5 in. | 125 | 75 | 49 | 23 | 13 | | | 4948 | 100 | 86 | 54 | 37 |
| | | 76 | 79 | 51 | 23 | 12 | | | 4988 | 100 | 85 | 50 | 37 |
| | | 105 | 74 | 47 | 23 | 12 | | | 4881 | 100 | 84 | 49 | 35 |
| | | 115 | 68 | 47 | 22 | 12 | | | 5011 | 100 | 85 | 54 | 38 |
| | | 125 | 75 | 47 | 23 | 12 | | | 5024 | 100 | 86 | 55 | 38 |
| | Bottom 2.5 in. | 76 | 72 | 42 | 17 | 8 | | | 5085 | 100 | 86 | 59 | 43 |
| | | 105 | 69 | 44 | 20 | 12 | | | 5118 | 100 | 84 | 55 | 41 |
| | | 115 | 69 | 44 | 22 | 12 | | | 4911 | 100 | 83 | 51 | 33 |
| | | 125 | 74 | 47 | 20 | 11 | | | 4948 | 100 | 85 | 55 | 37 |
| I80S | Top 2.5 in. | 4911 | 98 | 80 | 51 | 35 | | | 4988 | 100 | 84 | 49 | 35 |
| | | 4948 | 100 | 88 | 57 | 38 | | | 4881 | 100 | 84 | 51 | 35 |
| | | 4988 | 100 | 88 | 57 | 41 | | | 5011 | 100 | 82 | 52 | 37 |
| | | 4881 | 100 | 84 | 56 | 39 | | | 5024 | 100 | 85 | 57 | 40 |
| | | 5011 | 100 | 81 | 54 | 39 | | | 5085 | 100 | 84 | 51 | 38 |
| | | 5024 | 100 | 87 | 56 | 40 | | | 5118 | 100 | 83 | 55 | 39 |

for assistance in further work along that line will be made. At this time, records of field tests for entrained air at the test sections have been assembled for comparison with results on a limited number of core samples. The work that has been completed so far indicates that heavy vibration does not reduce the entrained air content to any appreciable extent.

Effect of Vibration on Pressure in the Grout

As a means of investigating the pressure in the mortar beneath the pavement surface on this project, a simple pressure measuring device was constructed with a rubber tip and glass tubing for readings of pressure in inches of water. The tests confirmed the findings of Ore and Straughan (1), U. S. Bureau of Reclamation engineers, that good vibration merely transforms a nonfluid mixture into a mixture that may become fluid enough to allow the behavior or effects of hydrostatic pressure. In other words, with this rather simple instrument, it was possible to detect the hydrostatic pressure of 0.66 psi (8 in. of concrete) in the immediate vicinity of an operating vibrator, but not when the vibrator was turned off or when it was several feet away from the sensor. The application of a new $\frac{1}{16}$ -in. diameter pressure cell, now being produced by a technical industry in California, may make it possible to more accurately determine pressure between vibrators and grout. Work along this line is under way in a cooperative effort.

Effect of Vibration on the Durability of the Concrete

An accurate evaluation of the effect of vibration on the durability of the concrete pavements placed for this study will depend on several years of observation. However, an attempt was made to predict the outcome by use of ASTM Test C 418 that makes use of a standard sandblast apparatus. After the surface of the concrete is exposed to the sandblast, a determination is made of the volume of the abraded material per square centimeter of surface area.

The Colorado Department of Highways had no previous experience with this abrasion test, and correlation of the results with actual pavement surface durability appears to be meager. However, technicians had no trouble performing the test approximately 400 times on surfaces of cores of the fine-aggregate mix on project I80S and the coarse-aggregate mix on project I270.

Results of the tests are shown in Figures 28 and 29. The fine-aggregate mix on project I80S showed very uniform abrasion coefficients of approximately 0.21 on concrete throughout the slabs, without apparent effect of the internal vibrators. Concrete in the bottom 4 in. of the slabs appeared to be slightly more durable than the concrete in the top 4 in.

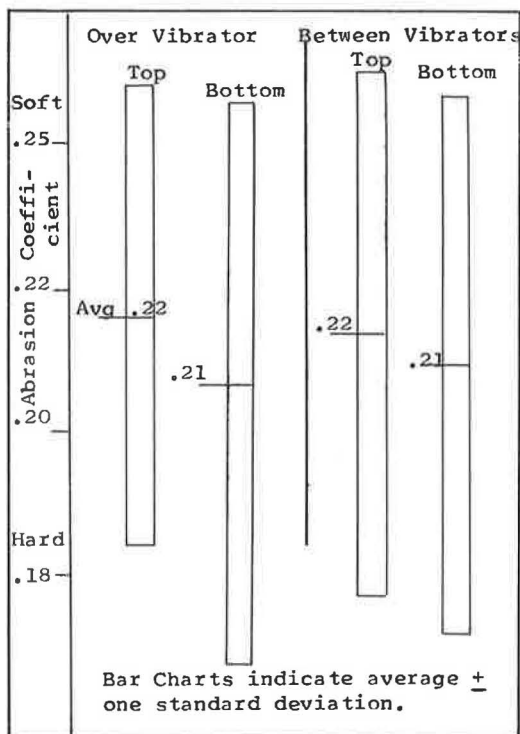


Figure 28. Abrasion coefficients for top 4 in. and bottom 4 in. on project I80S.

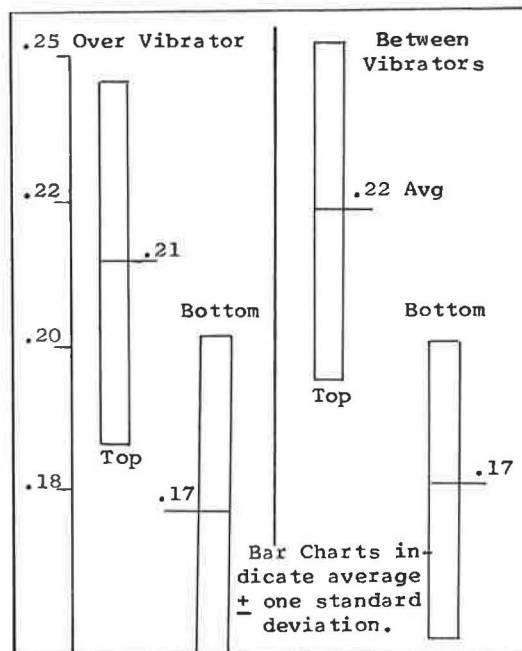


Figure 29. Abrasion coefficients for top 4 in. and bottom 4 in. on project I270.

A possible explanation for the increased hardness of the bottom of the 8-in. slabs from the coarse-aggregate mix on project I270 is that the vibration allowed the coarse aggregate to settle. It is a fact that, for both paving projects, the aggregate is more resistant to abrasion than is the mortar. However, the gradation tests taken separately at the upper third, middle third, and lower third of the slabs showed that only 2 percent less aggregate passed the $\frac{3}{8}$ -in. sieve for the bottom third than for the top third of the slab, and so this does not back up the above explanation. Another possible explanation might be the ideal curing condition of the bottom of the slab as opposed to the top. However, project I80S did not show such a difference between tops and bottoms, and the curing on that project should have been similar to the curing on project I270.

On greatest concern is the fact that, although abrasion coefficients generally averaged 0.21 to 0.22 for interior surfaces of the concrete, the exposed or roadway surfaces generally averaged 0.40, which indicates much softer concrete. If the sandblast test is truly indicative of resistance to wear, the top $\frac{1}{4}$ to $\frac{3}{8}$ in. of the pavement is considerably softer than the interior. Future work on this project will be to investigate more thoroughly vibration effects on the wearing surface of the test sections.

Effect of Vibrator Spacing

A general summary of the results of various vibrator spacings is shown in Figure 11. Data for the coarse-aggregate mix were obtained from project I270 during August 1970, and data for the fine-aggregate mix were obtained from project I80S in June 1970 and I70 in October 1970.

Because of the surface plate vibrator attached to the coarse-aggregate paver (and perhaps the characteristics of coarse-aggregate mixes in general), the loss in density with increasing distance from the vibrator paths was greater for the fine-aggregate mix than for the coarse-aggregate mix. The loss in density for the first 6-in. distance is

only about 1 pcf, but from the 6-in. to the 12-in. distance the loss is about 2 pcf for the fine-aggregate mix.

The research work on this project seems to show that, when the 1 $\frac{7}{8}$ -in. eccentric (0.0266-in. amplitude) vibrators were run at 10,800 rpm and spaced 22 to 24 in. apart, the coarse-aggregate mix was adequately consolidated. On the other hand, the slump had to be very closely controlled between 1 $\frac{1}{2}$ and 2 in. for the same vibrator setup to obtain 97 percent compaction on the fine-aggregate mix. Similar findings for the 1 $\frac{5}{8}$ -in. diameter (0.0176-in. amplitude), 1 $\frac{7}{16}$ -in. diameter (0.0124-in. amplitude), and 1 $\frac{1}{4}$ -in. diameter (0.0086-in. amplitude) were harder to assess because those vibrators were not used on project I70. However, the data given in Tables 2 and 3 show that at 10,800 rpm a 15-in. spacing is quite satisfactory for 1 $\frac{5}{8}$ -in. vibrators, 11-in. spacing for 1 $\frac{7}{16}$ -in. vibrators, and 8-in. spacing would be adequate for 1 $\frac{1}{4}$ -in. diameter eccentric vibrators.

VALUE OF NUCLEAR TESTING EQUIPMENT FOR RAPID TESTS ON CONSOLIDATION

Although primarily designed for density tests on subgrades, bases, and hot-asphaltic concrete, the nuclear device proved to be a very effective instrument for measuring the density of plastic portland cement concrete immediately behind the paver. It was calibrated in the Denver laboratory by using specially prepared concrete blocks of various density, but the device had never been used in the field for density control of concrete paving.

The operator worked from a small movable bridge between the paver and the burlap drag. His order of procedure was as follows:

1. Measure from the edge of the concrete to find the exact location for the test;
2. Place a small wooden jig on the fresh concrete surface so that the dowel on the jig makes a clean vertical hole 8 in. deep and $\frac{3}{4}$ in. in diameter (the size of the probe on the nuclear device);
3. Remove the jig and replace it with the nuclear device by inserting the probe in the prepared hole;
4. Turn on the device with the timer set for 1 min;
5. Take the reading at the end of 1 min, compare it with the previously prepared calibration curve, and calculate the density; and
6. Fill the hole with a pat of concrete and trowel it over slightly (the burlap drag will completely obscure the marks left by the device).

The test results given in Table 8 show almost as many high readings as low readings, indicating that the results are only slightly skewed to the left. The nuclear readings were overall 0.37 lb lighter than the average of all the readings. The operator was forced to take readings for this study with great haste, because of the desire to interfere with the contractor's operations as little as possible. Obviously a set of 3 readings at each location and an average of these readings would have resulted in values with a smaller standard deviation than the overall average of 1.3 pcf determined for this study. Actually, considering the variable slump and characteristics of the aggregates under field conditions, it is very possible that the density of the concrete actually varied within the range of 1 to 2 pcf from place to place within the test section.

The excellent performance of the nuclear device suggests the possibility of its use to control consolidation during construction of concrete pavements in the future. If, for instance, on some project a certain minimum density should be established, it would be well within the ability of a trained technician to determine, within 3 min of laydown, the density of the concrete. If changes in vibratory energy or placement were necessary, the need could be expressed before the paver had moved more than 100 ft.

An acceptance specification based on nuclear density relative to laboratory density appears to be practical. Under this type of specification, it would not be necessary to specify construction equipment and procedure for vibration of the concrete.

TABLE 8
RESULTS OF NUCLEAR DENSITY TESTS

| Fine Aggregate on Project I80S | | | | | | Fine Aggregate on Project I70 | | Coarse Aggregate on Project I270 | | | | | |
|--------------------------------|----------|------|----------|------|----------|-------------------------------|----------|----------------------------------|----------|------|----------|------|----------|
| Mean ^a | Nuclear | Mean | Nuclear | Mean | Nuclear | Mean | Nuclear | Mean | Nuclear | Mean | Nuclear | Mean | Nuclear |
| 140 | 140 (0) | 139 | 139 (0) | 138 | 137 (-1) | 136 | 136 (0) | 145 | 145 (0) | 146 | 146 (0) | 143 | 146 (+3) |
| 139 | 138 (-1) | 139 | 139 (0) | 138 | 138 (0) | 137 | 137 (0) | 145 | 144 (-1) | 144 | 144 (0) | | |
| 140 | 139 (-1) | 139 | 139 (-2) | 138 | 138 (0) | 137 | 138 (+1) | 144 | 145 (+1) | | | 142 | 143 (+1) |
| 136 | 131 (-5) | 136 | 136 (0) | 138 | 138 (0) | 137 | 136 (-1) | 144 | 143 (-1) | 145 | 146 (+1) | | |
| | | 136 | 133 (-3) | 136 | 136 (0) | 137 | 137 (0) | 144 | 143 (-1) | 144 | 142 (-2) | 143 | 144 (+1) |
| 139 | 139 (0) | | | 136 | 135 (-1) | 136 | 136 (0) | | | | | 142 | 143 (+1) |
| 135 | 135 (0) | 138 | 135 (-3) | | | | | 141 | 143 (+2) | 144 | 143 (-1) | | |
| 139 | 139 (0) | 138 | 138 (0) | 135 | 135 (0) | | | | | 143 | 140 (-3) | 142 | 142 (0) |
| 139 | 138 (-1) | 138 | 139 (+1) | 134 | 134 (0) | | | 143 | 143 (0) | | | 140 | 139 (-1) |
| 135 | 132 (-3) | 136 | 136 (0) | 134 | 135 (+1) | | | 143 | 144 (+1) | 144 | 142 (-2) | | |
| | | 136 | 133 (-3) | 135 | 132 (-3) | | | 143 | 143 (0) | 144 | 141 (-3) | 141 | 140 (-1) |
| 140 | 140 (0) | 136 | 138 (+2) | 134 | 135 (+1) | | | 141 | 141 (0) | | | 139 | 139 (0) |
| 140 | 139 (-1) | | | | | | | | | 145 | 144 (-1) | | |
| 135 | 133 (-2) | 133 | 132 (-1) | 136 | 136 (0) | | | 144 | 143 (-1) | 145 | 145 (0) | 143 | 143 (0) |
| 135 | 135 (0) | 133 | 133 (0) | 136 | 138 (+2) | | | | | | | 141 | 143 (+2) |
| 135 | 133 (-2) | 133 | 134 (+1) | 135 | 136 (+1) | | | 144 | 141 (-3) | 146 | 146 (0) | | |
| | | 132 | 132 (0) | 135 | 136 (+1) | | | 140 | 141 (+1) | 146 | 142 (-4) | 144 | 142 (-2) |
| 139 | 138 (-3) | 134 | 134 (-3) | | | | | | | | | | |
| 139 | 138 (-1) | 134 | 131 (-3) | 134 | 134 (0) | | | 140 | 138 (-2) | 144 | 144 (0) | 143 | 142 (-1) |
| 135 | 132 (-3) | | | 134 | 134 (0) | | | 140 | 140 (0) | 144 | 144 (0) | 142 | 141 (-1) |
| 135 | 133 (-2) | 137 | 137 (0) | 133 | 135 (+2) | | | 140 | 140 (0) | | | | |
| | | 137 | 138 (+1) | 133 | 134 (+1) | | | | | 145 | 145 (0) | 142 | 141 (-1) |
| 139 | 138 (-1) | 134 | 134 (0) | | | | | 143 | 142 (-1) | 145 | 146 (+1) | 141 | 141 (0) |
| 136 | 134 (-2) | | | 134 | 136 (+2) | | | | | | | | |
| 136 | 132 (-4) | 137 | 135 (-2) | 133 | 134 (+1) | | | 143 | 141 (-2) | 143 | 144 (+1) | 140 | 138 (-2) |
| 136 | 134 (-2) | 133 | 133 (0) | 133 | 131 (-2) | | | 138 | 139 (+1) | 140 | 140 (0) | | |
| | | 133 | 132 (-1) | | | | | | | | | | |
| 137 | 138 (+2) | | | 136 | 137 (+1) | | | 144 | 144 (0) | 142 | 141 (-1) | | |
| 136 | 136 (0) | 133 | 133 (0) | 135 | 135 (0) | | | 139 | 141 (+2) | 140 | 139 (-1) | | |
| 136 | 136 (0) | 133 | 133 (0) | 136 | 135 (-1) | | | 144 | 145 (+1) | | | | |
| 137 | 139 (+2) | 133 | 133 (0) | 135 | 137 (+2) | | | | | 143 | 142 (-1) | | |
| | | | | | | | | 140 | 142 (+2) | 143 | 142 (-1) | | |
| 135 | 136 (+1) | 137 | 137 (0) | 135 | 134 (-1) | | | | | 144 | 144 (0) | | |
| 135 | 137 (+2) | | | 133 | 131 (-2) | | | 140 | 142 (+2) | | | | |
| 135 | 135 (0) | 135 | 135 (0) | 135 | 135 (0) | | | | | 144 | 144 (0) | | |
| 135 | 135 (0) | 133 | 133 (0) | 135 | 135 (0) | | | 144 | 144 (0) | | | | |
| 135 | 134 (-1) | 133 | 134 (+1) | 133 | 132 (-1) | | | 141 | 141 (0) | | | | |
| 135 | 135 (0) | 133 | 133 (0) | | | | | 145 | 146 (+1) | 144 | 144 (0) | | |
| | | | | 134 | 134 (0) | | | | | 145 | 146 (+1) | | |
| 135 | 134 (-1) | 135 | 135 (0) | 133 | 134 (+1) | | | | | 141 | 141 (0) | | |
| 135 | 133 (-2) | 133 | 134 (+1) | 134 | 134 (+2) | | | | | | | | |
| 134 | 134 (0) | | | 134 | 132 (-2) | | | | | | | | |
| 134 | 133 (-1) | 137 | 138 (+1) | | | | | | | | | | |
| 134 | 133 (-1) | 135 | 135 (0) | | | | | | | | | | |
| 134 | 133 (-1) | 135 | 134 (-1) | | | | | | | | | | |
| | | 136 | 138 (+2) | | | | | | | | | | |

Note: Standard deviation = 1.45 pcf for project I80S; 0.57 pcf for project I70; 1.34 pcf for project I270; 1.3 pcf overall.

^aDensity of an area within a test section as indicated by the average of core samples, beam samples, and nuclear test results.

CONCLUSIONS

Two of the paving projects used to provide test sections for vibration studies were completed in October 1970 and opened to traffic. Snowstorms began less than a week after the sections were opened, and the number of vehicles with studded tires was estimated at 25 percent by November 1970. Some indication of the durability of each test section may be evident by the spring of 1971.

Meanwhile, the study has provided the following general conclusions:

1. A new construction specification for pavement consolidation is needed. Cores from pavements with poor abrasion records show high void contents, and their density is less than 97 percent of laboratory rodded density.

2. Good consistent consolidation (97 percent relative consolidation or more) requires considerable vibratory effort when concrete pavements are placed with less than a 2-in.

slump. However, concrete with less than a 2-in. slump is stronger than concrete with more than a 2-in. slump, and it is usually preferred when slip-form pavers are used.

3. Good surface plate vibrators operating at or above 4,000 vibrations per minute plus some internal vibration may effectively consolidate coarse-aggregate mixes for 8-in. thick pavement if the slump is between $1\frac{1}{2}$ and $2\frac{1}{2}$ in.

4. Internal vibrators with $1\frac{7}{8}$ -in. eccentrics turning 10,800 rpm in air and spaced 24 in. or less will effectively consolidate concrete for 8-in. thick pavements if the slump is between $1\frac{1}{2}$ and $2\frac{1}{2}$ in. Concrete with a slump less than 1 in. can entrap enough large air voids to reduce the relative consolidation below 97 percent.

5. The angle and height of internal vibrators are not critical if the entire vibrator is submerged. The vibrators used on this project appeared to provide the best consolidation when they were in a horizontal position midway between the base and the top of the surcharge.

6. Internal vibrators with $1\frac{7}{8}$ -in. eccentrics operating at 10,800 rpm in air at 24-in. spacing (or less) in 8-in. thick pavement may be moved at speeds up to 19 ft/min by the paver and still provide acceptable consolidation.

7. Reductions in amplitude below the $1\frac{7}{8}$ -in. eccentric size do not affect consolidation as much as reduction in rpm below the frequency of 10,800 rpm in air. The internal vibrator with a $1\frac{7}{8}$ -in. eccentric turning at 10,800 rpm in air appeared to be a very satisfactory size for 2-ft spacing. Internal vibrators with smaller eccentrics and lower vibrator rates should be spaced closer together to obtain at least 97 percent consolidation in $1\frac{1}{2}$ - to $2\frac{1}{2}$ -in. slump concrete.

8. "Dead" vibrators will leave low density areas, especially in low slump concrete.

9. Tests on cores indicate that the compressive strength of concrete pavements is as much as 10 percent lower between vibrator paths than it is over vibrator paths. Strength of the concrete on both of the observed projects was well above the minimum requirements, however.

10. Modulus of elasticity readings determined by sonic and pulse wave velocity tests show that the soundness of concrete is improved by good vibration.

11. Segregation of aggregate particles in concrete pavement mixes is not a matter of concern for the low slump concrete now being used in form paving or with slip-form pavers. Cores taken from all test sections showed no evidence of segregation and only a very slight evidence of settlement of coarse particles under heavy vibration.

12. The nuclear testing device is an invaluable aid in the control of consolidation immediately after concrete pavement is placed. The device shows good reliability and will provide density values within 3 min after laydown.

SUMMARY

One of the factors affecting the performance of concrete pavement appears to be the consolidation of the plastic mix by the paving machine. This study is an attempt to determine how strength and durability of pavements are affected by factors such as the speed of the paver, the type, height, angle, frequency, and amplitude of the vibrators, and the type of aggregate in the mix.

Seventy test sections at 3 different test sites were established in eastern Colorado with these variations in consolidation. Because it will take many winter seasons and millions of studded-tire traverses to really determine durability of each test section, it may be 5 or 10 years before the final results are available. However, tests for density, soundness by impact wave velocity, sonic modulus of elasticity, segregation, entrained air retention, strength, and deflection have been completed.

The results indicate that low slump concrete (usually used with slip-form pavers) requires considerable vibration effort to liquidize the mix and allow the escape of air bubbles. However, the proper combination of vibratory amplitude, frequency, and slump will secure the 97 percent of rodded density that seems necessary to obtain sound, durable concrete. This consolidation effort will not seriously reduce the entrained air content or cause segregation of the aggregate particles.

One of the most significant by-products of the study was the development of a procedure to measure density of the plastic mix immediately after laydown by means of a direct transmission type of nuclear gage. With the use of this nondestructive testing device, reliable pavement density values may be obtained before the paver has traveled more than 100 ft from the sampled area.

REFERENCE

1. Ore, E. L., and Straughan, J. J. Effect of Cement Hydration on Concrete Form Pressure. ACI Jour., Feb. 1968.

DEVELOPMENT AND TRIAL USE OF ACCEPTANCE SAMPLING PLANS FOR COMPACTED EMBANKMENTS

James L. Jorgenson, North Dakota State University

The purpose of this research was to develop and put in trial use statistically based acceptance sampling plans for compacted embankments based on the existing variability in acceptable construction. The first phase of this research indicated that the trial specifications should have the following characteristics: (a) Decisions should be based on the average of a number of in-place density tests; (b) sample locations should be determined for random numbers; (c) specifications should provide an incentive for the contractor to produce uniform compaction; and (d) specifications should call for the same average degree of compaction as is accepted under the current specifications. Specifications incorporating these characteristics were developed and used along with current highway department specifications on 3 construction projects. The lot acceptance plan used a sample size of 3 to 5 tests. A nuclear moisture-density instrument was used and resulted in a testing and calculation time of from 40 to 90 min per lot. A comparison of the current specifications that are based on representative sampling with a specified minimum density indicated that (a) the average lot test results for the representative sampling deviated from the random test results depending on the judgment of the inspector, (b) of the 60 lots accepted under the current specifications, 2 lots would have been rejected under the trial specifications, and (c) 9 of the 67 lots accepted by the trial specifications had one or more tests below the current specification limits indicating that they could have been rejected by the current specifications.

•AN EVOLUTION is in progress. The evolution is in the updating of construction specifications to account for the variability in the constructed product. Although variability has always been present in highway construction, it has been dealt with primarily by judgment of inspectors or engineers. Although this process has produced many miles of adequate highways, it has recently been complicated by many problems such as lack of adequately trained inspectors; the need to explain specification results to federal agencies, auditors, politicians, and the public; the variations in judgment among engineers and inspectors; and the increased rate of construction. All of these factors have put additional emphasis on the need for specifications that will clearly account for the variability.

This problem is of national concern; approximately 40 states have recently conducted research in this area. In 1967, the North Dakota State Highway Department sponsored research to measure the variability present in acceptable compacted embankments (1). This research is a continuation of that work. The purpose of this research was to develop and put into trial use acceptance sampling plans for compacted embankments based on the previously measured variability.

This report starts with an examination of the results of the variability study to note the implications for improving the specifications. This is followed by the trial speci-

cations. Next, the collection of data and their application to the trial specifications are discussed. A brief section indicates some statistically based construction specifications in use by other states, and the last section examines the question of quality assurance, i. e., a way of measuring the effectiveness of the different sampling plans.

VARIABILITY RESULTS AND IMPLICATIONS OF TRIAL SPECIFICATIONS

The research on variability of compacted embankment in North Dakota brought to light many characteristics of currently acceptable construction. They are as follows: (a) One of every 3 adjacent in-place density tests will deviate from the average by at least 3 to 5 percent compaction depending on the construction project; (b) the nuclear moisture-density gage when used in the direct transmission mode is a much more reliable indicator of field density than when used in the backscatter mode and slightly more reliable than the conventional water-balloon method; and (c) there was a significant difference in field density as measured by representative and random sampling. The following paragraphs in this section expand on these observations and discuss their implications on developing trial specifications.

The reason for measuring the density of the compacted embankment is to obtain information from which a decision will be made on the acceptability of the compaction. For each decision made, there is a probability that the decision is incorrect (i. e., acceptable material may have been rejected or poor material may have been accepted). The probability of the decision being incorrect increases with the variability of the information on which the decision is based. The variability of this information can be reduced by basing the decision on the results of more than one in-place density test. Therefore, the probability of making a correct decision on the acceptability of the compaction in the trial specifications is increased by basing this decision on the average of a number of in-place density tests.

Those experienced in compaction checking know that the test results are very much dependent on the judgment of the inspector in his selection of the representative sample. In most cases (and this depends on the degree of experience of the inspector), the inspector can select locations where moisture or density is high or low. All inspectors are not able to exercise this judgment uniformly, nor is a single inspector always consistent. For example, the average percentage compaction on the 3 projects in the variability research was from 2.8 to 4.4 pcf greater for the representative sampling than for the random sampling. It is, therefore, necessary to select sample locations by some means that eliminates (or reduces to a high degree) the bias of the inspector. Herein lies the advantage of random sampling, for unbiased numbers are used to select the sample locations. In the trial specifications, sample locations should be determined by random numbers.

Currently, the variability in construction not only increases the chance of making a wrong decision on its acceptability but also is a source of future maintenance problems with the roadway. There is often disagreement among engineers as to the amount of compaction desirable in a given roadway, but all will agree that the compaction should be uniform throughout the roadway. The road surface is like a continuous beam; if all the supports under the road surface settle the same amount, there is no distortion to the road surface. However, if the subgrade settlement is not uniform, the roadway surface will tend to break up. The variability is due partly to the use of heterogeneous soils and partly to the way the soil is processed. The processing part of the variability can be reduced by the contractor. The trial specifications should provide an incentive for the contractor to produce uniform compaction. This is accomplished by having the required average compaction for a series of tests increase as the range in the test results increase.

The final characteristic of the trial specification is that it should call for the same degree of compaction as was measured in the variability study. For example, when the highway department specifications called for 90 percent of T-180, through random sampling a density greater than 90 percent was actually obtained in about one-half the samples. The average of the test results was near 90 percent, while 80 percent of the samples had a density greater than 85 percent of T-180. Similarly, from the research

data when 95 percent of T-99 compaction was called for, that obtained was 80 percent of the samples with greater than 95 percent compaction. The trial specifications should be written so as to obtain the same degree of compaction under new specifications as is now being accepted.

TRIAL SPECIFICATIONS

The trial specifications were tailored to satisfy the requirements stated in the previous section. The portion pertaining to density and acceptance is as follows:

Density Requirements:

The target value for controlled density of the material shall be 85% of AASHTO T-180 (or 95% of AASHTO T-99). The Quality Index, Q , of each lot for the density requirement based on 5 random samples and the methods of ACP-1 (see below) shall be 0.36. Any AASHTO standard test method or an approved equivalent method may be used in determining in-place density.

The lot size shall normally be a day's run. However, a lot of any size may be selected by the Engineer when the factors affecting compaction exhibit variation, when an area is obviously defective, when approval is needed prior to placing another lift, or when the rate of construction is greater than 10,000 cubic yards per day.

Acceptance Testing, Density:

Acceptance testing will be performed as soon as practicable after the contractor has informed the engineer that the lot or sections within the lot are ready for test. Lots failing to meet the Quality Index requirement shall be reworked by the contractor at his expense and be resubmitted for acceptance testing. Retesting of reworked lots will be at the expense of the contractor at the stated unit cost per test.

ACCEPTANCE PLAN NO. 1 (ACP-1)

1. Locate five sampling positions on the lot by use of a table or random numbers.
2. Make an in-place density and moisture test measurement at each location, and using the proper maximum dry density, calculate the percent compaction at each location.
3. Add the measurements and divide by five to obtain \bar{X} .
4. Find the range in measurements, R , by subtracting the smallest value from the largest value in the group of measurements.
5. The lot is acceptable if: $\bar{X} \geq L + R Q$ where L is the target-value percent compaction and Q is the Quality Index given in the specification. i.e. accept if $\bar{X} \geq 85 + 0.36 R$ for T-180 compaction and $\bar{X} \geq 95 + 0.36 R$ for T-99 compaction.

OTHER TRIAL SPECIFICATIONS

The 2 types of acceptance sampling plans are inspection by variables and inspection by attributes. Inspection by variables applies when the characteristic measured is given a numerical value. Inspection by attributes applies when the characteristic is just given a suitable or defective classification. There are situations where inspection by attributes must be used; however, more information about the material in question can be derived through inspection by variables than by inspection by attributes. Hence, the further discussion will be on inspection by variables.

Acceptance sampling plans by variables can be divided into 3 cases (2). The case to use for a particular material or construction depends on the information known about that material or construction. Case 1 is a plan to estimate the percentage of the material within tolerance. It can be used when both the mean and standard deviation of the measured property are unknown or are known to vary widely. Case 2 is a plan to provide fixed protection against accepting poor material. To use this plan, one must know the average value of the measured property for unacceptable construction. Case 3 is a plan to provide fixed protection against accepting poor material or rejecting good material. However, this plan can be used only when acceptable or unacceptable material or construction can be defined in terms of its average value and the standard deviation is known.

Which of these 3 cases is applicable to subgrade compaction? Case 3 calls for the standard deviation to be known, meaning that the standard deviations of the lots in a construction project should be about equal. The range in lot standard deviations for the 3

projects were project 1, 2.0 to 13.5; project 2, 2.0 to 7.0; and project 3, 0.5 to 9.9. The large ranges in standard deviation values suggest that case 3 is not applicable. In order to use case 2, one must know the average value of unacceptable construction. In reference to subgrade compaction, this value is unknown. Research on current construction has indicated the average value of that being accepted but not the average value of that being rejected. This value could possibly be developed on the basis of a laboratory investigation of subgrade stability. In the absence of an average value for unacceptable subgrade compaction, the case 2 acceptance plan cannot be used. The remaining acceptance sampling plan is case 1, which is the type of plan used in the trial specifications.

DATA COLLECTION

One man using a direct transmission nuclear instrument carried out the trial specifications. Through discussion with the state inspector, he determined the largest size lot that could be tested. The lot size was usually limited by the contractor's desire to place material on the lot. Ten random numbers were selected that, when multiplied by the lot size, determined the 5 sample locations.

The sample locations were stepped off and 1-min moisture and density count readings were taken at each location. The readings, along with the calibration charts for the instrument, were used to obtain the in-place moisture and density of the soil. The maximum dry density for the soil in question was determined on advice of the state inspector. The percentage compaction for each sample along with the average and range in values was calculated. The required average percentage compaction for the 5 tests was calculated according to the specification requirements. If the average of the 5 samples was greater than the required average, the lot was accepted.

The time to carry out the testing for a sample size of 5 was as follows: 10 min to determine sample locations, 30 to 60 min to do the testing (depending on the ease with which the sample locations can be leveled by hand shovel), and 15 min to do the calculations necessary to determine whether the area is accepted. The testing was carried out while the contractor was working on the lot. For a few of the smaller lots, there was insufficient time to perform all 5 tests. For the 70 lots, 9 had 4 tests, 1 had 3 tests, and the remainder had 5 tests.

RESULTS AND DISCUSSION

The numerical results for the 3 projects in which the trial specifications were simulated are given in Tables 1, 2, and 3.

The lots varied in size from as small as 15 by 400 ft in project 1 to 60 by 3,000 ft in project 3. Lot sizes in project 1 were limited to a 15 ft width because the testing was done during the construction of the second stage of a 2-stage grading project. The compaction of all the lots was accepted by the current specifications.

For an individual lot, a considerable range in compaction values is observed, especially those for the trial specifications. This range brings a number of individual compaction values below the limits of the current department specification. For project 1, 8 of the 14 lots have one or more compaction values below 90 percent. Yet, only one of those lots was rejected according to the trial specifications. This comparison is made to point out that individual compaction values are not significant, and only information derived from a number of test results has meaning.

A better understanding of the degree of compliance with the trial specifications can be obtained if a plot is made of the measured average compaction and the required average compaction for each lot. This plot for project 3, shown in Figure 1, illustrates an interesting point. The compaction was more than adequate until lot 14 and the next 3 lots where the compaction was near or below the required value. At this point, the contractor overreacted and produced much more than the required compaction for the next 5 lots.

A comparison of the average percentage compaction for each lot as obtained from highway department and trial specifications is shown in Figure 2 for project 3. The research data are from random sample locations, and the department data are what the inspector considered to be representative samples. The comparison can be considered a measure of the inspector's ability to select a sample, or samples, representing the

average compaction of the lot. It can be observed that the major changes in density were detected by the inspector. However, as the lot numbers increase, there is a widening gap between the research and inspector's data. This indicates the inspector's inability to consistently select average compaction values.

STATISTICALLY BASED SPECIFICATIONS IN USE BY OTHERS

In a 1968 report, the California Division of Highways summarized its research on statistical quality control for a 4-year period (3): "The report proposed quality control by moving averages using control charts. It is anticipated that this procedure will provide control without increasing cost while at the same time supplying management information in the form of charts and graphs."

TABLE 1
RESULTS OF PROJECT 1 IN LEEDS

| Lot | | | Trial Specifications | | | | | | | Current Specifications | | | | |
|--------|------------|-------------|---|-------|-------|------|----------------------------|----------------------------|------------------|-------------------------------|------------------------------|---|------|------|
| Number | Width (ft) | Length (ft) | Individual Compaction Results (percent) | | | | Average Obtained (percent) | Average Required (percent) | Accept or Reject | Compaction Required (percent) | Average Compaction (percent) | Individual Compaction Results (percent) | | |
| L01 | 15 | 1,000 | 102.2 | 97.3 | 93.4 | 95.7 | 93.7 | 96.6 | 88.2 | A | 90 | 95.6 | 94.7 | 96.6 |
| L02 | 15 | 400 | 96.8 | 97.5 | 100.5 | 96.0 | 96.8 | 97.5 | 86.4 | A | 90 | 97.9 | 97.9 | |
| L03 | 15 | 500 | 95.0 | 69.0 | 97.5 | 90.0 | 95.0 | 89.3 | 95.2 | R | 90 | 94.0 | 94.0 | |
| L04 | 15 | 3,000 | 93.4 | 92.2 | 90.8 | 90.8 | 89.4 | 91.3 | 86.4 | A | 90 | 90.6 | 90.4 | 90.8 |
| L05 | 15 | 500 | 91.6 | 94.2 | 87.5 | | | 91.1 | 88.3 | A | 90 | 91.5 | 91.5 | |
| L06 | 15 | 1,000 | 101.0 | 96.6 | 90.3 | 88.5 | | 94.1 | 90.0 | A | 90 | 90.5 | 90.7 | 90.4 |
| L07 | 15 | 500 | 87.4 | 92.0 | 90.0 | 86.7 | 93.2 | 89.9 | 87.3 | A | 90 | 90.4 | 90.4 | |
| L08 | 15 | 500 | 92.0 | 88.4 | 93.0 | 86.7 | | 90.0 | 87.5 | A | 90 | 91.8 | 91.8 | |
| L09 | 15 | 500 | 95.5 | 92.0 | 88.4 | 85.0 | 91.4 | 90.5 | 88.8 | A | 90 | 96.3 | 95.4 | 97.3 |
| L10 | 15 | 500 | 94.7 | 93.7 | 98.0 | 84.0 | 87.0 | 91.5 | 90.0 | A | 90 | 91.7 | 91.7 | |
| L11 | 15 | 500 | 91.6 | 92.3 | 96.5 | 91.6 | 97.0 | 93.8 | 87.0 | A | 90 | 90.4 | 90.4 | |
| L12 | 15 | 500 | 92.4 | 104.0 | 104.0 | 96.5 | 99.0 | 99.2 | 89.2 | A | 90 | 99.6 | 99.6 | |
| L13 | 15 | 500 | 98.7 | 103.0 | 100.5 | 97.5 | 101.5 | 100.2 | 86.9 | A | 90 | 92.7 | 92.7 | |
| L14 | 15 | 500 | 98.3 | 92.4 | 98.8 | 96.4 | 94.3 | 96.0 | 87.3 | A | 90 | 96.8 | 96.8 | |

TABLE 2
RESULTS OF PROJECT 2 IN WILLISTON

| Lot | | | Trial Specifications | | | | | | | Current Specifications | | | | | |
|--------|------------|-------------|---|-------|-------|-------|----------------------------|----------------------------|------------------|-------------------------------|---|-------|-------|-------|-------|
| Number | Width (ft) | Length (ft) | Individual Compaction Results (percent) | | | | Average Obtained (percent) | Average Required (percent) | Accept or Reject | Compaction Required (percent) | Individual Compaction Results (percent) | | | | |
| W01 | 48 | 1,000 | 100.0 | 111.0 | 107.5 | 108.5 | 106.0 | 106.0 | 99.0 | A | 95 | 104.4 | 109.2 | 102.0 | 102.0 |
| W02 | 48 | 900 | 105.0 | 99.6 | 100.0 | 96.1 | 107.2 | 101.6 | 99.0 | A | 95 | 97.0 | 95.6 | 98.4 | |
| W03 | 48 | 2,500 | 105.0 | 117.0 | 101.5 | 116.0 | 110.5 | 110.0 | 100.6 | A | 95 | 98.2 | 100.8 | 95.6 | |
| W04 | 48 | 2,800 | 101.5 | 104.5 | 100.9 | 99.4 | 100.0 | 105.3 | 99.1 | A | 95 | 99.4 | 100.0 | 98.8 | |
| W05 | 48 | 600 | 105.0 | 105.5 | 106.0 | 101.5 | 110.8 | 105.7 | 98.3 | A | 95 | 101.0 | 101.0 | | |
| W06 | 48 | 600 | 106.0 | 110.0 | 108.0 | 100.0 | 100.6 | 104.9 | 98.6 | A | 95 | | | | |
| W07 | 48 | 600 | 98.2 | 99.6 | 99.0 | 102.0 | 112.0 | 102.2 | 99.9 | A | 95 | 99.8 | 100.0 | 99.6 | 98.2 |
| W08 | 48 | 1,300 | 111.0 | 93.5 | 98.5 | 108.0 | 103.0 | 102.8 | 101.3 | A | 95 | 101.3 | 98.7 | 104.0 | |
| W09 | 48 | 750 | 105.5 | 104.2 | 105.3 | 104.2 | 109.2 | 105.7 | 96.8 | A | 95 | | | | |
| W10 | 48 | 2,800 | 104.2 | 106.4 | 94.3 | 104.2 | 113.0 | 104.4 | 101.6 | A | 95 | 99.5 | 96.1 | 102.9 | |
| W11 | 48 | 2,000 | 108.0 | 103.8 | 108.3 | 99.5 | 104.3 | 104.8 | 98.2 | A | 95 | | | | |
| W12 | 48 | 600 | 104.5 | 99.8 | 100.0 | 108.0 | 104.3 | 103.3 | 97.9 | A | 95 | 100.0 | 100.0 | | |
| W13 | 48 | 910 | 104.0 | 105.0 | 99.3 | 104.0 | 100.9 | 102.6 | 97.1 | A | 95 | | | | |
| W14 | 48 | 3,200 | 93.7 | 106.2 | 99.3 | 105.5 | 106.8 | 102.3 | 99.7 | A | 95 | | | | |
| W15 | 78 | 600 | 108.0 | 104.0 | 100.0 | 103.0 | 98.2 | 102.6 | 98.5 | A | 95 | 103.0 | 99.7 | 106.4 | |
| W16 | 66 | 600 | 103.0 | 105.0 | 106.0 | 98.0 | 99.3 | 102.2 | 97.9 | A | 95 | 101.0 | 97.0 | 105.0 | |
| W17 | 60 | 700 | 107.0 | 103.0 | 100.0 | 100.7 | 98.0 | 101.7 | 98.2 | A | 95 | 104.5 | 104.5 | | |
| W18 | 60 | 1,000 | 109.0 | 107.4 | 110.2 | 105.8 | 105.7 | 107.6 | 96.6 | A | 95 | 101.2 | 100.3 | 100.0 | 103.3 |
| W19 | 60 | 1,000 | 110.1 | 103.4 | 108.5 | 109.4 | | 107.6 | 97.7 | A | 95 | 100.6 | 98.0 | 99.8 | 104.0 |
| W20 | 48 | 1,500 | 99.5 | 103.0 | 104.8 | 97.2 | 100.3 | 100.9 | 97.8 | A | 95 | | | | |

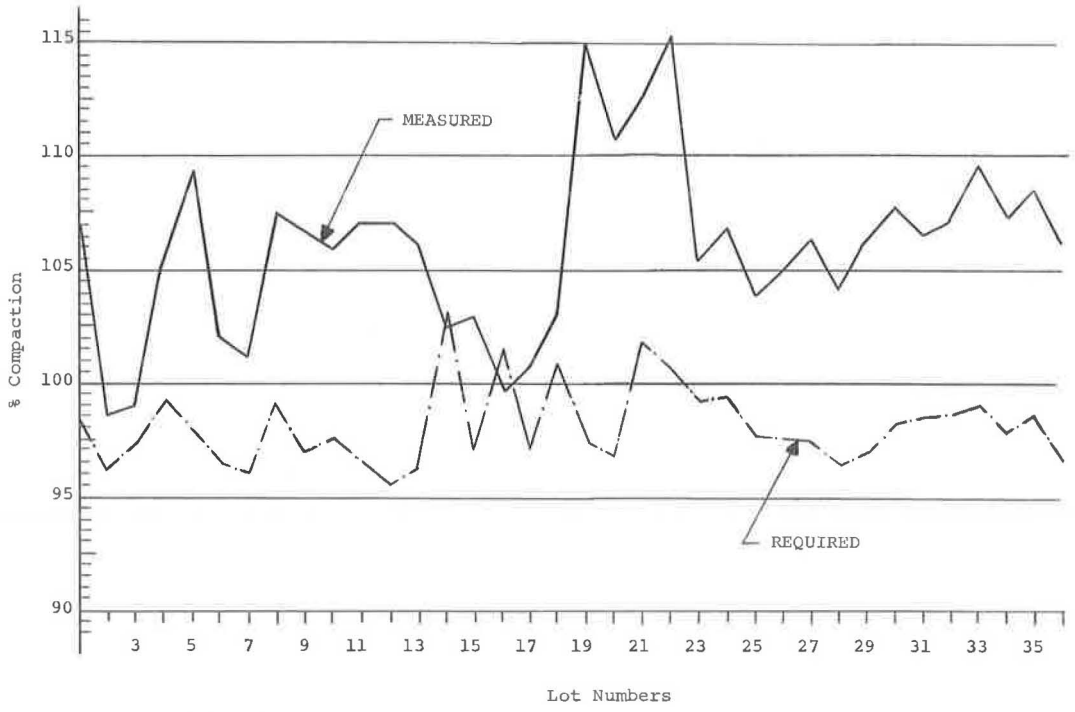


Figure 1. Required and measured average compaction values for project 3.

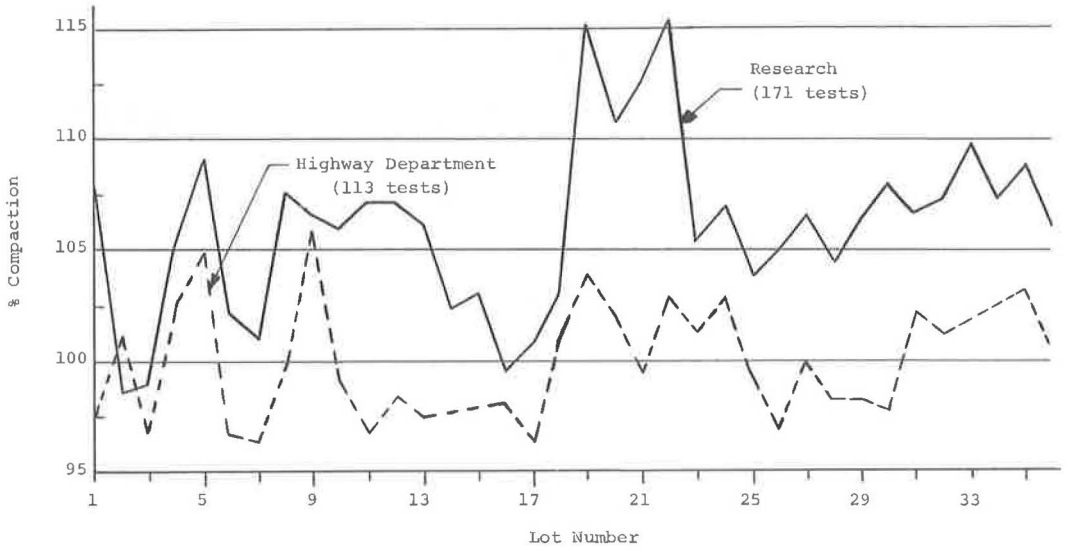


Figure 2. Average compaction values obtained from highway department and research data for project 3.

Briefly, the moving average method works as follows: (a) Sample locations are determined through the use of nonbiased sample cards (a form of random numbers); (b) individual density tests are performed; (c) the moving average is calculated where the moving average is the average of the 4 most recent test results representing acceptable material plus the test result from the material being considered for acceptance; and (d) the moving average is plotted on a control chart that provides visual evidence of the quality of construction. Generally, specifications will require both individual test and moving average test results to be within specified limits; if these limits are exceeded, the material does not meet the specifications. By noting trends on the control chart, the contractor is able to correct his process prior to test points exceeding the limits.

The report (3) lists the following advantages of using the moving average approach:

1. More information is available to the engineer than is available when only a single test result is considered;
2. Randomly occurring "extreme" values are identified;
3. When plotted on a control chart, trends in material quality or uniformity can be readily observed;
4. When compared to a full statistical specification, fewer tests are required for control; and
5. The precision of the test result can be controlled depending on the number of tests included in the running average, thereby increasing the reliability of the decision-making process.

The 1967 construction specifications for the Mississippi State Highway Department employ a compaction specification based on the moving average (4). The roadway area is divided into lots of sizes specified in the contract or in the department's standard operating procedures. Tests are taken at predetermined random sample locations. However, the engineer on visual inspection may take other samples as he deems necessary. The program is set up to have a number of tests at the start of the project and, if the compaction is satisfactory, to be followed by minimum testing.

The test area concept has been used in Virginia and California for density control of subgrade and base course work. In California (5) the procedure is as follows: An area to be tested is divided into 3 subareas; in each subarea, 2 nuclear density tests are taken at random locations determined by the engineer; the percentage compaction is determined for each test based on a single maximum dry density value for the area; and for the compaction of each subarea to be accepted, the average of the 2 percent compaction values must be more than the required compaction value, and no more than one of the individual tests may be less than the required compaction. Therefore, one or both of the subareas may fail.

The area concept is limited to locations where the soil is uniform. However, if it is modified to the extent that for each variation in soil a new maximum dry density test was performed, then it can be applied to locations of nonuniform soil.

QUALITY ASSURANCE: COMPARISON OF CURRENT AND TRIAL SPECIFICATIONS

The reason for securing samples of compacted soils is to gain information on the adequacy of the compaction. It has been established that there is considerable variation in the degree of compaction in any lot. This variability will cause the data to take the form of any one of the bell-shaped frequency distribution curves shown in Figure 3, which is included to show possible levels of compaction in the lot. A specification limit is indicated. The percentage of the lot with compaction less than the specification limit is the percentage defective and is shown in the figure.

When samples are drawn from lot a, the probability of accepting the lot would be 100 percent because all the values are above the specification limit. Likewise, the probability of accepting lot e would be 0 percent because none of the samples is above the specification limit. For lots b, c, and d, the probability of accepting the lots lies between 0 and 100 percent. This depends on the percentage of the lot that is defective and the number of samples taken from the lot for the purpose of determining the acceptability.

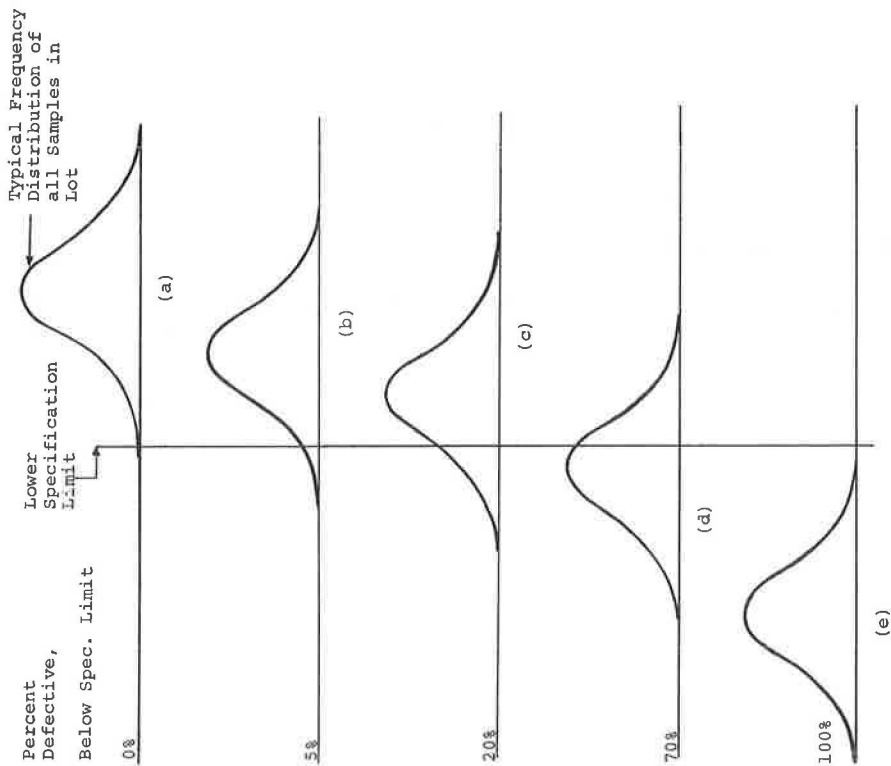


Figure 3. Examples of levels of compaction.

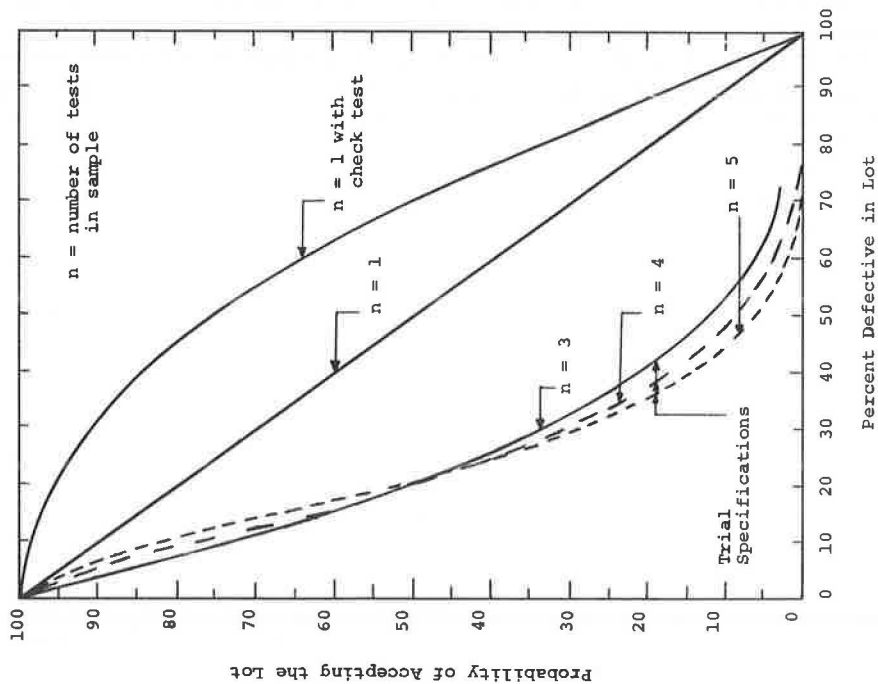


Figure 4. Comparison of sampling plans.

The relationship between the probability of accepting a lot and the percentage defective in the lot can be developed for different sampling plans. This has been done for the trial specifications and other sampling plans and is shown in Figure 4. The 3 lower lines are for the trial specifications of sample sizes 3, 4, and 5. They are based on Military Standard 414 with 20 percent defective material (6). The straight diagonal line is for a sampling plan based on a single test. The upper line is for a sampling plan that states that, if the first test is below specifications, take another test; if that is also below specifications, reject the lot.

When sampling plans are compared, it should first be noted that all sampling plans give the same results if the lot is either less than 0 percent or more than 100 percent defective. The trial specifications will accept 50 of 100 lots that are 20 percent defective, while $n = 1$ and $n = 1$ with check test will accept 80 and 96 of 100 lots respectively. There is less than a 5 percent chance of accepting lots with 70 percent defective when the trial specifications are used; however, the other sampling plans give probabilities of 30 and 50 percent. In conclusion, the trial specifications provide less chance of accepting defective material.

SUMMARY AND RECOMMENDATIONS

The investigation has brought out the following advantages in using a statistically based construction specification for embankment compaction as compared to the current method based on representative samples and a specified minimum compaction.

1. Random sampling produces an unbiased estimate of average compaction, while representative sampling produces results above or below the true average depending on the judgment of the inspector (Fig. 2).

2. The statistically based specifications produce greater uniformity of contract enforcement, and this results in better contractor relations.

3. There is a much greater chance of accepting poor quality construction from a sampling plan based on a single test result or a single test with check test than from a statistically based acceptance sampling plan (Fig. 4).

4. The proposed statistically based specification provides an incentive for the contractor to reduce the variability in the construction in that the required level of compaction is dependent on the range in test results.

5. The compaction level under the statistically based construction specifications is at the same level as that currently produced in acceptable construction in North Dakota.

6. The trial application of the statistically based specifications indicated that, of 60 lots accepted by the state, 2 lots would be rejected under the trial specifications. This does not mean there will be more rejections under the trial specifications, because 9 of the 67 lots accepted by the trial specifications could have been rejected by the state in that they contain compaction values below the specification limit (Tables 1, 2, and 3).

Certain problems will be encountered in a change in specifications. These include the following:

1. There will be a change in the frequency of sampling. This will depend on the acceptance plan employed, size of project, rate of construction, and testing equipment available to the inspector. For example, if a statistically based specification employing a sample size of three were used, this would have doubled the highway department's sampling frequency on project 1 and reduced the sampling frequency by 10 percent on project 3.

2. Equipment needs will increase if nuclear moisture-density instruments are employed. However, manpower needs will decrease in that 1 man with a nuclear instrument can carry out more tests than 2 men with water-balloon in-place density equipment.

3. An educational effort is necessary for inspectors as well as for administrative engineers not on the mechanics of employing the specification, for this is relatively simple, but rather on the concepts that a number of random samples are of far more value than representative samples and that variability is a normal characteristic of acceptable construction.

ACKNOWLEDGMENTS

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REFERENCES

1. Jorgenson, J. L. The Statistical Approach to Quality Control in Highway Construction, Phase I, Measuring the Variability, Part A, Compacted Embankments. Eng. Exp. Station, North Dakota State Univ., Fargo, No. 15, Nov. 1968.
2. Development of Guidelines for Practical and Realistic Construction Specifications. NCHRP Rept. 17, 1965, p. 27.
3. Sherman, G. B., and Watkins, R. O. Statistical Quality Control of Construction Materials. California Division of Highways, Sacramento, Research Rept. M and R 631133-9, May 1968.
4. Standard Specifications for Road Construction. Mississippi State Highway Department, Jackson, 1967.
5. Weber, W. G., Jr., and Smith, T. Practical Application of the Area Concept to Compaction Control Using Nuclear Gages. Highway Research Record 177, 1967, pp. 144-156.
6. Bowker, A. H., and Lieberman, G. J. Engineering Statistics. Prentice-Hall, Englewood Cliffs, N.J., 1959, p. 471.

DEVELOPMENT AND TRIAL USE OF ACCEPTANCE SAMPLING PLANS FOR ASPHALT CONSTRUCTION

James L. Jorgenson, North Dakota State University

ABRIDGMENT

•THE research on variability in asphalt construction in North Dakota brought to light many characteristics of current acceptable construction. They are as follows: (a) Single test results exhibited a large variability causing many of the measurements to be outside the specifications; (b) the current gradation band was only partially effective in controlling aggregate gradation; and (c) payments to the contractor were independent of the quality of his work. The following paragraphs expand on these observations and discuss their implications for improving specifications.

Field measurements are conducted to obtain information, which serves as a basis for making a decision on the acceptability of the construction. For each decision made, there is a probability of its being incorrect; that is, poor material is accepted or good material is rejected. The probability of the decision being incorrect increases with the variability of the measurements on which the decision is made. The primary source of variability is testing error. This variability is reduced by using the average of a number of tests. Therefore, an essential step in increasing the reliability of the decision and, hence, in improving the specifications is to base all decisions on the average of a number of test results.

The gradation limits in the highway department's specifications should be a means of controlling the gradation of the aggregate. Currently, the department does this in only a limited way because of the practice of resampling and because of the attitude that test results just within the limits are as acceptable as those midway between the limits. When the process has a target value just inside the specification band, because of the variability of the process, it is natural to find half of the readings outside the specification limit. The specification limits would be more effective if target values were chosen either at the center of the band or in 2 or 3 standard deviations from the current specification limits. When gradation target values based on the variability of the material are used and enforced, the specification limits will take on real meaning.

A study of the report on variability in asphalt construction reveals a large variation in contractor performance. For example, on one project, 5 percent of the hot bin gradation readings were outside the specification limits; on another project, 17 percent were outside. For asphalt contents, the difference between the design value and the plant consumption value was 0.05 percent for one project and 1.67 percent for another project. In pavement thickness, one contractor produced an average thickness within 0.01 in. of the design value, another 0.46 in. above the design thickness, and another 0.20 in. below the design thickness. Yet, in spite of these variations in performance, all contractors received the same full payment for their work. At least within the latitude of the projects studied, there is little or no incentive for the contractor to do a good job. His pay has been set by the bidding process, so his incentive now is to complete the job for the least cost.

Under the current specifications, the job can often be completed with relative ease. There are specification limits for gradation of aggregate, but they can be stretched considerably through the practice of resampling. Although there are target values for important things such as asphalt content, pavement density, and pavement thickness, the

specifications do not place limits on these variables. The permissible range in these variables is left up to the engineer. The project engineer has the option of shutting down the contractor if, in his judgment, the work is unacceptable. However, this option is seldom used, primarily because the contractor assures the engineer that he is working to correct the deficiency, and partly because the project engineer is also responsible for getting the project completed.

This discussion points out the real need for construction specifications that require the contractor to bear a greater responsibility for the quality of his construction. This will come about when acceptance criteria are specifically stated and substandard work is either not permitted to be placed or, if placed, paid for under an adjusted price.

During the summer of 1968, data were collected from 5 asphalt construction projects for the purpose of simulating the use of statistically based construction specifications. The lot size was taken as a full day's construction. It varied in size from about 1,200 to 2,500 tons.

At the plant, 5 aggregate samples were taken from dry batches at random times throughout the day. The gradation was determined by the percentage passing 4 sieves: $\frac{3}{4}$ in., No. 4, No. 30, and No. 200. A mix sample was taken for the purpose of making a Marshall test specimen to determine the maximum density of the mix. The following morning after the mat had cooled, 5 randomly located courses were taken and measured for density, thickness, and asphalt content.

Compliance with statistically based specifications is based on the difference between the test values and the target values. In the current North Dakota specifications, the target values are not clearly defined. Consequently, for the simulation, 3 different target values were used for gradation. They were from the Marshall mix design, from the average gradation as determined from samples taken during crushing of the aggregate, and from the average values of gradation during the simulation. The target value on asphalt content was that recommended from the Marshall mix design. The target value on density was 95 percent of the maximum density for the mix as determined by a daily test on the mix. The target value for pavement thickness was taken as the thickness called for in the specifications.

A number of lot payment schedules were tried; the one recommended is given in Table 1. The left column gives the different levels of payment as a percentage of the contract price. The second column gives in general terms the variation permitted in the sample average. A standard deviation, σ , from previous research in North Dakota and a sample size, n , of five are used to obtain the permissible variation of the average from the target values for the different items given in the remaining columns.

When the payment schedule was applied to the data collected from the 5 construction projects, the resulting components of lot payments were calculated and are given in Table 2. The lot payment is the average of the thickness, density, and mix contents payments. The mix content payment is the average of the asphalt and 4 gradation payments. For most lots, the premium payments were for gradation and the lower payments were a large variation in asphalt content. It is interesting to note the lot or daily variation in contract payments. This is shown in Figure 1. For projects 1 and 4, the

TABLE 1
LOT PAYMENT SCHEDULE FOR AVERAGE OF 5 TESTS

| Percent Payment | General | Variation of Average From Target Values | | | | Asphalt Content (percent) | Thickness (in.) | Density (percent) |
|-----------------|---------------------------|--|-------|--------|---------|---------------------------|-----------------|-------------------|
| | | Gradation ^a (percent passing) | | | | | | |
| | | $\frac{3}{4}$ In. | No. 4 | No. 30 | No. 200 | | | |
| 103 | $\leq \sigma/\sqrt{n}$ | 0.45 | 2.24 | 2.24 | 0.90 | 0.25 | 0.134 | 96.3 |
| 100 | $\leq 2(\sigma/\sqrt{n})$ | 0.90 | 4.48 | 4.48 | 1.80 | 0.50 | 0.268 | 95.0 |
| 97 | $\leq 3(\sigma/\sqrt{n})$ | 1.35 | 6.72 | 6.72 | 2.70 | 0.75 | 0.402 | 93.7 |
| 90 | $\leq 4(\sigma/\sqrt{n})$ | 1.80 | 8.96 | 8.96 | 3.60 | 1.00 | 0.536 | 92.4 |
| 80 | $\leq 4(\sigma/\sqrt{n})$ | 1.80 | 8.96 | 8.96 | 3.60 | 1.00 | 0.536 | 92.4 |

TABLE 2
COMPONENTS OF LOT PAYMENTS

| Project | Date | Gradation ^a (percent passing) | | | | Asphalt | Average for Mix Contents (percent) | Density (percent) | Thickness (percent) | Lot Payment (percent) |
|---------|------|---|-------|--------|---------|---------|--|----------------------|------------------------|-----------------------------|
| | | ¾ In. | No. 4 | No. 30 | No. 200 | | | | | |
| 1 | 7-11 | 103 | 103 | 103 | 97 | 90 | 99.2 | 97.0 | 96.4 | 97.53 |
| | 7-12 | 103 | 103 | 103 | 97 | 80 | 97.2 | 100.0 | 97.2 | 98.13 |
| | 7-15 | 103 | 103 | 100 | 90 | 97 | 98.6 | 100.0 | 93.2 | 97.26 |
| | 7-16 | 103 | 103 | 100 | 90 | 97 | 98.6 | 100.0 | 97.0 | 98.53 |
| | 7-18 | 103 | 103 | 103 | 80 | 90 | 95.8 | 100.0 | 100.0 | 98.60 |
| | 7-19 | 103 | 100 | 103 | 97 | 80 | 96.6 | 103.0 | 100.0 | 99.88 |
| | 7-22 | 103 | 100 | 100 | 80 | 80 | 92.6 | 90.0 | 95.0 | 95.88 |
| | 7-23 | 103 | 103 | 100 | 90 | 90 | 97.2 | 97.0 | 78.0 | 90.73 |
| | 7-24 | 103 | 103 | 100 | 90 | 97 | 98.6 | 97.0 | 103.0 | 99.53 |
| | 7-25 | 103 | 100 | 103 | 90 | 97 | 98.6 | 80.0 | 100.0 | 92.88 |
| | Avg | | | | | | | | | |
| 2 | 7-1 | 103 | 103 | 103 | 103 | 100 | 102.4 | 97.0 | 103.0 | 100.80 |
| | 6-20 | 103 | 100 | 103 | 103 | 97 | 101.2 | 103.0 | 103.0 | 102.40 |
| | 6-21 | 103 | 100 | 100 | 100 | 97 | 100.0 | 103.0 | 100.0 | 101.00 |
| | 6-27 | 103 | 103 | 100 | 103 | 100 | 101.8 | 100.0 | 100.0 | 100.60 |
| | 6-28 | 103 | 103 | 100 | 103 | 100 | 101.8 | 100.0 | 103.0 | 101.60 |
| | Avg | | | | | | | | | |
| 3 | 6-25 | 103 | 103 | 100 | 100 | 100 | 101.2 | 103.0 | 100.0 | 101.40 |
| | 6-26 | 103 | 103 | 100 | 100 | 97 | 100.6 | 103.0 | 100.0 | 101.20 |
| | 6-27 | 103 | 100 | 103 | 100 | 80 | 97.2 | 103.0 | 100.0 | 100.66 |
| | 6-28 | 103 | 100 | 103 | 103 | 97 | 101.2 | 103.0 | 100.0 | 101.40 |
| | 6-29 | 103 | 100 | 100 | 100 | 103 | 101.2 | 97.0 | 98.3 | 98.83 |
| | Avg | | | | | | | | | |
| 4 | 8-02 | 103 | 103 | 100 | 100 | 103 | 101.8 | 103.0 | 88.5 | 97.76 |
| | 8-05 | 103 | 103 | 100 | 103 | 97 | 101.2 | 97.0 | 87.0 | 95.06 |
| | 8-06 | 103 | 103 | 100 | 103 | 90 | 99.8 | 100.0 | 100.0 | 99.93 |
| | 8-07 | 103 | 103 | 103 | 97 | 90 | 99.2 | 103.0 | 99.5 | 100.56 |
| | 8-08 | 103 | 103 | 100 | 97 | 97 | 100.0 | 100.0 | 92.5 | 97.50 |
| | 8-09 | 103 | 103 | 100 | 103 | 103 | 102.4 | 100.0 | 100.0 | 100.80 |
| | 8-10 | 103 | 103 | 97 | 103 | 97 | 100.6 | 100.0 | 100.0 | 100.20 |
| | 8-12 | 103 | 103 | 97 | 103 | 97 | 100.6 | 103.0 | 98.0 | 100.53 |
| | 8-13 | 103 | 103 | 100 | 103 | 100 | 101.8 | 103.0 | 88.5 | 97.76 |
| | Avg | | | | | | | | | |
| 5 | 7-29 | 103 | 90 | 97 | 103 | 103 | 99.2 | 80.0 | 100.0 | 93.06 |
| | 7-30 | 103 | 90 | 97 | 103 | 90 | 96.6 | 97.0 | 103.0 | 98.86 |
| | 7-31 | 103 | 90 | 97 | 100 | 100 | 98.0 | 100.0 | 100.0 | 99.33 |
| | 8-1 | 103 | 97 | 97 | 100 | 100 | 99.4 | 100.0 | 100.0 | 99.80 |
| | Avg | | | | | | | | | |

^aTarget value is Marshall mix design.

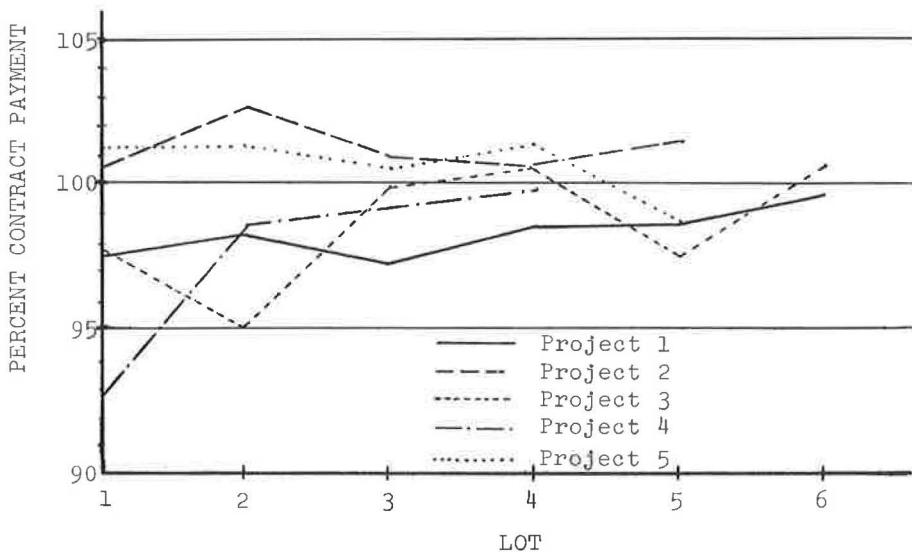


Figure 1. Simulated daily payments.

daily payments are all below 100 percent; however, for projects 2, 3, and 5, the payments fluctuate around 100 percent.

In summary, simulation of statistically based construction specifications on 5 asphalt construction projects in North Dakota shows that normal plant operations based on realistic target values will result in payments at or near 100 percent of contract price. Further advantages in the statistically based construction specifications were listed in the early part of this report. The following recommendations are made: (a) target values for all significant variables must be specified in the plans and specifications; (b) acceptance limits with appropriate price adjustments must be specified in the plans and specifications; and (c) it is necessary to rewrite current specifications to take out many of the restrictive control rules, thus allowing the contractor a freer hand in conducting his work.

STATISTICAL SPECIFICATION FOR THE ACCEPTANCE OF PUG MILL-MIXED BASE AND SUBBASE MATERIALS

M. C. Anday, Virginia Highway Research Council

This paper discusses the system currently used in Virginia for accepting pug mill-mixed base and subbase materials and a proposed system based on a job-mix band and a process tolerance concept. In developing the proposed system, we used a random sampling plan to collect samples from 14 plants in Virginia that produce subbase and base materials. The samples were tested to determine their gradation and liquid and plastic limits. From the results, average standard deviations for each property were determined and used for the selection of the job-mix band and the process tolerances. The proposed specifications, which permit acceptance or rejection of 2,000-ton lots, are believed to have the following advantages: The use of a job-mix band should result in more uniformly graded materials; job-mix bands developed by use of average standard deviations will allow flexibility in the operations of the plants; and the process tolerances developed, which are based on the averages of 4 samples, will allow for day-to-day variations in materials.

•OFFICIALS of the Virginia Department of Highways have long realized the need for statistical specifications in highway construction and maintenance operations and have encouraged the Virginia Highway Research Council's efforts to develop such specifications. As a result, the council has developed several specifications that have been adopted by the department, and the study reported here is an extension of this developmental work.

PURPOSE AND SCOPE

The purpose of the study was to develop statistically based specifications for pug mill-mixed materials. In short, the study attempted to accomplish the following:

1. Develop, on the basis of an analysis of the present system, a sampling, testing, and acceptance procedure that would lend itself to statistical analyses and yet require as few disruptions as possible in current procedures;
2. Determine the variabilities in the gradation, water content, and liquid and plastic limits of pug mill-mixed materials; and
3. Suggest a package procedure for sampling, testing, and accepting the materials, and offer recommendations regarding factors such as the point of sampling and process tolerances.

Though it was originally planned that the study would include all types of aggregate pug mill-mixed materials used in Virginia, only materials 21A and 21 were produced during the sampling period.

PROCEDURE

Analysis of the Present System

At present the Virginia Department of Highways requires that all aggregate base and subbase materials be pug mill-mixed (1). The properties of the pug mill-mixed

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materials are determined at the plant by the plant inspector. The rate of sampling is a minimum of 1 test sample per 1,000 tons of material, taken from the pug mill chute or from the truck. Also, at least 2 tests per day are required.

The samples taken are "representative" in the sense that the inspector looks for a portion of the material that he thinks is representative of the whole. The samples are split to provide the amounts of materials needed for testing; usually they are quartered. One quarter is used for a gradation test; another is slowly air-dried and used for determining the liquid and plastic limits; and the remainder is discarded. Each 1,000 tons of material is accepted or rejected on the basis of 1 test and, in a sense, is bought at the plant as far as the mix constituents are concerned.

The present specification is a product of many years of experience and numerous modifications. It seems to provide the highway department with the materials desired, but in the author's opinion it could be improved in the following areas:

1. Representative sampling—A great fallacy in representative sampling is that it assumes that a person can visually select a portion of the material that represents the whole. Though in some cases this is possible, one cannot safely assume that every plant inspector can do it day after day. An unbiased method such as random sampling or stratified random sampling would relieve the inspector of the burden of selecting the sample and thus would be a great improvement.

2. Checking for compliance on the basis of individual test values—That the present system of accepting or rejecting materials on the basis of individual test values can be misleading is demonstrated by the distributions shown in Figure 1. An individual test value (Fig. 1a) may pass, be recorded, and, because acceptance or rejection is based on an individual test, then be forgotten. If, however, data are accumulated, including occasional failing test values, they may plot as shown in Figure 1b. If the population of the data is then plotted in the form of a normal curve, as in Figure 1c, one can see that there is a great deal of material that actually does not meet the specification. Limited sampling will not disclose most of these failing values because of the laws of probability. They are, however, there. Failing test values do not necessarily indicate the presence of bad materials because "failing" is determined by comparing a value to the specification limits. Thousands of miles of roads have been built in Virginia with

very few failures. One, therefore, cannot help concluding that the unnoticed failing values were obtained from material that was actually good. The questions that then arise are, Why does not the specification encompass these materials? Why does not the compliance system show that they are actually there? Unless these questions are answered, the legal defensibility of the system is not very sound. The author believes that a compliance checking system based on the means of several samples is much sounder. An acceptance or rejection system based on a specified lot size of material and the average of several samples can give the tester a much better insight as to where the mean of the population of that material is located.

3. Use of a master band—The present Virginia specification for the gradation of pug mill-mixed materials consists of a master band within which all test values must fall to be classified as "passing." This master band is formed around Fuller's curve, and the mean as well as the limits have evolved from many years of experience. However, because compliance is based on individual samples, the gradation of the materials

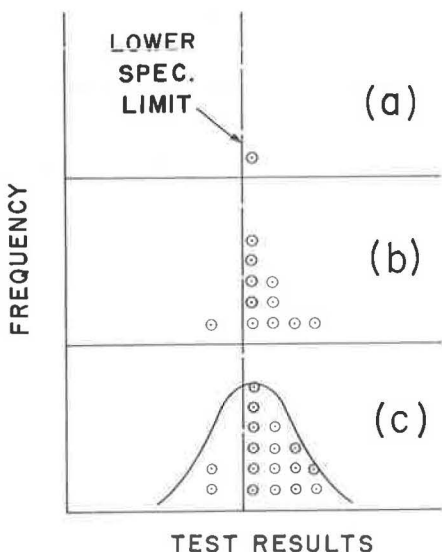


Figure 1. Accumulation of individual test values.

can meander as shown in Figure 2. Technically this meandering of the gradation within the master band is considered undesirable, but with the present specification there is no way to avoid it. It is believed that a compliance check based on means with set limits can lessen this meandering and result in improved materials.

The New Approach

From the analysis of the present system, it is apparent that a statistical approach will be helpful; this is discussed in the following subsections.

Random Sampling—The new approach is based on statistical concepts. The sampling, therefore, has to be of a random nature and based on a definite lot size. Implementation of this concept probably will be very difficult because the present system has been in effect for such a long time and inspection personnel have become accustomed to representative sampling. It is believed, however, that random sampling will relieve the inspectors of the task of selecting the sample and will eliminate a great deal of confusion. Tests for compliance of pug mill-mixed materials are time consuming. Ordinarily by the time the tests are completed the material is already in the road. The present system, based on penalty points, allows some below-specification materials up to a certain number of accumulated penalty points (1). If the accumulated penalty points are above the limit set, then the material is to be removed from the road. In enforcing this specification, the project inspectors keep a record of where each truck load of material is placed. To permit location of the material in case removal is necessary, the new approach will use a stratified random sampling procedure. That is, the lot size will be divided by the number of samples and one random sample will be taken from each subplot.

Testing Based on Means—As explained earlier, acceptance and rejection will be based on the mean of several samples rather than on individual values.

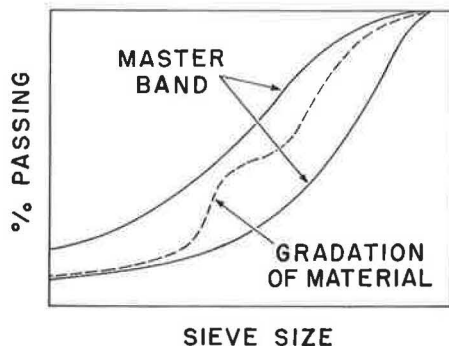


Figure 2. Possible gradation based on individual samples and a master band.

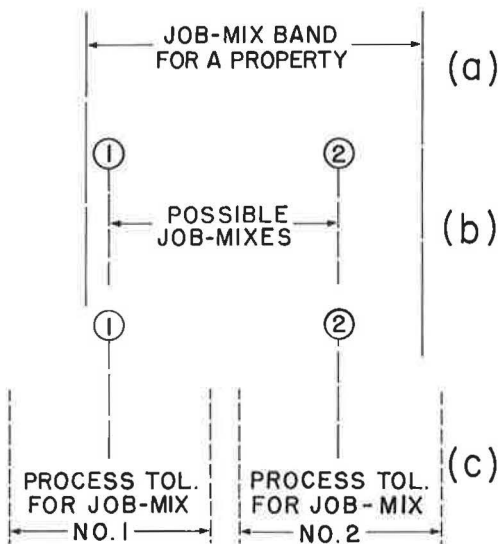


Figure 3. Job mix and process tolerances.

Job-Mix Band and Process Tolerances—A new concept, the job mix, will be included in the specifications for pug mill-mixed materials. The purpose will be to try to keep the means of properties in the middle of a job-mix band. Each producer will be required to submit a job mix that will fall in this band. Once the job mix is approved, the job-mix band will be removed from consideration and the producer will be allowed process tolerances around the job mix. This is shown in Figure 3. Figure 3a shows the job-mix band for a property. From this band the producer will be allowed to choose the location of the property involved, such as location 1 or 2 (Fig. 3b). Once the location of this property as submitted by the producer is approved, he will be given the process tolerances (Fig. 3c). The allowance of process tolerances is also thought to be an improvement to the system because it recognizes that the same test values cannot be attained day after day because of the variabilities inherent in the process.

TABLE 1
MEANS AND STANDARD DEVIATIONS OF MATERIAL 21A

| Property | Mean | | | | | | Standard Deviation | | | | | |
|--|---------|---------|---------|---------|---------|---------|--------------------|---------|---------|---------|---------|---------|
| | Plant 1 | Plant 2 | Plant 3 | Plant 4 | Plant 5 | Plant 6 | Plant 1 | Plant 2 | Plant 3 | Plant 4 | Plant 5 | Plant 6 |
| Percent passing 2-in. sieve ^a | 99.0 | 99.9 | 92.3 | 100.0 | 99.9 | 99.9 | 0.8 | 0.3 | 3.5 | 0.0 | 0.3 | 0.3 |
| 1-in. sieve | 65.2 | 68.4 | 65.2 | 77.6 | 75.2 | 69.8 | 4.9 | 6.3 | 3.8 | 8.5 | 4.7 | 5.2 |
| 3/8-in. sieve | 40.2 | 29.9 | 42.1 | 34.4 | 34.4 | 39.4 | 4.9 | 4.5 | 3.7 | 4.3 | 4.5 | 5.2 |
| No. 10 sieve | 25.0 | 15.9 | 25.8 | 17.4 | 14.3 | 23.0 | 3.9 | 2.4 | 2.7 | 2.5 | 2.4 | 2.7 |
| No. 40 sieve | 12.5 | 10.7 | 8.7 | 9.1 | 7.5 | 10.9 | 2.0 | 1.8 | 1.2 | 1.6 | 1.4 | 1.3 |
| Liquid limit, percent | 22.1 | 15.7 | 17.8 | 19.4 | 13.7 | 21.1 | 1.5 | 0.9 | 1.0 | 1.3 | 0.9 | 2.2 |
| Plasticity index, percent | 0.5 | 0.0 | 0.0 | 0.0 | 0.0 | 0.8 | 1.3 | 0.0 | 0.0 | 0.0 | 0.2 | 1.9 |
| Water content, percent | 6.4 | 6.4 | 5.4 | 5.5 | 4.9 | 6.0 | 1.0 | 1.8 | 0.8 | 0.9 | 0.8 | 1.1 |

^aTop size for all plants.

Field Work—To develop the new approach required that the variabilities of the gradation, as determined by different sieve sizes, and liquid and plastic limits be determined. In Virginia water content is not a pay item; that is, no penalty is imposed if the water content is too high or too low. It was included in this study, however, for purposes of information. With the cooperation of the materials engineers from the 8 construction districts in the Virginia Department of Highways, the author chose major plants in each district for study. At each plant written instructions on the new testing procedures were provided the plant inspector and the district materials technician in charge of pug mill mixes and discussed with them. In accordance with the instructions, the samples were taken by the inspectors in a random fashion by using a table of random numbers and were identified by the use of index cards. The samples were then sent to the district laboratories for testing, and the test data obtained were sent to the author for analysis. The inspectors were asked to take 30 or more samples from all aggregate materials produced by each plant. However, the Virginia Department of Highways generally uses materials designated as 21A and 21, and these were the only materials produced during the study period.

ANALYSIS OF RESULTS

The Data

The data obtained from the district laboratories were analyzed by the use of a computer. In the analysis the mean, \bar{x} , and the standard deviation, σ , for each property—that is, the gradation on each sieve, liquid limit, plasticity index, and water content—were determined. These are given in Tables 1 and 2. Data given in the tables are

TABLE 2
MEANS AND STANDARD DEVIATIONS OF MATERIAL 21

| Property | Mean | | | | | | | | Standard Deviation | | | | | | | |
|--|---------|---------|---------|---------|---------|---------|---------|---------|--------------------|---------|---------|---------|---------|---------|---------|---------|
| | Plant 1 | Plant 2 | Plant 3 | Plant 4 | Plant 5 | Plant 6 | Plant 7 | Plant 8 | Plant 1 | Plant 2 | Plant 3 | Plant 4 | Plant 5 | Plant 6 | Plant 7 | Plant 8 |
| Percent passing 2-in. sieve ^a | 83.9 | 91.0 | 90.6 | 94.7 | 90.6 | 97.5 | 85.4 | 92.0 | 5.7 | 4.2 | 3.8 | 2.5 | 2.7 | 4.0 | 5.3 | 3.1 |
| 1-in. sieve | 58.6 | 60.0 | 58.5 | 70.2 | 70.2 | 62.0 | 69.1 | 57.2 | 5.5 | 7.3 | 6.6 | 5.1 | 6.6 | 9.0 | 6.7 | 6.3 |
| 3/8-in. sieve | 41.6 | 38.2 | 35.6 | 41.3 | 30.8 | 30.4 | 36.7 | 27.2 | 4.4 | 6.1 | 5.2 | 4.5 | 4.1 | 5.6 | 4.8 | 4.5 |
| No. 10 sieve | 25.3 | 22.7 | 20.9 | 25.6 | 16.3 | 14.0 | 16.5 | 12.0 | 3.4 | 3.9 | 3.4 | 3.6 | 2.1 | 3.2 | 2.8 | 1.9 |
| No. 40 sieve | 14.0 | 8.0 | 5.0 | 11.3 | 10.0 | 7.7 | 9.3 | 6.7 | 1.7 | 2.5 | 1.6 | 1.5 | 1.5 | 2.5 | 2.3 | 1.2 |
| Liquid limit, percent | 20.0 | 17.6 | 18.2 | 20.5 | 16.5 | 14.7 | 15.0 | 14.9 | 1.0 | 0.7 | 1.4 | 1.5 | 0.9 | 1.3 | 0.9 | 1.6 |
| Plasticity index, percent | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 | 0.0 |
| Water content, percent | 4.6 | 5.9 | 5.1 | 6.0 | 6.3 | 5.6 | 4.5 | 4.3 | 1.1 | 0.8 | 0.8 | 1.2 | 1.1 | 1.5 | 0.9 | 0.7 |

^aTop size for all plants.

TABLE 3
AVERAGES AND RANGES FOR MEANS AND STANDARD DEVIATIONS FOR MATERIALS 21A AND 21

| Property | Material 21A | | | | Material 21 | | | |
|---------------------------|--------------|--------------|--------------------|------------|-------------|--------------|--------------------|------------|
| | Mean | | Standard Deviation | | Mean | | Standard Deviation | |
| | Average | Range | Average | Range | Average | Range | Average | Range |
| Percent passing | | | | | | | | |
| 2-in. sieve | 100 | 0 | 0 | 0 | 100 | 0 | 0 | 0 |
| 1-in. sieve | 98.5 | 92.3 to 100 | 0.9 | 0 to 3.5 | 89.9 | 83.9 to 94.7 | 3.9 | 2.5 to 5.7 |
| $\frac{3}{8}$ -in. sieve | 70.2 | 65.2 to 77.6 | 5.6 | 3.8 to 8.5 | 63.0 | 56.5 to 70.2 | 6.6 | 5.1 to 9.0 |
| No. 10 sieve | 36.7 | 29.9 to 42.1 | 4.5 | 3.7 to 5.2 | 35.2 | 30.4 to 41.6 | 4.9 | 4.1 to 6.1 |
| No. 40 sieve | 20.2 | 14.3 to 25.8 | 2.8 | 2.4 to 3.9 | 19.2 | 12.0 to 25.6 | 3.2 | 1.9 to 3.9 |
| No. 200 sieve | 9.9 | 7.5 to 12.5 | 1.6 | 1.2 to 2.0 | 9.2 | 5.0 to 14.0 | 1.9 | 1.2 to 2.5 |
| Liquid limit, percent | 18.3 | 13.7 to 22.1 | 1.3 | 0.9 to 2.2 | 17.2 | 14.7 to 20.5 | 1.2 | 0.7 to 1.6 |
| Plasticity index, percent | 0.2 | 0 to 0.8 | 0.6 | 0 to 1.9 | 0 | 0 | 0 | 0 |
| Water content, percent | 5.8 | 4.9 to 6.4 | 1.1 | 0.8 to 1.8 | 5.3 | 4.3 to 6.3 | 1.0 | 0.7 to 1.5 |

sufficiently uniform to enable one to assume average mean and standard deviation values for each property. One exception is the plasticity index values. Because most of the test results for this property were zero, good measures of the mean and standard deviation values were not obtained.

Average mean and standard deviation values for each property were obtained by tabulating the averages and ranges of each property of each material. These are given in Table 3. Based on the averages and ranges of the mean and standard deviation values given in Table 3 and the distribution of the data given in Tables 1 and 2, average standard deviation values were selected for each property of the materials. These are given in Table 4. Water content values are not included in Table 4 because, as was mentioned earlier, it is not a pay item and was determined only for information purposes. The standard deviation values for the plasticity index are based on only a very few samples.

Development of the Job-Mix Band

The current specifications for materials 21A and 21 are given in Table 5. In order for the materials to pass, all test values are required to be within the ranges given. Statistically, this would imply that the mean of the population of the material produced has to be 3 standard deviations from both the upper and lower limits. An extreme case is shown in Figure 4. It is apparent from the data shown in Figure 4 that it would be impossible to produce material that would fall within the specification limits if the variability or standard deviation is 6.5 percent. Yet from the study it was found that the average standard deviation is around 6.5 percent for the $\frac{3}{8}$ -in. sieve.

One should not lose sight of the fact that the sources of the standard deviations obtained in this study—that is, the materials sampled—were all accepted materials. They were "good" and technically desirable materials apparently representative of those with which the Virginia Department of Highways has built excellent roads with very few failures.

At this stage of the study it was concluded that with the current specifications, which are based on individual samples, one has very little idea of where the population mean and extremes are located. From this fact, it was in turn concluded that some "good" but "out of specification" materials can be accepted under the present system.

The next step then was to determine, for each property, the lower and upper limits that actually could exist in today's production.

TABLE 4
SELECTED AVERAGE STANDARD DEVIATIONS
FOR MATERIALS 21A AND 21

| Property | Material 21A | Material 21 |
|---------------------------|--------------|-------------|
| Percent passing | | |
| 2-in. sieve | Top size | Top size |
| 1-in. sieve | 2 | 4 |
| $\frac{3}{8}$ -in. sieve | 5.5 | 6.5 |
| No. 10 sieve | 4.5 | 5.0 |
| No. 40 sieve | 2.8 | 3.0 |
| No. 200 sieve | 1.6 | 2.0 |
| Liquid limit, percent | 1.3 | 1.0 |
| Plasticity index, percent | 0.6 | — |

TABLE 5
CURRENT SPECIFICATIONS, JOB-MIX BANDS, AND PROCESS TOLERANCES
FOR MATERIALS 21A AND 21

| Property | Current Specifications | | Job-Mix Bands | | Process Tolerances ^a | |
|---------------------------|------------------------|-------------|---------------|-------------|---------------------------------|-------------|
| | Material 21A | Material 21 | Material 21A | Material 21 | Material 21A | Material 21 |
| Percent passing | | | | | | |
| 2-in. sieve | 100 | 100 | 100 | 100 | Top size | Top size |
| 1-in. sieve | 90 to 100 | 71 to 95 | 94 to 100 | 79 to 87 | ±3 | ±6 |
| 3/4-in. sieve | 50 to 85 | 50 to 80 | 63 to 72 | 61 to 69 | +9 | +10 |
| No. 10 sieve | 25 to 50 | 25 to 50 | 34 to 41 | 34 to 41 | ±7 | ±7 |
| No. 40 sieve | 12 to 30 | 12 to 30 | 18 to 24 | 18 to 24 | +4 | +4 |
| No. 200 sieve | 5 to 15 | 5 to 15 | 8 to 12 | 8 to 12 | +2 | +2 |
| Liquid limit, percent | Max. 25 | Max. 25 | Max. 23 | Max. 23 | +2 | +2 |
| Plasticity index, percent | Max. 3 | Max. 3 | Max. 2 | Max. 2 | +1 | +1 |
| Water content, percent | 6 ± 2 | 6 ± 2 | — | — | — | — |

^aFor the average of 4 samples.

This was accomplished by taking each mean of each property and adding to it (or subtracting from it) its variability, that is, 3 standard deviations. By plotting all the populations in this manner, the statistically existing lower and upper limits were obtained.

An attempt was made to locate the job-mix band by moving in from the statistically existing lower and upper limits a value corresponding to three times the assumed average standard deviation for each property. As this was done, it was noticed in many instances that the current specifications will be at a 1 standard deviation (using an average standard deviation) distance from the statistically existing high or low values. Therefore, it was decided to use this method, as shown in Figure 5, to calculate job-mix bands for all properties. It is admitted that the calculations in some cases were tempered with engineering judgment. This was necessary, for example, in the gradation calculations where materials 21A and 21 had the same limits but not necessarily the exact same average standard deviations (Tables 4 and 5). The job-mix bands calculated and adjusted with engineering judgment are also shown in Table 5. It should be noted that the liquid limit and plasticity index bands are one-sided.

Development of Process Tolerances

As was mentioned earlier, the author endorses the concept that the same test values cannot be obtained day after day. For this reason process tolerances have to be allowed. It is noted that they are directly related to the number of samples tested. If a single sample is used for the acceptance or rejection of materials, then the entire population limits, that is, ±3 standard deviations, should be used as process tolerances. Because the use of a single sample is not desirable, then process tolerances should be adjusted by using the standard error concept. The relation between the standard error and the standard deviation is as follows:

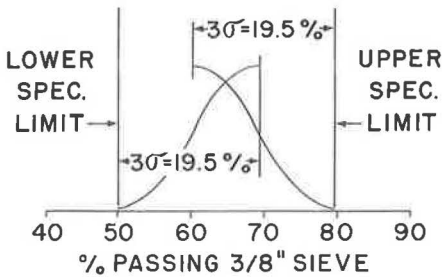


Figure 4. Location of the mean of the 3/8-in. material produced as it relates to specification limits.

$$\sigma_{\bar{x}} = \sigma / \sqrt{N}$$

where

- $\sigma_{\bar{x}}$ = standard error,
- σ = standard deviation, and
- N = number of samples averaged.

Another point that one has to consider in determining process tolerances concerns the acceptance of material that is outside the specification. This is, if no material outside the specification is to be accepted, then process tolerances can be set at ±3 standard errors. If

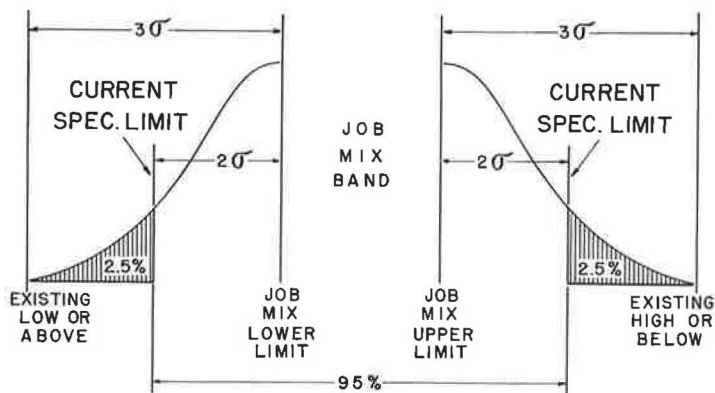


Figure 5. Job-mix band versus existing population, using average standard deviations.

5 percent can be allowed to fall outside specifications, then the limits can be set at ± 2 standard errors. Any other percentage, of course, can be chosen.

On the basis of experience and for purposes of simplicity, the author decided to use ± 3 standard error limits and a sample size of four for the acceptance or rejection of a lot size. The process tolerances are, therefore, calculated from the following formula:

$$\text{Process tolerance} = \pm 3\sigma_{\bar{x}} = \pm(3\sigma/\sqrt{N}) = \pm(3\sigma/\sqrt{4}) = \pm 1.5\sigma$$

In the case of the liquid limit and the plasticity index, a one-sided process tolerance of 1.5 standard deviations was used. The process tolerances calculated with this formula are also given in Table 5. They are based on the averages of 4 samples.

Lot Size

As mentioned in other parts of this report, in the proposed system the acceptance or rejection of material will be based on an average of 4 samples to be taken in a stratified random manner from a lot. The size of the lot to be sampled is generally established on the basis of practicality. It depends on how much one can spend on testing and the degree of certainty desired.

At present the Virginia Department of Highways requires a rate of testing of a minimum of 1 test per 1,000 tons of material. Therefore, if a lot size of 4,000 tons is chosen and 4 samples are taken, then the number of tests required by the proposed system will result in the same amount of testing. For practicality and economy this is desirable. However, it was learned from interviews with the district materials engineers that, although the specifications require a minimum of 1 test per 1,000 tons of material, many plant inspectors run more than this number and the rate of testing is closer to 1 test per 500 tons. It was, therefore, decided to propose a lot size of 2,000 tons.

DISCUSSION OF RESULTS

During the development of the proposed system, several assumptions were made. These are summarized as follows:

1. Because of the system currently in use, the Virginia Department of Highways accepts materials that are actually out of specification. These materials are assumed to be good road-building materials because the system has been accepting them for many years and the construction that includes these materials has performed satisfactorily.

2. The study sampled 14 major plants in the 8 construction districts of the Virginia Department of Highways. This constitutes about 10 to 15 percent of all plants in Virginia and might be considered a small sample. From the uniformity of the data collected, however, the author believes that more sampling will not change the results significantly.

3. It was also assumed that the chosen average standard deviations can be achieved by all plants that produce materials 21A and 21. This might not be possible in all cases, and some plants might have to improve their methods of operation. For example, for material 21, an average standard deviation of 6.5 percent was used for the variability of the $\frac{3}{8}$ -in. sieve. Data given in Table 2 show plants 2 and 6 have a variability much above this value and will experience difficulty in meeting the proposed specification. The other plants, however, will not have any difficulty. This requirement is believed to be desirable because it will encourage the producers to upgrade their operations.

CONCLUSIONS AND RECOMMENDATIONS

Based on the data collected on materials 21A and 21 for this study the following conclusions were drawn:

1. Although the current specification is doing a fair job, it can be improved so as to make the system more defensible from legal and technical standpoints.
2. The current specification, because of the system, accepts a certain amount of material that is outside of the specification limits. This material is, however, good material and should be encompassed in the specification.
3. The variabilities determined are fairly uniform, and the use of average standard deviations seems warranted (Table 4).
4. The use of a job-mix band should result in more uniformly graded materials.
5. Job-mix bands developed by use of average standard deviations will allow flexibility for each plant (Table 5).
6. The process tolerances developed, which are based on the averages of 4 samples, will allow for day-to-day variations in material (Table 5).
7. A lot size of 2,000 tons seems to be adequate for plants in Virginia.

It was, therefore, recommended that the proposed specification, which is based on a job-mix band and process tolerances, be tried on a pilot study basis in Virginia.

RECENT DEVELOPMENTS

Before the report on this study was written, the data and the findings were submitted to the Virginia Department of Highways with recommendations for the adoption of the proposed statistical specification on a trial basis. The materials engineer in charge of statistical specifications translated the recommended method into standard specification form. The recommended specification was then reviewed by the Materials Division, and as a result of 2 discussion meetings several modifications were made to permit more practical application. Because the recommended specification is not a part of but a result of this study, the final version of it is included in the Appendix. No interpretations of this specification and no comments on the modifications made are offered. It should be sufficient to say that the modifications are very minor and will aid the implementation of the recommendations made.

ACKNOWLEDGMENTS

The author thanks all the district materials engineers, their assistants in charge of pug mill mixing, and the plant inspectors for their excellent cooperation in providing the data for this study. Thanks are also extended to S. N. Runkle, head of the Data Systems and Analysis Section of the Virginia Highway Research Council, for his help with the computer analysis of the data; and special thanks go to J. H. Dillard, state highway research engineer, for offering the benefits of the experience he gained in developing statistical bituminous specifications. The study was financed under HPR funds administered through the Federal Highway Administration. The opinions, findings, and

conclusions expressed are those of the author and not necessarily those of the sponsoring agencies.

REFERENCE

1. Central Mix Aggregate Reference Guide, 1969-1970. Virginia Department of Highways.

Appendix

VIRGINIA DEPARTMENT OF HIGHWAYS
SPECIAL PROVISIONS FOR
SUBBASE AND AGGREGATE BASE COURSES
(STATISTICAL QUALITY CONTROL SPECIFICATIONS) 4-1-70

Material for subbase and aggregate base courses on this contract shall be furnished in accordance with the applicable requirements of the 1966 edition of the Road and Bridge Specifications as amended herein below.

Sections 209 and 210 of the Specifications are completely replaced by the following:

Description -- Material for subbase course shall consist of natural or artificial mixtures of natural or crushed gravel, crushed stone, slag, natural or crushed sand, with or without soil mortar.

Aggregate base course will be designated as Type I or Type II Aggregate Base Material.

Type I aggregate base material shall consist of crushed stone, crushed slag, or crushed gravel combined with soil mortar, with or without other admixtures. Gravel shall consist of particles of which a minimum of 90 percent, by weight of the material retained on the No. 10 sieve, shall have at least one fractured face by artificial crushing.

Type II aggregate base material shall consist of sand-clay mixtures; gravel, stone, or slag screenings; sand and crushed coarse aggregate; or any combination of these materials combined with soil mortar, with or without other admixtures.

Detail Requirements --

Aggregate subbase course shall conform to the following requirements:

- (a) Grading shall conform to Table VI (attached) for Size 21, 21A or 22. Aggregate size to be used will be specified in the contract.
- (b) Atterburg Limits: Liquid limit shall not be more than 21; plasticity index shall be not more than 4.
- (c) Soundness shall conform to Table IV, Section 203.

TABLE VI
DESIGN RANGE

| Size No. | Amount finer than each laboratory sieve (Square Openings*), Percentage by Weight | | | | | |
|----------|--|----------|---------|---------|---------|---------|
| | 2 | 1 | 3/8 | No. 10 | No. 40 | No. 200 |
| 21 | 100 | 79 - 89 | 61 - 69 | 32 - 41 | 16 - 24 | 8 - 12 |
| 21A | 100 | 94 - 100 | 63 - 72 | 32 - 41 | 16 - 24 | 8 - 12 |
| 22 | | 100 | 62 - 78 | 39 - 56 | 26 - 34 | 8 - 12 |

* In inches, except where otherwise indicated. Numbered sieves are those of the U. S. Standard Sieve Series.

Aggregate base course shall conform to the following requirements:

- (a) Grading shall conform to Table VI (attached) for Size 21, 21A or 22. Aggregate size to be used will be specified in the contract.
- (b) Atterburg Limits: Liquid limit shall not be more than 21; plasticity index shall be not more than 1 for Type I and not more than 4 for Type II.
- (c) Soundness shall conform to Table IV, Section 203.
- (d) Abrasion Loss shall be not more than 45 percent.

Admixtures — Chemicals or other admixtures to be used with subbase or aggregate base course shall meet the requirements of the current specifications. Chemicals or other admixtures not covered by current specifications may be used on written approval of the Engineer.

Job-Mix Formula — The Contractor shall submit, for the Engineer's approval, a job-mix formula for each mixture to be supplied for the project, prior to starting work. The job-mix formula shall be within the design range specified in Table VI, Design Range (see attached) for the particular size number specified. The job-mix formula shall establish a single percentage of aggregate passing each required sieve size, and shall be in effect until modified in writing by the Engineer. When unsatisfactory results or other conditions make it necessary, the Contractor shall prepare and submit a new job-mix formula for approval. Approximately one week may be required for the evaluation of a new job-mix formula.

Mixing — The materials for subbase or aggregate base course shall be mixed in an approved central mixing plant of the pugmill or other mechanical type, unless otherwise specified. The materials shall be blended prior to or during mechanical mixing in such a manner that will insure conformance with the specified requirements. In the production of these materials, optimum moisture, plus or minus two (2) percentage points, will be required.

Plant Inspection — The preparation of subbase and aggregate base course material shall be subject to inspection at the plant. For this purpose, the Contractor shall provide a suitable building to be used as a field laboratory in accordance with the requirements of Supplemental Specifications for Section 539. The Contractor shall furnish, maintain, and replace as condition necessitates, the following equipment:

- 1 Motorized screen shaker for coarse and fine aggregate gradation analysis.

- 1 Set of sieves for the motorized shaker. The screen sizes shall include those necessary for testing the material being produced.
- 1 Sample splitter capable of handling material from sand up to 6 inch particles.
- 1 Motorized soil grinder, bench or floor model. The grinder shall be constructed using a 15 to 20 inch bench or floor model drill press.
The press shall be equipped as follows: Hand fed type with 6-inch stroke; variable shaft speed of 300 rpm (plus or minus 100 rpm); powered by an electric motor of $\frac{1}{2}$ hp or larger; machined steel adapter tapered on one end to fit drill and threaded on the opposite end for proper length to receive rubber mall attachment.

The above mentioned equipment shall be installed and ready for operation in the specified field laboratory.

Note: Cast iron grinding pots and rubber malls will be furnished by the Department.

The Department's representative shall have ready access to all parts of the plant for checking the accuracy of the equipment in use, inspecting the condition and operation of the plant, and for any purpose in connection with the materials and their processing.

Acceptance — Sampling for determination of gradation, liquid limit, and plasticity index will be performed at the plant, and no further sampling will be performed for these properties. However, should visual examination reveal that the material in any load is obviously contaminated or segregated, that load will be rejected without additional sampling or testing of the lot. In the event it is necessary to determine, quantitatively, the quality of the material in an individual load, one sample (taken from the load) will be tested and the results compared to the "process tolerance for one test" as described herein. The results obtained in the testing of a specific individual load will apply only to the load in question. Gradation, liquid limit, and plasticity index determinations will be performed in the plant laboratory furnished by the Contractor; however, the Department reserves the right to discontinue the use of the plant laboratory for acceptance testing in the event of mechanical malfunctions in the laboratory equipment and in cases of emergency involving plant inspection personnel. In the event of such malfunctions or emergencies, acceptance testing will be performed at the District or Central Office laboratory until the malfunctions or emergency has been satisfactorily corrected or resolved.

Acceptance for gradation, liquid limit, and plasticity index will be based upon a mean of the results of four tests performed on samples taken in a stratified random manner from each 2000 ton lot. A lot will be considered to be acceptable for gradation if the mean of the results obtained from the four tests fall within the following process tolerances allowed for deviation from the job-mix formula:

| Sieve | Process Tolerance, % Passing |
|----------|---------------------------------|
| Top Size | ± 0.0 |
| 1" | 5.0 |
| 3/8" | 9.5 |
| #10 | 7.0 |
| #40 | 4.0 |
| #200 | 2.0 |

A lot will be considered to be acceptable for liquid limit and plasticity index if the mean of the results obtained from the four tests fall within the following process tolerances allowed for deviation from the values given in Detail Requirements Section:

| <u>Atterburg Tests</u> | <u>Process Tolerance,</u> <u>%</u> |
|------------------------|---------------------------------------|
| Liquid Limit | + 2.0 |
| Plasticity Index | + 1.0 |

Should the liquid limit exceed 30 or the plasticity index exceed 6 for Type I base material or 9 for Type II base material or subbase material on any individual sample, the 500 ton portion of material from which the sample was taken will be considered a separate part of the lot and shall be removed from the road, unless otherwise directed by the Engineer.

In the event that the job requires less than 2,000 tons of material; or that the amount of material necessary to complete the job is less than 2,000 tons; or that the job-mix formula is modified within a lot, or a portion of the lot is rejected for excessive liquid limit or plasticity index, the mean results of samples taken will be compared to a new process tolerance, computed as follows:

$$\text{Process tolerance for one test} = \frac{\text{Process tolerance for mean of four tests}}{0.5}$$

$$\text{Process tolerance for mean of two tests} = \frac{\text{Process tolerance for mean of four tests}}{0.7}$$

$$\text{Process tolerance for mean of three tests} = \frac{\text{Process tolerance for mean of four tests}}{0.9}$$

Individual test results and lot averages obtained from acceptance testing will be plotted on control charts as the information is obtained. Standard deviations, when computed, will be made available to the Contractor. However, the Inspector will in no way attempt to interpret test results, lot averages or standard deviations for the Contractor in terms of needful plant or process adjustments.

Adjustment System — An adjustment of the unit bid price will not be made for the value of one test result or the mean value of two or three test results, unless circumstances as stated in Acceptance Section above require that the lot size be less than 2,000 tons. Should the value of one test result or the mean value of two or more test results, as required by Acceptance Section above, fall outside the allowable process tolerance, an adjustment will be applied to the unit bid price as follows:

| <u>Sieves</u> | <u>Adjustment points for each one (1) %</u> <u>that the gradation is out of process</u> <u>tolerance</u> |
|-------------------------|--|
| 2" | 1 |
| 1" | 1 |
| 3/8" | 1 |
| #10 | 1 |
| #40 | 3 |
| #200 | 5 |
| <u>Atterburg Limits</u> | <u>Adjustment points for each point that</u> <u>the Atterburg limits are out of process</u> <u>tolerance</u> |
| Liquid Limit | 3 |
| Plasticity Index | 7 |

In the event the total adjustment for a 2,000 ton lot is greater than twenty-five points, the failing material shall be removed from the road. In the event the total adjustment is twenty-five points or less and the Contractor does not elect to remove and replace the material, the unit price paid for the material will be reduced 1% of the unit price bid, for each adjustment point. The adjustment will be applied to the tonnage represented by the sample or samples.

The Contractor shall control the variability of his product in order to furnish the project with a uniform mix. When the contract item is greater than 1,000 tons and an adjustment is necessary as indicated in the following table, it shall be for the entire quantity of that type material on the project based upon its variability as measured by the standard deviation.

| Sieve Size | Standard Deviation | | |
|------------|--|---|---|
| | 1 adjustment point for each sieve size | 2 adjustment points for each sieve size | 3 adjustment points for each sieve size |
| 2" | 0.6 - 1.5 | 1.6 - 2.5 | 2.6 - 3.5 |
| 1" | 4.6 - 5.5 | 5.6 - 6.5 | 6.6 - 7.5 |
| 3/8" | 7.1 - 8.0 | 8.1 - 9.0 | 9.1 - 10.0 |
| #10 | 5.6 - 6.5 | 6.6 - 7.5 | 7.6 - 8.5 |
| #40 | 3.6 - 4.5 | 4.6 - 5.5 | 5.6 - 6.5 |
| #200 | 3.1 - 4.0 | 4.1 - 5.0 | 5.1 - 6.0 |

The unit bid price shall be reduced by 0.5% for each adjustment point applied.

The disposition of material having standard deviations larger than those shown in the table shall be determined by the Engineer.

Referee System

- (a) In the event the test results obtained from one of the four samples taken to evaluate a particular lot appear to be questionable, the Contractor or the Engineer may request that the results of the questionable sample be disregarded; whereupon, tests will be performed on five additional samples taken from randomly selected locations in the roadway where the lot was placed. The test results of the three original (unquestioned) samples will be averaged with the test results of the five road samples and the mean of the test values obtained for the eight samples will be compared to the following process tolerance:

$$\text{Process tolerance for mean of eight tests} = \frac{\text{Process tolerance for mean of four tests}}{1.4}$$

- (b) In the event the Contractor elects to question the mean of the four original test results obtained for a particular lot, he may request additional testing of that lot. Upon receipt of written request for additional testing, the Department will test four samples taken from randomly selected locations in the roadway where the lot was placed. The test results of the original four samples will be averaged with the test results of the four additional road samples and the mean of the test values obtained for the eight samples will be compared to the "process tolerance for mean of eight tests" as described hereinabove.

In the event the mean of the test values obtained for the eight samples is within the process tolerance for the mean of the results of eight tests, the material will be considered acceptable. In the event the mean of the test values obtained for the eight samples is outside of the process tolerance for the mean of the results of eight tests, the lot will be adjusted in accordance with the adjustment rate specified hereinabove.

Additional tests, requested by the Contractor under the provisions of Referee System Section (a) and (b), shall be paid for by the Contractor in the event the mean of the test values obtained for the eight samples falls outside of the process tolerances. Such additional tests shall be paid for at a rate of five times the bid price per ton of material per sample.

In the event that cement or other admixtures which would alter the characteristics of the material are used, the Referee System does not apply.

SUMMARY OF SURFACE FACTORS INFLUENCING THE FRICTION PROPERTIES OF CONCRETE PAVEMENTS

J. G. Rose, Kentucky Department of Highways, Lexington; and
W. B. Ledbetter, Texas Transportation Institute, Texas A&M University

This paper summarizes the contributions of surface finish, fine aggregate, mix proportions, and construction practices to the resulting friction properties of portland cement concrete pavements. This summary involves both macrotexture and microtexture effects on friction. Surface finish should generally impart the necessary macrotexture and is extremely important to adequate friction for high-speed traffic under wet pavement conditions. On the other hand, the fine aggregate should impart the necessary microtexture, which in turn establishes the friction level on dry to moderately wet pavement surfaces and on flooded surfaces provided primary drainage is achieved with macrotexture. Mix proportions, as well as construction practices, must be sound to ensure that the designed surface characteristics are actually built and that the surface remains skid-resistant throughout its service life.

●HIGHWAY engineers have been studying the vehicle skidding problem, particularly resistance to skidding offered by pavement surfaces, for several decades. In fact, initial technical papers (1, 2) written on the subject 30 to 40 years ago include many present-day ideas. However, the problem was not considered to be serious because most pavement types were adequate for the low-volume, low-speed traffic that existed at that time. Today, as a result of increasing traffic volumes and speeds, the skidding problem is rapidly gaining in significance. A recent HRB survey (3) stated that "slippery pavements were recognized as a problem of major concern in 22 states, moderate concern in 24 states, and minor concern in 2 states." Several studies, which have analyzed accident records, indicate that the incidence of total accidents, as well as accidents directly involving vehicle skidding, increases significantly with decreasing friction coefficients between the pavement and the tire (4, 5). The HRB survey (3), with 48 states reporting, also revealed that "42 states were using accident data for the detection or selection of slippery pavements and that 32 states were using skid test data as a criterion for resurfacing or deslicking."

The shorter the distance required to stop a vehicle in emergency situations and the higher the force to provide adequate cornering are, the better the resultant chance will be to avoid or reduce the severity of accidents. Because stopping distance and cornering capability are directly functions of friction coefficient, a high value of friction coefficient is becoming more and more important.

It is generally recognized that the 2 forms of pavement roughness, or texture, that contribute to friction resistance (6) are macrotexture and microtexture. Macrotexture refers to asperities, or rough areas, generally visible to the individual eye and is a result of factors such as the roughness built into the pavement surface, the shape of the aggregate particles, and the type of surface. On the other hand, microtexture is the roughness that is inherent in an aggregate itself and that enhances or prohibits its polishability under traffic. These terms will be discussed in more detail later in the report.

Research has determined that certain factors primarily affect the wear-resistant and, thus, friction qualities of concrete pavements. These factors can be grouped under the following 4 general headings: type of surface finish, type of fine aggregate, mix proportions, and construction practices.

The type of surface finish determines the level of large-scale or macroscopic texture (macrotexture). A certain macrotexture level is required in order to provide water-escape channels between the tire and pavement and also to contribute a high hysteresis friction component. The type of fine aggregate and the degree of polish determine the level of small-scale or microscopic texture (microtexture). Harsh microscopic texture is required to aid in puncturing the thin water film, thus providing for essentially dry contact between the tire and pavement. This is a necessity because a high adhesion friction component is available.

Mix proportions and construction practices have been found to have bearing on friction properties. Control of water-cement ratio, sand content, and curing practices are particularly important. Compliance with job specifications will normally provide for adequate control of these items.

In the following sections of this report the factors of surface finish, fine aggregate, mix proportions, and construction practices are summarized in relation to their effect on the friction properties of concrete surfaces. The reader is cautioned against generalization of the findings reported here beyond the data.

SURFACE FINISH

Importance

The importance of surface finish on the friction properties of portland cement concrete roads has been recognized for many years (7). Sculptured mortar, or mortar with marks left by finishing, provides the large-scale texture (macrotexture). Researchers have found that adequate channels and escape paths among the asperities are necessary to facilitate the escape of the lubricating water from tire-pavement contact patch (8). This effect is definitely significant for smooth tires; with treaded tires, however, the macrotexture is of less importance because the tire itself can, to some extent, facilitate drainage. In addition, the macrotexture serves to increase frictional characteristics of the pavement when vehicles are traveling at high speeds by deforming the tire rubber and thus increasing the hysteresis friction component. Macrotexture is particularly important where surface drainage is relatively slow and vehicle speeds are high.

Measurement

Macrotexture measurement techniques include the sand patch (9, 10), grease smear (11), drainage meter (12), stereophotography (13), profilograph (10, 14), texture meter (10), and putty impression (10). Most of these methods utilize either a volumetric measure of the voids below the asperity tops or a profile of the asperities for calculating the average texture depth or peak height.

The macrotexture level of a surface has been found to primarily affect the degree of friction decrease with speed, this decrease being reported as the gradient or slope of the friction speed curve (10, 15). A minimum texture depth of 0.025 in., as obtained with the sand-patch method, was reportedly required if the decrease was to be held at 25 percent (15). The friction level between tire and pavement at a given speed is primarily a function of microtexture, provided adequate drainage capability is available to assist in removing water. It is the decrease in friction with speed, rather than the actual level, that is mainly affected by coarse texture.

Methods

Finishing is generally accomplished with a broom, belt, or burlap drag. In 1963, approximately 60 percent of the highway departments used a burlap drag, 12 percent used either a burlap drag or a broom, 8 percent used a combination of a broom and a

burlap drag, 6 percent used a belt, 6 percent used a broom, and the remainder specified that either a belt or a burlap drag should be used (16).

It has been found that best results with a burlap drag are obtained when at least 3 to 3½ ft of burlap are in contact with the pavement surface and when there are 2 to 4 passes of the drag. Texturing is halted as soon as the desired texture is obtained because additional passes seem to reduce the depth and grittiness of the surface. Timing is particularly important because texture depths vary considerably with the time of finishing. In general, the best burlap textures are obtained by dragging soon after the final finishing operation while the pavement is fairly moist. Burlap with an open weave usually provides a grittier surface with deeper striations, although, some doubt still exists as to optimum burlap weights and combinations. Uniformity of the texture should be carefully watched and large grout buildups should be removed. Normally the dragging operation is contained in the paving train (17).

Brooms usually are either steel bristle or coarse bristle made from various materials. It has been found that better textures result when the mechanical broom is inclined from the direction of motion rather than vertically. Best results are obtained when brooming closely follows the paving operation. Brooming should be completed before the concrete is in such a condition of set that the surface is torn or unduly roughened when textured.

Belting is done by the use of a narrow canvas or rubber belt that is moved longitudinally along the surface with a slight transverse back-and-forth motion. Two men handle the belt, one on either side of the slab.

Research directed toward determining the surface texture produced by various finishing methods has been performed (16). Specimens were prepared, and various finishing devices were used in the laboratory to texture the surface 30 min after casting. Texture depths as measured by the sand-patch method (16) are given in Table 1. The largest deviation in depth from the average of any individual specimen in a set of four was only 0.003 in. This indicated that the texture depth for each method of finish was unique for the concrete mix and time interval between casting and finishing. Other data were obtained where the time interval was a variable. In general, only minor differences in texture depth were obtained with a broom for time delays between ½ and 1½ hours or with a belt for time delays between 30 to 50 min. In contrast, texture depths obtained with a burlap drag varied considerably with the time of finishing. Dragging the surface 20 min after casting resulted in depths about 25 percent larger than those given in Table 1. Thus, the time delays investigated for the laboratory tests may not be valid in the field where changing temperature, humidity, and wind velocity must be considered; however, the trends should remain the same.

A related laboratory experiment was also carried out to determine the effect of type of finish on the friction of concrete cast with an aggregate principally containing 55 percent dolomite and 39 percent siliceous materials (18). A "wear" index was used for the measurable parameter. In the wear test, water is fed continuously to an ASTM tire held against a concrete specimen with a normal force of 600 lb, while the tire is rotated at a constant speed of 250 rpm (20 mph). After 75 min of wear, a second test phase begins during which a fine Ottawa sand is blown onto the specimen with the stream of water passing between the rotating tire and concrete. This abrades the specimen and accelerates wear. After 2 hours of sand abrasion, the test is continued for 75 min without the addition of sand to evaluate the worn pavement. The power in kilowatts required to rotate the wheel against the specimen is referred to as the wear index and is considered a comparative measure of the friction; that is, the higher the wear index is, the higher the friction

TABLE 1
TEXTURE DEPTHS MEASURED BY
SAND-PATCH METHOD

| Specimen | Method of Finish | Texture Depth (in.) |
|----------|--------------------------|---------------------|
| 1 | Wood float | 0.014 |
| 2 | Light belt | 0.015 |
| 3 | Light burlap drag | 0.017 |
| 4 | Heavy belt | 0.020 |
| 5 | Steel wool | 0.022 |
| 6 | Heavy burlap drag | 0.025 |
| 7 | Wallpaper brush | 0.026 |
| 8 | Medium paving broom | 0.029 |
| 9 | Door mat (cocoa matting) | 0.032 |
| 10 | Wire drag | 0.036 |
| 11 | Heavy paving broom | 0.037 |
| 12 | Flexible wire brush | 0.051 |
| 13 | Stiff wire brush | 0.075 |

coefficient will be. The resisting force and consequent power requirement decrease as the surface is polished by wear.

It was recognized that differences in surface texture might be more apparent prior to wear; therefore, data were obtained after only 15 min of testing (Fig. 1, 18). Tests were conducted on a steel-troweled surface for comparing a smooth finish surface with the textured surfaces obtained by conventional finishing techniques. The wear index for the troweled surface was only 4 as compared with values between 8.7 and 9.6 for the textured surfaces. Contrary to most test results, it was found that the wear index of the troweled surface increased with wear as the test was performed until the final value was 5.6. This occurred because the wear exposed more of the sharp edges of the sand grains.

Higher textured surfaces were obtained with the burlap drag, broom finish, and belt finish. The data from specimens on which the finish marks were transverse to the direction of wheel rotation were slightly higher to those on which the finish mark where parallel to the finish; however, the textures produced in both directions and by all 3 methods were deemed adequate. At the end of 270 min of testing, the finish marks from the 3 conventional textures were still visible, and the wear indexes averaged about 6.8.

Although broom and burlap-drag finishes are regarded to result in good initial friction, in one study they have been found to be inferior to a belt finish in maintaining good frictional properties under traffic and wear (19).

In another investigation, friction coefficients were obtained for concrete surfaces with various types of finish (20). The surfaces finished by dragging burlap had lower wet friction coefficients than those finished by brooming or by a combination of brooming and dragging burlap. When dry, the surface with the combination transverse broom-burlap drag finish gave the lowest friction coefficient, and there were only slight differences in the dry friction coefficients among the other surfaces.

Grooving of plastic concrete has also been used as a means of obtaining adequate texture during construction. A vibrating unit is mounted on a float equipped with projections to groove the plastic concrete. The appearance of grooves formed in this manner is similar to those produced by sawing hardened concrete. The cost of grooving with vibrator is estimated at 10 to 25 cents/sq yd compared to \$1.50 or more/sq yd for sawing (21).

It has been suggested that macrotexture could be added to the surface of concrete pavements by rolling a nonpolishing friction-textured aggregate into the fresh concrete (22). It has also been suggested that the aggregate should be impregnated with a cement grout in order to satisfy the aggregate's demand for water and to ensure an adequate bond between the concrete mortar and the aggregate. The aggregate would need to be embedded in the concrete with a rolling screed. Preliminary investigations have shown that wear indexes can be improved by spreading abrasives such as aluminum oxide and silicon of 12 grit and smaller over test specimens and floating them into the plastic concrete (16). Figure 2 (16) shows that the 1 lb/sq yd application of aluminum oxide increased the wear resistance of a concrete. Silicon carbide, applied in the same amount, was even more effective. Abrasives in concrete would be advantageous in

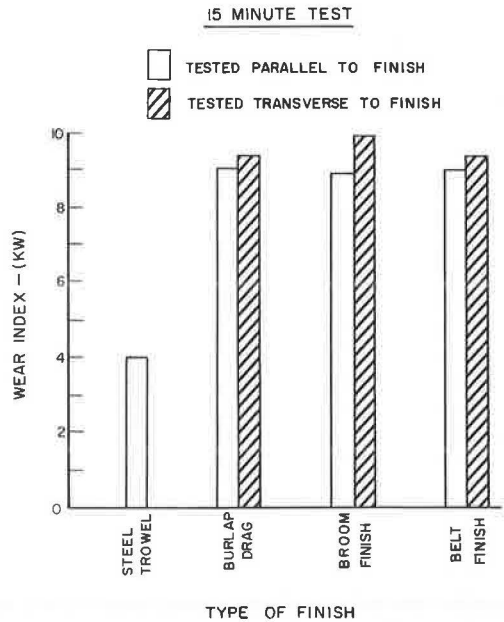


Figure 1. Influence of concrete finish on skid resistance.

critical areas such as at toll plazas, near busy intersections, or in areas where frequent braking, traction, or cornering occurs.

Because the majority of paving materials used in concretes wear or polish under excessive traffic volumes and, thus, lose friction resulting from microtexture, it is obvious that, with other factors being equal, a pavement with an initial deep texture will retain relatively flat friction-speed gradients resulting from macrotexture longer than a pavement with a shallow texture. This does not imply, however, that the texture should be as deep as possible; instead the finishing method selected should provide a texture level compatible with the urban and rural environment, traffic speed and density, and pavement topography and geometrics.

Road noise resulting from a high macrotexture level can be very disturbing in urban areas. Road noise can be especially annoying to occupants of vehicles traveling at high speeds. In addition, tire wear rates are increased,

and vibrations can be destructive to vehicles.

FINE AGGREGATE

Importance

In relation to the size of aggregate used in surfaces, friction is primarily controlled by the fine aggregate. This stems from the fact that in concrete surfaces microtexture is contained in the sand-cement mortar layer at the surface. The coarse aggregate is not exposed at the surface and, thus, generally does not contribute favorably or adversely to the friction properties until the mortar is worn away.

It is generally accepted that the fine asperities impart the adhesion-friction component in the tire-pavement interaction. Adhesion (microtexture effects) is of primary importance on dry pavements and on damp to moderately wet pavements but assumes a lesser role on poorly drained or flooded pavements where hysteresis (macrotexture effects) becomes dominant because the mass of water must be removed before the adhesion component can be effective.

The fine aggregate must exhibit high microtexture or harshness. Investigations (23) have shown that, if the contact patch is originally covered with an approximately 0.020 in. water film, drainage under the tire is relatively rapid until a thickness of 0.001 to 0.002 in. remains. At this stage, drainage becomes extremely difficult and high pressures are required to squeeze out the remaining water. Elastic theory shows that, when projections (such as those making up the road surface) are forced into an elastic solid (such as the tire rubber), the localized pressure distribution over the surface of the projections depends essentially on the shape of the projections and the hardness of the solid. Localized pressure peaks are determined by the degree of sharpness of the projections. Consequently, the harsher the road surface is, the easier it is to squeeze out the intervening water. This permits more direct contact between the tire and the pavement surface and thus provides higher frictional resistance.

Measurement

The amount of friction developed by a surface with fine asperities has been correlated with the "degree of texture" as measured by various test procedures. Some procedures

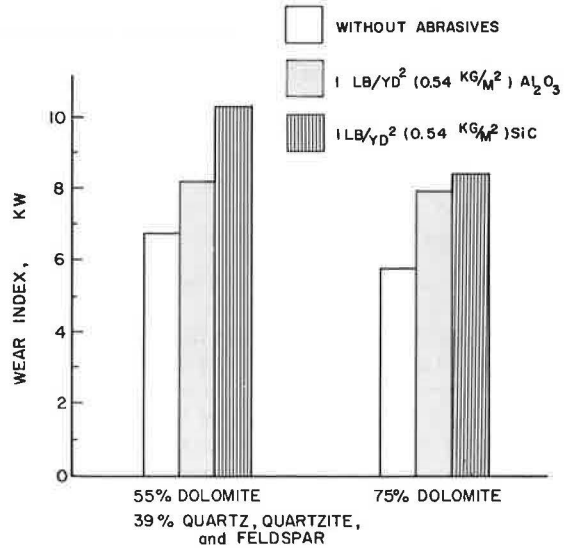


Figure 2. Use of abrasives to develop superior friction surfaces.

use techniques to obtain a cast reproduction in a plaster or resin, an ink print, or a stereophotographic view of the surface (24, 25). Current investigations are focused on using a "surfindicator" to measure surface roughness in the range from 1 to 1000 microinches (26). Another procedure, "foil-piercing," was used by Gillespie (27). However, research results using this technique have shown some disagreement (16), and thus it appears that this test procedure is not sufficient for predicting friction coefficients on concrete surfaces.

Evaluations

Procedures for evaluating wear of fine aggregate-cement mortar have been developed (18, 28, 29). One study showed that the wear rate and, hence, the friction of concrete pavements depend largely on the mineral composition of the fine aggregate (18). Wear tests were made with fine-aggregate samples obtained from 20 different sources. These aggregates had been used to construct concrete pavements; therefore, the wear index could be correlated with in-service pavement performance. In addition, the wear index was compared with results from an acid-insoluble residue laboratory test (13). A description of each fine aggregate used is given in Table 2 (16) together with the siliceous particle content, the laboratory wear index, and the field performance of the pavements constructed with each aggregate (18). The increase in wear resistance with increasing siliceous particle content suggests that beneficiation of poor aggregates could be obtained by a partial replacement of the fines with a siliceous sand (Table 3, 16). Twenty-five percent replacement was found satisfactory for most of the aggregates; however, the acceptable percentages of replacement depended on the siliceous particle content of the blend.

A plot of wear index values as a function of siliceous particle content for the basic field aggregates and laboratory combinations is shown in Figure 3 (18). Although there was no abrupt change in the curve to indicate a separation between poor and excellent field performances, aggregates with a wear index greater than 6.2 were classified excellent. The authors concluded that a fine aggregate with a siliceous particle content of 25 percent or greater would provide excellent field performance.

To determine the effect of the hardness of aggregates on the friction coefficient, Finney and Brown (31) conducted skid tests on concrete pavements made with natural sand and those made with stone sand. The results indicated that pavements made with natural sand had wet friction coefficients higher than those made from stone sand. The average friction coefficient for 23 natural-sand projects was 0.51 compared to 0.28 for 19 stone-sand projects. Limestone particles in stone sand, on the other hand, came from a sedimentary rock that had a hardness of 3. Thus, it was reasonably suspected that stone sand became smooth more rapidly than natural sand under traffic.

In an effort to increase the friction of concrete mixtures containing soft, polish-susceptible aggregates, Shup and Goetz (32) studied the effect of blending polish-resistant aggregates with polish-susceptible limestone (Fig. 4, 32). In general, the relative resistance values, or skid resistances, increased with increases in the percentage of substitution of fine aggregate. They concluded that the blending of polish-resistant fine aggregates to improve friction of polish-susceptible limestone in portland cement concrete was very effective and that the fine aggregate mortar made an important contribution even after wear progressed to a point where an appreciable amount of coarse aggregate was exposed.

Ottawa sand has been used to study the influence of particle size on the wear or friction of concrete (18). This material was selected because it had a greater uniformity of hardness throughout the various particle sizes than could be found in most natural sands. Mortar mixtures in which the maximum and minimum particle sizes varied were used to apply a $\frac{1}{2}$ -in. thick surface to the specimens. The data shown in Figure 5 (18) indicate an optimum range of particle size that resulted in the least wear.

It was noted that the better wear values occurred with the larger size of sand particles, although it was significant that a specimen containing Ottawa sand graded to include the total range of particle sizes from the No. 20 to the No. 200 mesh sieves gave a wear index of 7.5, which was near the optimum 8.2.

TABLE 2
EFFECT OF AGGREGATE CONSTITUENTS ON WEAR INDEX

| Fine Aggregate | Principal Constituents (percent) | Rating of Field Performances | Wear Index (kw) | Siliceous Particle Content (percent) |
|----------------|------------------------------------|------------------------------|-----------------|--------------------------------------|
| 1 | 90 calcite | Poor | 3.6 | 2 |
| 2 | 70 calcite | Poor | 4.4 | 2 |
| 3 | 24 dolomite | Poor | 4.0 | 3 |
| 4 | 90 calcite | Poor | 5.6 | 6 |
| 5 | 80 dolomite | Poor | 5.8 | 9 |
| 6 | 75 dolomite | Poor | 5.4 | 13 |
| 7 | 60 calcite | Poor | 5.3 | 17 |
| 8 | 16 silt and clay | Poor | 5.3 | 17 |
| 8 | 80 calcite | Fair | 6.2 | 19 |
| 9 | 15 quartz | Poor | 5.9 | 22 |
| 9 | 65 calcite | Poor | 5.9 | 22 |
| 10 | 12 dolomite | Poor | 5.9 | 22 |
| 10 | 50 calcite | Poor | 5.9 | 22 |
| 10 | 33 mica and quartz | Excellent | 6.8 | 33 |
| 11 | 55 dolomite | Excellent | 6.8 | 33 |
| 11 | 39 quartz, quartzite, and feldspar | Excellent | 6.8 | 39 |
| 12 | 55 calcite and dolomite | Excellent | 6.8 | 39 |
| 12 | 40 quartz, mica, and epidote | Excellent | 6.7 | 40 |
| 13 | 45 calcite | Excellent | 6.7 | 40 |
| 13 | 42 quartz and feldspar | Excellent | 7.3 | 42 |
| 14 | 50 dolomite | Excellent | 7.3 | 42 |
| 14 | 44 quartz | Excellent | 7.0 | 44 |
| 15 | 45 dolomite | Excellent | 6.8 | 50 |
| 15 | 45 quartz | Excellent | 6.8 | 50 |
| 16 | 45 dolomite | Excellent | 7.2 | 53 |
| 16 | 45 quartz | Excellent | 7.2 | 53 |
| 17 | 30 graywacke | Excellent | 7.0 | 96 |
| 17 | 55 quartz | Excellent | 7.0 | 96 |
| 18 | 75 quartz | Excellent | 7.3 | 97 |
| 18 | 17 feldspar | Excellent | 7.3 | 97 |
| 19 | 72 quartz | Excellent | 7.2 | 98 |
| 19 | 20 feldspar | Excellent | 7.2 | 98 |
| 20 | 99 quartz | Excellent | 7.5 | 99 |

TABLE 3
IMPROVEMENT OF FRICTION BY BENEFICIATION

| Basic Aggregate | Replacement Aggregate | Final Blend (percent) | | Siliceous Particle Content (percent) | Wear Index (kw) |
|-----------------|-----------------------|-----------------------|-------------|--------------------------------------|-----------------|
| | | Basic | Replacement | | |
| 1 | 17 | 100 | 0 | 2 | 3.6 |
| | | 75 | 25 | 26 | 5.9 |
| | | 50 | 50 | 49 | 6.5 |
| | | 25 | 75 | 72 | 6.8 |
| | | 0 | 100 | 96 | 7.0 |
| 2 | 19 | 100 | 0 | 2 | 4.4 |
| | | 50 | 50 | 50 | 7.0 |
| | | 25 | 75 | 74 | 7.1 |
| | | 0 | 100 | 98 | 7.2 |
| 4 | 18 | 100 | 0 | 6 | 5.6 |
| | | 75 | 25 | 29 | 6.3 |
| | | 60 | 40 | 42 | 6.6 |
| | | 50 | 50 | 52 | 7.0 |
| | | 0 | 100 | 97 | 7.3 |
| 5 | 11 | 100 | 0 | 9 | 5.8 |
| | | 50 | 50 | 24 | 6.3 |
| | | 0 | 100 | 39 | 6.8 |
| 6 | 18 | 100 | 0 | 13 | 5.4 |
| | | 75 | 25 | 34 | 6.5 |
| | | 50 | 50 | 55 | 7.2 |
| | | 25 | 75 | 76 | 7.7 |
| | | 0 | 100 | 97 | 7.3 |
| 7 | 11 | 100 | 0 | 17 | 5.3 |
| | | 75 | 25 | 22 | 6.4 |
| | | 50 | 50 | 28 | 6.6 |
| | | 25 | 75 | 34 | 6.7 |
| | | 0 | 100 | 39 | 6.8 |
| 10 | 18 | 100 | 0 | 33 | 6.8 |
| | | 75 | 25 | 49 | 7.2 |
| | | 60 | 40 | 59 | 7.4 |
| | | 50 | 50 | 65 | 7.6 |
| | | 0 | 100 | 97 | 7.3 |

MIX PROPORTIONS AND CONSTRUCTION PRACTICES

The proper mixture of materials is important to ensure high friction on concrete surfaces.

Effects

Sawyer (33) obtained data demonstrating the effect of variations in cement content and water-cement ratio on concrete wear. He found that, for 2-in. slump concrete after 14 days of moist-curing, the wear increased approximately 40 and 130 percent as the cement content was decreased from 705 (7.5 bags) to 564 (6.0 bags) and 432 (4.5 bags) lb/cu yd respectively. A comparison of the data for the 564 (6.0 bags) lb/cu yd mixes with 2- and 6-in. slumps after 14 days of curing indicated that the increased water-cement ratio accompanying the change in slump resulted in an increase of wear of 20 percent.

The data shown in Figure 6 (16) also show the effect on wear of changes in the water-cement ratio. These tests were made by using the rotating wheel equipment described previously (18).

Moyer reported that concrete surfaces finished with excess cement paste were very slippery (34). Coefficients of friction as low as 0.06 were measured.

Water added to concrete surfaces during finishing operations was found to have detrimental effects on the wearing ability and friction (16). Duplicate specimens were cast at a water-cement ratio of 0.47, and 25 cm³ of water were sprinkled on the 4-sq ft surface area of the plastic concrete. Wear tests on these specimens showed that initial power requirements needed to drive the rotating wheel were decreased by about 35 percent from the values on specimens without added water, and the final wear values averaged 6.2 instead of 6.7.

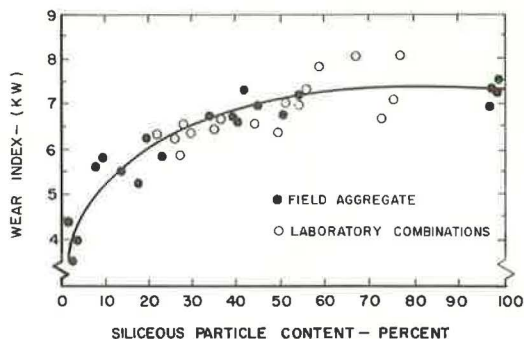


Figure 3. Effect of siliceous particle content on wear index.

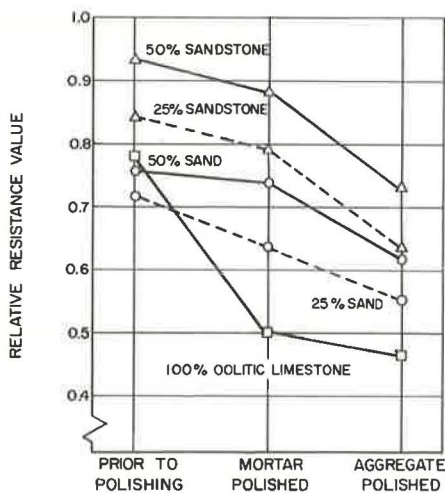


Figure 4. Fine aggregate replacement in portland cement concrete.

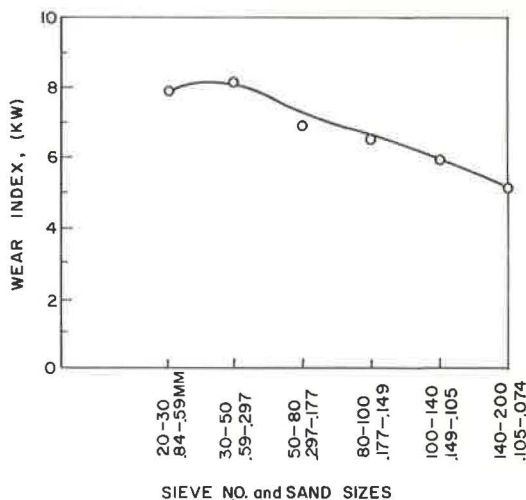


Figure 5. Effect of particle size on skid resistance.

Wear tests have also been conducted with the percentage of sand as the principal variable (16). Results of these tests were found to vary considerably, but in general the wear resistance was increased as the percentage of sand was increased. It was theorized that this was probably due to a combination of a larger surface area of wear-resistant fine aggregate at the tire-pavement interface and the increase in the cement content to maintain a constant water-cement ratio and slump when the percentage of sand was increased. The Portland Cement Association suggests that its procedures (35) be used to determine mix proportions and that the amount of sand should be based on the most economical combination of available aggregates that will produce the necessary workability in the fresh concrete and the required qualities in the hardened concrete.

As discussed earlier in this paper, tests have shown that a concrete pavement composed of fine aggregate with a high siliceous particle content is best suited to withstand polishing and wearing effects over a long period of time. This siliceous particle content is very important, and a high percentage should be used.

The durability and wear resistance of a concrete surface have also been found to be influenced to a large extent by the effectiveness of curing (17). Much of the value of curing is lost if the curing process is not started immediately after the disappearance of the water sheen following texturing. Certain curing materials or compounds can be harmful to the texture if care is not exercised. It was found that linseed oil compounds applied to old and new concrete surfaces decreased the friction during the first few hours, but rapid recovery of friction was achieved (36).

SUMMARY

The range of coefficient of friction values found on portland cement concrete can be sufficient magnitude to render some concrete pavements slippery when wet. Because of high initial construction cost, economics necessitate that concrete pavements maintain an acceptable friction coefficient for several years.

Factors that contribute most to friction properties of concrete pavement surfaces are surface finish, type of fine aggregate, mix proportions, and construction practices.

Macrotexture, needed to ensure minimum drop of skid resistance with speed, can effectively be provided only by a proper surface finish. Because a majority of concrete pavements contain polishable limestones and river gravels, a suitable finish is one that will wear as slowly as possible. Microtexture, needed to ensure high skid resistance after the bulk of the water has been removed, also is solely a function of the surface finish. Thus, the use of proper surface finishing cannot be overemphasized.

Characteristics of the fine aggregate determine the microtexture level of the surface, which is the main contributor to permanence of a high friction level under heavy traffic volumes. A hard, harsh-textured sand should be used. Composition of the fine aggregate is also the major factor controlling wear rate on concrete pavements.

Mix proportions and construction practices are generally of secondary importance provided that mix design and construction procedures meet specifications and if specifications are adequate.

REFERENCES

1. Agg, T. R. Tractive Resistance and Related Characteristics of Roadway Surfaces. Eng. Exp. Station, Iowa State College, Bull. 67, Feb. 1924.

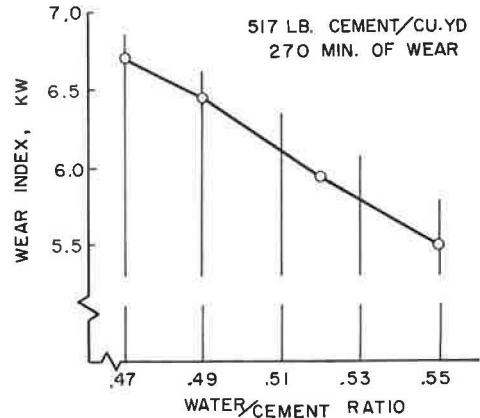


Figure 6. Effect of water-cement ratio on wear.

2. Moyer, R. A. Skidding Characteristics of Road Surfaces. HRB Proc., Vol. 13, Pt. 1, 1934, pp. 123-168.
3. HRB Committee D-B4 Task Group. An Inventory of Existing Practices and Solutions to Slippery Pavements—1969. Presented at the HRB 49th Annual Meeting, Jan. 1970.
4. Giles, C. G. The Skidding Resistance of Roads and the Requirements of Modern Traffic. Inst. of Civil Engineers, Proc. Vol. 6, 1957.
5. Csathy, T. I. A Study of the Skid Resistance of Pavement Surfaces. Ontario Dept. of Highways, Rept. 32, 1963.
6. Sabey, B. E. The Road Surface in Relation to Friction and Wear of Tyres. Road Traffic, Vol. 23, No. 1, March 1969.
7. Fleming, E. M. Coefficients of Friction Between Rubber Tires and Concrete Road Surfaces. HRB Proc., Vol. 14, Pt. 1, 1934. pp. 214-216.
8. Csathy, T. I., Burnett, W. C., and Armstrong, M. D. State of the Art of Skid Resistance Research. HRB Spec. Rept. 95, 1968, pp. 34-48.
9. Instruction for Using the Portable Skid Resistance Tester. Road Note 27, Road Research Laboratory, Her Majesty's Stationery Office, London, 1960.
10. Rose, J. G., Hankins, K., and Gallaway, B. M. Macrotecture Measurements and Related Skid Resistance at Speeds From 20-60 mph. Paper presented at the HRB 49th Annual Meeting, Jan. 1970.
11. Leland, T. J., Yager, T. J., and Joyner, U. T. Effects of Pavement Texture on Wet-Runway Braking Performance. National Aeronautics and Space Administration, NASA TN D-4323, 1968.
12. Moore, D. F. Prediction of Skid-Resistant Gradient and Drainage Characteristics for Pavements. Highway Research Record 131, 1966, pp. 181-203.
13. Schultz, K. G., and Beckmann, L. Friction Properties of Pavements at Different Speeds," ASTM, Spec. Tech. Publ. 326, 1962.
14. Ashkar, B. H. Development of a Skid Test Trailer. Texas Highway Department, Res. Rept. 45-1, April 1965.
15. Sabey, B. E. Road Surface Texture and Change in Skidding Resistance with Speed. Road Research Laboratory, Harmondsworth, England, RRL Rept. 20, 1966.
16. Colley, B. E., Christensen, A. P., and Nowlen, W. J. Factors Affecting Skid Resistance and Safety of Concrete Pavements. HRB Spec. Rept. 101, 1969. pp. 80-99.
17. Paine, J. E. Skid Resistance of Concrete Pavements. Concrete Construction, Vol. 14, No. 10, Oct. 1969.
18. Balmer, G. G., and Colley, B. E. Laboratory Studies of the Skid Resistance of Concrete. Jour of Materials, Vol. 1, No. 3, 1966.
19. Close, W. Locked-Wheel Friction Tests on Wet Pavements. HRB Bull. 302, 1961, pp. 18-34.
20. White, A. M., and Thompson, H. O. Test for Coefficients of Friction by the Skidding Car Method on Wet and Dry Surfaces. HRB Bull. 186, 1958, pp. 26-34.
21. Christensen, A. P. Private Communication to B. M. Gallaway, June 6, 1969.
22. Hargett, E. R. Design of Friction Textured Surfaces for Highway and Airport Pavements. Paper presented at meeting of Texas and New Mexico ASCE, Oct. 1969.
23. Giles, C. G. Skidding and the Skid-Resisting Properties of Roads. Road Research Laboratory, Her Majesty's Stationery Office, London, 1959.
24. Sabey, B. E., and Lupton, G. N. Measurement of Road Surface Texture Using Photogrammetry. Road Research Laboratory, Crowthorne, England, RRL Rept. LR 57, 1957.
25. Schonfeld, R. Skid Numbers from Stereo-Photographs. Ontario Dept. of Highways, Rept. RR155, Jan. 1970.
26. Gallaway, B. M., and Tomita, H. Micro-Texture Measurements on Pavement Surfaces. Texas Transportation Institute, Texas A&M Univ., Res. Rept. 138-1, Feb. 1970.
27. Gillespie, T. D. Pavement Surface Characteristics and Their Correlation with Skid Resistance. Pennsylvania Dept. of Highways—The Pennsylvania State Univ. Joint Friction Program, Rept. 12, 1965.

28. Whitehurst, E. A., and Goodwin, W. A. A Device for Determining Relative Potential Slipperiness of Pavement Mixtures. HRB Bull. 186, 1958, pp. 1-7.
29. Maclean, D. J., and Shergold, F. A. The Polishing of Roadstone in Relation to the Resistance to Skidding of Bituminous Road Surfacing. Dept. of Scientific and Industrial Research, London, RRTP 43, 1958.
30. Gray, J. E., and Renninger, F. A. The Skid Resistance Properties of Carbonate Aggregates. Paper presented at the HRB 44th Annual Meeting, Jan. 1965.
31. Finney, E. A., and Brown, M. G. Relative Skid Resistance of Pavement Surfaces Based on Michigan's Experience. First Internat. Skid Prev. Conf., Proc. Part 2, 1959.
32. Shupe, J. W., and Goetz, W. H. A Laboratory Investigation of Pavement Slipperiness. HRB Bull. 219, 1959, pp. 56-73.
33. Sawyer, J. L. Wear Tests on Concrete Using the German Standard Method of Test and Machine. Paper presented at the ASTM 60th Annual Meeting, 1957.
34. Moyer, R. A. Effect of Pavement Type and Composition on Slipperiness, California Experience. First Internat. Skid Prev. Conf., Proc., Part 2, 1959.
35. Design and Control of Concrete Mixtures, 10th Ed. Portland Cement Association.
36. Kubie, W. L., Gast, L. E., and Cowan, J. C. Preliminary Report on Skid Resistance of Linseed Oil-Coated Concrete. Highway Research Record 214, 1968, pp 42-49.
37. Interim Recommendations for the Construction of Skid Resistant Concrete Pavement. American Concrete Paving Association, Tech. Bull. 6, 1969.
38. The State of the Art of Traffic Safety, Arthur D. Little, Inc., 1966.
39. Kummer, H. W., and Meyer, W. E. Tentative Skid-Resistance Requirements for Main Rural Highways. NCHRP Rept. 37, 1967.

AN ELECTRICAL METHOD FOR EVALUATING BRIDGE DECK COATINGS

Donald L. Spellman and Richard F. Stratfull, Materials and Research Department,
California Division of Highways

An electrical method for evaluating bridge deck coatings is being experimentally used on California highway bridges. Field and laboratory tests have indicated that the electrical resistance of a bridge deck coating can be related to the voids and, thus, sealing ability of the coating. This nondestructive method for evaluating bridge deck coatings may be an additional tool for evaluating the performance of membranes used to prevent the ingress of de-icing salts that cause corrosion of the steel.

•IN RECENT years, an increasing amount of attention is being devoted to the problem of bridge deck deterioration (1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11). Most recently, the Highway Research Board has published a finding that indicates that one of the most significant causes of bridge deck deterioration is spalling of the concrete resulting from the use of de-icing chemicals (12). In general, the spalling of the concrete has been found to be the result of corrosion of reinforcing steel (10, 11, 12). One method to prevent the corrosion of the steel caused by de-icing salt is to apply a waterproof membrane to the bridge deck (12) before any salt is used. However, the authors have not found any literature that describes a technique for the field evaluation of the waterproofing ability of bridge deck seals. It has been reported that a measure of the performance of a membrane is its ability to remain in place on the deck surface (3, 12, 13). In this study, one additional criteria for performance of a bridge deck coating is that it be a waterproof membrane. In the case of a dielectric material being used in the seal, it is assumed that the electrical resistance of the coating should be a measure of the waterproofing ability of the coating. For example, it is assumed that, if a coating is porous and water can pass through these pores, then the coating should have a low electrical resistance because of the multiple paths that are available for current flow. Conversely, if the coating is not porous and is of a dielectric nature, then the electrical resistance of the coating should be high. Although the authors are not aware of any specific reports concerning measuring of the electrical resistance of bridge deck coatings, they are aware that such techniques have been previously utilized on buried pipelines (14). Therefore, the concept of the measurement of electrical resistance of a coating is not considered to be new, but the use of this technique as applied to the measurement of the electrical resistance of bridge deck coatings may be unique.

INSTRUMENTATION

The basic concept for the instrumentation is to connect one lead of the ohmmeter to a plate or contact that could be placed on the surface of the bridge deck and thus measure gross electrical resistance. This arrangement would permit the measurement of the electrical resistance from the reinforcing steel through the concrete, through the membrane or surface coatings or both, and then to the contact placed on top of the deck surface. The measurement of the electrical resistance on the surface of the bridge was facilitated by the use of a moist sponge as a conducting medium that will electrically complete the current (Fig. 1). In the construction of the contact, a piece of copper

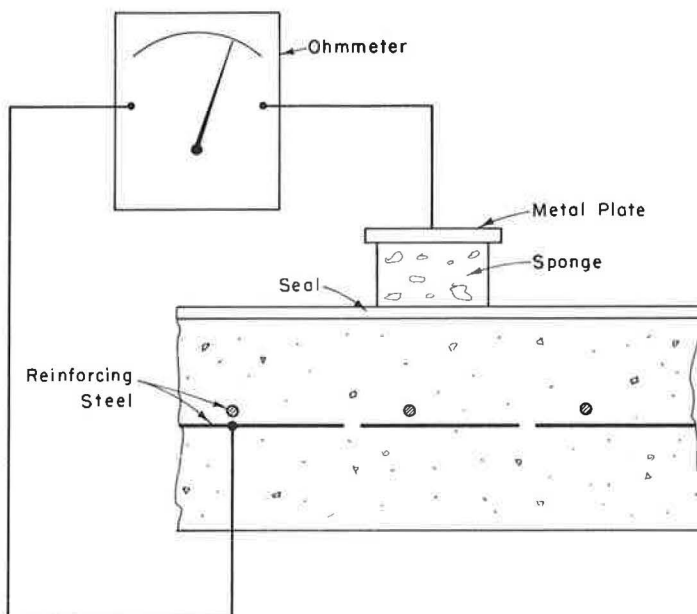


Figure 1. Method for measuring resistance.

plate, 7 by 9 by $\frac{1}{8}$ in. thick, was used and an electrical connection was made to the plate. A nonmetallic handle was attached to the plate for convenience in moving and placing the plate at various points on the bridge surface. Two sponges totaling about 63 in.² of area on the bottom of the plate facilitate contact with the surface. These sponges are attached by means of wooden dowels that are inserted into the sponges and secured to the plate. Because many bridge deck surfaces contain an asphaltic concrete wearing surface, water containing a wetting agent was utilized to increase the rapidity of the penetration through the asphaltic concrete. The wetting agent used is a nontoxic, nonvolatile, practically odorless ester of a sulphonated dicarboxylic acid. The wetting agent is mixed with water at the ratio of about 95 ml of wetting agent to 5 gal of water and has a specific resistance of 2,350 ohm/cm.

The ohmmeter used was an ordinary general purpose voltmeter having an input impedance of 100,000 ohms/volt in the dc voltage ranges and a maximum readable resistance value of 200 million ohms. Figure 2 shows the metal plate contact assembly that is touched to the surface of the concrete to measure the gross electrical resistance. Figure 3 shows the general setup and operations for measuring the electrical resistance of the coating.

In the use of direct current ohmmeters, a problem of nonreproducible values has developed when low electrical resistance values are measured. The cause of this problem is the generation of an external voltage that results from the galvanic coupling of the copper plate to the reinforcing steel. Normally, when external galvanic voltages are present, they cannot be balanced out by shorting of the instrument leads as is normally done. Therefore, depending on the magnitude of the external galvanic voltages that exist, gross errors can occur in the low resistance ranges. For example, with the leads connected with one polarity, the apparently measured values can be in the order of 1,000 ohms. By reversing the leads or polarity, the resistance values can be in the order of 3,000 or 4,000 ohms.

Two techniques for measuring coating resistance have been utilized. One is by obtaining at least 20 resistance measurements at random across the deck. The values are then plotted on probability paper as shown in Figure 4. The other method is to systematically measure the resistance values on approximately 5-ft interval grid across the bridge

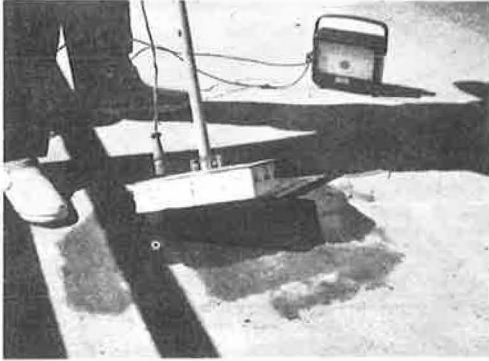


Figure 2. Volt-ohmmeter and apparatus to measure bridge deck coating.



Figure 3. Measuring electrical resistance of bridge deck coating.

deck. Then by making a contour map of equal resistance values, areas of low resistance contours could indicate the location of significant larger perforations of the coating as shown in Figure 5.

Greater precision can be obtained by reversing polarity and averaging the resistance values. Using impressed voltages (15) could result in even more accurate measurements. However, at this time, it is not considered necessary to go to more sophisticated instrumentation because these errors are significant only in the low resistance

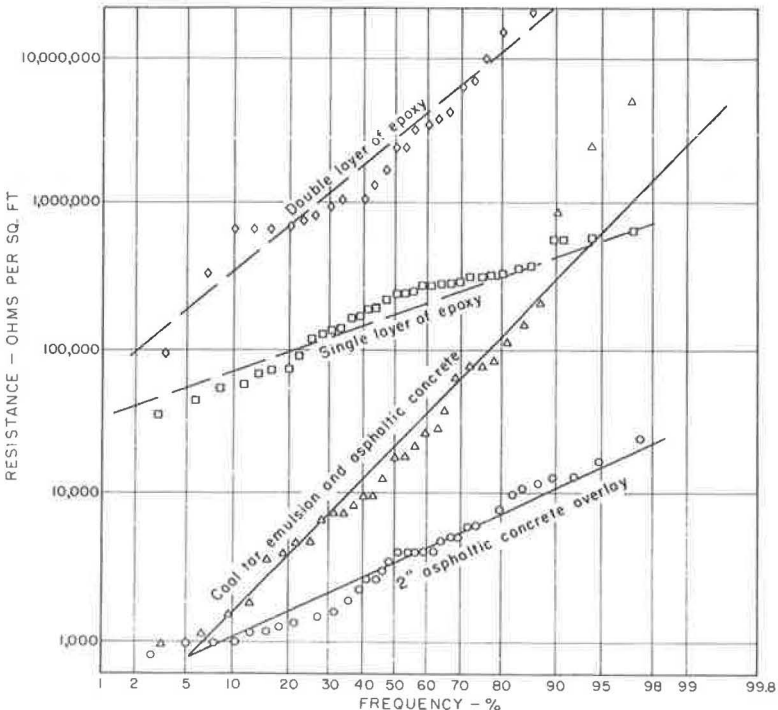


Figure 4. Gross resistance of deck coatings.

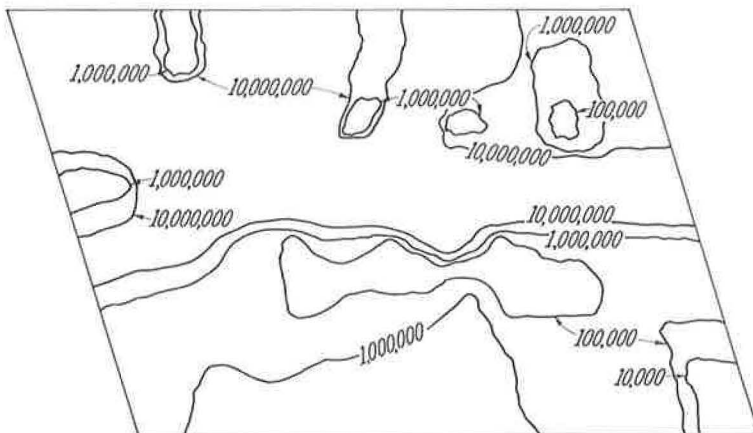


Figure 5. Equireistance contours of bridge deck membrane in ohms/sq ft.

range where readings would indicate a conducting or highly permeable membrane. In the areas of high dielectric strength of, say, greater than 1 million ohms, an error of 2,000 or 3,000 ohms in most cases is not significant or even readable on the instrument scale.

SEALANT VOIDS AND ELECTRICAL RESISTANCE

In order to determine what effect perforations in a coating would have on the electrical resistance, studies were made on 2 bridge decks that were coated with epoxy (California state specification 35). Initially, locations on the sealant were selected where the gross electrical resistance was in excess of 8 million ohms. Then, by means of drill bits of various sizes, holes were drilled into the coating, and the electrical resistance was remeasured by repeatedly placing the copper over the hole. By the method of least squares, an equation was derived that related the area of the drilled holes to the measured resistance. In one case, the resulting equation was

$$A = 79.6R^{-0.76}$$

where

A = area of the holes in coating in in.², and
R = ohms resistance.

The coefficient of correlation for this equation was -0.989, the number of observations was 31, and the standard error of estimate was 0.091 log₁₀.

For the second bridge, the same procedure of drilling holes and measuring resistance of the coating was repeated. The resulting equation and correlation are shown in Figure 6a. When the area of the hole is reduced by about one-half, the electrical resistance approximately triples. A further graphic representation of the influence of the perforations made in the coating to the gross electrical resistance is shown in Figure 6b. In what might be considered a large area of holes or openings in the coating (approximately 0.1 in.²), the measured electrical resistance would be about 30,000 ohms. In what might be considered as a small area of perforation (0.02 in.²), the measured resistance would be approximately 250,000 ohms. Fortunately, in the area of holes we are interested in, the sensitivity to electrical resistance is greatest.

From the preceding relationships, it is apparent that there is a significant value to measuring the electrical resistance of bridge deck sealants, and this gross figure, although not precise, can be an indicator of the porosity of the coating of de-icing salts. To further demonstrate the influence of the gross electrical influence on bridge deck coatings, Figure 6a shows some of the values that were measured on various types of

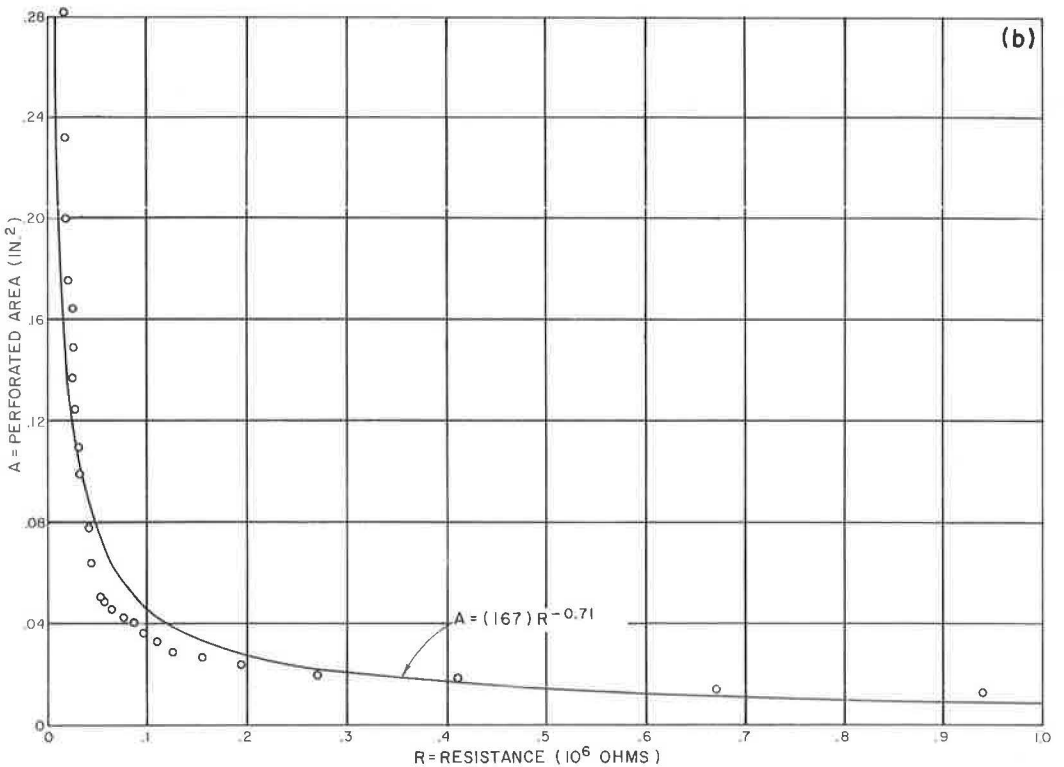
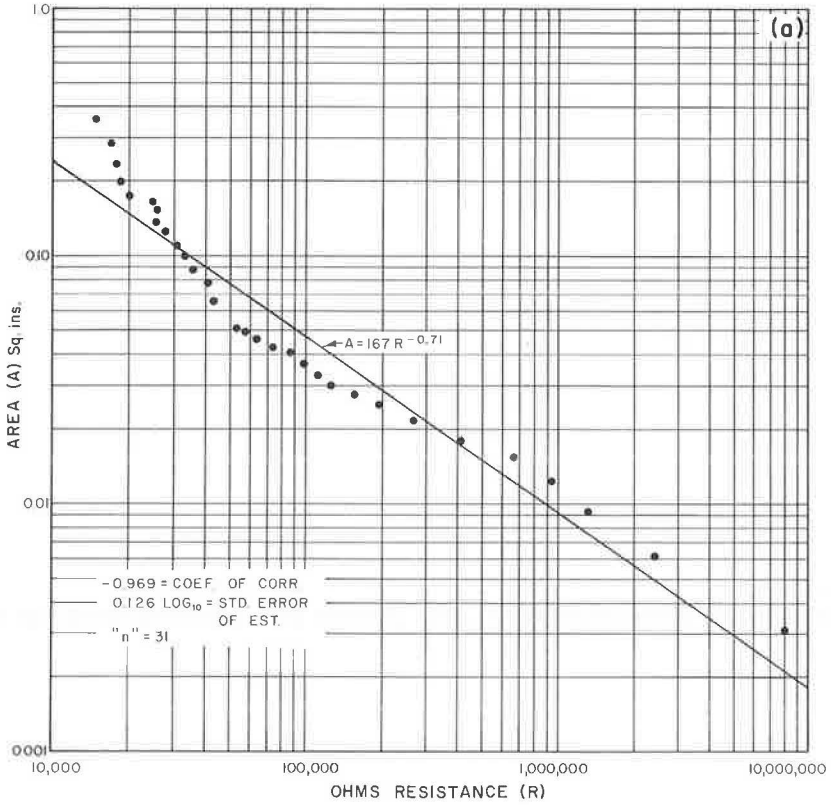


Figure 6. Area of perforations versus ohms resistance with epoxy coating.

TABLE 1
GROSS RESISTANCE OF BRIDGE DECKS

| Bridge | Coating | Average Resistance (ohm/sq ft) | Standard Deviation Factor | Number of Observations |
|---------------------------|---|--------------------------------|---------------------------|------------------------|
| Grizzly Creek | None | 300 | 2.251 | 24 |
| | Coal tar emulsion + 2 in. of asphalt concrete | 22,000 | 7.799 | 31 |
| Sacramento River No. 2-02 | None | 2,600 | 1.174 | 42 |
| | Epoxy 35 | 21,000 | 2.458 | 42 |
| Sims Road No. 6-111 | None | 1,700 | 1.689 | 19 |
| | Coal tar emulsion | 2,638,000 | 7.722 | 19 |
| | Coal tar emulsion + 1 in. of asphalt concrete | 18,000 | 2.020 | 14 |
| Yuba Pass No. 17-23 R | None | 1,300 | 2.963 | 23 |
| | Epoxy 35 | 114,000 | 3.141 | 20 |
| Yolo Bypass No. 22-124R | None | 2,000 | 1.304 | 22 |
| | Epoxy 91 (8 by 16) | 73,000 | 1.660 | 14 |
| | Epoxy 91 (4 by 8) | 82,000 | 1.776 | 18 |
| | Epoxy 45 (4 by 8) | 178,000 | 2.138 | 35 |
| | Epoxy 45 (8 by 16) | 679,000 | 3.933 | 42 |
| | Epoxy, 2 layers, 35 (4 by 8) under 45 (8 by 16) | 2,505,000 | 5.097 | 29 |
| | Epoxy 35 (4 by 8) | 195,000 | 2.990 | 47 |
| Epoxy 35 (8 by 16) | 541,000 | 5.881 | 61 | |
| Yolo Causeway | Asphalt emulsion shoulder | 12,000 | 1.677 | 25 |
| | None | 1,700 | 1.463 | 43 |
| | 2-in. asphalt concrete overlay | 3,500 | 2.512 | 38 |
| Towle OH No. 19-40 | None | 1,000 | 1.380 | 33 |
| | Thermoplastic | 97,180 | 2.485 | 22 |

bridge deck sealants. As will be noted, the average gross resistance of a 2-in. asphalt concrete overlay was in the order of 3,500 ohms/sq ft.

In one case, the average electrical resistance of a reinforced coal tar emulsion coating having an asphalt concrete overlay was about 22,000 ohms/sq ft. A single layer of epoxy was approximately 110,000 ohms/sq ft, while a double layer of epoxy was in excess of 2,500,000 ohms/sq ft. In calculating the resistance on a square foot basis, we assumed that the holes in the membranes that were tested were randomly and normally

distributed. Therefore, because our apparatus had approximately 63 in.² of contact surface, the reported gross ohms/sq ft was directly calculated as an inverse proportion of a square foot to the 63 in.² on contacting electrode. Table 1 gives the results of a number of tests on bridges on which electrical resistance measurements were made. These gross resistance values were further checked insofar as a tool that may be applicable to laboratory work by constructing coatings on 3- by 18- by 24-in. concrete blocks (Fig. 7). In order to measure the electrical resistance of the coating, a metal plate was first placed on the bench, on top of that was placed a sponge wetted with water containing a wetting agent, and above that but in contact with the sponge the block was placed. The metal plate and sponge assembly as shown in Figure 2 was placed on the sealed



Figure 7. Laboratory test of bridge deck coating on 3- by 18- by 24-in. concrete block.

concrete block surface. In constructing these membranes in the laboratory, we attempted to create, as far as possible, conditions similar to those that would be encountered in the field. The sealants were placed on the concrete and, if required, hot asphalt concrete was compacted in the laboratory by means of a roller. In general, in the laboratory, the asphalt concrete was compacted to a density of approximately 90 percent or more.

Table 2 gives the gross resistance tests of laboratory specimens and also some field test results. As will be noted, the values are not exactly the same; however, if reference is made to the magnitude of the resistance values shown in Figure 6b, then we would consider that there is a close approximation between the laboratory and the field tests of the gross electrical resistance of the coatings.

DISCUSSION OF FINDINGS

A technique for comparing the electrical resistance of bridge deck sealing coatings has been presented. Although no correlation between measured resistance and sealant performance in the field has been possible because of the short time these sealants have been in place, it is hoped that with time such data may become available. However, if these measurements are made on new sealants and on a periodic basis, it is considered that this technique may enable researchers to have a common tool whereby they can report on the apparent porosity of bridge deck coatings as related to the penetration of de-icing chemicals.

Because of seasonal and climatic variations, it is obvious that there may be variable moisture conditions on the surface of a coating and within the matrix of an asphaltic mix overlay that can affect electrical measurements. For this reason, the specific values for gross resistance will not be closely reproducible except in broad terms. For example, it is speculated that an excellent waterproof coating for bridges would always have an average electrical resistance greater than 500,000 ohms/sq ft, while a poor or perforated coating would never have an average resistance greater than about 100,000 ohms/sq ft. Uniformity of measurements can be improved in some cases by thoroughly and repeatedly wetting the overlay (asphaltic concrete) at the locations to be measured and allowing time for the water to permeate the layer before making measurements. This may require different waiting periods depending on the permeability of the asphaltic concrete layer. For example, on dry asphaltic concrete overlays about 4 in. thick, it has taken as long as 1½ hours for the applied water to penetrate the asphalt to the concrete deck surface. In addition, seal coats are also applied to the surface of asphaltic concrete that greatly impede the rate of permeation of the wetting fluid. As a result, the electrical measurements could be misleading in that high values on a dry asphalt concrete overlay would be recorded that would imply the presence of a "waterproof" membrane seal.

Because of the observed and measurable time element for water to penetrate a "dry" asphalt concrete, further work is being considered in evaluating the applicability of resistance measurements as an empirical permeability type of test for asphalt concrete and soils.

ACKNOWLEDGMENT

This project was performed in cooperation with the Federal Highway Administration. The authors wish to acknowledge the cooperation of W. O. Comella, W. S. Mendenhall, Jr., S. P. LaHue, R. Brink, and H. Lindberg of the Federal Highway Administration during the initial demonstration of this technique to the District of Columbia Department of Highways and Traffic. The authors also wish to acknowledge the able assistance of

TABLE 2
GROSS RESISTANCE TESTS OF COATINGS

| Coating | Laboratory Tests (ohm/sq ft) | Field Tests (ohm/sq ft) |
|---|---------------------------------|----------------------------|
| Bare concrete | 1,100 | 1,300 |
| Reinforced coal tar emulsion, no asphalt concrete | 43,800,000 | 2,600,000 |
| Reinforced coal tar emulsion + 1½ in. of asphalt concrete | 15,000 | 18,000 to 43,800 |
| Thermoplastic + 1½ in. | 660,000 | 350,000 |

S. Dukelow and R. Trimble of the Materials and Research Department, California Division of Highways. The opinions, findings, and conclusions expressed in this report are those of the authors and not necessarily those of the Federal Highway Administration.

REFERENCES

1. States Escalate Bridge Deck Battle. Eng. News-Record, May 4, 1967.
2. Maun, V. P., and Britton, H. Examples of Repairs to Concrete in Bridges. HRB Bull. 353, 1962, pp. 66-71.
3. Hughes, R. D., and Scott, J. W. Concrete Bridge Decks—Deterioration and Repair, Protective Coatings, and Admixtures. Kentucky Dept. of Highways, Res. Rept., June 1966, 283 pp.
4. Larson, T. D., Cady, P. D., and Price, J. T. Review of a Three-Year Bridge Deck Study in Pennsylvania. Highway Research Record 226, 1968, pp. 11-25.
5. Durability of Concrete Bridge Decks—A Cooperative Study of Ten States. Federal Highway Administration and Portland Cement Association, Repts. 1, 2, 3, 4, 5, and 6, 1965, 1966, 1967, 1968, 1969, and 1970.
6. A Study of Deterioration in Concrete Bridge Decks. Division of Materials and Research, Missouri State Highway Commission, Oct. 1965.
7. Crumpton, C. F., Pattengill, N. G., and Badgley, W. A. Bridge Deck Deterioration Study: Part 8. Special Study of Blue Rapids Bridge Deck. Planning and Development Department, State Highway Commission of Kansas, 1969.
8. Axon, E. O., Murray, L. T., and Rucker, R. M. A Study of Deterioration in Concrete Bridge Decks. Highway Research Record 268, 1969, pp. 80-88.
9. Hilton, M. H., Newlon, H. H., Jr., and Shelburne, T. E. Research Relating to Bridge Decks in Virginia. Virginia Department of Highways, 1965.
10. Spellman, D. L., and Stratfull, R. F. Chlorides and Bridge Deck Deterioration. Highway Research Record 328, 1970, pp. 38-49.
11. Riley, O. Bridge Deck Repair Techniques on the New Jersey Turnpike. Highway Research Record 11, 1963, pp. 50-61.
12. Concrete Bridge Deck Durability. NCHRP Synthesis of Highway Practice 4, 1970.
13. Riley, O. Development of a Bridge Deck Protective System. Highway Research Record 173, 1967, pp. 13-24.
14. Kemp, W. E. Coal Tar Enamel Coatings for Underground Pipelines. Materials Protection, Vol. 9, No. 6, June 1970, p. 14.
15. Parker, M. E. Pipeline Coating Conductance Materials Protection, Vol. 6, No. 8, Aug. 1967, p. 25.

ANALYSIS OF ELASTOMERIC PAVEMENT SEALS

Ravindra K. Vyas, University of Utah, Salt Lake City

The objective of this paper is to present a method for predicting the load deflection characteristic and maximum stresses in elastomeric seals. The basic problem is reduced to the structural analysis of V-shaped web members for large flexural deformations. The analysis is presented in terms of dimensionless quantities, with sample results in tabular and graphical form. The dimensionless results are used to construct the predicted load deflection curve for an actual sample and to compare it with the experimental load deflection curve for the same sample. The values of elastic moduli necessary for constructing the curve were obtained by laboratory experiments. Results indicate that a successful prediction of load deflection behavior and maximum stresses can be made theoretically. The prediction gives a good estimate of load deflection behavior up to 60 percent deformation; beyond this value the sample ceases to respond as a structure and becomes more like a mass of neoprene. The predicted maximum stresses for a particular example are given in graphical form. The results show that maximum shear stress is very low, so that shear deformations may be neglected; the tensile and compressive stresses, however, do have significant values.

•THE ELASTOMER polychloroprene, more commonly known as neoprene synthetic rubber, is a product suited to a variety of uses. Various types of neoprene, with different compositions and properties, are commercially available. Neoprene is an important chemical component of most of the elastomeric pavement seals available today.

The problem of effectively sealing concrete pavement joints is perhaps as old as the concrete pavement itself. To the designer it presents a complex problem because of the many variables that must be taken into account. Tons (1) and Cook (2, 3) have dealt with some important aspects of the joint design problem. Neoprene seals combine flexibility with resilience, 2 prime factors necessary for satisfactory performance of any pavement joint seal. This fact perhaps explains the encouraging findings of Hiss et al. (4) as well as the gain in popularity of elastomeric seals in recent years.

Neoprene seals for pavement joints are available in long coils of tubelike structures with varying section geometry. The interior of the tube is strengthened by means of a gridwork of thin flexible members. Figure 1 shows one such seal section in detail. The manufacturers of elastomeric seals have been constantly experimenting with the chemical composition and also with the section geometry in order to improve the product. The performance of these seals is currently being studied by a group of research and academic institutions. This paper contains a portion of such an investigation in progress under the direction of the author at the University of Utah.

An important criterion in assessing the performance of a neoprene seal is its load deflection characteristic. The load deflection curve of a given sample can be obtained experimentally. However, it is desirable to develop the analytical technique for predicting this characteristic before the product is manufactured. The main objective of this paper is to present such a technique. In order to make the results more useful, the analysis is carried out in general terms; dimensionless forces and dimensionless displacements are used. Sample results of the theoretical study are presented in tab-

ular and graphical form. From the dimensionless results the theoretical load deflection curve for the section shown in Figure 1 is plotted by using the actual values of the elastic moduli and the geometric parameters α , t , and l . The results of experiments on the same sample are also plotted for the purpose of comparison. An advantage of the analytical technique is that it enables the designer to evaluate the maximum stresses.

The physics and chemistry of rubber and rubberlike materials are adequately described in standard works (5, 6, 7). Our experimental work indicates, as would be expected, that neoprene is a viscoelastic solid. The stress-strain behavior of the material can most probably be simulated by a Kelvin chain (8). The simplest possible model of this type would, of course, be the 3-parameter solid. The viscoelastic character of the material complicates the task of the analysis. However, the experiments also indicate that, for short durations of time, the stress-strain behavior of the material can be treated as time-independent.

In order to make it as simple as possible, this presentation of the initial attempt is restricted to the short time load deflection characteristic. It is assumed that, in the range of deformations of interest, the strain is small enough so that the stress-strain curves for the material in tension and compression can be treated as reasonably linear. The corresponding values of the elastic moduli of the material in tension and in compression have to be obtained from laboratory experiments. Such experiments were conducted by the Utah State Department of Highways to determine these moduli for the material of some samples. These experiments were performed at room temperature, with adequate repetitions to ensure reliability, on flat strips cut out from actual samples. The results of these efforts are given in Table 1. The material properties vary considerably, which is not surprising in view of the empirical efforts to improve the product and the fact that the material properties are also affected by the process of extrusion and subsequent curing.

An advantage of the analytical approach in general terms is that one need not worry about these variations in material properties at the outset. The values of elastic moduli are needed only when one wants to predict the behavior of a given sample. For research purposes these values can be obtained from laboratory experiments; for design purpose it would be desirable to have these values furnished by the manufacturers as a matter of course.

ANALYSIS

Any analysis, to be of value to the practical engineer, has to be as simple as possible. An effort has been made in this paper to concentrate on the essentials of the problem and to focus the attention on important patterns. An inevitable consequence of such

an effort is the simplifying assumptions necessary in the process. Two simplifying assumptions have already been stated in the introductory remarks: (a) only short-time load deflection response is considered, and (b) the strains are small enough to justify a linear stress-strain relationship. It must be noted, however, that small strain does not imply small deformations; indeed, the problem being investigated involves large deformations.

In addition to these assumptions, the present analysis is confined to seal sections that have a specific geometric pattern. The sec-

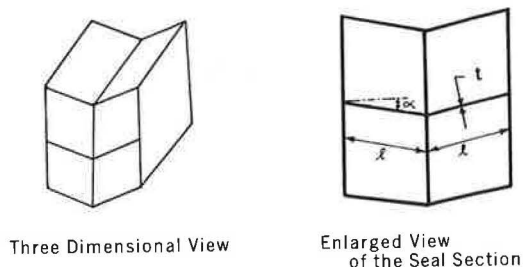


Figure 1. A neoprene pavement seal.

TABLE 1
SAMPLE VALUES OF MODULI OF ELASTICITY
AT ROOM TEMPERATURE

| Material | E_T (psi) | E_C (psi) | E_T/E_C |
|----------|-------------|-------------|-----------|
| A | 808 | 531 | 1.522 |
| B | 603 | 780 | 0.773 |
| C | 717 | 903 | 0.794 |

Note: E_C = modulus of elasticity in compression; and
 E_T = modulus of elasticity in tension.

tion shown in Figure 1 gives one of the typical patterns in its simplest form. Basically the pattern may be described as a symmetrical system of 2 vertical side walls and an internal vertical diaphragm. In addition, there are a series of V-shaped members, called web members, joining the side walls and intersecting the diaphragm. All the web members are assumed to be identical and of constant thickness. During load deflection experiments, the vertical side walls are forced to move toward each other while remaining mutually parallel. The side walls are at all times vertical and straight. The resistance to displacement, then, is derived from the web members. Two of the following assumptions are based on the preceding discussion:

1. The seal section is perfectly symmetrical about the central diaphragm;
2. The resistance to deformation is proved entirely by the web members, and all the web members are assumed to be identical and of constant thickness; and
3. In addition, it is assumed that the deformations are produced by bending only (the effects of axial compression and shear are treated as negligible in the present analysis).

With these assumptions, the task of analysis reduces to the study of flexural deformations of the typical configuration shown in Figure 2. When subjected to a load P , the system deforms into the configuration $A'B'C'$ shown in this figure. Corresponding to a given end load P , a correct end moment M_0 must be applied as shown to provide complete restraint against end rotation. The object of the analysis is to find the deflections Δa and Δb , the end moment M_0 , and the maximum stresses for a given load P .

Because of symmetry, it is sufficient to restrict our attention only to the left member AB . Also because of symmetry, the point of zero curvature is located at a point D midway along the length AB . The entire problem, then, may be solved by confining the analysis to the length AD (Fig. 3).

The moment curvature relationship for large flexural deformations of this member is given by

$$M(s) = -EIK(s) \quad (1)$$

where

- $M(s)$ = bending moment at s ;
- E = reference elastic constant (either E_C or E_T);
- I = geometric constant (modified moment of inertia corresponding to the chosen elastic constant E); and
- $K(s)$ = curvature at s .

From geometry, kinematics of deformation, and equilibrium respectively, we have

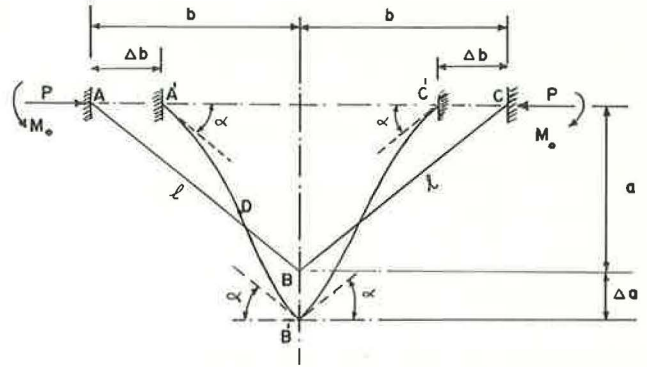


Figure 2. V-shaped web member.

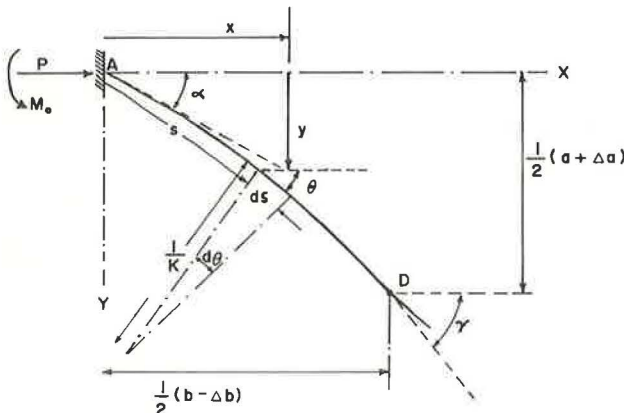


Figure 3. Kinematics of deformation.

$$dx = ds \cos \theta, \text{ and } dy = ds \sin \theta \quad (2)$$

$$K(s) = d\theta/ds \quad (3)$$

$$M(s) = Py - M_0 \quad (4)$$

Substitution from Eq. 3 and 4 into 1 and then differentiation with respect to s yields

$$(d^2\theta/ds^2) + k^2 \sin \theta = 0 \quad (5)$$

where

$$k^2 = P/EI \quad (6)$$

Finally integrating Eq. 5 with respect to s and imposing the condition that as $s = 0$, $\theta = \alpha$, and $d\theta/ds = -(M_0/EI)$, we obtain

$$d\theta/ds = \sqrt{(M_0/EI)^2 - 2k^2 \cos \alpha + 2k^2 \cos \theta}; \quad 0 \leq s \leq \ell/2 \quad (7)$$

From Eq. 7, the angle γ at inflection point is obtained as

$$\cos \gamma = \cos \alpha - (M_0/EI)^2 (1/2k^2) \quad (8)$$

Also from Eq. 7, we obtain

$$\int_0^{\ell/2} ds = \int_{\alpha}^{\gamma} \frac{d\theta}{\sqrt{(M_0/EI)^2 - 2k^2 \cos \alpha + 2k^2 \cos \theta}} \quad (9)$$

Next, the following dimensionless quantities are introduced:

$$u = k\ell = \sqrt{P/EI} \ell \quad (10)$$

$$v = M_0\ell/EI \quad (11)$$

$$\Delta = \Delta b/b \quad (12)$$

$$\Delta' = \Delta a/a \quad (13)$$

In Eqs. 10 through 13, u and v are dimensionless force and dimensionless end moment respectively; and Δ and Δ' are dimensionless horizontal and vertical deformations respectively. The discussion that follows will be in terms of these dimensionless quantities. Next, let

$$p = \sin \gamma/2 \quad (14)$$

and

$$p \sin \phi = \sin \theta/2 \quad (15)$$

then, integrating Eq. 9 and using Eqs. 8 through 13, we obtain

$$u = 2 \int_{\phi_0}^{\pi/2} \frac{d\phi}{\sqrt{1 - p^2 \sin^2 \phi}} \quad (16)$$

where

$$\phi_0 = \sin^{-1} [(\sin \alpha/2)/(\sin \gamma/2)] \quad (17)$$

In terms of the dimensionless quantities, the angle γ is expressed by

$$\gamma = \cos^{-1} [\cos \alpha - (v^2/2u^2)] \quad (18)$$

The displacements of any point of the member ABC can be obtained by integrating Eq. 2 and using symmetry principle. The quantities of interest here are the deformations Δ and Δ' . Of these, Δ can be obtained by integrating the first of Eq. 2, and Δ' can be obtained from the requirement of zero bending moment at the point of inflexion. The final expressions for Δ and Δ' are as follows:

$$\Delta = [1 + (1/\cos \alpha)] - (4/u \cos \alpha) \int_{\phi_0}^{\pi/2} \sqrt{1 - p^2 \sin^2 \phi} \, d\phi \quad (19)$$

$$\Delta' = [2v/(u^2 \sin \alpha)] - 1 \quad (20)$$

Equations (1 through 20) present briefly the essential points of the analysis. It is basically a variation of the well-known problem of elastica, first investigated by Euler. Euler's elastica, which refers to the large flexural deformations of a thin straight elastic rod, is described in sufficient detail in standard sources (9, 10). The present problem differs from Euler's elastica in that it deals with the large flexural deformations of a V-shaped rod. Hence, trigonometric functions of the characteristic configuration angle α appear in the preceding expressions. The load deformation curves are much influenced by this angle.

The integrals in Eqs. 16 and 19 are incomplete elliptic integrals of the first and second kind (11). Hence, it is necessary to use numerical integration procedures to evaluate the required quantities. From Eqs. 17 through 20, it can be seen that for a given u we can evaluate all the quantities, provided we know the value of either the end moment v or the angle γ at the inflection point. It is sufficient to know only one of these 2 quantities. Therefore, the problem can be looked on as statically or kinematically indeterminate to the first degree, depending on whether v or γ is treated as the unknown quantity. It can be seen that the integral in Eq. 16 will be equal to the given value of u only for the correct choice of v or γ . The correct values of either of these quantities

have to be obtained by the process of numerical interpolation. This discussion, of course, applies only to the case where one wants to solve the problem for a specific value of u . The task of constructing the theoretical load deflection curve is numerically much easier. This is done by using the kinematic method, which relies on finding u for progressively increasing values of γ .

The kinematic method can be briefly described as follows: As the load u is increased from zero, the system deforms progressively, and the angle γ at the inflection point increases as u increases. Hence, it is only necessary to select progressively increasing values of γ ($> \alpha$) in suitable increments and to evaluate the corresponding values of u , v , Δ , and Δ' . The numerical work is easily done with the digital computer, and the process is ter-

TABLE 2
SAMPLE COMPUTATIONS FOR A CONFIGURATION
ANGLE $\alpha = 8.5$ DEG

| γ (deg) | u^2 (= PL^2/EI) | v (= M_0L/EI) | Δ (percent) | Δ' (percent) |
|-------------------|-------------------------|-----------------------|-----------------------|------------------------|
| 9.5 | 0.8611 | 0.0686 | 0.1813 | 0.0775 |
| 11.5 | 2.1981 | 0.1999 | 0.5888 | 0.2306 |
| 15.5 | 3.965 | 0.4487 | 1.5834 | 0.5311 |
| 19.5 | 5.0975 | 0.6876 | 2.8156 | 0.8252 |
| 22.5 | 5.7221 | 0.8634 | 3.8940 | 1.0416 |
| 27.5 | 6.5161 | 1.1530 | 5.9805 | 1.3942 |
| 32.5 | 7.1256 | 1.4406 | 8.4200 | 1.7356 |
| 37.5 | 7.6292 | 1.7279 | 11.2015 | 2.0645 |
| 42.5 | 8.0708 | 2.0158 | 14.31 | 2.379 |
| 47.5 | 8.4776 | 2.305 | 17.73 | 2.679 |
| 52.5 | 8.8674 | 2.5969 | 21.46 | 2.962 |
| 57.5 | 9.2511 | 2.8913 | 25.47 | 3.228 |
| 67.5 | 10.04 | 3.491 | 34.26 | 3.700 |
| 77.5 | 10.93 | 4.1097 | 43.95 | 4.087 |
| 87.5 | 11.95 | 4.753 | 54.37 | 4.381 |
| 97.5 | 13.17 | 5.431 | 65.35 | 4.578 |
| 107.5 | 14.68 | 6.154 | 76.71 | 4.671 |
| 110.5 | 15.20 | 6.328 | 80.17 | 4.678 |

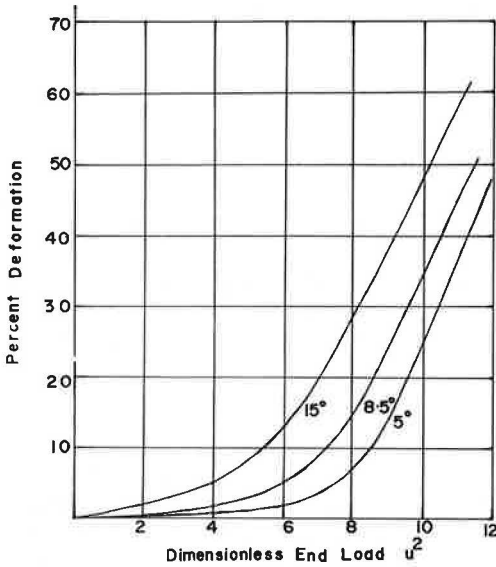


Figure 4. Dimensionless force u^2 for some values of the configuration angle α .

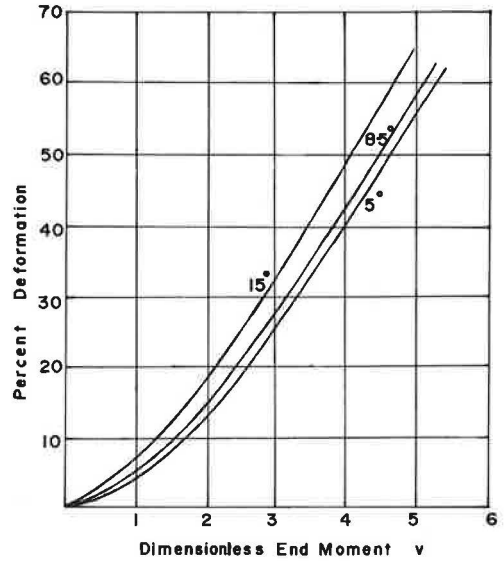


Figure 5. Dimensionless end moment v for some values of the configuration angle α .

minated when the deflection exceeds a certain predetermined value, say 80 percent. Some sample values of computer work for a configuration angle $\alpha = 8.5$ deg are given in Table 2. The numerical integration leading to these tabulated values was performed with Simpson's rule (12) by dividing the interval from ϕ_0 to $\pi/2$ into 1,000 equal steps. The error of numerical integration in these values is of the order of ± 1 in the fourth significant digit. The dimensionless results for this and some other values of angle α are shown in Figures 4 and 5. Such plots are quite adequate for practical purposes, as the values of Δ and v for given u , or the values of u and v for given Δ , can be easily read off from these characteristic curves.

PREDICTION AND DISCUSSION

The results presented in the preceding section can be used to predict the actual load deflection curve for a sample that conforms with the structural assumptions made in the analysis. The sample shown in Figure 1 agrees very closely in geometry with one of the samples received for our experiments. The average values of the parameters t , l , and α are 0.064 in., 0.372 in., and 8.5 deg respectively. The elastic moduli of the material of this sample are given as properties of material A in Table 1. In the analysis, the choice of the reference modulus of elasticity was left open. If we choose the modulus of elasticity in tension, E_T , as the reference elastic constant, the corresponding modified moment of inertia, I , per inch in the length of a web member is given by

$$I = (\beta^2 t^3) / [3(1 + \beta)^2] \quad (21)$$

where

$$\beta = \sqrt{E_c / E_T} \quad (22)$$

Hence, from Eq. 10 the load per inch in the length of the web member is given by

$$P = (E_T \beta^2 t^3 u^2) / [3l^2 (1 + \beta)^2] \quad (23)$$

For the numerical values of E_T , t , l , and so on, referred to earlier in this section, Eq. 23 reduces to the form

$$P = 0.1023u^2 \text{ lb/in.} \tag{24}$$

The dimensionless load deflection curve for $\alpha = 8.5$ deg, shown in Figure 4, gives the value of u^2 for any chosen percentage deformation. When this value of u^2 is used in Eq. 24, the value of P obtained represents the end load, in pounds per inch in the length of the web member, required to produce the same value of percentage deformation. Because the sample consists of 3 web members, the load required to produce the same deformation in the sample is 3 times the value given by Eq. 24. Then, the predicted load deflection curve at room temperature is easily constructed as follows: From the curve shown in Figure 4, for angle $\alpha = 8.5$ deg, read off the value of u^2 for a selected percentage deformation, and then multiply this value by 0.307 to obtain the value of load per inch in the length of the sample for the same deformation. The predicted load deflection curve shown in Figure 6 was constructed in this manner.

The research currently in progress involves a substantial amount of experimental work. The experiments of interest in the present context are those dealing with load deflection curves, particularly those for the sample for which the theoretical prediction is shown in Figure 6. The average of two most reliable experiments out of three performed at room temperature is shown in the same figure for the purpose of comparison.

In comparing the theoretical prediction and the experimental results shown in Figure 6, it is important to take several factors into account. The theoretical prediction is made by using the average dimensions. The approximate variations in the quantities t , l , and α are ± 10 percent, ± 5 percent and ± 0.5 deg respectively. When these factors are taken into account, the theoretical prediction gives a satisfactory estimate of the actual load deflection curve, as can be seen from the closeness of the 2 curves. It was pointed out earlier that the samples are supplied in the form of coils of tubelike structure. Consequently, the side walls of the samples tested, instead of being perfect planes, are slightly curved. This fact is perhaps responsible for the sharper rise in the experimental curves in the early stages of deformation. At about 60 percent deformation, the sample becomes more like a solid mass of neoprene and does not behave like a structure. This is seen in the sharp increases in load with small increases in the deformation. The theoretical prediction, of course, does not apply to this latter situation.

Although the analysis in this case gives a reasonably good estimate of the load deflection behavior, it can be further improved by taking into account the nonlinearity of the stress-strain relationship and the viscoelastic effects. The lack of sufficient information in these respects, especially for the material of an actual sample, suggests the need for more detailed experimental research in basic material properties.

The theoretical study, once the required values of the numerical constants have been ascertained, enables us to predict the maximum stresses. These occur in the web members and are evaluated from the following expressions:

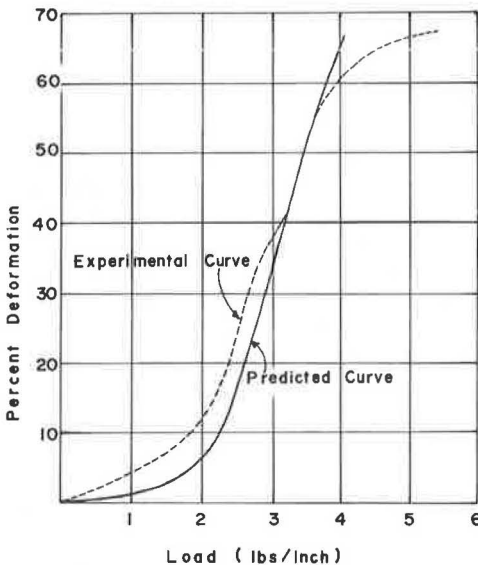


Figure 6. Predicted and experimental load deflection curves.

$$\sigma_c(\text{max}) = [(\lambda\beta^2 E_T)/(1 + \beta)] \left(v + \{(\lambda \cos \alpha u^2)/[3(1 + \beta)]\} \right) \tag{25}$$

$$\sigma_T(\max) = [\lambda\beta E_T/(1 + \beta)] \left(v - \{(\lambda\beta \cos \alpha u^2)/[3(1 + \beta)]\} \right) \quad (26)$$

$$\tau_{\max} = \{(\lambda^2\beta^2 E_T)/[2(1 + \beta)^2]\} \sqrt{u^4 \sin^2 \alpha + u^2 v^2 \cos \alpha - (v^4/4)}; \gamma \leq 90 \text{ deg} \quad (27)$$

or

$$\tau_{\max} = (\lambda^2\beta^2 E_T u^2)/[2(1 + \beta)^2]; \gamma \geq 90 \text{ deg} \quad (28)$$

where

$$\lambda = t/l \quad (29)$$

E_T in these equations refers to the modulus of elasticity in tension; β has been defined in Eq. 22. Equations 25 and 26 refer respectively to the maximum compressive and tensile stresses. These maximum stresses occur in the web member at sections A, B, and C (Fig. 2). If the end moment v exceeds $3(1 + \beta)/(\lambda\beta)$, the maximum tensile stress will occur at cross sections different from those mentioned. In practical situations, v is unlikely to exceed this value within the range for which the theoretical prediction is valid. The maximum shear stress occurs at the inflection point and is given by Eq. 27, provided the angle at inflection point is less than or equal to 90 deg. If the angle at inflection point is greater than 90 deg, the maximum shear stress given by Eq. 28 occurs at points where the neutral axis makes an angle of 90 deg with the horizontal axis.

The plots of maximum tensile and compressive stresses are shown in Figure 7. The maximum shear stress is shown in Figure 8. The shear stresses, for the example considered, are quite low, justifying the assumption made at the outset that shear deformations may be neglected. An interesting feature is the closeness between the maximum compressive and tensile stresses, as shown in Figure 7. The near closeness of these curves is a peculiarity of this particular sample. The value of E_T for this sample is substantially larger than that of E_C . This means that the tensile stress induced by bending alone is larger than the compressive stress. When a uniform direct compressive stress due to the axial force is added to these stresses, the resulting effect is a reduction in the tensile stress and an increase in the compressive stress. The net effect is a near equalization of the 2 stresses. The situation would be much different if

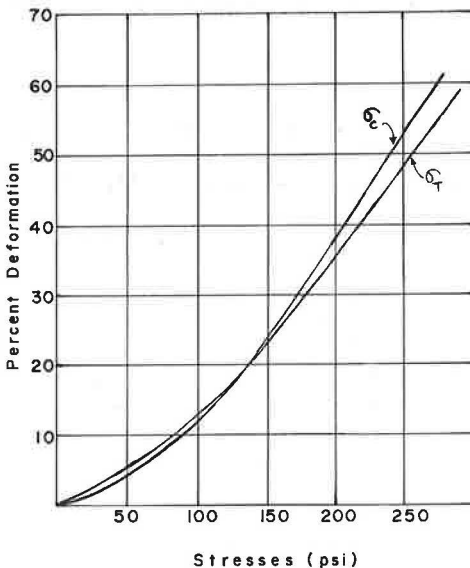


Figure 7. Predicted maximum tensile and compressive stresses.

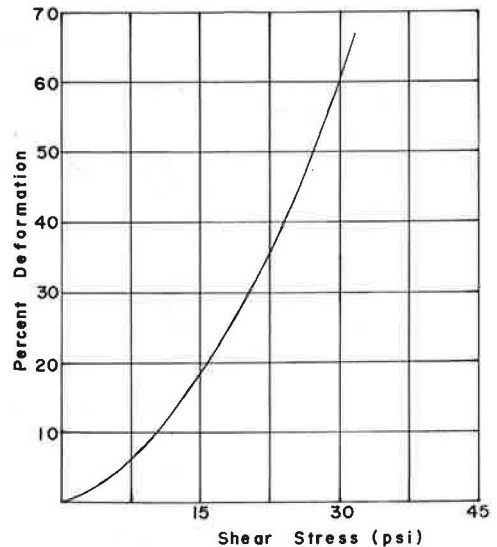


Figure 8. Predicted maximum shear stress.

this sample were made from material B or material C (Table 1). Then the compressive stress would be significantly larger.

CONCLUDING REMARKS

The theoretical technique presented here, simple as it is, gives a good estimate of the actual short time load deflection characteristics of elastomeric seals. Furthermore, it also enables prediction of maximum tensile, compressive, and shear stresses, which are of value to the designer. The dimensionless approach makes the results quite general in application and may be used to study other aspects of the performance of elastomeric seals. Specifically, one can study the effects of changes in material properties and the changes in section geometry on the stresses and stiffness of the sample. The author is currently investigating this aspect of the problem. The study may lead to some of the criteria that may be useful in producing the best design.

It is important to note that the analysis assumes linear stress-strain relationship and a typical geometry that consists of V-shaped web members. Sections with X-shaped web members, another typical arrangement, can be studied by using the same technique, but additional conditions must be incorporated in the analysis. Further refinements of the technique can be made by using nonlinear stress-strain relationship and including the fact of viscoelastic behavior in the study. Such refinements will indeed complicate an already nonlinear problem. At present, it seems that more experimental work is necessary to assess the basic properties of actual seal materials. A knowledge of these properties would be useful in making theoretical predictions regarding the short-time and long-time performance of the elastomeric seals.

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REFERENCES

1. Tons, E. A Theoretical Approach to Design of a Road Joint Seal. HRB Bull. 229, 1959, pp. 20-53.
2. Cook, J. P. A Study of Polysulfide Sealants for Joints in Bridges. Highway Research Record 80, 1965, pp. 11-35.
3. Cook, J. P., and Lewis, R. M. Evaluation of Pavement Joint and Crack Sealing Materials and Practices. NCHRP Rept. 38, 1967.
4. Hiss, J. G. F., Jr., Lambert, J. R., and McCarty, W. M. Joint Seal Materials: Final Report. Bureau of Physical Research, New York State Department of Transportation, Albany, Res. Rept. 68-6, Dec. 1968.
5. Treloar, L. R. G. The Physics of Rubber Elasticity. Oxford Univ. Press, 1958.
6. Tobolsky, A. V. Properties and Structures of Polymers. John Wiley and Sons, 1967.
7. Flory, P. J. Principles of Polymer Chemistry. Cornell Univ. Press, Ithaca, New York, 1967.
8. Flügge, W. Viscoelasticity. Blaisdell Publishing Co., 1967.
9. Love, A. E. H. A Treatise on the Mathematical Theory of Elasticity. Dover Publications, New York, 1944.
10. Timoshenko, S. P., and Gere, J. M. Theory of Elastic Stability. McGraw-Hill, 1961.
11. Flügge, W., ed. Handbook of Engineering Mechanics. McGraw-Hill, 1962.
12. Hildebrand, F. B. Introduction to Numerical Analysis. McGraw-Hill, 1956.