PRACTICAL USES OF SPECTRAL ANALYSIS WITH SURFACE DYNAMICS ROAD PROFILOMETER

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This paper presents some practical uses of power spectral analysis and coherence analysis of data obtained from the surface dynamics road profilometer. A brief description of spectral and coherence analyses is provided, along with some practical examples of their use. The first application is an investigation of differences between an inexpensive replacement road-following wheel and the standard wheel that comes with the profilometer. The second example involves construction control and identification of differences between 2 methods for laying asphaltic base materials. Both of these investigations involved statistically designed experiments so that more reliable conclusions could be obtained and confidence limits defined. Slope variance and roughness index statistics were also examined for the work and are compared with the spectral and coherence analyses results. These methods appear to be a practical use of spectral techniques. Extension of these methods may provide the best approach yet available for development of adequate road profile specifications and construction control.

•DEVELOPMENT of the General Motors surface dynamics (SD) profilometer has made it possible to rapidly obtain road profile data. In addition, the data provided by the SD profilometer are better than those provided by other profilometers in that long wavelength information is included (1, 2). This new device, however, brought many problems of how to process and use the large quantities of data obtained. Initially, research efforts at the Center for Highway Research at The University of Texas at Austin were primarily directed toward computing various summary statistics, such as slope variance and roughness index values from the digitized road profile data. These values were then used for correlations with ratings made by a panel in order to develop equations to predict pavement serviceability index (PSI) (3). Recent work has been expanded to include the use of spectral analysis for analyzing these data.

Spectral analysis, which separates road profile data into the various frequencies contained in the data, has been discussed by Quinn and Hagen, who used rod and level measurements for obtaining profile data (4), and briefly by Whittemore and others, who used the SD profilometer as the measuring device (5). In the studies by Quinn and Hagen, problems in obtaining a standardized method for computing power spectra were discussed. Some of these problems still exist although the profile data obtained by Quinn and Hagen for these studies were not obtained with the profilometer, and the fast Fourier transform (FFT) was not used to compute the power spectral estimates. (The fast Fourier transform is an algorithm that provides Fourier coefficients directly. Use of this method is much faster for computing power spectral estimates than the mean-lagged product method that was most commonly used up until the past 10 years.)

The General Motors report (5) includes several power spectral plots of road profile data obtained with the profilometer. In this study, however, the investigators towed

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the profilometer behind a test truck at 3 mph so that usable data could be obtained for their investigations. Wheel bounce at higher speeds of the road-following wheel was said to be detrimental to their studies. (The profilometer was towed so that approximately the same wheelpath used by the towing or test truck would be measured as required by the experiment.)

Consideration of this brief background would seem to suggest rejection of the use of power spectrum or spectral analysis as an analysis tool for examining road profile data obtained with the SD profilometer. In this paper, however, a way to avoid many of the problems in the reports mentioned previously is discussed, and some practical methods of using spectral analysis on data obtained with the SD profilometer are given. The Appendix briefly discusses spectral analysis and some problems that should be avoided when computing power spectral and coherence estimates. Accurate power spectral estimates require certain statistical assumptions about the data to be analyzed. This paper discusses the conditions imposed on road profile data to satisfy these assumptions.

STATISTICAL ASSUMPTIONS FOR SPECTRAL ANALYSIS METHODS OF ROAD PROFILE DATA

Accurate power spectral estimates of road profile data require data from a stationary Gaussian random process. An ensemble of random time functions (or a random process) is stationary if any translation of the time (or distance) origin leaves its statistical properties unchanged. Because a power spectrum may be thought of as a second-moment spectrum (see spectral analysis discussion in Appendix), its first and second moments fall under this category. The profilometer filters out all low-frequency and direct current components. Data characteristics of the first moment or mean approximately meet this definition (see also trend removal discussions in Appendix). The second moment or variance requirement is not generally satisfied. Darlington, however, has found that road profile variance is reasonably constant on newer concrete and bituminous pavements (6). The total problem of nonstationarity can be somewhat ignored by a change in viewpoint from a local to an overall or averaging effect. That is, from the overall viewpoint, an averaging of several regions of rough and smooth pavements is of primary concern. For this viewpoint, if stationarity is not met, the variance values would be too high for the smooth regions and too low for the rough regions. It should be noted that the stationarity problems are not confined to spectral analysis. They also affect slope variance or other summary statistics and must always be watched for in any such statistical analysis. In most analyses of this type, the overall viewpoint is usually assumed.

Darlington (6) provides a good discussion concerning the random characteristics of road profile data. These discussions lead to the assumption that typical highway profile data, as obtained with the SD profilometer, are usually Gaussian or near-Gaussian because they are an ensemble of random time functions and have a mean approximately equal to zero.

These statistical restrictions are not so serious as they might first appear because it has been shown (7) that the power spectral estimates are fairly robust with respect to non-Gaussian signals. Furthermore, using the combination of data from the profilometer, spectral analysis from an overall viewpoint, and trend removal techniques (7) alleviates the problems of stationarity.

Coherence analysis (Appendix) has been found to be quite useful for current road profile analysis because of its capacity to detect differences between 2 different road profiles on a frequency basis. In addition, it also has several other advantageous features that should be noted. First, Foster and Guinzy (8) found that coherence is also fairly insensitive to non-Gaussian signals. Second, a comparison was made of run-to run profiles, or right wheelpath versus left wheelpath, for the same road section, and profiles are stationary or nonstationary in the same manner. Because each coherence value is a statistic, confidence limits can be applied; and statistical tests on these coherence values can be made. It is in the use of coherence analysis that recent progress has been made at the Center for Highway Research.

Some of the problems that one must avoid in using spectral analysis are discussed in the Appendix. The next 2 sections describe how a combination of coherence and statistics has been used in detecting road-following wheel characteristics of the profilometer and differences in construction methods for laying an asphalt base material.

SD PROFILOMETER—REPLACEMENT WHEEL STUDIES

The SD profilometer (1, 2) was developed so that road profile data could be obtained at high speeds without causing undue traffic interference. A potentiometer mounted to a road-following or sensor wheel is used to detect sensor-wheel and vehicle-body displacements (high-frequency roughness). The weakest link in the overall system has been this sensor or road-following wheel for the following reasons:

1. The mechanical equipment causes most of the system troubles, at least more than the electronic equipment;
2. The usable life of these wheels is too short in relation to their high cost; and
3. Wheel bounce is not uncommon, and has been noted at speeds as low as 10 mph on relatively good roads with a PSI > 4.0.

Many of the mechanical problems mentioned can probably be solved only by use of a noncontact probe, which should also greatly enhance operation of the SD profilometer.

The limited usable life of the sensor wheel, its susceptibility to cutting, and its high cost (about $500) prompted investigations for a substitute. K. J. Law Engineers, Inc., is now selling a less expensive wheel for about $300, and recent but incomplete investigations have proved it to be acceptable, thus far, in terms of its measurement capabilities. Indications of its usable life have yet to be obtained.

Several inexpensive wheels were carefully examined and tested (e.g., in terms of balancing, construction, and visual measuring quality), and all but one were eliminated as a potential replacement wheel candidate. A test was then conducted (9) to determine whether any differences could be discerned between the standard $500 wheel delivered with the system (from now on referred to as the control wheel) and the replacement wheel candidate.

At the beginning of this experiment, the various measurement characteristics that could be used to discern possible differences were defined. Because a set of PSI prediction equations was developed (3), it was decided to use the independent variables in this set of equations as 1 group of characteristics, i.e., log slope variance and roughness index. Significant differences in these statistics would then indicate significant differences in PSI measurements. As indicated previously, these 2 variables, however, provide only 1 index for the total profile wave form. Thus, it was also decided to use coherence values between repeat runs for each frequency range as a further check on possible wheel differences. The following experiment was then designed (Fig. 1):

1. Wheel type—2 levels (control wheel and replacement wheel);
2. Roughness—3 levels (PSI values of 4.0, 2.5, and 1.7);
3. Speed—2 levels (50 mph and 20 mph); and
4. Replications—6 replications.

The analysis of variance for the log slope variance and roughness index statistics is given in Table 1. There is a significant difference only in roughness type, as expected.

From these findings, it can be concluded that the inexpensive wheel could replace the control wheel for computing PSI. Examination of the marginal means (Table 2) seems to provide further evidence to support using the relacement wheel because the marginal mean of the replacement wheel is less than that of the control wheel in the log slope variance analysis. However, just the reverse is true in the roughness index analysis. Thus, it might be suspected, particularly with the large number of degrees of freedom, that there are no significant differences between the replacement wheel and the control wheel. However, an examination using spectral analysis revealed that the characteristics of the 2 wheels had certain significant differences.

				Wheel Type	
				Control Wheel	Replacement Wheel
Roughness	Rough (PSI=1.7)	Speed	50		
			20		
	Medium (PSI=2.5)	Speed	50		
			20	6 Replications per Cell	
	Smooth (PSI=4.0)	Speed	50		
			20		

Figure 1. Experiment design for replacement wheel experiment.

For the spectral analysis, coherences between repeat runs were used as the dependent variable. Table 3 gives the general analysis of variance that was then run on all spectral frequencies. For this experiment, the frequency spectrum was divided into 128 frequency bands; hence, there were 128 analysis-of-variance runs. As noted from this table, the third-order interaction term is used as the experimental error with only 2 degrees of freedom. To get a better test, the other interaction terms were tested and pooled if not found significant at the 75 percent confidence level. This yields a possible maximum of 7 degrees of freedom as noted. In all cases, at least some terms were pooled.

A few comments should be made in regard to the assumptions of normality necessary for the analysis of variance tests. The coherence samples do not come from a normal distribution, as may be noted elsewhere (10). However, for coherence about 0.25 or more and 1.0 or less and for 10 or more degrees of freedom, these curves are near normal or at least fairly symmetrical. In addition, the F-test in the analysis of variance has been found to be fairly robust for some symmetrical distributions. The restriction of normality, however, is a stringent restriction in many cases and should be considered when conclusions are drawn about the analysis of variance results.

From the results of the coherence analysis of variance test, roughness was found significant as expected in the range from the 99 percent confidence level, in most cases, down to wavelengths of about 4.5 ft. Speed was found significant at the 97.5 percent level at the 83- and 43-ft wavelengths. This is logical because these frequencies are affected by the filtering action of the analog computer because of the 20- and 50-mph

TABLE 1

ANALYSIS OF VARIANCE FOR LOG SLOPE VARIANCE AND FOR ROUGHNESS INDEX

Source	Degrees of Freedom	Log Slope Variance		Roughness Index	
		Sums of Squares	Mean Squares	Sums of Squares	Mean Squares
Wheel type, 1	1	0.04289	0.04289	3,870.11278	3,870.11278
Roughness, 2	2	20.74885	10.37443[a]	2,997,657.24426	1,498,828.62213[a]
Speed, 3	1	0.04714	0.04714	11,002.62726	11,002.62726
12	2	0.87306	0.43653	43,553.67089	21,776.83545
13	1	0.11598	0.11598	359.85474	359.85474
23	2	0.61547	0.30773	9,182.44606	4,591.22303
123	2	0.00036	0.00018	745.69207	372.84603
Experimental error	60	8.55918	0.14265	114,013.74619	1,900.22910
Total	71	31.00293		3,180,385.39424	

[a]Significant at 99 percent confidence level.

TABLE 2

MARGINAL MEANS FOR LOG SLOPE VARIANCE AND FOR ROUGHNESS INDEX

Factor	Category	Log Slope Variance Mean	Roughness Index Mean
1	1, replacement wheel	1.96151	417.04149
	2, control wheel	2.01032	402.37841
2	1, smooth	1.22690	136.82475
	2, medium	2.35175	464.90328
	3, rough	2.37909	627.40182
3	1, 50 mph	2.01150	422.07176
	2, 20 mph	1.96033	397.34814

TABLE 3

ANALYSIS OF VARIANCE FOR COHERENCE

Source	Degrees of Freedom	
Wheel type, 1	1	
Roughness, 2	2	
Speed, 3	1	
12	2	Pool when possible
13	1	Pool when possible
23	2	Pool when possible
Error, 123	2	Pool when possible

speed differences. Most important, the wheel type was found significant at the 99 percent confidence level at wavelength bands corresponding to ½, 2, and 3 times the sensor wheel circumference. There was a small mean square error term corresponding to the sensor wheel circumference, but it was not found to be significant at the 90 percent confidence level. Finally, speed, roughness, and wheel type were all significant at the 99 percent confidence level at the third harmonic of the wheel circumference. From these results, it can be concluded that both wheels bounce (as was noted from the power spectral plots) but that the bounce of the replacement wheel significantly affects the variance amplitudes at harmonics of the wheel circumference and that, furthermore, this bounce is a function of roughness, type, and speed. These conclusions appear to be quite reasonable and yield more confidence to the analysis of variance assumptions.

The results of this experiment make the use of the replacement wheel tempting when PSI is computed, particularly where many runs are required over rough sections and might result in more rapid wear of the sensor wheel. On the other hand, the experiment may indicate the robustness of the log slope variance and roughness index statistics and recommend that the control wheel be used for accurate profile measurements.

Repeat runs on many different road sections reveal that, at 20-mph and greater speeds, wavelengths less than 4.5 ft (which correspond to the third harmonic of the wheel circumference) are difficult to measure twice unless perhaps the exact same wheelpath is rerun. That is, these coherence values (including their respective confidence limits) tend to drop below 0.5. This result can be accounted for by considering, first, the wheel bounce problems at these speeds and, second, the failure to drive the vehicle over the exact same wheelpath. On roads, the longer wavelengths are usually more uniform, whereas the short wavelengths tend to be more localized. In addition, the ability of the vehicle to measure the very small amplitude roughness, particularly on smooth roads, becomes a problem and in most roads cannot be detected.

An example of wheel bounce with the control wheels on a smooth section run at 10 mph is shown in Figures 2 and 3 for both left and right sensor wheels. The PSI computed for the section was 3.7. The section is located on I-45 near Buffalo, Texas. As may be noted, the spectral peaks at 0.645 cycles per foot, which is in the same frequency band as the wheel circumference. A more pronounced effect of this bounce exists for the left wheel; however, further plots indicate that this is simply a function of the road traveled.

Coherence between repeat runs seldom yields high coherence values less than about 4.5-ft wavelengths that are much greater than 9 in. or 0.75 ft (the base length is used in computing slope variance). Because wheel bounce occurs at 0.645 ft, slope variance is clearly biased by both wheel bounce and the inability of the vehicle to travel over the same wheelpath (i.e., without the special concerted effort of driving over the exact same wheelpath). Furthermore, it should be noted that much of the right portion of the spectrum is near horizontal, or that the energy in these frequencies is all about equal. This characteristic can be considered system noise (coherence is 1 measure of its randomness) and is similar to white noise, i.e., an ensemble whose spectral density is sensibly constant through the frequencies of interest. (System noise will include not only electronic noise but also the failure of the system to obtain the exact same profile

twice. This may be due to the electronic noise, the measuring limits of the profilometer, or simply the exact wheelpath not being repeated.) Slope variance is, thus, lower bounded by the system or random noise and probably measures roughness indirectly through wheel bounce in many cases of smooth sections at high speeds.

These general observations seem to further support the use of the cheaper wheel, particularly when PSI is measured and when high speeds and longer wavelength results are desired. For precise road profile measurements and short wavelengths, it would seem though that the control wheel should be used and then at a very low speed. This is consistent with the earlier comments on use of a profilometer (5).

With spectral analysis, measuring characteristics of the system for particular road types can be investigated by studying the coherence of repeat runs. Additional use of spectral analysis for determining differences between construction methods is discussed in the next section.

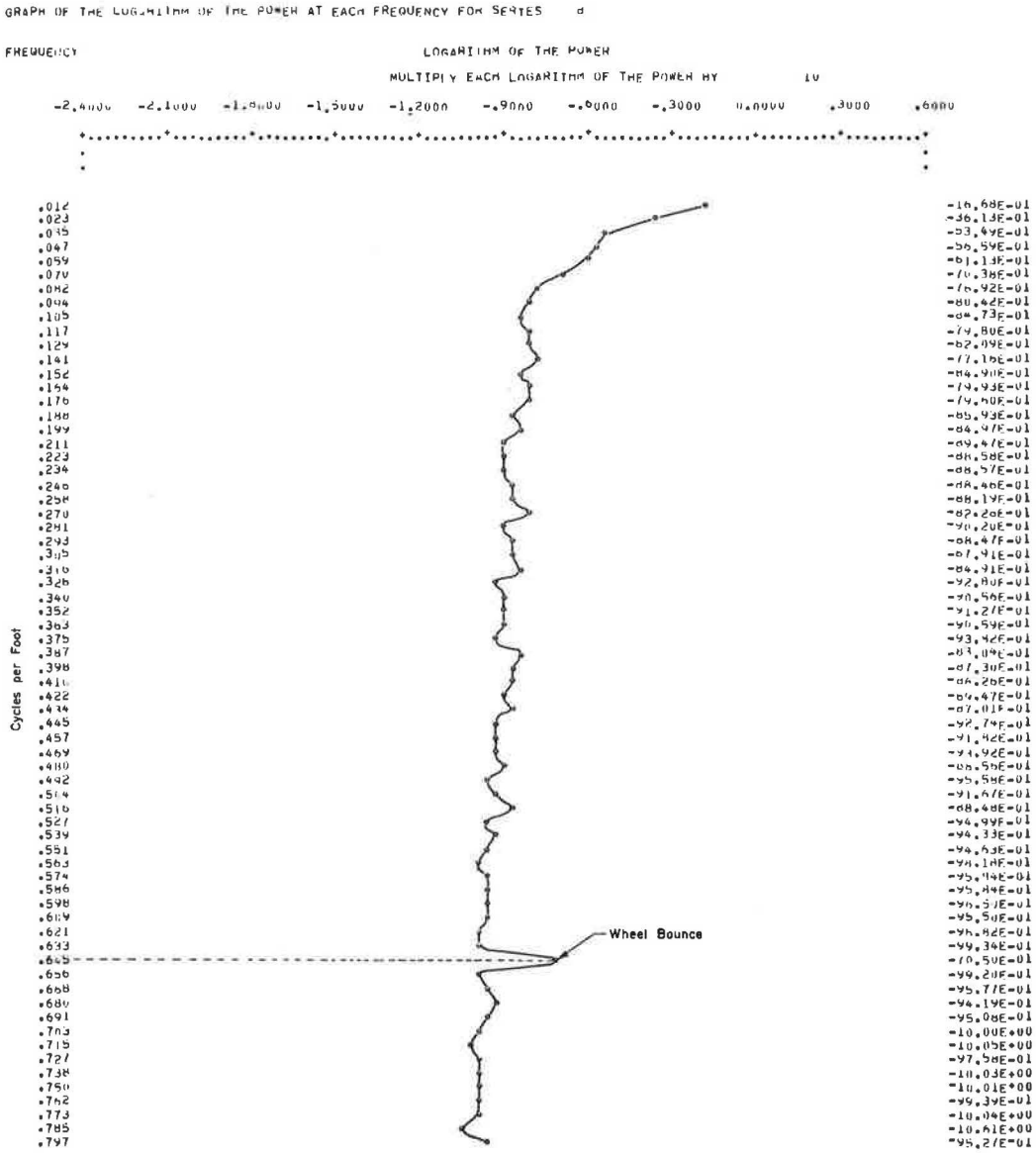

Figure 2. Power spectral plot for left wheelpath.

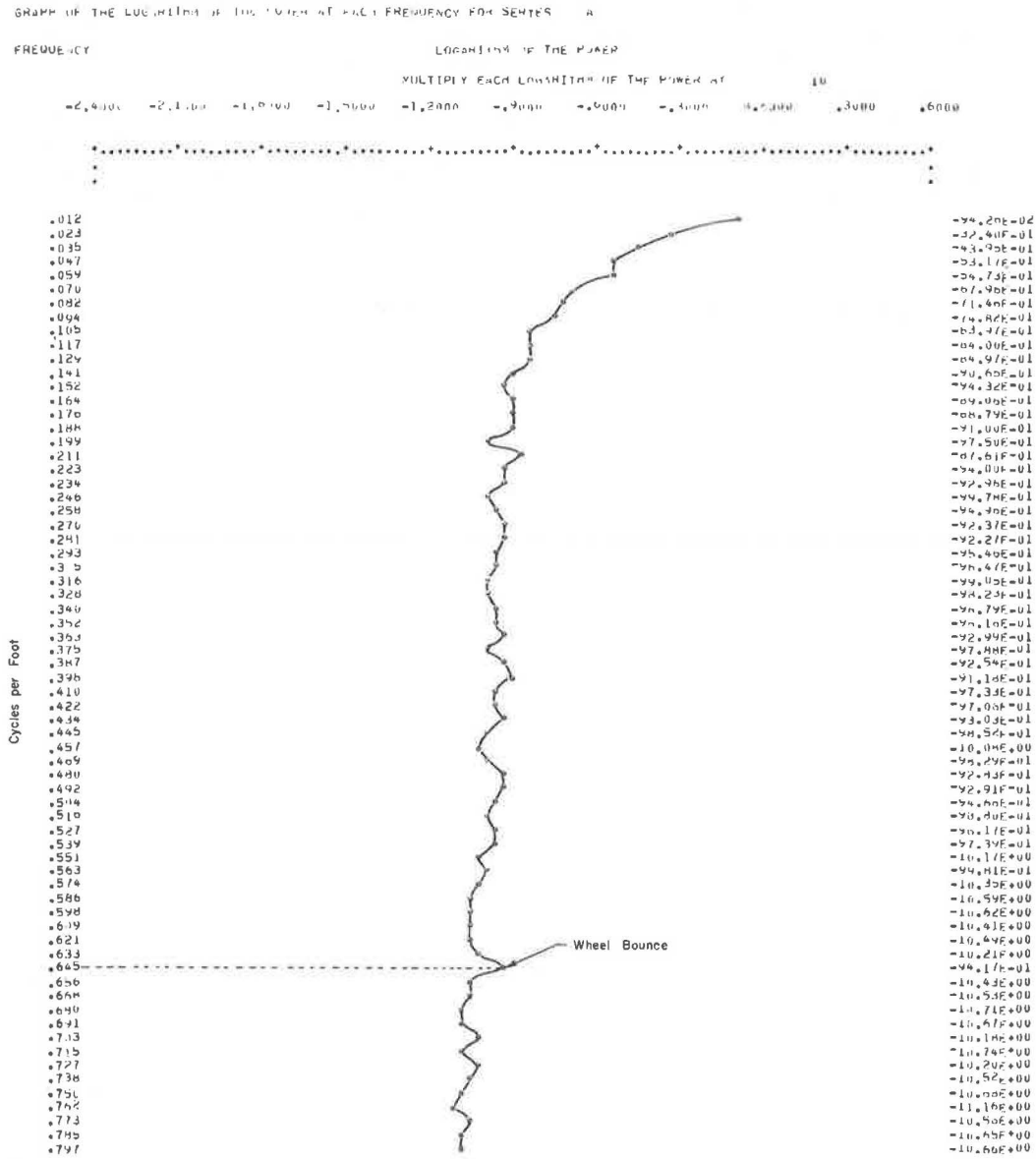

Figure 3. Power spectral plot for right wheelpath.

USE OF SPECTRAL ANALYSIS IN CONSTRUCTION CONTROL

In the preceding section, the use of spectral analysis for investigating system characteristics of the SD profilometer was discussed. This section describes some recent results in investigating differences between 2 methods of laying an asphalt base material on an Interstate highway (I-45) in Texas. The 2 methods are the traveling straightedge and the stretched-wire methods. The stretched-wire method had been used in this particular area, but, because it costs more than the traveling straightedge method, there was interest in determining whether any differences between the methods could be found and what conclusions could be drawn from these differences, if any. In the following discussions, it will be shown that differences between these methods were found (10). However, what effects these differences may yield are yet to be determined.

For the experiment, the SD profilometer was driven over 2 sample sets: 1 set was driven in July and 1 set in August for each construction method; the sample set or sections in July were geographically different from the set run in August. For each method, each sample set consisted of 4 randomly selected 1,200-ft sections of about 2 miles of road in the July runs and of about 1 mile of road in the August runs. The 2 methods were used side by side on the northbound and southbound lanes respectively.

TABLE 4

AVERAGE PSI VALUES (BASE MATERIALS)

Run Set	Traveling Straightedge	Stretched Wire
July, sets 1 and 2	3.7	3.4
August, sets 3 and 4	3.6	3.4

Note: PSI values were computed from 20-mph equation.

From the road profiles measured with the SD profilometer, slope variance, cross-slope variance, roughness index, PSI, and spectral analysis were all computed. The slope variance, cross-slope, roughness index, and PSI values all revealed that the traveling straightedge method yielded a less rough road, and, in many of the cases, this statistic was found significant at the 95 percent level. These findings would all seem to indicate that the traveling straightedge should be used not only because it costs less but also because it actually yields a less rough road. A few facts, however, should be noted. The material being laid, over which the runs were made, was only the base material; portland cement concrete is used on top of this base, and it changes the short wavelength roughness. Also, the base length used for computing slope variance (9 in.) is very sensitive to the small-amplitude, short wavelengths.

Because these small-amplitude bumps are probably taken out, or at least changed, by the concrete top layer, the use of slope variance and roughness index is not too useful as a basis for deciding that the first method is better than the second for laying an asphaltic base material. However, as far as indicating the small roughness differences, they do provide some measurements that might be useful for other problems. Table 4 gives the average PSI values for the 4 sets of runs. For smooth roads such as these, the PSI is almost completely determined from the log slope variance.

Possible uniform wavelength differences, which 1 method might consistently introduce over the other, were discerned by computing coherence between right and left wheelpaths or profiles. t-tests were then made on these values for each frequency range to see whether 1 method had higher coherence for a particular frequency. These tests indicated that there were differences at the 99 percent confidence level for wavelengths in the 24- to 34-ft range and at the 95 percent confidence level for wavelengths in the 55- to 100-ft range. Tests to ensure that the frequencies on the 2 common sets were not significantly different were made to provide further validation. These yielded the proper indications. Furthermore, the 10-mph runs yielded the same set of results for the 24- to 34-ft results. The 55- to 100-ft band could not be called significantly different at the 95 percent confidence level; however, this band has begun to be affected by analog computer filtering in the profilometer. The coherence means were higher for the 10-mph runs, as was expected. Table 5 gives the results of these runs.

These findings, of course, are dependent on how well the 2 sets of runs represent all the sections in which these 2 construction methods were used. The consistency in the 2 run sets as well as in the 10- and 20-mph runs, however, lends strong support to the measurement accuracies.

TABLE 5

WAVELENGTH ANALYSIS

Wavelength Bands (ft)	Mean Coherence at 20 mph		Notes
	Traveling Straightedge	Stretched Wire	
24 to 34, 20 mph	0.751 (0.851 at 10 mph)	0.524 (0.610 at 10 mph)	Significant at 99 percent
55 to 100, 20 mph	0.960 (0.877 at 10 mph)	0.815 (0.806 at 10 mph)	Significant at 95 percent

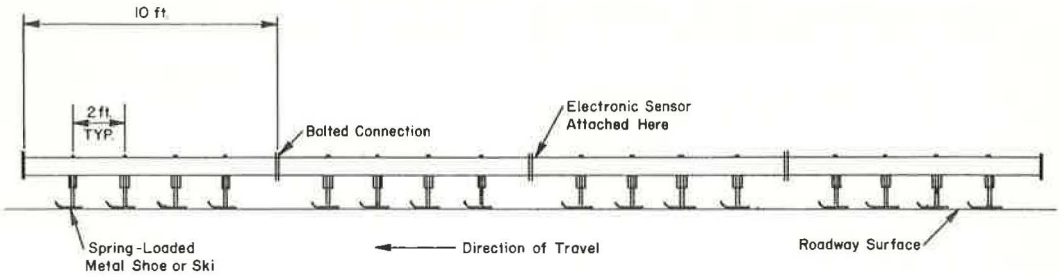

Figure 4. Longitudinal grade-reference ski.

With regard to the statistical characteristics of coherence, the coherence values were obtained at about 30 degrees of freedom from a population that is fairly symmetrical. The normality assumption of this population, however, is not now as important because of the central limit theorem, particularly with the peaks in these distributions and the number of degrees of freedom used (14 for the 20-mph runs) in the t-tests.

Further investigation of the physical characteristics of the longitudinal grade reference ski, which was used in the northbound lane where the frequency band, 24 to 34 ft, was found to be significant, revealed a definite relationship between the ski dimensions and this band. Figure 4, a schematic of the ski dimensions, shows that, although the

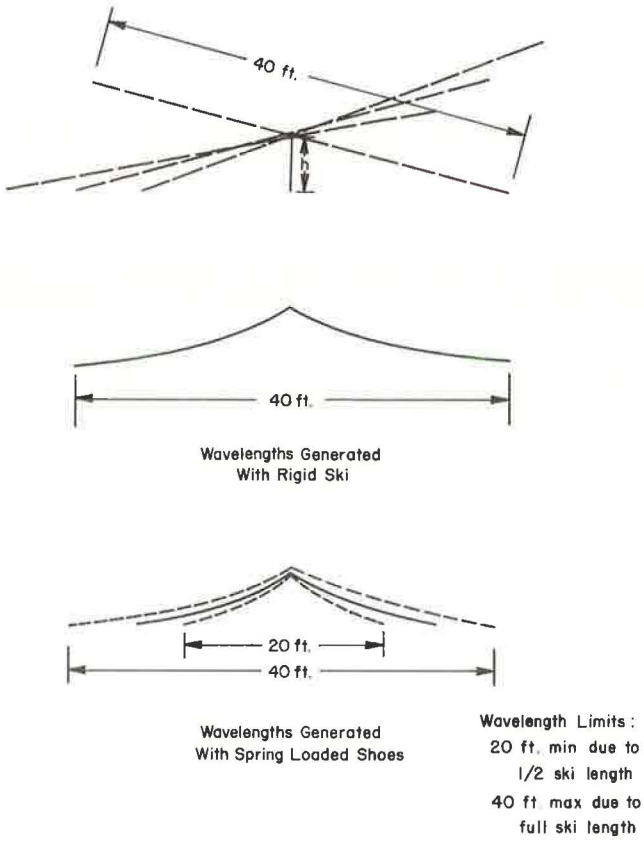

Figure 5. Wavelengths generated by longitudinal grade-reference ski transversing abrupt bump.

40-ft frame is rigid, spring-loaded metal shoes are used for roadway contact. Without these shoes, the ski would tend to generate 40-ft wavelengths, as shown in Figure 5. With the shoes, however, the ski frame deflects only when the spring for the contacting shoe reaches its maximum contraction. This tends to reduce the width of these bumps to values between 20 to 40 ft, and these most likely tend to be distributed around 30 ft or in the 24- to 34-ft wavelength bands. The other band found significant is in the vicinity of the second harmonic.

Spectral analysis was used to discern differences between roads constructed by the 2 methods. Determining such differences is 1 problem; relating these differences to pavement ridability or deterioration is another. For example, even though the traveling straightedge method might have introduced these wavelength differences, their effect on PSI might be negligible. The next section discusses a method of relating PSI to wavelengths in a road with the use of coherence.

USE OF SPECTRAL ANALYSIS FOR COMPUTING PSI

As mentioned in the previous section, identifying wavelength differences in construction methods is 1 area of concern. Determining how these differences affect pavement ridability and wear is an additional area of concern. Investigations are currently in progress to obtain a PSI prediction equation with wavelength coherences as the independent variables. The procedure being used to find such an equation involves computing coherence values for the right versus left and for repeat runs from profile data obtained during 2 large-scale rating sessions held in Texas in 1968 (3). Regression analysis will be run on the results, and PSI values will be determined from the rating panel that is used as the dependent variable. If the procedure is successful, the results of the regression analysis not only will yield a new method for computing PSI as a function of wavelengths in a road but also will provide a measure of wavelengths that are most undesirable to a rider. This information could be used as a scale for specifications in construction control. It would be much more meaningful than slope variance because wavelength bands are physically more easily taken into account than a single statistic, such as slope variance or roughness index measurement.

SUMMARY AND CONCLUSIONS

This paper discusses some uses of spectral analysis as a tool for analyzing road profile data obtained with the SD profilometer. Assumptions necessary for using spectral analyses are summarized briefly; the uses of spectral analysis in the identification of system characteristics of the SD profilometer and in the control of construction are described, and some general comments on current investigations of its use in obtaining a new PSI prediction equation are given. The Appendix briefly defines power spectrum and coherence and discusses some of the more common computational problems that must be avoided in using the FFT for spectral analysis.

Some of the more important conclusions drawn from this paper are as follows:

1. Spectral analysis is a useful tool for analyzing road profile data obtained with the SD profilometer;
2. The combined use of spectral analysis and slope variance provides conditions in which an inexpensive sensor wheel can be used in place of the much more expensive wheel delivered with the system;
3. Spectral analysis is useful for establishing measurement accuracies of the SD profilometer for the various operating speeds; and
4. Spectral analysis can be successfully used for discerning differences in various construction techniques not discernible with other common statistics such as slope variance.

It may be possible to use spectral analysis to obtain a measurement of the wavelengths that are most bothersome to the rider. Work is currently proceeding in this area.

ACKNOWLEDGMENTS

The findings discussed in this paper are those obtained from research on the feasibility of high-speed road profilometer equipment. Support is gratefully acknowledged from the Texas Highway Department and the Federal Highway Administration. The opinions, findings, and conclusions expressed in this publication are those of the authors and not necessarily those of the Federal Highway Administration.

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APPENDIX

BRIEF DISCUSSION OF SPECTRAL ANALYSIS

The term spectral analysis, as it is used in this paper, includes all techniques for summarizing time series functions by separating these functions into their frequency components. A detailed discussion of spectral analysis techniques such as Fourier transformations, power spectrum, and coherence is not provided here, but information on these analysis tools can be found in the listed references. A brief description of these terms is given to supplement the paper.

In 1807, Fourier discovered that an "arbitrary" function could be expressed as a linear combination of sine and cosine terms. The mathematical transformation, which performs this operation on a function to transform data from the time domain to the frequency domain, was appropriately named a Fourier transformation.

The following equation provides the formula for this transformation for a smooth function:

$$G(t) = \frac{a_o}{2} + \sum_{n=1}^{\infty} (a_n \cos nt + b_n \sin nt) \tag{1}$$

where

$$a_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \cos nt \, dt, \text{ and}$$

$$b_n = \frac{1}{\pi} \int_{-\pi}^{\pi} f(t) \sin nt \, dt \quad -\pi \le t \le \pi.$$

Figure 6, which consists of a very simple wave form composed of only 2 sine waves, shows a simple example of this transformation. The more complicated wave forms can, of course, consist of an infinite number of these terms.

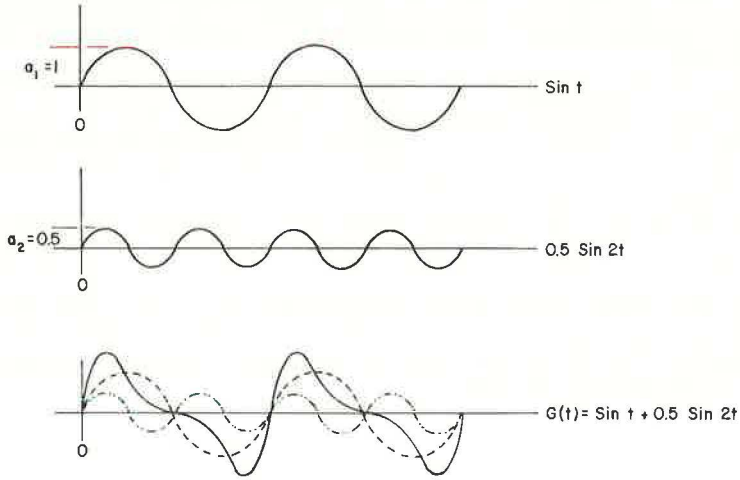

Figure 6. Wave consisting of first and second sine terms of its Fourier coefficients.

Equation 1 is one of several formulations of the Fourier transformation. Another formulation, which at first appears less meaningful, is given by

$$S(f) = \int_{-\infty}^{\infty} G(t) \times e^{-i2\pi ft} dt \tag{2}$$

$$G(t) = \int_{-\infty}^{\infty} S(f) \times e^{i2\pi ft} df \tag{3}$$

The exponential terms can be easily derived by using trigonometric identities. For road profile analysis, the function G(t) is the road profile as measured with the SD profilometer, where t is the time or distance variable.

Transforming profile data from the time or distance domain to the frequency domain is 1 form of spectral analysis. However, although this form may have certain uses, it is of limited value because of its dependence on time or distance. That is, a profile wave form of constant shape will have the same energy or variance at any 1 frequency, but the distribution of this energy or variance between the sine and cosine terms depends on the phase shift or time position of the profile wave form. The energy or variance of the profile wave form at each frequency is obtained by squaring the amplitude of each sine and cosine term for each frequency with the phase angle being obtained from the arc tan of the ratio of the amplitudes. This spectrum, consisting only of amplitude and phase angles, is referred to as the power spectrum.

The autocovariance of a function x(t) at lag λ may be given as

$$C(\lambda) = \lim_{T \to \infty} \frac{1}{T} \int_{-\frac{T}{2}}^{\frac{T}{2}} x(t) \times x(t + \lambda) dt \tag{4}$$

It can be shown that the power spectrum is also the Fourier transform of the autocovariance function, or

$$P(f) = \int_{-\infty}^{\infty} C(\lambda) \times e^{-i2\pi f\lambda} d\lambda \tag{5}$$

These equations have been discussed because use of a power spectrum as a means of analyzing road profile data may be relatively unfamiliar to some readers. From the foregoing interpretation, it can be seen that a power spectrum, so commonly used in communications engineering, geophysics, and other sciences, can also be referred to as a covariance spectrum (7). Thus, P(f)df represents the contribution to the variance of the road profile wave form from frequencies f and (f + df). A power spectrum, therefore, is another statistic, like slope variance, except that it provides a set of spectral values or variance densities for a road profile section, whereas slope variance or simple variance yields only 1 such value. It is this fact that prompted an interest in the investigation of spectral analysis as a means of providing some measure of roadway roughness.

Information on energy differences between 2 or more time series can be obtained with cross-spectral analysis. Whereas the power spectrum is the Fourier transform of the autocovariance, the cross-power spectrum is the Fourier transform of the cross-covariance function between 2 separate time signals. Coherence can be thought of as a kind of normalized cross-power spectrum where the values range from zero to one. The coherence function is defined by the following equation:

$$\partial xy = \begin{cases} \dfrac{|P_{xy}(w)|}{\sqrt{P_{xx}(w)P_{yy}(w)}} & \text{when } P_{xx}(w) \text{ and } P_{yy}(w) > 0 \\ 0 & \text{when } P_{xx}(w) \text{ and } P_{yy}(w) = 0 \end{cases} \tag{6}$$

where

$P_{xy}(w)$ = cross-power spectrum between x(t) and y(t),
$P_{xx}(w)$ = power spectrum of x(t), and
$P_{yy}(w)$ = power spectrum of y(t).

Also associated with coherence is a phase lage between x(t) and y(t).

Multiple coherence is analogous to the multiple-correlation matrix in statistics, and, just as significance levels are used in correlation analysis, confidence levels may be used in cross-spectral analysis. Goodman (11) discusses the theory and practices of cross-spectral analysis. Foster and Guinzy (8) and Gossard (12) provide good discussions of some of the relationships and uses of cross-spectral analysis.

COMPUTATIONAL PROBLEMS IN USING SPECTRAL ANALYSIS FOR ANALYZING ROAD PROFILE DATA

Some of the common problems that must be avoided if accurate measurements in spectral analysis are to be obtained are discussed here. Greater details of these problems may be found elsewhere (4, 7-23).

Aliasing

The problem of aliasing is probably the best known of the pitfalls that will be discussed. It results from the fact that high-frequency components of a time function, such as a profile signal, can imitate low frequencies if the sampling rate is too low. In Figure 7, which shows this effect, high-frequency and low-frequency signals are sharing identical sample points. Once sampled, there is no way of filtering this high-frequency imitation out of the data. The solution to the problem is to ensure that the sampling rate is at least twice as high as the highest frequency present.

Figure 7. Aliasing effect of high-frequency wave.

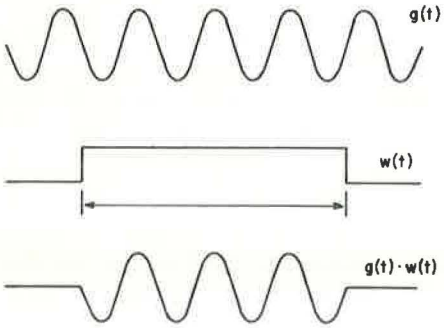

Figure 8. Rectangular data window result when finite data record is used.

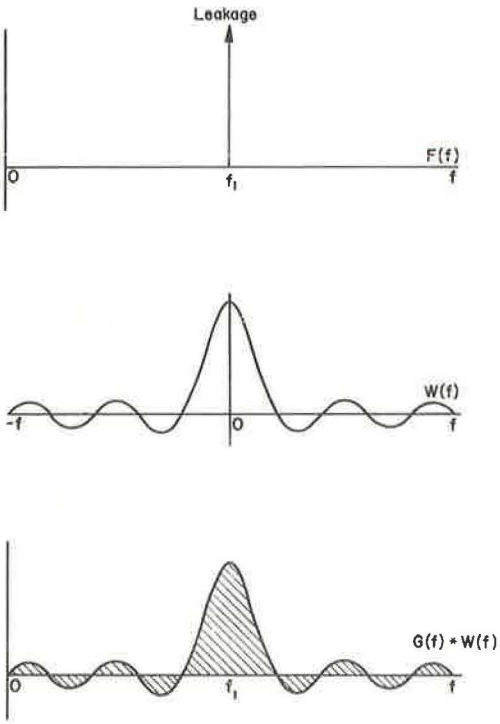

Figure 9. Results of convoluting rectangular data window with sine wave.

Leakage

The problem of leakage occurs because of the use of a road profile signal of finite length. This use may be thought of as multiplying the actual road profile signal by a rectangular data window that limits the infinite profile to finite lengths, as shown in Figure 8. Because multiplication in the time domain is equivalent to convolution in the frequency domain, the Fourier transform of the finite profile signal results in the transformed profile signal being convolved with a sin x/x function. For example, had the profile signal been a pure cosine wave, its Fourier transform would have been limited to only 1 point on the frequency axis (Fig. 9). However, because of its finite length, which is caused by the rectangular data window, the actual result is as shown in Figure 9. As a result, a loss of energy due to these side lobes occurs (Fig. 9). The problem may be alleviated by using a different type of data window, one that when transformed appears more as a rectangular function (7).

Picket-Fence Effect

Because of the multiplication of the profile signal by a finite-length data window, the effect of the FFT algorithm is similar to the use of a bank of bank-pass filters as shown in Figure 10, depicting the main lobes of the spectral window. The width of each main lobe is inversely proportional to the original profile length. To reduce this ripple distortion requires that the record length analyzed be extended by adding a set of samples identically zero before making the FFT transformation. This procedure causes an overlapping of the Fourier coefficient terms and considerably reduces the amount of ripple, as shown in Figure 10.

Trend Removal

As discussed previously, the filtering in the SD profilometer attenuates the low-frequency and direct-current components from the profile signal and, thus, yields a mean profile signal of approximately zero. It is the fact that this mean may not be identically zero that causes some distortion in the low-frequency spectral estimates. Blackman and Tukey (7) illustrate the effects on the spectral coefficients when this mean is only near zero, and, as noted, the effects can be significant. This is the reason for

their statement that it is almost always wise to use some type of trend-removal function in the spectral analysis.

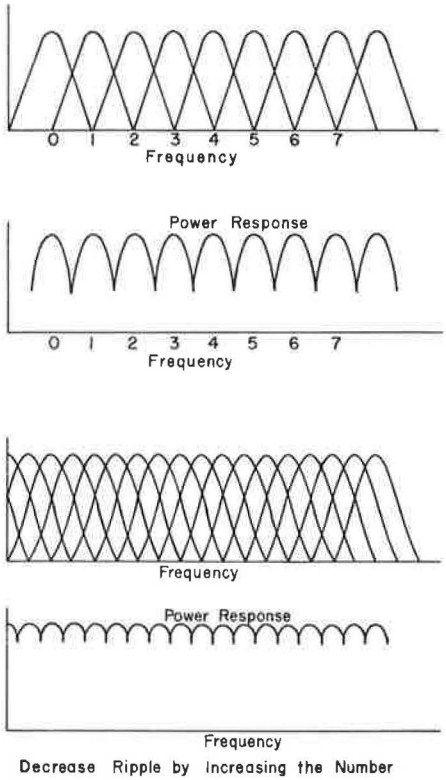

Figure 10. Picket-fence effect.

Degrees of Freedom

As in all statistical analyses, a reasonable number of degrees of freedom should be used when the spectral estimates are computed. The power spectral estimate for a given frequency will vary about the population spectrum according to the chi-square distribution. The number of degrees of freedom, which is a function of the spectral window used, should then be large enough so that usable confidence limits can be obtained. In actual practice, use of 20 or more degrees of freedom has been found desirable for road profile data. The number of degrees of freedom is also extremely important in coherence analysis. Coherence quickly becomes unreliable if the number of degrees of freedom is insufficient and may even be unity if the number of degrees of freedom is too low. Too many degrees of freedom, however, can result in too low coherence values if the spectral estimates are not constant within the spectral data interval (8). [The establishment of confidence intervals on the coherence values can be made by using the tables in the paper by Quinn and Hagen (4) for up to 21 degrees of freedom. The paper by Amos and Koopmans (26) provides additional graphs and tables that can be used for obtaining confidence limits on samples with up to 200 degrees of freedom. Graphs produced by Walker provide coherence distribution data for a large variety of population coherence, degrees of freedom, and confidence limit combinations (10).]

Prewhitening

Prewhitening, as defined by Blackman and Tukey (7), is the process of prefiltering the profile data so as to make the spectral density more nearly constant. This prefiltering, for example, can be used in the preceding so that more degrees of freedom can be obtained for coherence computation. Prewhitening is discussed in detail by Blackman and Tukey and in many of the listed references.