PREDICTION OF POTENTIAL STRENGTH OF CONCRETE FROM THE RESULTS OF EARLY TESTS

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A series of 3 designed experiments was conducted to investigate the practicality of using early tests of concrete cylinders made at the job site as a means of quality control of concrete. The scope of the experiments included concrete of 3 cement constants, 3 brands of cement, and 4 methods of conditioning specimens prior to test. The test results, when analyzed by statistical methods, indicate a definite linear relationship between the logarithm of the maturity of the concrete in degree-hours and the relative compressive strength. When specimens were tested at spaced values of maturity, an equation capable of predicting the 28-day strength of concrete to within about 500 psi was derived. Predicted results are suitable for constructing one-sided confidence limits for quality control purposes. It was found practical to construct a nomograph to expedite the computation of predicted values. A limited amount of simulation was conducted on 8 sets of test cylinders made at the job site and tested at 23 to 42 hours of age. The average difference between the predicted 28-day strengths and the routine 28-day test results was about 300 psi. Practical application of this method requires that a definite early test procedure be established and that equation constants for commonly used concrete mixtures be established by experiment using this procedure. Predicted results from early tests on cylinders made at the job site can then be used for quality control purposes and to provide prompt warning of the production of concrete of unacceptable strength.

•IT IS EVIDENT that effective quality control of concrete requires some means of assessing the potential strength of concrete within several hours of manufacture instead of 4 weeks. The 28-day test is really only a measure of potential strength and, by the time the results are available, they are useful only for historical records. Even 14- and 7-day tests are not satisfactory for quality control purposes because, with current rates of production, large quantities of concrete can be placed and other large amounts superimposed before an indication can be obtained that corrective measures are necessary.

Since the 1930's, a great deal of work has been done in this area in the United States and in other countries. A report by the Ontario Department of Highways (1) lists 87 references relating to methods for use in estimating the later strength of concrete from early test results. Currently, Subcommittee 11-i of ASTM Committee C-9 is writing procedures for estimating potential strength at significantly early times. However, none of these procedures or methods has come into general use in connection with highway construction in this country.

In view of the critical need for an expedient means of detecting concrete of inferior quality, the Materials Control, Soils and Testing Division of the West Virginia Department of Highways conducted a series of designed experiments to investigate the

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possibility of obtaining reliable estimates of the potential strength of concrete from the results of early tests. These results were analyzed by statistical methods, and the equations that have been developed indicate a high probability of success with respect to the ability to classify the quality of concrete at very early ages.

OBJECTIVES

The objectives of the work described here were to evaluate alternate methods of preparing concrete specimens for early tests, to determine the effects of various factors on the relationship between early and potential strength of concrete, and to develop reliable means of predicting minimum potential strength from the results of tests on concrete cylinders made at the job site or laboratory and cured over a range of times and conditions.

RESEARCH APPROACH

A review of the literature indicated that most previous approaches to the problem of relating the strength of concrete at an early age to that at 28-days contained restrictions that should be eliminated or modified, if possible. Procedures required that the early tests be made at fixed times after the forming of the specimens so that they were not very adaptable to conditions associated with highway construction, where there may be differences in early curing temperatures and variations in the time required to transfer specimens from the job site to the location of the testing equipment. The methods of preparing specimens for test were sometimes inconvenient or required extensive equipment. Finally, the mathematical equations that had been previously used did not appear to be suitable for prediction of potential strengths of specimens formed at the job site, and the degree of confidence that could be placed in the prediction under these conditions was unknown.

However, in view of the normal variation in 28-day tests of concrete made and cured under standard conditions and knowledge that, for quality control purposes, all that is required is a prediction of minimum potential strength with an acceptable degree of confidence, it was believed that a satisfactory method could be developed that could be used in evaluating concrete quality at early ages and that would be comparable in significance with 28-day tests.

Designed Experiments

The designed experiments set up for the purposes of obtaining data required for deriving the desired relationships included making and testing 3 series of specimens. All experiments were so designed that test specimens from the same batch were tested at spaced intervals. These equal increments of maturity form the basis to this approach for determining the line of prediction.

Series 1—The purpose of series 1 was to investigate the effect of variation of cement content over a practical range. It consisted of 3 groups of 30 cylinders each with groups made with 4.00, 5.50, and 7.25 bags/cu yd of the same brand of cement.

The order of making the batches was arranged to be pseudo-random so that no consistent pattern of batching occurred. In each batch, duplicate cylinders were designated for testing at the same degree of maturity for the purpose of estimating testing error under the conditions of the experiment. Brand M cement was used in all batches with Ohio River sand and gravel. Air-econ admixture was used to obtain the desired air content. The 2.7 cu ft batches were mixed in a Müller 6-S (6 cu ft) mixer in general accordance with ASTM Method C 192 (Standard Method of Making and Curing Concrete Test Specimens in the Laboratory).

After it was mixed, the concrete from all batches was tested for slump and air content. Concrete used for these purposes was discarded. Cylinders for test purposes were made by the "group" method in the curing chamber and were not disturbed for 24 hours.

At the end of the curing period, the cylinders for early test were stripped and placed in a water bath consisting of half a 55-gal oil drum filled with water. Heat was supplied by a thermostatically controlled Chromalox electric immersion unit. The bath was agitated with compressed air. The temperature of the bath was maintained at approximately 200 F for the 3½-hour conditioning of the 6 to 7 cylinders placed in the bath at 1 time. The cylinders were placed and removed from the bath by the use of a specially designed tool to avoid accidental injury to the test specimens or to personnel.

At the end of the conditioning period, the cylinders were removed from the bath and allowed to cool in air for 45 min. They were then capped with a commercial sulfur composition capping compound that was allowed to harden before the cylinders were tested in compression.

Comparison cylinders for the 28-day tests were cured and prepared for test in the routine manner. All cylinders were tested in compression by the use of a 450,000-lb Baldwin machine. The test results are shown in Figure 1.

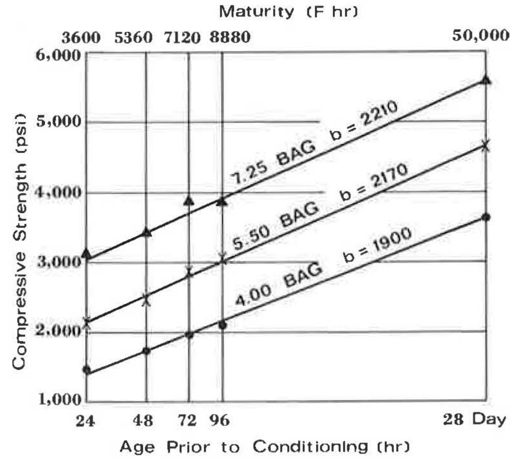

Figure 1. Series 1—relationship of early to 28-day strength.

The lines of relationship shown in Figure 1, computed by the use of Eq. 3, are essentially parallel and indicate an increase in strength of about 2,500 psi over the range of maturity covered. The percentage increase in strength is from about 40 percent to about 55 percent depending on the level of the 28-day test results.

Series 2—Series 2 also consisted of 3 groups of 30 cylinders each. Each group was made with a different brand of cement in common use in West Virginia, but all groups were made with 6.00 bags/cu yd of cement. The purpose was to determine the effect of cement source on strength relationships. Curing, conditioning, and testing of the test specimens were identical to the procedures of series 1. The results of the tests are shown in Figures 2 and 3.

The lines of relationship were computed in the same way as those shown in Figure 1, and they indicate that there is some difference in the level of 28-day results and in the rate of gain of strength when different brands of cement are used under identical

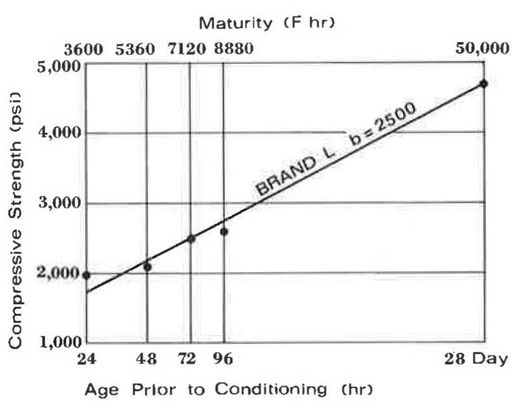

Figure 2. Series 2—relationship of early to 28-day strength.

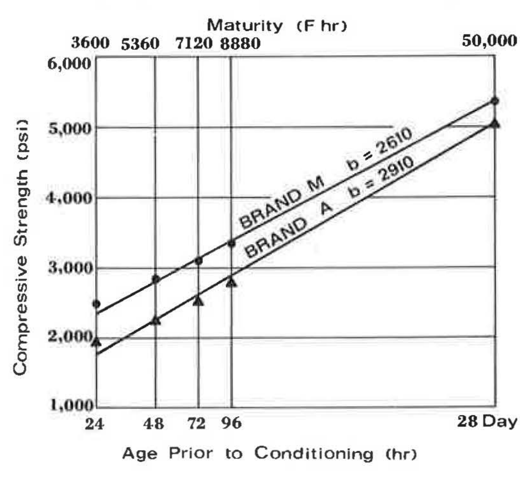

Figure 3. Series 2—relationship of early to 28-day strength.

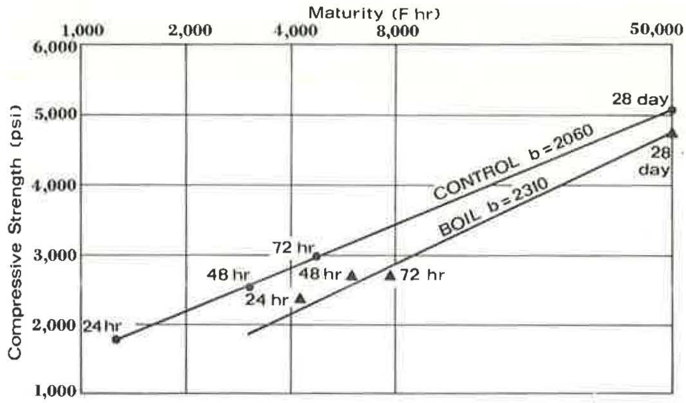

Figure 4. Series 3—relationship of early to 28-day strength.

conditions. The lines are essentially parallel and indicate an increase in strength of about 3,000 psi over the range of maturity covered. The percentage increase in strength is from about 34 percent to about 43 percent but is not directly related to the level of the 28-day test results shown in Figures 2 and 3.

Series 3—The primary purpose of series 3 was to investigate the effect of different methods of preparation for early test on strength relationships. The effect of conditioning 28-day cylinders immediately prior to test was also investigated. This series included 4 groups of 30 cylinders each. Each group was made with 6.00 bags/cu yd of the same brand of cement.

Test results are shown in Figures 4 and 5. Early test cylinders designated as "autoclave" were conditioned, after normal curing, by being heated in a pressure cooker maintained at 15 psi for a period of 3 hours. The temperature of the water in the cooker at time of immersion of the cylinders was about 100 F and the maximum temperature was probably about 245 F. Early test cylinders designated as "boil" were conditioned after normal curing by a $3\frac{1}{2}$-hour immersion in a 200 F water bath. This is the same procedure as used for series 1 and 2. Cylinders designated as "control" were tested after normal curing. No heating or other conditioning was applied to these specimens. Early test cylinders designated as "oven" were conditioned after normal curing by heating in a convection oven for 6 hours. The temperature of the test

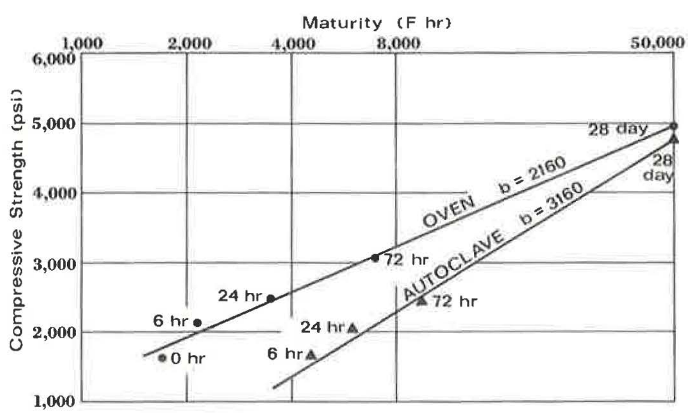

Figure 5. Series 3—relationship of early to 28-day strength.

cylinders was determined by measurements on a dummy cylinder. The temperature varied from about 76 F at the start of heating to 199 F for the last hour with an average of about 175 F. The 28-day boil cylinders were heated for 3½ hours in the 200 F bath at the end of the normal curing for 28 days. All of the heated cylinders were allowed to cool for a period of 45 to 60 min, were capped with sulfur composition caps, and were tested about 30 min after the end of the cooling period. Interior temperature of the concrete at time of test was about 130 F.

The lines of relationship shown in Figures 4 and 5 were computed by the use of Eq. 3, and C-values were determined by iterative graphical methods as described in the Appendix. The lines shown in Figure 4 for the control cylinders and for the boiled cylinders that were conditioned the same as those for series 1 and 2 are essentially comparable and the previous remarks apply. The oven-conditioned cylinders (Fig. 5) also show a relationship comparable to the previous series. However, the cylinders conditioned under pressure in the autoclave appear to have a significantly different rate of gain of strength, as could be expected from the experience of prior investigators (2).

Mathematical Analysis

Several methods of deriving an equation that would predict 28-day strength from the results of early tests were applied to the experimental data, and the results were evaluated. The criterion was the size of the standard deviation of the differences exposed by comparing the value predicted from each early test result with the corresponding 28-day test on a cylinder from the same batch.

Many prior investigators have attempted to develop a factor by which to multiply early results to estimate 28-day results. Others have derived equations in which early results are expressed as a percentage of strengths at later ages. These approaches were investigated but, when applied to the experimental data, the results were not of acceptable accuracy.

The maturity concept advocated by Plowman (3) and others was next investigated. It was found that a reasonably close fit to a straight line was obtained when compressive strength in pounds per square inch (psi) was plotted against maturity in degree-hours on semilog graph paper. This indicated that an equation of the general form $Y = a + bX$, where the slope b of the regression line is found by the method of least squares, could be fitted to the data. Accordingly, the experimental data were first analyzed by fitting the equation

$$S_M = b\,(\log m) - a \tag{1}$$

to the data. In this equation,

S_M = predicted compressive strength in psi,
m = maturity in degree-hours at time of test,
b = regression coefficient (slope of line), and
a = a constant.

Parameters for Eq. 1 were computed by machine calculation and computer printout, and a good fit was obtained. However, when the exposures were calculated, it was found that Eq. 1 was not entirely satisfactory with respect to accuracy of prediction.

It seemed that accuracy could be improved by projecting the prediction line from the result of each early test rather than from the intercept a, which is a fixed point. This resulted in Eq. 2 as follows:

$$S_M \approx S_m + b\,(\log M - \log m) \tag{2}$$

where

S_M = predicted normal compressive strength (psi) at maturity M;
S_m = measured compressive strength at maturity m;
M = degree-hours of maturity under standard conditions (i.e., when cured at 73.4 F);
m = degree-hours of maturity of specimen at time of early test after conditioning [(hours of age × ambient temperature) + C];

C = degree-hours of maturity determined by autogenous heating, method of preparation or conditioning, residual temperature at time of test, and possible unknown factors; and
b = slope of prediction line.

In this case, the value of b was found by the ratio of the averages, sometimes called the method of zero sum, as follows:

$$b = \frac{\Sigma (S_M - S_m)}{\Sigma (\log M - \log m)} \quad (3)$$

Because, in general, the standard deviation is proportional to the compressive strengths, it is theoretically possible to obtain a better value of b from the average of individual ratios. However, on trial, no large difference was observed, so the zero sum method was used as a matter of convenience.

For series 1 and 2, a pooled value for C of 1,840 degree-hours was used. The value of the maturity m was then the number of hours of age at 73.4 F × 73.4 + 1,840. The regression coefficients and related prediction error terms are given in Table 1.

Because the mean of the differences between predicted and actual values of compressive strengths was zero (or very nearly so), the error standard deviation was computed by the use of the equation

$$\sigma_e = \sqrt{\frac{\Sigma (S_M - S_{28})^2}{2\eta}} \quad (4)$$

This error term can be used to construct a one-sided confidence interval for the predicted value. For example, if

$$S_M - 1.645\ \sigma_e > L \quad (5)$$

there is a 95 percent probability that the true average strength is greater than the designated lower limit L.

Comparisons of predicted and measured values were made by substitution in Eq. 2, as shown in the following example:

$$\begin{aligned} S_M &\approx S_m + b\,(\log M - \log m) \\ &\approx 1{,}540 + 1{,}900\,(4.699 - 3.556) \\ &\approx 1{,}540 + 1{,}900\,(1.143) \\ &\approx 1{,}540 + 2{,}170 \\ &\approx 3{,}710 \text{ psi} \\ S_{28} &= \underline{3{,}790 \text{ psi}} \\ \text{Difference} &= -\ 80 \text{ psi} \end{aligned}$$

An estimate of the maximum error to be expected over a wide range of conditions was obtained by using the pooled values of 2,410 for b and 1,840 for C to compute the exposure of the differences between predicted and measured strengths for all of the series 1 and 2 results. The results in histogram form are shown in Figure 6. This analysis indicates that, over the range of cement constants and cement brands covered in the 2 series, there is a 95 percent probability that the strength predicted from the results of early tests on specimens conditioned for 3½ hours in the 200 F water bath will not exceed corresponding, measured 28-day strengths by more than 800 psi.

TABLE 1

RESULTS OF COMPUTATIONS FOR SERIES 1, 2, AND 3

Series	Batch	Value of Slope b	Degree-Hours of Maturity C	Prediction Error σ_e, psi
1	4.00 bag	1,900	1,840	162
	5.50 bag	2,170	1,840	317
	7.25 bag	2,210	1,840	282
2	Brand M	2,610	1,840	128
	Brand A	2,910	1,840	185
	Brand L	2,550	1,840	392
3	Oven	2,160	1,740	201
	Autoclave	3,160	4,200	142
	Boil	2,310	2,480	292
	Control	2,060	-500	190

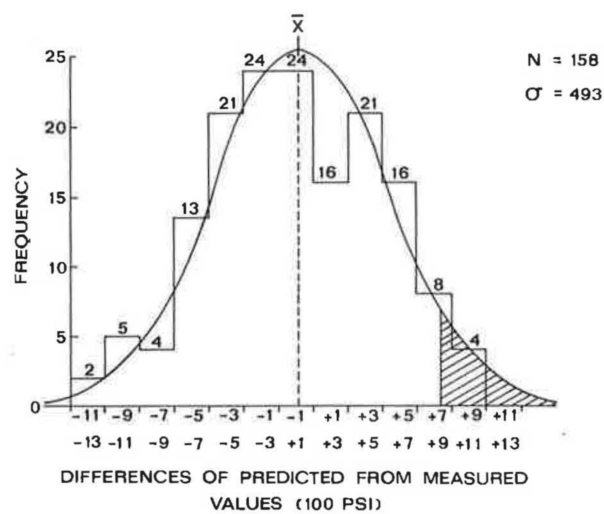

Figure 6. Frequency distribution of differences of predicted value from measured value (28-day concrete compressive strength).

DISCUSSION OF RESULTS

The excellent fit of Eq. 1 to the data confirms the original mathematical assumptions and indicates that, for a specific set of conditions, reasonably accurate predictions of potential strength can be made from the results of early tests of concrete. The accuracy of these predictions is limited by the size of the standard deviation (or coefficient of variation) normally found among 28-day tests.

For general application, over a range of cement constants and brands of cement, accuracy is considerably reduced because these factors influence the slope b of the prediction line. However, use of Eq. 2 should provide sufficient accuracy for a quality control test where the objective is to ensure a minimum potential strength rather than to predict an exact value. This can be accomplished by applying a one-sided confidence interval to the predicted value. For example, if $S_M - 1.645\,\sigma_e > L$ (Eq. 5), there is a 95 percent probability that the true average strength is greater than the designated lower limit L.

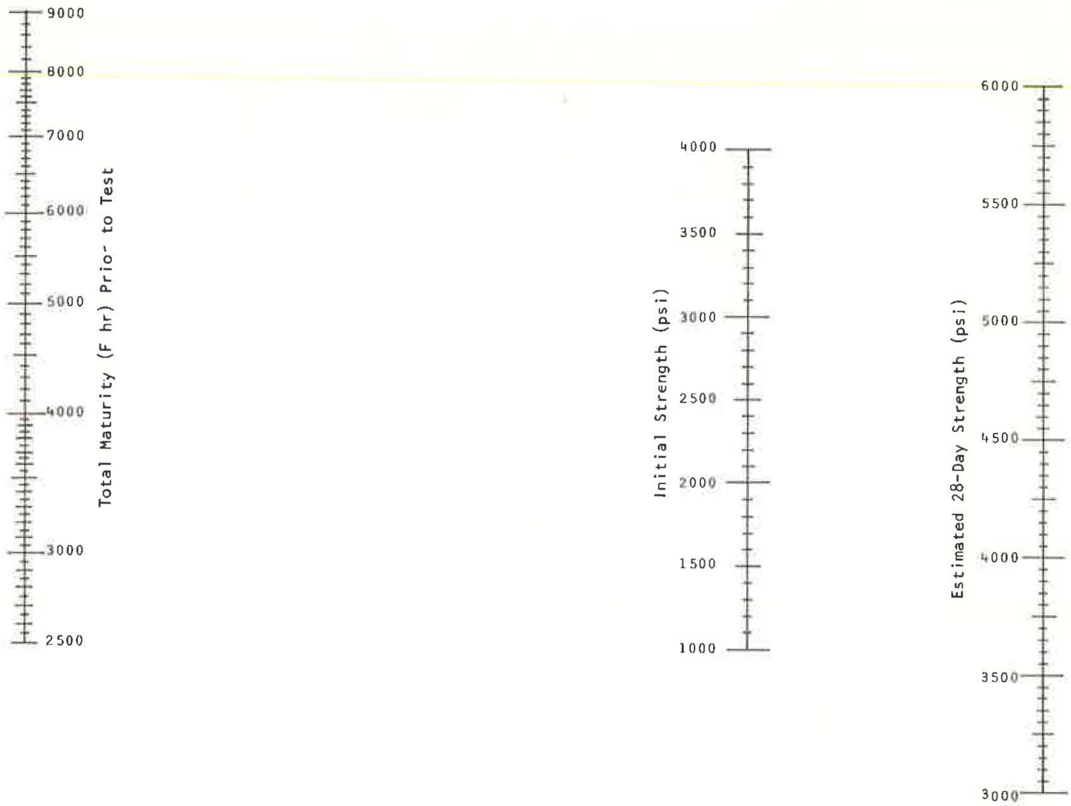

Figure 7. Nomograph for estimating potential 28-day strength from early tests (normal curing).

The necessity of using logarithms may appear to be a disadvantage, but this is easily overcome by the use of a nomograph. A trial nomograph, as shown in Figure 7, has been constructed and found to be practical. A finalized version can be produced after further simulation and acceptance of the philosophy associated with Eq. 5.

CONCLUSIONS

Within the range of the conditions included in the experiment, all test results indicate that a linear relationship exists between the logarithms of the maturity in degree-hours of concrete and the relative compressive strength. This relationship is accurately expressed by Eq. 2, provided that proper values of the constants b and C are used. Reliable values of b and C for a particular combination of conditions can be determined by designed experiment wherein early tests of the concrete are made at uniformly spaced intervals of maturity. Trial values of b and C can be obtained by graphical methods as shown in the Appendix. Final values of b are best obtained by the method of zero sum.

For the purpose of determining potential strength of concrete 24 hours or more of age, prior conditioning does not appear to be necessary. Where results are required at earlier ages, conditioning with either a convection oven or with an autoclave appear to be about equally satisfactory. However, it is advisable to allow concrete to reach the state of final set before conditioning.

When values of b and C have been determined for a particular set of conditions, the potential 28-day strength of concrete can be predicted by the use of Eq. 2 to within about 500 psi 19 times out of 20.

Computations of predicted strengths can be expedited by use of a nomograph that can be easily constructed for any particular value of b.

ACKNOWLEDGMENT

The opinions, findings, and conclusions expressed in this paper are those of the authors and not necessarily those of the West Virginia Department of Highways or of the Federal Highway Administration.

REFERENCES

1. Smith, P., and Tiede, H. Earlier Determination of Concrete Strength Potential. Highway Research Record 210, 1967, pp. 29-66.
2. Budnikov, P. P., and Erschler, E. Y. Studies of the Process of Cement Hardening in the Course of Low-Pressure Steam Curing of Concrete. HRB Spec. Rept. 90, 1966, pp. 431-446.
3. Plowman, J. M. Maturity and Strength of Concrete. Mag. of Concrete Research, London, Vol. 8, No. 22, 1956, pp. 13-22.

APPENDIX

PROCEDURE FOR ESTABLISHING AN EQUATION FOR PREDICTING POTENTIAL STRENGTH FROM EARLY TESTS ON CONCRETE CYLINDERS

1. Prepare laboratory batches large enough to make 6 test cylinders of a design mix to which equation will apply. Repeat on 5 different days. To minimize variation, make cylinders in place where they can remain undisturbed for 24 hours at 73 F ± 3 F.

2. At the end of 24 hours, test 1 cylinder from each batch with or without conditioning such as 3-hour immersion in 200 F bath or 200 F oven. Place remaining cylinders under standard moist cure at 73.4 F ± 3 F.

3. Test 1 cylinder from each batch at end of 48, 72, and 96 hours with or without conditioning, as tested for 24-hour cylinder. Test 2 cylinders for 28-days without conditioning.

4. Average test results ($\bar{X}_5$) for each age.

5. Prepare a sheet of semilog graph paper, 2 cycles by 70 divisions (K and E 46-4970, or similar) by numbering 1-in. divisions in thousands of psi and letting the log scale represent maturity m in thousands of degree-hours at time of test.

6. Draw horizontal lines across graph from psi scale for each average strength. Plot point for 28-day psi at 50,000 degree-hours of maturity.

7. Points for each strength should lie on their respective lines in positions such that a straight line from the 28-day strength point will pass through or near all the early strength points. Because both the slope of the line and the maturity values of the points are unknown, the equation must be found graphically by trial and error. It is known that the maturity values of the points will be 1,760 degree-hours (24 × 73.4 F) apart. Then each point will have a maturity value of some multiple of 1,760 plus some value C determined by conditioning (if any), autogenous heating, and unknown factors. The iterative method of finding the line of prediction is shown in Figure 8. The synthetic data on which the graph is based is given in Table 2. Figure 8 shows that the first series of points, ●, have too flat a slope to intersect the 28-day point. The second trial, X, points have too steep a slope. The third trial, ▲, provides points falling very closely on the line projected to the 28-day point, so the best value of C is 1,240.

8. The slope of b of this line is the vertical scale distance between the intercept on the 10,000 m-line and the intercept on the 100,000 m-line or about 2,850 in the case of the example.

9. The general form of the prediction equation (Eq. 2) is

$$S_M \approx S_m + b\,(\log M - \log m) \tag{2}$$

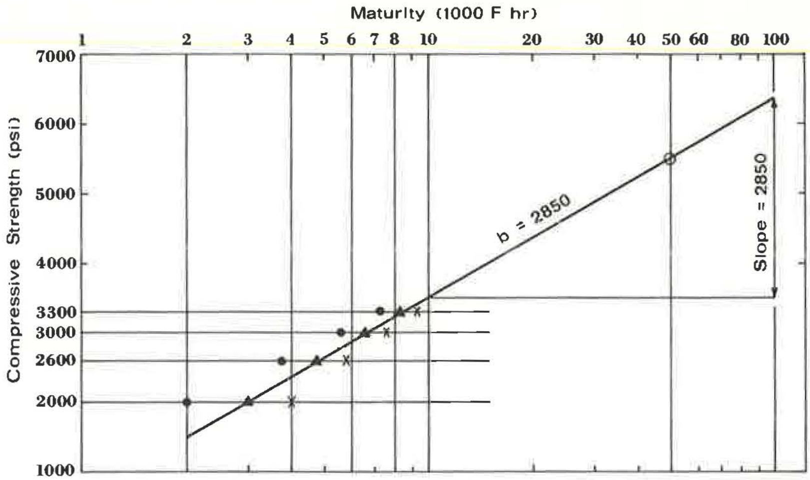

Figure 8. Determining line of prediction.

10. In the case of the example, the potential 28-day strength projected from the 24-hour test would be

$$\begin{aligned} S_{28} &= 2{,}000 + 2{,}850\ (4.699 - 3.477) \\ &= 2{,}000 + 2{,}850\ (1.222) \\ &= 5{,}480 \text{ psi} \end{aligned}$$

11. The graphical method described in the preceding provides an approximate prediction equation for averages of groups of compressive test results. A better estimate of the value of slope b can be obtained by the method of zero sum using the following equation:

$$b = \frac{\Sigma\,(S_M - S_m)}{\Sigma\,(\log M - \log m)} \qquad (3)$$

When this method is used, the difference between each early test result and the corresponding 28-day tests on cylinders from the same batch are noted along with the matching difference in the logarithms of the maturities. The total of the differences in compressive strengths is then divided by the total of the difference in the logarithms of the maturities.

TABLE 2

SYNTHETIC DATA FOR FIGURE 8

Age (hour)	S_c (psi)	m′	m = m′ + C		
			C = 240	C = 2,240	C = 1,240
24	2,000	1,760	2,000	4,000	3,000
48	2,600	3,520	3,760	5,760	4,760
72	3,000	5,280	5,520	7,520	6,520
96	3,300	7,040	7,280	9,280	8,280
28[a]	5,500	50,000	50,000	50,000	50,000

[a]Days.

12. The prediction equation is tested by computing an exposure table. The slope b obtained by the method of zero sum (item 11 preceding) is used to predict the 28-day strength from each individual early test result. The algebraic difference between each predicted result and the corresponding, measured 28-day test results are listed and totaled. The total should be zero or nearly so.

13. The prediction error is computed by the use of the formula

$$\sigma_e = \sqrt{\frac{\Sigma (S_M - S_{28})^2}{2\eta}} \tag{4}$$

The best values of b and C are indicated by the lowest value of σ_e. A one-sided confidence limit can be established by

$$S_M - 1.645\ \sigma_e > L \tag{5}$$

which indicates that 95 times out of 100 the true average strength is greater than L.

14. The testing error is computed by the use of the equation

$$\sigma_t = \sqrt{\frac{\Sigma |X_1 - X_2|^2}{2\eta}} \tag{6}$$

where X_1 and X_2 are the paired 28-day test results on cylinders from the same batch.

15. The actual prediction error is

$$\sigma_p = \sqrt{\sigma_e^2 - \sigma_t^2} \tag{7}$$

16. Equation 6 may be used to predict potential strength at any age from a test result of a cylinder of any age, provided that the temperature history of the cylinder prior to test can be estimated. For example, the potential 14-day strength of concrete can be estimated from a cylinder 41 hours old at time of test when the average temperature of the cylinder was 90 F as follows:

$$
\begin{aligned}
b &= 2{,}850 \\
C &= 1{,}240 \\
M &= 14 \times 24 \times 73.4 = 24{,}640;\ \log = 4.39164 \\
m &= (41 \times 90) + 1{,}240 = 4{,}930;\ \log = 3.69285 \\
\log M - \log m &= 0.69879 \\
S_m &= 2{,}210 \text{ psi} \\
S_{14} &= 2{,}210 + 2{,}850\ (0.699) \\
&= 2{,}210 + 1{,}990 \\
&= 4{,}200 \text{ psi}
\end{aligned}
$$