DEMAND FOR TRAVEL ON THE CANADIAN AIRWAY SYSTEM

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•A STATEMENT of future transportation needs is required so that transportation systems can be effectively and efficiently planned. Demand forecasts are fundamental planning variables and serve as a basis for generating long-term development plans, establishing investment priorities, and designing and implementing physical facilities.

However, many forecasts of demand, especially for air travel, have proved to be inaccurate. This has resulted in the provision of facilities that were incapable of serving actual traffic volumes. For example, the first aeroquay at Toronto International Airport, which was designed for 3 million passengers per year, was to provide adequate capacity until 1970 (1). However, in 1966, 2 years after the aeroquay was opened, there were almost 3-1/2 million passengers (2). On a larger scale, forecast and actual domestic air passenger-miles for the United States are shown in Figure 1.

The deficiencies of existing air travel demand models primarily result from the inability to quantify many of the underlying socioeconomic and transport system factors. For example, many models relate the total trips generated to total population. These models cannot account for changes in individual trip-making behavior. Other models do not explicitly include transport system factors and, therefore, cannot forecast traffic that will be generated because of technological advances. Furthermore, existing models examine only pairs of cities taken one at a time, and the competition of destination attractions cannot be taken into account.

The objective of this paper is to present an air travel demand model to overcome the preceding deficiencies. The modeling technique is based on systems theory and particularly on linear graph analysis. The technique is applied to business travel on the Canadian Domestic Airway System. The changes in traffic volumes as related to changes in cost and time of travel are derived for selected city pairs.

STATEMENT OF THE PROBLEM

The primary purpose of the linear graph model presented in this paper was to simulate the demand for intercity business travel on the Canadian Domestic Airway System. The problem may be stated as follows:

Given

1. An origin city consisting of people with specified incomes,
2. A travel network consisting of air links between the origin city and all destinations on the system, and
3. Destination cities consisting of various land use and employment types;

Find

1. The total number of annual business air trips originating at the origin city,
2. The assignment of these trips on the available travel links, and
3. The number of annual business trips from the origin city arriving at the destinations in the system.

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MODEL FOR BUSINESS TRAVEL ON THE CANADIAN DOMESTIC AIRWAY SYSTEM

The system examined in this model is shown in Figure 2. It includes 11 major Canadian airport regions as follows:

1. Vancouver, Victoria, and New Westminister;
2. Edmonton and Calgary;
3. Regina and Saskatoon;
4. Winnipeg;
5. Toronto;
6. London and Windsor;
7. Ottawa;
8. Montreal;
9. Quebec City, Trois Rivières, Bagotville;
10. The Atlantic Provinces; and
11. Newfoundland.

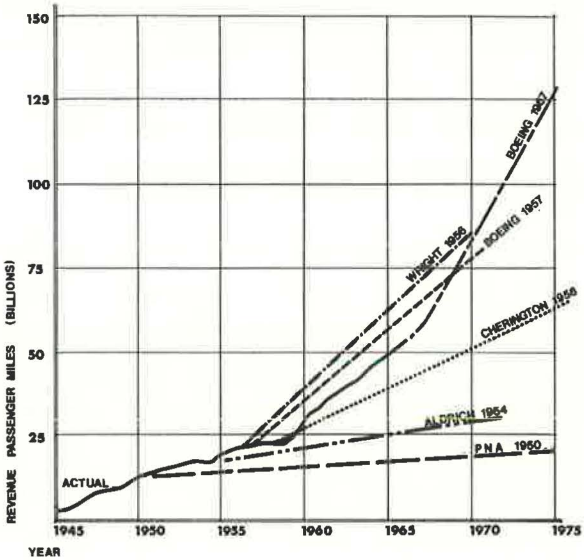

Figure 1. Comparison of forecast and actual air passenger-miles (U.S. domestic market, 3).

The components of the system included origin cities, destination cities, and all nonstop airway links between the origin and destination cities. The origin and destination cities are shown as links 001 and 011 in Figure 2. For any particular run of the model, 1 city acted as origin and all others as destinations. This allowed 1 line of an origin and destination table to be constructed per run. The origin or destination city included all major centers of population served by the airport. For example, link 006 is the London and Windsor region and included Woodstock, St. Thomas, Chatham, Sarnia, and Wallaceburg, as well as

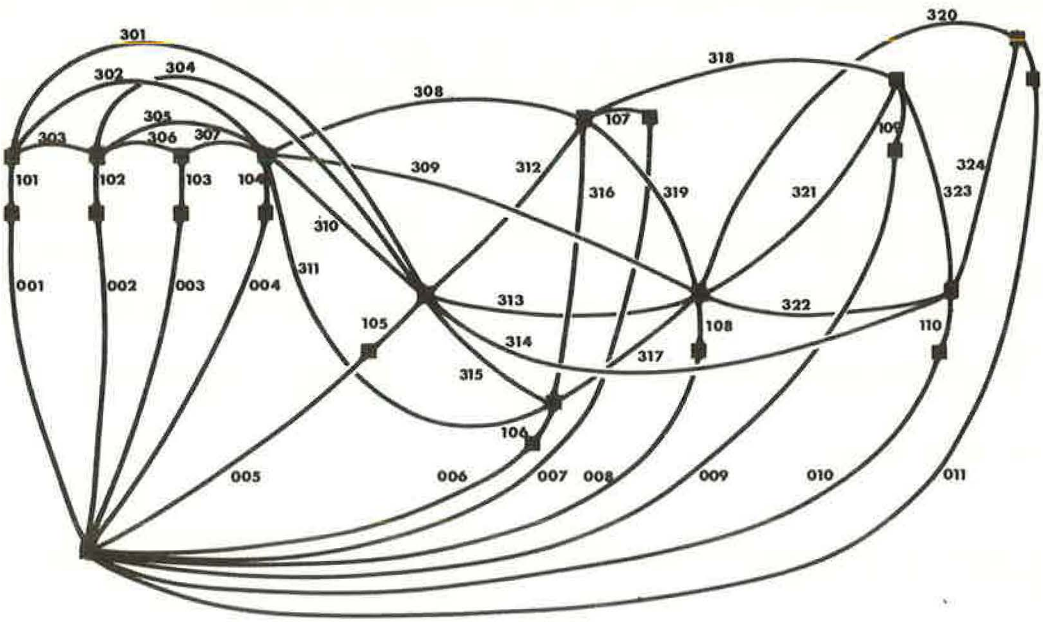

Figure 2. Systems graph for the Canadian domestic airway systems.

London and Windsor. Links 101 to 111 were measures of airport access and egress. Links 301 and 324 were the nonstop airway links between the cities.

Measurement on Components

Linear graph analysis requires that the following specifications be met:

1. The individual system components must be quantitatively describable by 2 fundamental variables. These variables are a flow variable y and a complimentary pressure variable x that causes flow.
2. The components are connected at their ends (vertexes) to yield a model for the entire system. The interconnected model must satisfy the 2 generalized Kirchoff laws. The first law states that the algebraic sum of all flows y at a vertex is zero. The second law states that the algebraic sum of all pressures around any closed loop of the system must be zero.
3. The flow and pressure variables must be related by a linear or nonlinear function.

The y-variable for the intercity air travel network is person trips per year. This satisfied the first Kirchoff law and eliminated the necessity of modeling for storage within the system. That is, all business travelers are assumed to return to their origin over the yearly period.

The x-variable is postulated to be a value measure used by the travelers in making an air trip. It is analogous to the portion of the travel potential of an origin area that is used up as a trip is made and thus is that pressure causing the flow.

The reasons for the preceding postulates are as follows:

1. If it is believed that the making of a trip can be simulated within a reasonable degree of accuracy, then it follows that there is some underlying process made by the traveler in making such a choice.
2. The traveler will act as a free agent and attempt to optimize his degree of satisfaction.
3. Relating reasons 1 and 2 to a value measurement used in travel allows the origin pressure to dissipate as the trip is made, thus satisfying the second Kirchoff law.

Terminal Equations of Components

Linear graph analysis requires that each link be individually described with a relationship between the flow and the pressure variables. This relationship is called the terminal equation of the component. For the airway model, 3 forms of terminal equations were required, one for each of the component types—destination, airway link, and origin.

Destination Area Components

The terminal equations of the destination cities can be expressed as

$$Y_k = A_k X_k \tag{1}$$

where

Y_k = number of business trips per year arriving at destination k,
A_k = attraction of city k, and
X_k = remainder of the perceived cost of travel that is used up across city k.

The attraction variable A_k for business trips was postulated to be a function of employment of the form

$$A_k = f\ (\text{employment}) \tag{2}$$

A recent study by Air Canada (4) provided information on the trip-making characteristics of various employment types. The study provided employment trip-making characteristics, and the following relationship was developed:

$$A = \phi \left(B_s \frac{e_{sk}}{e_{s,ave}} + B_H \frac{e_{Hk}}{e_{H,ave}} + B_L \frac{e_{Lk}}{e_{L,ave}} \right) \qquad (3)$$

where

A_k = relative attraction of a destination;
ϕ = calibration constant;
e_{sk}, e_{Hk}, e_{Lk} = employees of a destination city in the service, heavy, and light industries respectively;
$e_{s,ave}, e_{H,ave}, e_{L,ave}$ = number of employees in the service, heavy, and light industries respectively in the average city of the network;
B_s, B_H, B_L = trip attraction characteristics of each employment type (these were found to be 0.452, 0.362, and 0.185 respectively).

The employment factors $(e_{ik}/e_{i,ave})$ for the destination cities were derived from data of Dominion Bureau of Statistics (6).

The total number of employees in service, heavy, and light industries was calculated for each city. These were then divided by the average number of employees in the service, heavy, and light industries in the 11 cities in the system. These dimensionless numbers were then multiplied by the appropriate constant, and the attractions were derived. The attraction values are given in Table 1.

Air Link Components

The terminal equations of the air links were postulated to be of the form

$$Xij = R(y)\,Yij \qquad (4)$$

where

Xij = perceived value or cost used up by the business traveler in crossing the link,
Yij = flow in person per year across a link ij, and
R(y) = resistance to flow.

It is hypothesized that the resistance function can be expressed as

$$R(y) = C(y) + T(y) \qquad (5)$$

TABLE 1

DERIVED ATTRACTIONS FOR THE 11 AIRPORT REGIONS

Airport Region Code	Name	$0.452 \frac{e_s}{e_{s,ave}}$	$0.363 \frac{e_H}{e_{H,ave}}$	$0.185 \frac{e_L}{e_{L,ave}}$	Total Attraction
001	Vancouver	0.4342	0.1174	0.3599	0.9915
002	Edmonton	0.3838	0.1209	0.3179	0.8226
003	Saskatoon	0.1353	0.0566	0.0421	0.2340
004	Winnipeg	0.2454	0.1107	0.0600	0.4161
005	Toronto	1.4825	1.2356	0.2263	2.9444
006	London	0.2338	0.3361	0.0462	0.6161
007	Ottawa	0.1870	0.0157	0.1083	0.3110
008	Montreal	1.2101	1.6705	0.0462	3.0281
009	Quebec	0.1764	0.1525	0.0864	0.4153
010	Maritimes	0.4282	0.1624	0.4869	1.0775
011	Newfoundland	0.1001	0.0130	0.1190	0.2321

where

$C(y)$ = function of cost to cross a link in units of cents per person per mile; and
$T(y)$ = function of time to cross a link in units of minutes per person.

The functions $C(y)$ and $T(y)$ can incorporate the capacity and scheduling parameters of public carriers.

The cost function is of the form

$$C(y) = M\frac{c}{m} + \frac{c(q)}{m} \tag{6}$$

where

$C(y)$ = cost in cents per passenger per mile;

$\frac{c}{m}$ = cost in cents of the air fare per mile on a link;

$\frac{c(q)}{m}$ = quality of travel costs in cents for a link-mile; and

M = length of the link in miles.

It was assumed that travelers perceived cost directly. The cost term contains no model-calibration term. The quality of travel, $c(q)/m$, reflects the comfort and convenience of traveling by the air mode as compared to another mode. This cost at the present time is difficult to measure.

The time to cross a mile of travel link is given by the formula

$$t(y) = KM\frac{t}{m} + \frac{t(d)}{m} \tag{7}$$

where

$\frac{t}{m}$ = time in minutes per passenger to cross a link;

$\frac{t(d)}{m}$ = time delay in minutes per mile of travel link;

K = constant defining of the perceived travel time costs in cents per minute; and

M = length of the link in miles.

The term $M[t/m + t(d)/m]$ can be approximated with the mean journey time concept developed by Morlok (7).

A capacity constraint can be imposed on public carriers. The constraint states that, if available capacity is exceeded, no additional trips can occur on a particular link. The constraint can be stated in terms of available space as

$$ES = B_1 S_1 + B_2 S_2 + \ldots \tag{8}$$

where

ES = effective seats available; and
B_1 = proportion of available seats S_1 that are demanded in a given time period of the day.

The total resistance on a travel link is given by

$$\begin{aligned} R(y) &= C(y) + T(y) \\ &= M\left[\frac{c}{m} + \frac{c(q)}{m}\right] + KM\left[\frac{t}{m} + \frac{t(d)}{m}\right] \end{aligned} \tag{9}$$

or

$$R(y) = \infty \quad \text{for } y_{ij} > ES$$

where the terms are as previously defined. No capacity constraint problems were encountered in the networks examined in the project.

The airway link resistances were thus calculated by considering air fares, route travel times, and scheduled departure frequency. The values for these resistances are given in Table 2.

Airport Access Link Resistances

The airport access and egress link resistances (101 to 111) for business air passengers were calculated by considering travel costs, travel times, passenger processing times, and passenger insurance time (8). The resistances were calculated from Eq. 14 with K = 10.0 per minute. The choice of value of K is discussed in a following section. The access and egress resistance values are given in Table 3.

For destination airports within 300 miles of the origin area, it was necessary to modify the resistances of the airport egress links. Within this trip length range, there is considerable competition among air and other modes of travel. It was felt that some measure of competition should be included. Furthermore, this measure was included in the access links so that other traffic using the same airway link for longer trips would not be similarly penalized. The competition measure was a scalar that was multiplied by the access resistances. Thus, for the previously described egress links,

TABLE 2

RESISTANCES FOR CANADIAN DOMESTIC AIRWAY NETWORK

Link[a]	Time (min)	Cost (dollars)	$K_2 = 5$	$K_2 = 10$	$K_2 = 20$
301	972	109	15,760	20,620	30,340
302	696	63	9,780	13,060	20,220
303	270	32	4,550	5,900	8,600
304	870	89	13,250	17,600	26,300
305	606	43	7,330	10,360	16,420
306	408	29	4,940	6,980	11,060
307	294	26	4,070	5,540	8,480
308	876	63	10,680	15,060	23,820
309	876	63	10,680	15,060	23,820
310	408	52	7,240	9,280	13,360
311	912	50	9,560	14,120	23,240
312	144	19	2,620	3,340	4,780
313	138	23	2,990	3,680	5,090
314	834	48	8,970	13,140	21,480
315	246	15	2,730	3,960	6,420
316	798	28	6,790	10,780	18,760
317	852	40	8,260	12,520	21,040
318	540	22	4,900	7,600	13,000
319	132	11	1,760	2,420	3,740
320	864	60	10,320	14,640	23,280
321	192	13	2,260	3,220	4,140
322	408	28	4,890	6,930	1,010
323	792	22	6,160	10,120	18,040
324	864	37	7,020	12,340	20,980

[a]Figure 2 shows relative position of links.

TABLE 3

ACCESS AND EGRESS RESISTANCE VALUES FOR K = 10.0 CENTS PER MIN

Link	Access and Egress	Egress for < 300 Miles	Origin
101	1,510		
102	980		
103	1,020		
104	1,040		
105	1,115	13,500	London
106	1,320	13,500	Toronto
107	880	14,300	Montreal
108	1,160	14,300	Ottawa
109	1,710	12,000	Montreal
110	1,940		
111	1,940		

$$R_{egress} = SR_{link} \quad (10)$$

where

R_{egress} = resistance of the short-haul airport egress link (values are given in Table 3);
S = scalar that is postulated to account for the trade-offs made by business travelers in choosing air for a trip shorter than 300 miles (cost and time parameters were derived from Norhling 9); and
R_{link} = egress link resistance derived from Eq. 14.

Origin Area Components

The origin area can be characterized as a known flow driver of the form

$$Y_i = y_s \quad (11)$$

where

Y_i = flow from origin i in annual business trips; and
y_s = specified flow value taken from actual data.

The relationship is necessary in calibrating the model for a given network. The flow values were taken as the total flows originating in a city for destinations on the network.

As it will be shown, the origin areas are modeled as an individual link of the system. Thus, associated with the origin area flow is its complementary pressure variable. The magnitude of the pressure variable can be derived from the solution of the linear graph network. This pressure is related to the origin travel volumes because it is that pressure that was necessary to create the flows.

Therefore, the pressure variable of the origin area is postulated to be the travel potential that created the traffic volumes. As will be shown, the travel potential can be characterized by the following equation

$$X = A(IP) + B \quad (12)$$

where

X = travel potential in cost per year (which is the perceived cost used up as the trip is made);

I = annual average income of the origin area population P;
P = origin area population; and
A, B = constants (A implicitly describes the travel attributes required to make a business trip, and B implicitly describes the threshold value attached to the travel attributes before any trip can be made).

Systems Graph

The systems graph is a set of terminal graphs (i.e., a graphical representation of a component and its terminal equation) connected at the vertexes to form a one-to-one correspondence with the components of a physical system. Figure 2 shows a systems graph for the Canadian Domestic Airway System.

Systems Equations

To construct the air travel demand model required the derivation of both the chord and branch formulation equations of the system. Both of these formulation methods can be found elsewhere (11, 12, 13).

The graph procedure is illustrated by the following generalized results of the formulation techniques. From Figure 2, with city 001 as a known flow driver, the independent chord formulation equations can be written as

$$\begin{matrix} BRB^TY_c & + 0X_{c1} \\ Y_{c1} & U \end{matrix} = 0 \qquad (13)$$

where

B = matrix whose coefficients represent the manner in which the systems is connected (i.e., the coefficients of the cut-set equations);
B^T = transpose of B;
R = diagonal matrix (whose entries include the link resistances and the destination city attractions);
Y_{c1} = known flow value for city 001;
Y_c = column matrix whose entries are the unknown chord flows; and
X_{c1} = unknown pressure or travel potential for city 001.

Removing the last equation from the set makes it possible to derive the unknown chord flows. Substitution into the cut-set equations yields the branch flows. The unknown pressures are found by substitution into the terminal equations. The origin area travel potential can then be solved from the last equation of the set. The procedure is repeated for each city.

With the origin area travel potentials available, branch formulation equations can be derived. The branch equations with city 001 as the origin is given by

$$\begin{matrix} UY_{c1} & + ARA^TX_{B1} \\ 0 & XB \end{matrix} = 0 \qquad (14)$$

This system of equations can be solved for all pressures and flows. It is the branch model that is capable of expressing the generations of air trips in relation to the system parameters.

CALIBRATION OF THE MODEL

The model was calibrated to obtain a value of K for the resistance function. Three cities were chosen at random—one to represent a large city, one to represent a medium city, and one to represent a small city. The city names were grouped according to originating traffic volumes as follows: large volumes, Toronto and Montreal; medium volumes, Vancouver, Edmonton and Calgary, Ottawa, and Winnipeg; small volumes, Atlantic Provinces, Saskatoon and Regina, London and Windsor, Quebec, and Newfoundland. One name was then selected from each group: Montreal (large), Vancouver (medium), and London and Windsor (small).

TABLE 4

ONE-WAY AIR PASSENGER BUSINESS TRIPS IN 1964

Origin[a]	Destination[a]											Total
	001	002	003	004	005	006	007	008	009	010	011	
001	x	30,137	6,080	11,897	12,053	3,238	224	5,218	1,082	1,326	511	73,766
002	18,932	x	10,784	10,277	7,034	2,227	1,809	3,848	457	1,160	339	56,867
003	2,019	6,656	x	4,524	1,852	740	524	1,123	257	222	122	18,039
004	5,540	7,796	7,147	x	7,532	4,740	2,843	5,051	1,284	1,541	479	43,953
005	25,570	26,127	11,338	39,043	x	19,667	52,310	97,108	24,106	36,127	9,293	340,689
006	866	1,276	625	2,200	1,094	x	2,412	4,838	1,341	1,281	468	16,421
007	1,439	1,936	1,249	3,872	15,868	5,615	x	2,670	6,853	3,779	1,712	44,993
008	7,352	8,485	5,050	20,204	71,300	30,355	10,043	x	17,426	42,188	16,244	228,647
009	253	208	142	589	2,414	824	1,652	4,298	x	1,936	486	12,796
010	397	497	252	692	4,029	1,284	1,475	6,083	3,181	x	3,038	20,928
011	166	131	108	276	1,135	291	536	3,139	771	3,273	x	9,826

Source: Canada Yearbook, 1967 (18).

[a]Names of origins and destinations identified by 3-digit code are given in Table 1.

TABLE 6

MODEL RESULTS FOR K = 10.0

Origin	Destination											Total
	001	002	003	004	005	006	007	008	009	010	011	
001	x	29,838	6,069	12,729	11,552	3,451	2,377	4,736	1,198	1,396	416	73,762
002	17,571	x	10,878	10,624	7,943	2,570	1,755	3,500	522	1,000	304	56,867
003	2,042	6,218	x	4,954	1,709	833	550	1,099	273	271	90	18,039
004	5,697	8,075	6,588	x	7,672	4,299	2,814	5,624	1,392	1,331	455	43,947
005	24,669	28,966	11,298	38,724	x	19,079	49,957	98,928	25,516	34,230	9,314	340,682
006	964	1,189	605	2,278	1,075	x	2,489	4,837	1,226	1,343	415	16,421
007	1,531	1,920	1,053	4,051	14,608	6,352	x	2,346	6,631	4,741	1,781	44,993
008	7,558	9,481	5,204	20,021	72,022	30,644	11,278	x	15,839	40,182	16,416	228,645
009	202	252	136	524	1,964	813	1,583	4,726	x	2,091	499	12,790
010	351	426	203	747	4,021	1,209	1,530	6,327	3,113	x	3,006	20,933
011	114	141	73	277	1,170	432	577	2,967	808	3,264	x	9,823

TABLE 7

RESULTS OF STATISTICAL TESTS ON AIRWAY MODEL

Origin	Destination										
	001	002	003	004	005	006	007	008	009	010	011
001	x	-0.99	-0.18	6.99	-4.16	6.58	6.88	-9.24	10.72	5.28	-18.59
002	-7.19	x	6.87	3.38	12.92	15.40	-2.98	09.04	14.22	-13.79	-10.32
003	1.14	-6.58	x	9.50	-7.72	12.57	4.96	-2.14	6.23	22.07	-26.23
004	2.83	3.58	-7.82	x	1.86	-9.30	-1.02	11.34	8.41	-13.63	-5.01
005	-3.52	10.87	-0.35	-0.82	x	-2.99	-4.50	1.87	5.85	-5.25	0.23
006	11.32	-6.82	-3.20	3.55	-1.74	x	3.19	-0.02	-8.57	4.84	-11.32
007	6.39	-0.83	-15.69	4.62	-7.94	13.13	x	-12.13	-3.24	25.46	4.03
008	2.80	11.74	3.05	-0.91	1.01	0.95	12.29	x	-9.10	-4.75	1.06
009	-20.16	21.15	-4.22	-11.04	-18.64	-1.33	-4.18	9.96	x	8.01	2.67
010	-11.59	-14.29	-19.44	7.95	-0.20	-5.84	3.73	4.01	-2.14	x	-1.05
011	-31.33	7.63	-32.41	0.36	3.08	48.45	7.65	-5.48	4.70	-0.27	x

Note: Average percentage of error = 7.90; root mean square error = 11.01; and algebraic sum of errors = -0.18.

TABLE 8

AIRWAY LINK ASSIGNMENTS TOTAL ONE-WAY PASSENGER TRIPS FOR MODEL K = 10.0

Link	Origin											Total
	001	002	003	004	005	006	007	008	009	010	011	
301	13,787	1,651	136	175	20,506	600	838	4,130	113	239	68	92,243
302	20,050	984	763	5,374	4,237	346	652	3,224	83	106	42	35,861
303	39,928	20,207	1,415	498	74	18	41	203	5	4	2	62,395
304	2,768	8,709	634	373	24,051	696	968	4,771	130	279	79	43,458
305	2,536	12,749	156	6,491	5,384	426	798	3,948	102	131	52	32,773
306	4,784	15,364	8,113	2,456	544	84	194	964	24	19	11	32,557
307	1,284	4,484	9,929	9,044	10,753	689	1,248	6,169	161	222	84	44,067
308	1,959	1,697	864	4,877	5,656	210	3,916	9,017	323	263	100	28,882
309	2,137	1,828	905	5,086	1,860	12	1,449	17,691	360	472	236	32,036
310	2,417	2,284	1,377	7,955	39,606	846	944	4,640	133	382	91	60,675
311	2,059	1,784	908	5,122	11,976	2,672	439	2,015	54	88	29	27,146
312	2,117	1,307	70	107	84,540	1,402	15,038	27,769	1,085	123	198	133,756
313	2,649	1,723	230	756	92,265	2,084	3,551	60,709	1,136	970	738	166,811
314	975	661	130	559	29,603	839	585	773	237	4,407	581	39,350
315	1,679	1,008	8	376	50,110	7,545	646	3,689	117	578	108	65,864
316	39	34	18	104	7,785	3,206	4,896	9,958	379	250	101	26,770
317	247	187	65	341	11,269	2,999	1,248	19,010	371	468	251	36,456
318	631	467	148	761	13,088	676	5,811	4,033	3,142	863	126	29,746
319	1,106	816	254	1,299	23,624	1,233	15,354	53,992	229	1,304	850	100,061
320	304	226	73	377	6,093	314	1,584	15,411	267	639	4,675	29,963
321	659	490	159	821	13,137	669	2,177	31,059	7,244	1,056	340	57,811
322	441	339	124	659	7,138	485	2,995	30,771	152	7,841	709	51,654
323	92	76	34	190	708	119	1,358	11,189	2,409	5,033	594	21,802
324	112	77	17	77	3,220	100	197	1,004	232	3,645	5,148	13,829

TABLE 9

DATA FOR ORIGIN AREA TRAVEL POTENTIAL FUNCTION

Origin	Derived Travel Potential (millions)	Origin Area Travel Volumes (thousands)	Origin Area Average 1964 Weekly Salary[a]	Origin Area 1964 Population[b] (millions)	Product of income and Population (millions)
001	413.098	73.766	90.47	1.465	133.1
002	220.252	56.867	83.48	1.069	87.0
003	89.244	18.039	74.90	0.516	38.6
004	130.502	43.953	76.28	0.615	47.0
005	866.723	340.689	92.82	3.053	281.0
006	66.457	16.421	83.09	0.650	53.9
007	111.924	44.993	80.72	0.646	52.1
008	597.808	228.647	85.89	2.328	200.0
009	52.253	12.796	74.08	0.403	29.8
010	104.328	20.928	73.21	0.600	43.9
011	92.421	9.826	66.72	0.253	16.8

[a]See (18).
[b]See (19).

increase. This fact is more or less substantiated by the regression equation. The population term merely reflects the number of people available to make trips.

There are 2 points outside of the 95 percent confidence level envelopes and these are Newfoundland (011) and London and Windsor (006). Because of its physical location, Newfoundland would require a greater travel potential to produce the same amount of trips as other areas. The travel resistance to make a trip is higher because the trips are longer. London and Windsor are in close approximation to heavily populated areas of New York State and Michigan. It can be concluded that a disproportionate number of air trips from this area are attracted to the United States. Therefore, the travel potential suggested by the regression function would create more trips on the airway system than were actually made.

APPLICATION OF THE BRANCH FORMULATION MODELS

Air Business Travel Demands

Branch formulation models were constructed for each origin area on the Canadian Domestic Airway System. The origin areas were modeled as pressure drivers, as discussed previously. The values of the pressure drivers were taken as the traffic potentials derived from the chord formulation models (Table 9). The branch formulation models permit demand elasticities to be calculated and demand curves for any particular origin and destination pair to be derived.

Income Elasticities

Sensitivity tests on the travel potential (and the total annual air business trips) generated by the origin areas were conducted with respect to the income variable. The income variable was changed for a number of selected cities. The resultant changes in travel potential and traffic volumes are given in Table 10. The income elasticities of Table 10 were calculated from

$$TP, I = \frac{TP_1 - TP_2}{I_1 - I_2} \frac{I_1}{TP_1} \tag{17}$$

where

TP, I = elasticity of travel potential or travel volume with respect to the income variable (the reported elasticities are for business air trips);
TP_1, TP_2 = original and final values of travel potential (flows);
I_1, I_2 = origin and final values of the average weekly salary.

In each case, the income term was increased by 10 percent. The resultant elasticities of travel volumes (Table 10) range from 2.01 for the Maritime Provinces to 1.00 for the Toronto airport area. The interpretation of the variation in the elasticities of income is as follows:

1. The lower travel potentials are associated with areas of relatively low incomes and population. An increase of 10 percent in average income would result from a large increase in economic activity. Therefore, the effect on business air travel volumes would also be large.
2. In an area of high economic activity such as Toronto, an increase of 10 percent in average salary may not reflect as great an increase in business transaction. Therefore, the increase in travel would be relatively inelastic.

Elasticity of Air Travel Costs and Times

The cost and time elasticities of air business travel were derived from the branch formulation models. The cost and time elasticities were derived for the Toronto and Montreal city pair. The origin area travel potentials were used as pressure drivers, and the air fare for link 313 (air route between Toronto and Montreal, Fig. 2) was increased and decreased by 5 and 10 percent with travel time remaining constant. The procedure was then reversed and the air fare was fixed while the travel time was changed ±5 and ±10 percent.

The resultant changes in travel volumes are given in Table 11. As would be anticipated, a decrease in air fare produces an increase in travel volumes. The anticipated increase is 5,100 annual yearly passengers for a decrease of 10 percent in air fare. The calculation of elasticity of business air trips produces a value of A, p = -0.30. Therefore, business air travel is inelastic by this model. This is in accordance with studies by a number of authors (14, 16) that business air travel is inelastic.

Business air travel is also inelastic with respect to travel time. The change in volume is 4,100 annual air business passengers for a change of 10 percent in the travel time. The change in travel time could correspond to a change in departure schedules as well as decrease in actual running time. The value of the elasticity is A, T = 0.24 and again is inelastic. One author (16) concluded that the time elasticity was greater than the cost elasticity. However, a 10 percent decrease in travel time amounts to

TABLE 10

RESULTS OF SENSITIVITY TESTS ON INCOME

City	Original Income	Final Income	Travel Potential[a]		Travel Volumes[b]		Elasticity of Traffic Volumes
			Original[c]	Final[d]	Original	Final	
Atlantic	73.21	80.53	104.3	125.0	20.9	25.1	2.01
Winnipeg	76.28	83.91	130.5	140.0	44.0	53.8	1.85
Vancouver	90.47	99.52	375.1	419.8	73.8	82.6	1.20
Montreal	85.89	94.48	595.8	658.1	228.6	255.0	1.05
Toronto	92.82	102.10	860.7	944.7	340.7	374.8	1.00

[a]In millions of cost units.
[b]In thousands of annual trips.
[c]From regression relationship.
[d]Original value plus 10 percent.

TABLE 11

DERIVED CHANGES IN TOTAL ANNUAL ONE-WAY BUSINESS PASSENGER VOLUMES FOR THE TORONTO-MONTREAL AIR ROUTE

Change	Cost		Time	
	Volume Change[a]	Elasticity	Volume Change[a]	Elasticity
-0.10	+5,100	-0.30	+4,100	-0.24
-0.05	+2,550	-0.30	+2,050	-0.24
0.00	—	—	—	—
+0.05	-2,550	-0.30	-2,050	-0.24
+0.10	-5,100	-0.30	-4,100	-0.24

[a]The volume changes are the total of generated plus diverted traffic.

approximately 14 min on link 313. With a value of K = 10 cents per minute, the 14 min represents $1.40. However, a 10 percent decrease in air fare is equal to a savings of $2.30. Therefore, in consideration of the preceding cost factors, the elasticity measurement of the branch model appears valid.

The elasticities incorporated in the model result not from the resistance change of the particular link but from the change of equivalent resistance for the origin, access and egress, and the destination links. The elasticities are therefore more or less constant for a city pair. With large changes of cost and time, the model may be in error. The assumed resistance functions may deviate considerably from the real world in the area of large cost-of-time changes. However, the range of the sensitivity measures should be adequate for most planning purposes.

CONCLUSIONS

The travel demand simulation procedure described by the chord formulation does not differ significantly from existing models with respect to the following:

1. There are coefficients of the model that must be calibrated;
2. The calibration constants are assumed to remain constant over the planning horizon; and
3. The origin flow values (or travel potentials) for the planning horizon must be estimated through some type of regression formulation.

However, the chord equation model does offer some advantages and these are as follows:

1. Generation, distribution, and assignment are considered as interdependent and are completed simultaneously for each origin;
2. The model considers the competitive attractions of all destinations on the system with respect to each origin;
3. The travel links are mathematically described by their time and cost parameters;
4. The model quantifies the interconnections of many of the variables relating to demand, and the variable and interconnection measurements are achieved at the aggregate level; and
5. The model is analytic and, therefore, no interactions or balancing procedures are required to determine the travel volumes on each link.

The nature of the sensitivity attributes of the branch formulation model offer some important features to the transportation planner, including the following:

1. Elasticity measures for system or origin attributes can be derived; and

2. The generation of trips due to changes in the system attributes can be identified.

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