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THE n-DIMENSIONAL LOGIT MODEL: DEVELOPMENT AND APPLICATION

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This paper discusses the development, calibration, and testing of the n-dimensional logit model, a modal-split model that appears to appropriately respond to the n-mode situation and that, at least conceptually, addresses the new mode problem. The model is based on the assumption that the ratio of a small change in modal split of a given mode to that of a given transportation variable is proportional to the modal split of this mode and to a linear function of the modal split of all modes. In conjunction with the mathematical definition relating to modal split, these assumptions lead to a system of differential equations defining modal-market shares as functions of the transportation attributes. This formulation does not require that the same set of variables be used to define the transportation attributes of each of the modes. The model was applied to predict the market shares of 4 modes—private automobile, rental car, taxi, and limousine—used for access (or egress) to (or from) airports in the Baltimore-Washington area. Calibration results indicate that the coefficients estimated have the expected signs and relative magnitudes as well as low standard errors of estimate. Testing indicates that the model displays the appropriate sensitivities and adequately reproduces the aggregate trips for each mode. Overall, results of this initial work are sufficiently promising that further application of the model in both operational and experimental situations appears warranted.

•THE INCREASE IN RECOGNITION of the dollar and social costs associated with programs mainly advocating the construction of urban freeways have led planners and decision-makers to seek other strategies for alleviating the urban transportation problem. Many urban transportation concepts, ranging widely in the extent of technological and institutional innovation required for implementation, have been proposed in recent years.

However, it is not at all clear that current modal-split models are capable of adequately predicting passenger demands for these new transportation systems. Aside from the difficulties inherent to introducing a new mode in a model structure, many of the existing modal-split models are designed to deal with dual choices between the 2 current urban transportation modes, automobile and public transit. This limitation obliges the analyst to lump into the public-transit category 2 or 3 modes with widely differing characteristics. It requires the a priori definition of aggregate transit impedances and, in most cases, the breakdown of the nonautomobile market into specific transit categories.

In view of the preceding, it appears desirable to develop a modal-split model that has the capability to predict simultaneously the market shares of 2 or more modes operating in a competitive environment. This paper discusses the development, calibration, and testing of the n-dimensional logit model, a modal-split model that appears to appropriately respond to the n-mode problem and that, at least conceptually, addresses the new mode problem.

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The model discussed in this paper belongs to a general class of models known as stimulus-response models (1). Such models have been previously used in transportation studies. Two early examples of stochastic disaggregate modal-choice models are those of Lisco (2) and Stopher (3). Lisco used probit analysis to derive the value of time for commuters in the Chicago area. Stopher used logit analysis to model the modal choice of work trips in London, England. In a recent paper, Stopher and Reichman (4) have given an excellent discussion of the modal-split problem, focusing particularly on the potential of disaggregate stochastic modal-choice models.

MODEL DEVELOPMENT

Let w and x identify respectively the dependent variables (modal splits) and the independent variables (transportation or eventually dummy variables). Let m be an index identifying a given mode (m = 1, ..., j, ..., k, ..., M) and i be an index identifying a given transportation attribute, e.g., time or cost (i = 1, ..., I). Hence, the share of mode m is noted w_m and the ith attribute of mode j is noted x_{ij}.

Two types of assumptions are required. The first assumption pertains to the general nature of any modal-split problem: (a) The modal split of each mode included is between 0 and 1, and their sum is equal to unity; (b) modal splits are monotonic functions of the independent variables; and (c) if the transportation variables are expressed in units such that the disutility of traveling by a given mode is an increasing (decreasing) function of its transportation variables, then the share of that mode decreases (increases) when any of its transportation variables increases (decreases) and, all other things equal, those of the other modes increase (decrease). The second pertains to the specific premises postulated to structure the relationship between modal split and the explanatory transportation variables; that is, the ratio of a small change in modal split of a given mode to that of a given transportation variable is proportional to the modal split of this mode and to a linear (homogeneous) function of the modal splits of all modes. If it is assumed that these are the requisite continuity and derivability conditions, this relationship is expressed mathematically as

$$\frac{\partial w_m}{\partial x_{ij}} = w_m \sum_k \alpha_{ijmk} w_k \tag{1}$$

where α_{ijmk} is a coefficient to be determined. A total of (M x I x M) such equations can be written for all possible permutations of m, i, and j. Together they constitute a system of differential equations defining the functions w_m. However, before these equations are integrated, the nature of the coefficients α_{ijmk} should be examined in the context of the assumptions formulated previously. It can be shown that several classes of coefficients α are, in fact, equal to 0 and that there exist very simple relationships among those that are different from 0. Specifically, the coefficients α are such that $\alpha_{ijkm} = 0$ if $(j \neq k \neq m)$, $\alpha_{ijkj} = -\alpha_{ijjk}$ if $(j \neq k)$, $\alpha_{ijkk} = 0$, and $\alpha_{ijjk} = \alpha_{ijjm} = \alpha_{ij}$. When the preceding constraints are considered, the underlying assumption of the model as stated by Eq. 1 becomes

$$\frac{\partial w_m}{\partial x_{ij}} = \Delta_{jm} w_m \tag{2}$$

where

$$\Delta_{jm} = \begin{cases} -\alpha_{ij} w_j & \text{if } (m \neq k) \\ \alpha_{im}(1 - w_m) & \text{if } (m = k) \end{cases}$$

It can be shown that the solution of the differential system defined by Eq. 2 is

$$w_m = \frac{\exp\left(\sum_i \alpha_{im} x_{im} + a_m\right)}{\sum_j \exp\left(\sum_i \alpha_{ij} x_{ij} + \alpha_j\right)} \tag{3}$$

where α_j is a mode-specific (integration) constant.

It should be noted that the sign of the coefficient α_{ij} depends on the nature of the variable x_{ij} to which it is attached. For example, if i corresponds to frequency, then α_{im} must be positive because the share of mode m increases as its frequency increases. Conversely, if i corresponds to travel time, α_{im} must be negative in order for the modal split of m to increase when its travel time is improved. These prerequisite sign conditions must be met by the coefficients derived from calibration. If this were not the case, the prediction capability of the model would be poor to begin with, inasmuch as decreases in the level of service of a given mode would result in higher market share for that mode. Finally, it should be observed that no assumption must be made as to the number of attributes considered for each mode. This feature introduces into the analysis a degree of flexibility lacking in models incorporating ratios or differences of variables.

Equation 2 can be used to derive the elasticity of a given w_m with respect to a given x_{ik}, as in the following:

$$\frac{x_{ik}}{w_m}\frac{\partial w_m}{\partial x_{ik}} = \Delta_{km} x_{ik} \tag{4}$$

In other words, Eq. 4 shows that the elasticity of w_m with respect to x_{ik}, a transportation variable of another mode k, is proportional to x_{ik} and to the share w_k of that mode; the elasticity of w_m with respect to one of its own transportation variables x_{im} is proportional to x_{im} and to the market share that mode m can still gain $(1 - w_m)$.

CALIBRATION

Two alternatives are available for calibrating the proposed modal-split model—simultaneous least squares and maximum likelihood. Because of the additive structure of the denominator of Eq. 3, the model does not lend itself to least squares calibration. However, if we single out 1 mode, say mode u, and call it a base mode, then the natural logarithm of the ratio of the modal split of any given mode m to that of mode u is a linear function of the independent variables. If there are M modes competing for a given market, then (M - 1) such equations have to be calibrated, one for each mode except mode u. This leads to (M - 1) sets of coefficients for mode u. Because these coefficients may vary substantially, it is suggested that, instead of minimizing the square error for each equation considered separately, the square error of all the equations considered simultaneously be minimized.

The second alternative involves maximizing the likelihood function of the logit function. In this case, contrary to the preceding, individual rather than aggregate observations are required. When only aggregate observations are available, they can be converted to individual observations if the number of trips is known. If not, and this would imply giving equal weights to each zone pair, a common trip number can be used.

More specifically, the quantity maximized is the logarithm of the likelihood function. The coefficients are determined by successive iterations, the first estimate of the likelihood being that of the multinomial model, that is to say, the average modal splits of the sample. The method allows establishing constraints between any subset of the coefficients. However, because of its iterative nature and, in some cases, the large number of observations to be processed, this method is more time-consuming than the simultaneous least squares method that requires summing variables and cross products thereof, followed by the solution of a system of linear equations.

MODEL APPLICATION

Trip Stratification

The n-dimensional logit model was calibrated with data collected by the Washington-Baltimore airport access survey. Essentially, 2 trip-purpose stratifications were considered, namely, business and nonbusiness trips. This initial stratification is dictated by the behavior of business travelers who emphasize access time and somewhat disregard cost, whereas the opposite is true for nonbusiness travelers. A further breakdown within this initial stratification is justified by the markedly different overall trip patterns generated by the origin end of a trip. As can be expected, the private automobile is an even more convenient mode for departing travelers than for arriving travelers. Table 1 gives average modal splits for the stratifications (i.e., business and nonbusiness travelers going from and to the airport and for 5 modes: private car, rented car, taxi, limousine, and public bus. The business trip market is shared, to a very large extent, by the first 4 modes inasmuch as bus trips account for less than 2 percent of the total trips in either direction. Because of this situation, only 4 modes were retained in the analysis; that is, private car, rented car, taxi, and limousine. Similarly, in the case of nonbusiness trips, the rented car, which accounts for about 4 percent of the total trips in both directions, was not included in the analysis and, hence, 3 modes were retained—private car, taxi, and limousine.

The modal-split analysis performed in this study is, to a large extent, comparable to those analyses performed in classic urban transportation studies. These studies have generally attached a great significance to the nature of the trip end, e.g., by distinguishing between trip ends to the home and to other destinations. In the present analysis, the corollary is whether a traveler resides in the area under study. This distinction is important inasmuch as it generally determines the availability of a private car. This important information was not collected by the study. Had it been available, the stratification by residents and nonresidents would have precluded, for most cases, the use of private car for the nonresident traveler. An attempt was made to supplement this deficiency by assuming that, in all likelihood, most nonresident travelers would originate in or be destined to the central business districts and, therefore, that trips should be further broken down by central business districts and residential districts. However, this approach did not improve the quality of the business-to-airport model to which it was applied.

Data Preparation

Impedance estimation is open to a considerable range of detail and specification, particularly with respect to perceived impedances vis-a-vis actual-time measures and cost estimates based on marginal or average cost models. Component measures of the travel service provided by each mode were derived from network analysis, fare and

TABLE 1

AIR TRAVELER MODAL SPLIT BY DIRECTION

Mode	Percentage of Airport Trips			
	Business		Nonbusiness	
	To	From	To	From
Private automobile	45.4	32.6	63.5	57.1
Rented automobile	9.2	10.5	3.7	4.3
Taxi	32.1	31.2	22.7	19.9
Limousine or coach	12.3	23.9	8.3	16.3
Bus	1.0	1.8	1.8	2.4

time schedules, ground counts at airports, and data reported and processed from the Baltimore-Washington airport access survey. The component measures were selectively combined to form impedance measures for each mode as shown in Figure 1. The set of impedance measure equations used in the calibration are shown in Figure 2. The effects of changes in component service measures (e.g., parking rates) on aggregate impedance measures can be explicitly determined by evaluating these equations.

A data set was constructed for the travel survey period in 1966. This consisted of travel volumes and average impedance measures for each mode between each of the 78 zones and each of the 3 airports. For each mode, the time components were divided into main modal and submodal times (e.g., limousine riding time and access to limousine time) and in-vehicle and out-of-vehicle times. Nine different time measures were available from this classification, as given in Table 2. Travel cost was simply broken down into submodal cost, main modal cost, and total cost. Impedances used for calibration are average daily values and are the same for both purposes.

An observation should be made concerning the nature of the time and cost impedances that, in many instances, were derived from distance measures. As might be expected, time and cost are correlated and the resulting collinearity is a source of problems because it increases, sometimes substantially, the standard error of estimate of the coefficients.

Model Calibration

Several prototype models, involving different combinations of time and cost variables, were tested initially. Specifically, business models were calibrated by maximum likelihood, with the following variables:

1. Total times and costs for all modes;
2. Total time and parking cost for the private automobile and total times and costs for the remaining 3 modes;
3. Total times and costs for all modes except limousine for which in-vehicle time, out-of-vehicle time, and total cost were incorporated; and
4. Total times and costs for all modes except limousine for which in-limousine time, out-of-limousine time, and total cost were incorporated.

These models were calibrated for each directional stratification. In most cases, the signs of the coefficients were acceptable. However, the standard errors were often large and even, in some instances, exceeded the magnitude of the corresponding coefficient. To overcome these problems, the directional stratification was abandoned and, as mentioned earlier, the sample was stratified by central business districts and residential districts. Neither approach provided acceptable results.

TABLE 2

TRIP TIME COMPONENTS

Travel Classification	Modal Classification		
	Submode	Main Mode	Total
In vehicle	Submode in-vehicle time	Main mode in-vehicle time	Total in-vehicle time
Out of vehicle	Submode out-of-vehicle time	Main mode out-of-vehicle time	Total out-of-vehicle time
Total	Total submode time	Total main mode time	Total travel time

COMPONENT SERVICE MEASURES	No.	PC: 1 MAJOR MODE IN-VEHICLE TIME	PC: 2 MAJOR MODE OUT OF VEHICLE TIME	PC: 3 MAJOR MODE COST	RC: 4 MAJOR MODE IN-VEHICLE TIME	RC: 5 MAJOR MODE OUT OF VEHICLE TIME	RC: 6 MAJOR MODE COST	TX: 7 MAJOR MODE IN-VEHICLE TIME	TX: 8 MAJOR MODE OUT OF VEHICLE TIME	TX: 9 MAJOR MODE COST	LM: 10 MAJOR MODE IN-VEHICLE TIME	LM: 11 SUB MODE IN-VEHICLE TIME	LM: 12 MAJOR MODE OUT OF VEHICLE TIME	LM: 13 SUB-MODE OUT OF VEHICLE TIME	LM: 14 MAJOR MODE COST	LM: 15 SUB-MODE COST	LM: 16 FREQUENCY
IN-VEHICLE TIME - PC	1	●			●			●									
IN-VEHICLE DISTANCE - PC	2			●			●										
TOLL CHARGE - PC	3			●			●									●	
NON-APT TERM TIME - PC	4		●			●								●			
NON-APT PARK CHARGE - PC	5			●			●									●	
APT NON-PARK TERM TIME - PC - TO APT	6		●														
APT NON-PARK TERM TIME - PC - FROM APT	7		●														
APT PARK TERM TIME - PC - TO APT	8		●														
APT PARK TERM TIME - PC - FROM APT	9		●														
APT PARK RATE - PC	10			●													
APT PARK LOT DURATION PC	11			●													
APT METER PARK CHARGE - PC	12			●													
APT METER PARK TERM TIME - PC - TO APT	13		●														
APT METER PARK TERM TIME - PC - FROM APT	14		●														
APT VALET PARK TERM TIME - PC - TO APT	15		●														
APT VALET PARK TERM TIME - PC - FROM APT	16		●														
APT VALET PARK RATE - PC	17			●													
APT VALET PARK DURATION - PC	18			●													
Σ PARK RIDE - PC - TO APT	19		●	●			●										
Σ METER PARK - PC - TO APT	20		●	●			●										
Σ KISS RIDE - PC - TO APT	21		●	●			●										
Σ VALET PARK - PC - TO APT	22		●	●			●										
Σ PARK RIDE - PC - FROM APT	23		●	●			●										
Σ METER PARK - PC - FROM APT	24		●	●			●										
Σ KISS RIDE - PC - FROM APT	25		●	●			●										
Σ VALET PARK - PC - FROM APT	26		●	●			●										
OCCUPANCY - PC	27			●			●									●	
APT TERM TIME - RC - TO APT	28					●											
APT TERM TIME - RC - FROM APT	29					●											
TRIP COST 1PASS-TX-TO APT	30									●							
TRIP COST 2PASS-TX-TO APT	31									●							
TRIP COST 1PASS-TX-FROM APT	32									●							
TRIP COST 2PASS-TX-FROM APT	33									●							
APT TERM TIME - TX - TO APT	34								●								
APT TERM TIME - TX - FROM APT	35								●								
NON-APT TERM TIME - TX - TO APT	36								●					●			
NON-APT TERM TIME - TX - FROM APT	37								●					●			
PASS OCCUPANCY - TX	38									●							
IN-VEHICLE TIME - LM	39										●						
APT TERM TIME - LM - TO APT	40												●				
APT TERM TIME - LM - FROM APT	41												●				
WAIT TIME - LM	42												●				
IN-VEHICLE COST - LM	43														●		
LIMO ACC/EGR IN-VEHICLE TIME - PC SUB-MODE	44											●					
LIMO STOP TERM TIME-PKG.-PC SUBMODE	45													●			
LIMO STOP TERM TIME-NO PKG.-PC SUBMODE	46													●			
LIMO STOP TERM TIME-TO STOP-TX SUBMODE	47													●			
LIMO STOP TERM TIME-FR STOP-TX SUBMODE	48													●			
LIMO STOP PARK COST - PC SUB-MODE	49															●	
LIMO ACC/EGR IN-VEHICLE DISTANCE-PC SUB-MODE	50															●	
LIMO ACC/EGR IN-VEHICLE COST - BS SUB-MODE	51															●	
LIMO ACC/EGR. IN-VEHICLE TIME - BS SUB-MODE	52											●					
LIMO ACC/EGR. TRANSFER TIME - BS SUB-MODE	53													●			
NON-APT WAIT TIME - BS SUB-MODE	54													●			
LIMO ACC/EGR COST - TX SUB-MODE	55															●	
LIMO ACC/EGR WALK TIME - WALK SUB-MODE	56													●			
LIMO STOP WALK TIME - ALL SUB-MODES	57													●			
Σ LIMO ACC/EGR PARK RIDE - PC - SUB-MODE	58											●		●		●	
Σ LIMO ACC/EGR KISS RIDE - PC - SUB-MODE	59											●		●		●	
Σ LIMO ACC/EGR - TX SUB-MODE	60											●		●		●	
Σ LIMO ACC/EGR - BS SUB-MODE	61											●		●		●	
Σ LIMO ACC/EGR - WALK SUB-MODE	62													●			
FREQUENCY - LM	63																●
NON APT WALK TIME - BS	64													●			

PC = PRIVATE CAR
RC = RENTAL CAR
TX = TAXI
LM = LIMOUSINE
BS = BUS

Figure 1. Modal impedance measures.

$$Y_{1,t} = Y_{1,f} = X_1$$

$$Y_{2,t} = X_4 + (X_6)(X_{21}) + (X_8)(X_{19}) + (X_{13})(X_{20}) + (X_{15})(X_{22})$$

$$Y_{2,f} = X_4 + (X_7)(X_{25}) + (X_9)(X_{23}) + (X_{14})(X_{24}) + (X_{16})(X_{26})$$

$$Y_{3,t} = \left[(X_2)(\alpha_1) + X_3 + X_5 + (X_{10})(X_{11})(X_{19}) + (X_{17})(X_{18})(X_{22})\right]\left[\frac{X_{19} + X_{22}}{X_{27} - (X_{21} + X_{20})(2.0)}\right] + \left[(X_2)(\alpha_1) + X_3 + X_5\right](X_{20} + X_{21}) + (X_{12})(X_{20})$$

$$Y_{3,f} = \left[(X_2)(\alpha_1) + X_3 + X_5 + (X_{10})(X_{11})(X_{23}) + (X_{17})(X_{18})(X_{26})\right]\left[\frac{X_{23} + X_{26}}{X_{27} - (X_{25} + X_{24})(2.0)}\right] + \left[(X_2)(\alpha_1) + X_3 + X_5\right](X_{24} + X_{25}) + (X_{12})(X_{24})$$

$$Y_{4,t} = Y_{4,f} = X_1 \qquad Y_{5,t} = X_4 + X_{28} \qquad Y_{5,f} = X_4 + X_{29}$$

$$Y_{6,t} = \left[(X_2)(\alpha_2) + X_3 + X_5 + \alpha_3\right]\left[\frac{X_{19} + X_{22}}{X_{27} - (X_{21} + X_{20})(2.0)}\right]$$

$$Y_{6,f} = \left[(X_2)(\alpha_2) + X_3 + X_5 + \alpha_3\right]\left[\frac{X_{23} + X_{26}}{X_{27} - (X_{25} + X_{24})(2.0)}\right]$$

$$Y_{7,t} = Y_{7,f} = X_1 \qquad Y_{8,t} = X_{34} + X_{36} \qquad Y_{8,f} = X_{35} + X_{37}$$

$$Y_{9,t} = (X_{38} - 1.0)(X_{31})(0.5) + (2.0 - X_{38})(X_{30})$$

$$Y_{9,f} = (X_{38} - 1.0)(X_{33})(0.5) + (2.0 - X_{38})(X_{32}) \qquad Y_{10,t} = Y_{10,f} = X_{39}$$

$$Y_{11,t} = Y_{11,f} = (X_{44})(X_{58} + X_{59} + X_{60}) + (X_{52})(X_{61})$$

$$Y_{12,t} = X_{40} + X_{42} \qquad Y_{12,f} = X_{41} + X_{42}$$

$$Y_{13,t} = (X_4 + X_{45} + X_{57})(X_{58}) + (X_4 + X_{46} + X_{57})(X_{59}) + (X_{47} + X_{36} + X_{57})(X_{60}) + (X_{56} + X_{57})(X_{62}) + (X_{64} + X_{54} + X_{53} + X_{57})(X_{61})$$

$$Y_{13,f} = (X_4 + X_{45} + X_{57})(X_{58}) + (X_4 + X_{46} + X_{57})(X_{59}) + (X_{48} + X_{37} + X_{57})(X_{60}) + (X_{64} + X_{54} + X_{53} + X_{57})(X_{61}) + (X_{56} + X_{57})(X_{62})$$

$$Y_{14,t} = Y_{14,f} = X_{43}$$

$$Y_{15,t} = Y_{15,f} = \left[X_3 + X_5 + X_{49} + (X_{50})(\alpha_1)\right]\left[\frac{X_{58}}{X_{27} - (X_{59})(2)}\right] + \left[X_3 + X_5 + (X_{50})(\alpha_1)\right](X_{59}) + (X_{55})(X_{60}) + (X_{51})(X_{61})$$

$$Y_{16,t} = Y_{16,f} = X_{63}$$

X = component service measure

Y = modal impedance measure

α_1 = cost per mile for private car

α_2 = cost per mile for rental car

α_3 = fixed charge rental car

t = to airport

f = from airport

subscripts refer to figure 2

Figure 2. Impedance measures formulas.

Total time and total cost were correlated and combined for each mode to yield a single impedance measure. Two series of calibration runs were performed for each of the 4 stratifications. In the first series, no occupancy was taken into account. In the second series, occupancies of 1.67 and 1.40 were assumed for automobile and taxi respectively. In all cases, the standard errors of estimate of the coefficients were low and in many instances were under 10 percent, and coefficients always displayed the expected signs. In addition, these coefficients were always significantly different from 0, far beyond the 0.001 level according to the maximum likelihood ratio test (minus twice the logarithm of the likelihood ratio), which is distributed as χ^2 with as many degrees of freedom as there are coefficients estimated.

The choice of the final model was determined by the coefficient magnitude that directly influences the sensitivity of the model. Both types of models display relatively high sensitivities, particularly in the case of private car cost. This condition is accentuated when occupancy is not taken into account. For this reason, it was decided to choose the model series incorporating occupancy. The results are given in Table 3 for which the following observations can be made.

For each mode and each trip purpose stratification, travelers seem to be more sensitive, and hence more likely to change modes, when going (in terms of access) to rather than when leaving the airport. Because the calibration sample does not distinguish between residents and nonresidents, this cannot, strictly speaking, be explained by familiarity with the region. Among other factors, it is due to the importance attached by the traveler to "making his flight".

The high sensitivity of the private car to cost is an indicator showing that cost, as expected, can be a powerful deterrent from using a car, especially for long-duration trips. Most probably, this would have appeared were this mode broken into 3 classes: those travelers who are driven to the airport, those who park for a short duration (2 days or less), and those who park for a long duration. As noted earlier, parking cost is an average measure based on the average proportion of travelers who park their vehicles at the airports and the average duration of parking.

Between the 2 purpose stratifications, it is surprising to note that the private car cost coefficients, and therefore the sensitivities to this measure, are higher in the case of business trips, regardless of the trip direction. This unexpected result could be attributed to the fact that nonbusiness travelers are often accompanied to or met at the airport. This reason also suggests that the higher cost coefficients of the business trips reflect the cost of the unavailability to the rest of the household of a personal vehicle parked at an airport.

Rented car sensitivity to time is low, and, in fact, ranks third following taxi and private car. This probably suggests that the businessman renting a car does not use it solely for airport access reasons. In most cases, a taxi ride would be more convenient and faster, in particular when downtown parking is required. It can be conjectured that an automobile has the advantage of "flexibility" when several trips are projected in the area visited.

Taxi displays, somewhat markedly and regardless of the trip purpose, the highest sensitivity to time. Two related justifications to this could be found. National Airport "points" dominate the calibration sample, and taxi fares to and from the airport are relatively low, which makes this mode particularly attractive.

The relatively low sensitivity to taxi cost can be explained by the latter justification presented in the preceding, i.e., the relatively low fare of taxi. Most probably, were this fare significantly higher and, hence, not within the means of most travelers, the sensitivity might be higher.

As expected, limousine displays the least sensitivity to time and second highest sensitivity to cost after private car. This is due to, in all likelihood, the competition offered by taxi that is faster and, in absolute terms, not much more expensive (especially in group riding) to areas close to an airport. Hence, were limousine cost to increase, all other things equal, in any significant amount, one would expect the attraction of taxi to be "irresistible".

The least squares techniques described previously were also used in the calibration runs. When applied to models in which total times and costs were introduced for

TABLE 3

CALIBRATED TIME AND COST COEFFICIENTS

Trip Purpose Stratification	Private Car		Rented Car		Taxi		Limousine	
	Time (min)	Cost (dollars)	Time (min)	Cost (dollars)	Time (min)	Cost (dollars)	Time (min)	Cost (dollars)
Business trips to airports	-0.0401	-1.9097 (0.125)	-0.0265	-0.6030 (0.030)	-0.0624	-0.3962 (0.024)	-0.0224	-1.0646 (0.057)
Business trips from airports	-0.0276	-1.3031 (0.075)	-0.0150	-0.3505 (0.015)	-0.0642	-0.3447 (0.020)	-0.0106	-0.5482 (0.030)
Nonbusiness trips to airports	-0.0342	-1.6058 (0.203)	—	—	-0.0631	-0.4025 (0.030)	-0.0177	-0.7922 (0.142)
Nonbusiness trips from airports	-0.0144	-0.6767 (0.163)	—	—	-0.0573	0.3082 (0.025)	-0.0104	-0.5348 (0.110)

TABLE 4

TEST RESULTS OF BUSINESS TRIPS TO AND FROM AIRPORTS

		Unadjusted Results				Adjusted Results			
Mode	Direction	Observed Mean Trips	Estimated Mean Trips	Correlation Coefficient	Root Mean Square Error	Observed Mean Trips	Estimated Mean Trips	Correlation Coefficient	Root Mean Square Error
Private car	To	103.1	102.4	0.96	100.6	103.1	103.4	0.96	61.3
	From	75.4	63.2	0.91	61.5	75.4	75.3	0.92	61.1
Rented car	To	19.2	22.4	0.88	20.2	19.2	19.1	0.88	18.4
	From	20.7	0.7	0.81	33.7	20.7	21.2	0.88	18.4
Taxi	To	83.3	73.8	0.99	86.2	83.2	83.0	0.99	68.8
	From	79.9	144.5	0.99	152.1	79.9	80.2	0.99	41.3
Limousine	To	31.5	38.4	0.87	53.6	31.5	31.4	0.88	48.2
	From	61.4	29.0	0.83	122.0	61.4	60.7	0.96	63.1

TABLE 5

TEST RESULTS OF NONBUSINESS TRIPS TO AND FROM AIRPORTS

Mode	Direction	Unadjusted Results				Adjusted Results			
		Observed Mean Trips	Estimated Mean Trips	Correlation Coefficient	Root Mean Square Error	Observed Mean Trips	Estimated Mean Trips	Correlation Coefficient	Root Mean Square Error
Private car	To	57.8	51.0	0.95	35.9	57.8	57.6	0.96	35.7
	From	45.8	39.2	0.93	34.6	45.8	45.0	0.95	29.3
Taxi	To	22.9	30.2	0.95	28.8	22.9	23.1	0.95	28.1
	From	17.6	35.2	0.97	42.5	17.6	17.4	0.97	17.0
Limousine	To	8.2	7.6	0.65	15.6	8.2	8.1	0.67	15.6
	From	14.5	3.4	0.80	27.8	14.5	14.4	0.87	18.2

TABLE 6

COEFFICIENTS FOR SELECTED MODELS

Mode	Direction	Business Trips			Nonbusiness Trips		
		Time	Cost	Constant	Time	Cost	Constant
Private car	To	-0.0401	-1.9097	0.0	-0.0342	-1.6058	0.0
	From	-0.0276	-1.3031	0.0	-0.0144	-0.6767	0.0
Rented car	To	-0.0265	-0.6030	1.7187	—	—	—
	From	-0.0150	-0.3505	-0.9498	—	—	—
Taxi	To	-0.0624	-0.3962	2.0495	-0.0631	-0.4025	0.3610
	From	-0.0642	-0.3447	1.2519	-0.0573	-0.3082	1.2998
Limousine	To	-0.0224	-1.0646	1.6664	-0.0177	-0.7922	-1.0416
	From	-0.0106	-0.5482	-0.1488	-0.0104	-0.5348	-0.0237

each mode, the coefficients in many cases did not meet the sign condition. The transformation of time into cost was also applied as in the case of the maximum likelihood calibration technique. This transformation did actually yield fewer incorrect coefficients, but the results were still unacceptable. It might be of interest to note that the private car coefficients (which were negative) remained at the same order of magnitude as those derived by maximum likelihood.

Testing

The model predictions were compared to the observed values of the sample. This comparison was performed both at the modal-split level and the trip level for each mode and each stratification. The latter comparison is, in fact, a weighted formulation of the former inasmuch as the larger the total trip number is for a given observation, the higher is the importance attached thereto. For each of the 4 calibrated models, data given in Tables 4 and 5 display for each mode the observed and estimated average trips, the correlation coefficient between estimated and observed trips, and the root mean square error of the prediction.

In view of the prototypical nature of this model, the relative importance of this task within the overall objectives of the study, and the nature and quality of the calibration data, it was decided to select the models described in the preceding because these models display desirable attributes in terms of sensitivity. The modal-mean trips were matched and the dispersions were reduced for each mode by adding an empirical constant to the linear forms attached to each mode. Because of the structure of the model, it is sufficient to find three such constants for the business models and two for the nonbusiness models. These constants are given in Table 6 and the results of their incorporation into the model are given in Tables 4 and 5 parallel to those of the unadjusted models.

CONCLUSIONS

The work discussed in this paper represents a first step toward the development and application of a modal-split model that is not limited as to the number of competing modes. Although developed for an airport access application, it is equally valid for travel mode analyses at the intraurban or intercity levels.

In contrast to other formulations, the elasticity of the share of a given mode is not constant but rather is proportional to the share of that mode and to the value of the independent variable with respect to which the elasticity is considered.

Furthermore, the underlying assumptions in the formulation of the model appear desirable because a mode is most "vulnerable" or most "attractive" when it holds half of the market and because its "potential" of gaining (or losing) patronage is minimal when its share is already large (or small).

Other researchers have encountered difficulties in calibrating multimode modal-split models by relying on linear regression techniques. The present work suggests that use of maximum likelihood techniques can be helpful in alleviating these difficulties.

Experience derived from this study suggests that the model can be a useful tool for predicting modal split. It is felt, however, that additional data could improve the model's forecasting capabilities in an airport access application. In particular, as mentioned earlier, it appears important to know whether a traveler is a resident of the region under study. Several other measures also could be used. They include reliability of a mode as reflected by the standard deviation of travel times; income, which greatly determines the ability to pay; trip duration, which would give better information on parking cost; and group size, which is necessary for better cost estimation. Furthermore, smaller and more homogeneous zones could contribute to more accurate results.

Finally, consideration should be given to the dichotomy between the perceived and the observed values of impedances, particularly in terms of time and cost. This is all the more important in behavioral models inasmuch as the perception of impedances has necessarily some influence on user behavior and hence on model choice. A recent paper by Watson (5) gives an assessment of this problem and the implications on modal-split modeling resulting from biases in impedance data estimation.

In general, this initial investigation suggests that the n-dimensional logit model is a flexible and useful tool that can be applied operationally and that warrants further refinement. Despite the lack of certain key data elements, results of this airport access application were quite satisfactory. It is hoped that the results of further tests will confirm initial expectations.

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