CUMULATIVE FATIGUE DAMAGE CHARACTERISTICS OF PLAIN CONCRETE

Craig A. Ballinger, Federal Highway Administration,
U.S. Department of Transportation

A laboratory study was conducted to investigate the fatigue characteristics of plain concrete. The primary goal was to evaluate the effect of variable loads on the fatigue life—to determine whether the Miner hypothesis adequately represents cumulative damage. In the laboratory work, 6- by 6- by 64-in. beams were tested statically and also under monotonic or variable loadings. Most specimens were moist-cured for 7 days and then stored in normal laboratory air for an average of 425 days before testing. The sinusoidal fatigue tests were conducted at a loading rate of 450 cpm, with the ratio of minimum to maximum stress equal to 0.15. The specimens in the variable load series were subjected to 2 levels of loading. A range of conditions was provided by raising or lowering the stress level by varying the number of cycles at the first level and by varying the stress range. The test data indicated a wide scatter in strength for concrete subjected to extended drying conditions. The applied fatigue stresses was normalized by using a multiple linear regression procedure to predict the static strength. The variable load fatigue tests were evaluated against the basic S/N data. It is concluded that the Miner hypothesis represents the cumulative damage characteristics of plain concrete in a reasonable manner. Because of the variability of concrete strength, a more elaborate rationale is not warranted. The accuracy of the S/N diagram is very dependent on the accuracy of the prediction of the normalizing strength values.

•WHEN CONCRETE and other structural materials are subjected to high repetitive stresses, they are subject to failure by fatigue. Therefore, if it is known that structures are going to be subjected to heavy repetitive loads during their service lives, it is essential that this factor be considered in their design. Economics and other considerations also require that the structures not be overdesigned. Therefore, the designer must have accurate criteria for adequately estimating the fatigue life of a structure under the expected service loading.

In the past, numerous research studies have been conducted to evaluate the fatigue characteristics of concrete. However, most of these studies have been restricted to determining the influence of variations in the type or proportion of the component materials on the fatigue life. This basic information was obtained by subjecting individual specimens to repetitive loading at constant stress amplitudes until failure occurred. The results of many tests and studies are plotted on a S/N diagram, which relates the percentage of ultimate stress that the specimens were loaded at to the number of cycles to failure. The line of 'best fit', which represents the data for each variable being considered, is then evaluated. The following conclusions result from these studies: (a) The fatigue response, when expressed in terms of the static ultimate strength, is statistically independent of the nominal strength, air-entrainment, type of aggregate, or the frequency of the repetitions of load; (b) the inclusion of rest periods

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was found to extend the fatigue life, with the effect increasing to a maximum for rest periods of about 5 min; and (c) fatigue life is found to be affected by the range of the applied cyclic stress, and, for a given minimum stress, the life is increased as the stress range is reduced (1).

Unfortunately little research has been conducted to evaluate the effect of variations in repetitive stresses on the fatigue life of concrete. Because the service stresses in structures do vary, many designers feel that the bulk of the existing fatigue information on concrete is only of limited value. Because of this lack of definitive information on concrete, many structures are overdesigned so that service stresses will be below the fatigue limit, and therefore fatigue will not be a real factor. If, however, the cumulative damage due to repetitive stresses of varying magnitudes must be estimated, designers have almost traditionally used the Miner hypothesis. This method assumes that damage is proportional to the stress level and the number of cycles of load. However, Miner's theory is based not on fatigue tests of concrete but rather on notched aluminum specimens (2). In recent years research has shown that this method does not adequately represent the cumulative damage relationship for many metals. Accordingly, there is concern that the Miner equation may not adequately represent the fatigue characteristics of concrete.

OBJECTIVES

A laboratory study was conducted to develop information on the cumulative damage characteristics of plain concrete that is subjected to variations in repetitive loads. Because many designers currently use the Miner equation to estimate such damage, the data from this study were evaluated with respect to that theory to determine whether it represents such damage to a reasonable degree of accuracy. Also, to test a large number of specimens under a wide range of variations in fatigue loading was recognized as not being feasible. Therefore, rather than attempting to develop a new cumulative damage theory testing a limited number of specimens under variable fatigue loads and then determining whether the Miner equation could reasonably be used to predict the failure of these specimens were deemed to be more appropriate.

The laboratory study that provided the necessary data was conducted in 3 phases. Numerous static tests were made on beams and cylinders to provide basic strength information on each batch of concrete, 44 beams were subjected to constant cycle repetitive loads to develop the basic S/N relationship for this study, and 28 beams were subjected to a variety of 2-level cyclic loads to evaluate the Miner hypothesis.

LABORATORY TESTING PROGRAM

Test Specimens and Equipment

For the preceding evaluations, it was considered necessary that numerous specimens be tested to provide as much information on the concrete as possible. The batches of concrete were of sufficient size to make the following specimens: two 6- by 6- by 64-in. beams, three 6- by 6- by 21-in. beams, and three 6- by 12-in. cylinders. From each batch, one 21-in. beam and one cylinder were continuously moist-cured until tested at 28 days of age. The remaining specimens were divided into two test groups, including one cylinder, one 6- by 6- by 21-in. beam, and one 6- by 6- by 64-in. beam. Each of these groups is moist-cured for 7 days, and then moved into an air-drying laboratory storage area until time of test. The air-drying period ranged from about 175 to 550 days.

The concrete mix that was used for this study was designed for a 28-day compressive strength of 5,500 psi, a maximum slump of 3 in., and an air content of $5\frac{1}{2}$ percent. The coarse and fine aggregates were high-quality crushed limestone and a natural quartz sand respectively. The cement was a Type I non-air-entraining material. The air content was adjusted by adding Protex, a neutralized vinsol resin. All materials met the requirements of the appropriate ASTM specifications.

For most of the preceding static tests, conventional laboratory testing machines were used. However, special equipment was used for the static and fatigue tests of the

64-in. beams. The loading equipment was an electronically controlled hydraulic system, which was manufactured by the MTS Systems Corporation. A self-contained structural steel frame was fabricated for these third-point flectural tests. The fixtures conformed to the requirements of ASTM Method C 78 [Test for Flexural Strength of Concrete (using Simple Beam With Third-Point Loading)].

Phase 1—Static Tests

Initially all of the specimens from 32 batches of concrete were tested statically to develop basic strength information on the concrete and correlations between test methods. Phase 1 tests are shown in Figure 1. It may be noted that the halves of the broken 64-in. beams were tested in flexure and that the resulting pieces from these and the 21-in. beam tests were tested in compression by the modified cube method. Except for the tests of the 64-in. beams, all of these tests were also made on the fatigue test group specimens. Thus, a considerable amount of static test data was accumulated in this laboratory study.

Phase 2—Constant Cycle Fatigue Tests

In the phase 2 series of tests, the 64-in. beams from 44 specimen groups were subjected to constant cycle fatigue tests to provide data for developing the basic S/N

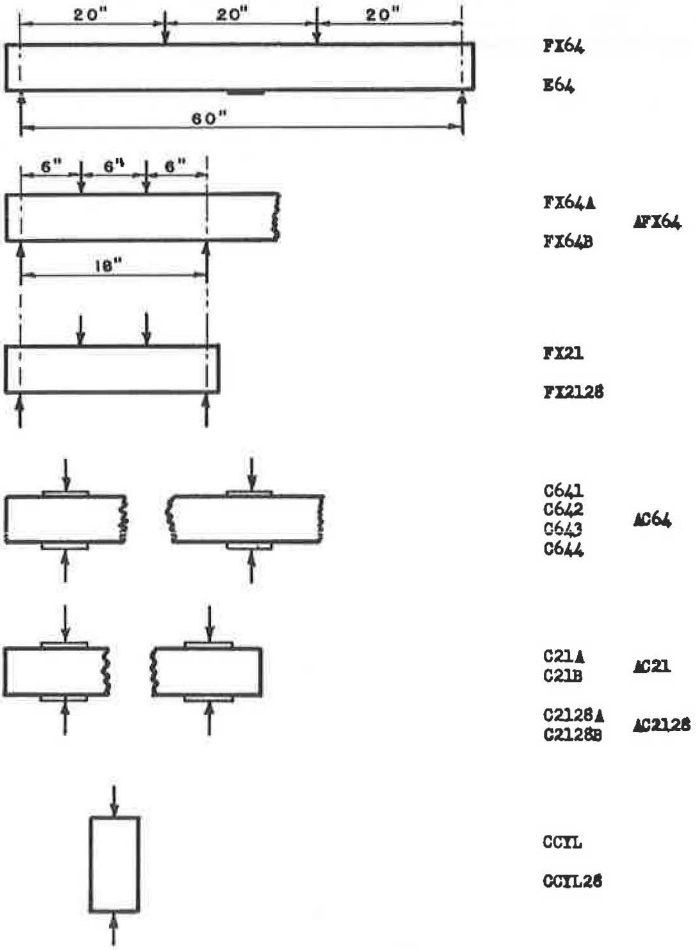

Figure 1. Schematic of static tests.

relationship for this study. The loads were applied in a sinusoidal pattern at a rate of 450 cycles per minute (cpm). Also, the ratio of the minimum to maximum loads was held constant at 0.15.

For these tests the level of the cyclic loads that were applied to each specimen was established arbitrarily. However, it was based somewhat on the age of the specimen and the number of cycles of load the previous specimens had sustained before failing. Because the static strength of these specimens varied, the actual range of applied stress levels (expressed as a percentage of the ultimate) was quite wide.

Phase 3—Variable Load Fatigue Tests

In the phase 3 series of tests, the 64-in. beams from 28 specimen groups were subjected to two levels of repetitive loads. A range of test conditions was provided by varying the magnitude and difference in the loads and the number of cycles at each level. Also, in 8 cases the second load level was lower than the first, whereas in 20 cases it was reversed. The cyclic load pattern, rate, and load ratio were the same as in the constant cycle tests of phase 2.

STATIC TEST DATA AND ANALYSIS

A considerable number of static tests were conducted in this research study to determine the fatigue characteristics of concrete and to provide the data for evaluating the fatigue test results.

As mentioned previously, results of fatigue tests are customarily reported on an S/N diagram. The effects of variables that are not of interest are masked out by normalizing and reporting the actual applied cyclic loads in terms of percentage of the ultimate strength of the specimens. It is therefore necessary to predict the ultimate strength of the specimens that are tested in fatigue, and this is the greatest weakness in the analysis of fatigue test data, especially for concrete. Concrete is not a homogeneous material but is rather a composite of several materials. Many physical and environmental factors cause variations in the strength of test specimens, even within the same batch of concrete. Data from a group of tests may be analyzed statistically to establish empirical relationships or evaluations of their strength. Unfortunately, it is one thing to derive such empirical relationships and yet another to relate that equation to specific test specimens (or to other specimens that were not included in the original set of data). Thus, it is obvious that the accuracy of the S/N relationship is completely dependent on the accuracy of the prediction of the static ultimate strength of the specimens that are tested in fatigue. One further step, as also mentioned, is that the S/N relationship that is developed will then be used to evaluate the results of the variable load fatigue tests.

The static tests that were conducted and the subsequent analysis of those data form a very important part of this study. So that the best possible results would be achieved for the study, several methods of analysis were evaluated and discarded. All were rational and have been used by other researchers. However, the appropriateness of a method had to be evaluated on the basis of the S/N line that it yielded. Certain boundary conditions on the nature of the required line necessitated the selection of the final method of analysis. A few of the methods of analysis that were tried will be briefly commented on before the final approach is discussed.

Single Regression Analysis

When the schedule of tests was set up for this study, the original plan was that tests of the specimens from the 32 static groups would yield an empirical relationship from which the static strength of the 64-in. beams could be predicted with reasonable accuracy. If this could be done, the stresses to be applied in the fatigue tests could be established prior to those tests. This approach proved to be unworkable because there was considerable scatter in the data from the tests of all types of specimens. An examination of the scatter diagrams of comparisons between the various test results revealed that the best one was between flexure tests of the 64-in. beams and the average for their halves. It was therefore concluded that the prediction of the ultimate strength of the 64-in. beams should be based on tests of the pieces remaining after the fatigue tests. A regression analysis was made on the data from tests of the 32 static specimen groups to develop an equation relating the strength of the 64-in. beams and the average flexural strength of their halves. This was then used with the actual strengths of the halves of a particular beam to predict the flexural strength of that beam after completion of the fatigue test. This value was then used to normalize the applied fatigue stress for developing the desired S/N relationship. When this approach was used, however, it was found that the resulting S/N line was almost flat and completely unreasonable.

Multiple Regression Analyses

The level of scatter within the data from every type of static test precluded the possibility of developing a simple relationship that could be used to predict accurately the static flexural strength of the 64-in. beams. The next approach to the problem was based on the assumption that, because several types of specimens were cast from a single batch of concrete, the data from tests of specimens within a group could be combined to form a better basis for predicting the strength of the 64-in. beams. Accordingly, a multiple linear regression analysis was made on all of the data from tests of specimens from the 32 static groups. The computer program that was used for this analysis computes a sequence of multiple linear regression equations in a stepwise manner. At each step one variable is added to the regression equation (3). Thus, a multiple-term equation was developed that related the data from many strength tests to the flexural strength of the 64-in. beams. This equation was then used to predict the flexural strength of the 64-in. beams tested in fatigue by using the data from many static tests of specimens from their groups. However, when this method was used to normalize the applied fatigue loads, the results indicated that most of the specimens had been subjected to fatigue stresses greater than 100 percent of their ultimate strength. Because this could not be true, it indicated that the relationships between the various types of test data and those of the 64-in. beams might be changing. It was decided to take a closer look at the static data from all phases of the study.

The data from the static tests conducted in all phases of this study are given in Table 1. One section gives the data from the 32 static specimen groups of phase 1, whereas the other section summarizes the data from each type of test from all of the specimen groups. Although it has only limited statistical value, the standard deviation for each test type is also expressed as a percentage of the mean strength value to permit comparison between flexural and compressive strength data.

In some cases there is considerable difference between the values in the 2 sections. One of the major causes for this variation may be attributed to the difference in the

TABLE 1

SUMMARY OF STATISTICAL MEASURES OF STATIC TEST DATA

Type of Test and Specimens Used	32 Static Tests			All Test Groups		
	Mean (psi)	Standard Deviation (psi)	Percentage of Mean	Mean (psi)	Standard Deviation (psi)	Percentage of Mean
Compressive Strength						
Cylinders at 28 days (CCYL28)	5,576	329	5.90	5,767	409	7.09
Halves of 21-in. beams at 28 days (AC2128)	5,927	402	6.78	6,134	352	5.74
Cylinders at test of 64-in. beams (CCYL)	6,235	331	5.31	6,472	464	7.17
Halves of 21-in. beam at test of 64-in. beam (AC21)	7,100	414	5.83	7,283	441	6.06
Quarters of 64-in. beams (AC64)	6,959	483	6.94	7,126	427	5.99
Flexural Strength						
21-in. beams, at 28 days (FX2128)	770	61	7.92	814	63	7.74
21-in. beams, at test of 64-in. beams (FX21)	741	69	9.31	793	70	8.83
Halves of 64-in. beams (AFX64)	694	49	7.06	788	78	9.90
64-in. beams (FX64)	623	65	10.43	—[a]	—[a]	—[a]
Modulus of Elasticity						
64-in. beams (E64)	5.51	0.36	6.53	5.58	0.45	7.65

[a] Not available for beams tested in fatigue.

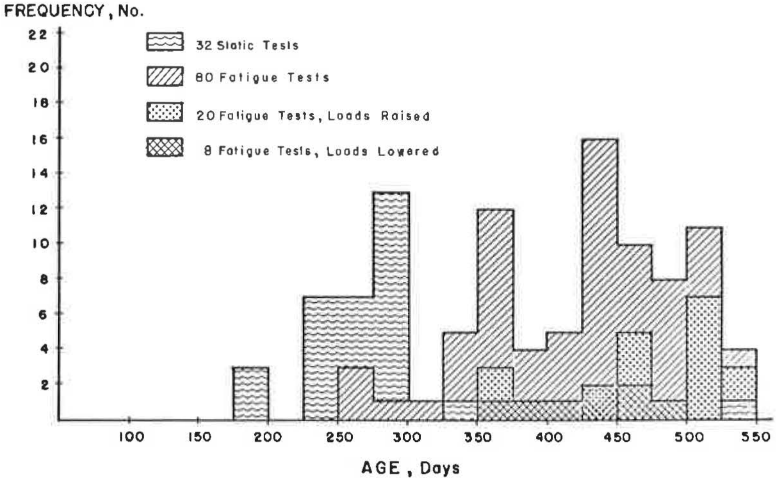

Figure 2. Frequency of tests conducted at various ages.

ages of the specimens. Figure 2 shows the distribution of specimen ages for the various testing plans. The primary cause of the wide variation in age at test was that the specimens were selected at random after all of the specimens were cast.

As a by-product of the computer analysis, the level of correlation among the various types of tests was also determined. The correlation coefficients for the 32 static tests groups are given in matrix form in Table 2. As expected, the degree of correlation among some of the tests is poor. The maximum correlation of any test with the static flexural strength of the 64-in. beams (FX64) is obtained for the average of the flexural strengths of the halves of the broken 64-in. beams (AFX64), which is expected. However, that correlation is only 0.477. This emphasizes the fact that even this correlation cannot be used to accurately predict the static ultimate strength of the 64-in. beams.

A similar correlation matrix relating the static test data for all 112 specimen groups is given in Table 3. A comparison of the 2 matrices shows that the level of correlations is improved considerably when all test values are considered, even though the range of test ages is widened. The strength of the 64-in. beams obviously could not be included in this table because most of these specimens were broken in fatigue. With regard to the average of the strength of the halves of the 64-in. beams, the maximum correlation is obtained for the strength of the companion 21-in. beams (FX21), i.e., 0.584.

Because of this situation, it was concluded that a somewhat indirect method of analysis would be necessary. It was still felt, however, that the multiple regression method

TABLE 2

CORRELATION OF VARIABLES FOR 32 STATIC TEST GROUPS

Specimen	AFX64	FX21	FX2128	AC64	AC21	AC2128	CCYL	CCYL28	E64	LAGE	FX64
AFX64	1.000	0.133	0.126	0.417	0.391	0.336	0.173	0.333	0.065	0.317	0.477
FX21		1.000	0.247	0.035	0.093	0.029	0.017	0.002	0.261	0.027	0.022
FX2128			1.000	0.530	0.248	0.491	0.070	0.374	0.351	0.165	0.165
AC64				1.000	0.742	0.736	0.277	0.680	0.465	0.187	0.394
AC21					1.000	0.732	0.522	0.726	0.431	0.184	0.342
AC2128						1.000	0.437	0.672	0.514	0.204	0.361
CCYL							1.000	0.575	0.430	0.028	0.373
CCYL28								1.000	0.542	0.127	0.240
E64									1.000	0.217	0.312
LAGE										1.000	0.399
FX64											1.000

TABLE 3

CORRELATION OF VARIABLES FOR ALL 112 TEST GROUPS

Specimen	AFX64	FX21	FX2128	AC64	AC21	AC2128	CCYL	CCYL28	E64	LLAGE
AFX64	1.000	0.584	0.480	0.413	0.323	0.435	0.361	0.360	0.094	0.745
FX21		1.000	0.351	0.276	0.276	0.283	0.261	0.214	0.094	0.517
FX2128			1.000	0.505	0.400	0.517	0.310	0.384	0.155	0.473
AC64				1.000	0.638	0.738	0.457	0.623	0.298	0.221
AC21					1.000	0.684	0.521	0.595	0.167	0.150
AC2128						1.000	0.558	0.640	0.248	0.279
CCYL							1.000	0.591	0.287	0.225
CCYL28								1.000	0.161	0.201
E64									1.000	0.073
LLAGE										1.000

of analysis had to be used to obtain the best possible prediction for the static flexural strength of the 64-in. beams. Because much higher correlations among the various types of tests were obtained when the data from all of the test groups were considered (Table 3), it was also concluded that this should form the basis for further analysis.

For this final method of analysis, the computer program was first used to develop a multiple-term regression equation relating the different static tests to the average flexural strength of the halves of the broken 64-in. beams (AFX64) by using the data from all 112 specimen groups. The program was then used to develop a simple linear equation relating the average strength of the beam ends (AFX64) to the strength of the 64-in. beam (FX64) by using the data only from the 32 static test groups. The predicted strength of the 64-in. beams was calculated as follows: The first equation was used to calculate an expected value for AFX64, and then this expected value of AFX64 was used with the second equation to predict the static strength of the 64-in. beam (FX64).

A summary of the first 6 steps of the regression analysis used to develop the first equation is given in Table 4. Table 4 gives the level of correlation (multiple R) that increases as each step is completed and as another variable is added to the equation. The benefit of adding variables drops as each variable is added. This is evident by the reduction in the "increase in R^2" value. To minimize the complexity of the computations and the resulting equations, we decided to terminate the regression equation after the fourth step, where the "increase in R^2" value drops from 0.0451 to 0.0034. The correlation coefficient for the resulting equation is 0.8481, which is a substantial increase over the values shown in the previous correlation matrices. The computer output for the fourth step of the regression analysis provided the coefficients for the desired equation:

$$\mathrm{AFX64} = -2720 + \underset{(\pm 0.1382)}{0.286\,(\mathrm{FX21})} - \underset{(\pm 0.5592)}{1.36\,(\mathrm{AGE})} + \underset{(\pm 0.0197)}{0.0405\,(\mathrm{C642})} + \underset{(\pm 2658)}{8580\,(\mathrm{LLAGE})}$$

where

AFX64 = average flexural strength of ends of 64-in. beam;
FX21 = flexural strength of companion 21-in. beam;
AGE = test age for 64-in. beam and companion specimens;
C642 = compressive strength, second strongest quarter of 64-in. beam; and
LLAGE = loglog of AGE.

The equation contains two "age" terms; however, one is negative. The combination of the two gives a curved relationship

TABLE 4

SUMMARY OF REGRESSION ANALYSIS

Step	Variable Entered	Multiple R	Multiple R^2	Increase in R^2
1	LLAGE	0.7451	0.5552	0.5552
2	C642	0.7898	0.6237	0.0686
3	AGE	0.8211	0.6743	0.0506
4	FX21	0.8481	0.7194	0.0451
5	C2128B	0.8502	0.7228	0.0034
6	CCYL28	0.8506	0.7236	0.0008

Note: Summary of statistical values for each step of regression analysis relating results of individual tests to the average flexural strength of the halves of the 64-in. beams (AFX64).

for the strength of the specimens with age, as drying and other changes occur. The 95 percent confidence limits for each coefficient are given in the equation in parentheses. Thus, each term in the following equation is significant.

The second equation, which relates the flexural strength of the 64-in. beam to the average for the halves, is given as follows:

$$FX64 = 159 + 0.667\ (AFX64)$$

where FX64 is static flexural strength of 64-in beam.

The effectiveness of the first equation to predict the average flexural strength of the halves of the 64-in. beams is shown in Figure 3. It is felt that the quality of this prediction is good, considering the level of scatter associated with the data from each type of test. It may be noted that the values for the 32 static tests fall together along the lower portion of the 45 deg comparison line.

S/N FATIGUE TEST DATA AND ANALYSIS

In the second phase of this study the 64-in. beams from 44 specimen groups were subjected to constant cycle fatigue tests to develop the basic S/N relationship necessary for evaluating the results of the variable load fatigue tests. A summary of the test conditions and the S/N prediction are given for each specimen in Table 5. The actual range in applied stresses was very wide. These data are plotted on the S/N diagram shown in Figure 4.

The S/N line was established by using two boundary conditions from previous concrete fatigue research. First, the main portion of the S/N relationship is represented as a straight, descending line when plotted on a semilog basis. Second, the initial portion of the S/N line is not straight, but rather it curves downward from 100 percent to meet the linear portion of the line. The exact number of cycles at which the two portions of the line should intersect is not well established. This initial portion of the S/N line represents the low cycle fatigue region. That is, specimens that are loaded in this range of their ultimate strength are subject not only to the effects of simple fatigue but also to other fracture phenomena as well. The explanation of the fracture mechanics involved will not be discussed, but the general nature of this situation is recognized to influence the analysis of the data.

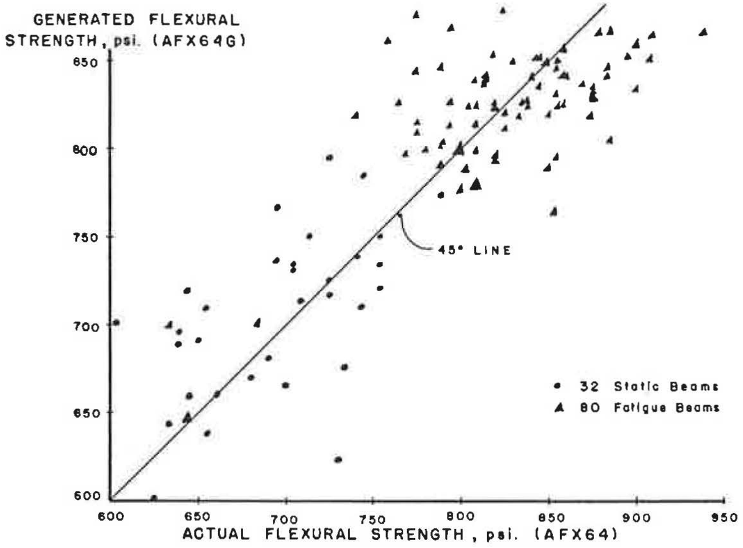

Figure 3. Average flexural strength of halves of 64-in. beam, generated versus actual.

TABLE 5

SUMMARY OF S/N FATIGUE TEST DATA

Beam	Stress Range (psi)	Maximum Stress (psi)	Maximum Stress S (percent)	Actual Cycles n	Predicted Cycles N	Damage d (n/N)
6	350	418	67	5,700	317×10^7	0.00
8	389	458	73	107,695	189×10^5	0.01
9	369	442	75	885	3,437,980	0.00
30	461	544	78	324,000	265,711	1.22
32	339	398	59	3,247,000	29×10^{11}	0.00
35	496	583	80	89,345	48,211	1.85
36	472	555	76	1,965,150	1,464,440	1.34
42	519	611	83	1,729	3,726	0.46
44	519	611	86	1,658	288	5.76
45	496	583	80	3,965	48,211	0.08
47	519	611	86	9,500	288	32.99
48	519	611	85	43,405	676	64.20
51	519	611	83	3,457	3,726	0.93
54	567	666	92	26	6	4.33
55	567	666	93	478	4	109.89
57	519	611	83	3,578	3,726	0.96
64	519	611	84	266	1,587	0.17
71	496	583	81	197,845	20,536	9.63
73	472	555	78	3,904,462	265,711	14.69
74	519	611	86	202	288	0.70
75	496	583	82	53,438	8,747	6.11
76	496	583	81	490	20,536	0.02
79	496	583	80	361,856	48,211	7.51
82	519	611	82	130	8,747	0.01
88	496	583	82	9,010	8,747	1.03
93	496	583	85	3,400	676	5.03
94	567	666	96	24	2	11.65
98	519	611	88	1,621	50	31.02
99	472	555	79	461	113,182	0.00
101	496	583	83	115,440	3,726	30.98
102	567	666	93	79	4	18.16
103	472	555	79	340	113,182	0.00
104	519	611	90	16	14	1.17
105	472	555	79	75,175	113,182	0.66
108	519	611	85	849	676	1.26
109	519	611	85	259,842	676	384.34
110	519	611	84	19,415	1,587	12.23
111	519	611	89	4,011	24	168.53
113	496	583	84	5,041	1,587	3.18
114	567	666	94	140	3	42.42
116	519	611	89	579	23	24.33
118	472	555	80	2,055	48,211	0.04
122	496	583	87	25,902	123	211.14
126	567	666	96	3	2	1.46

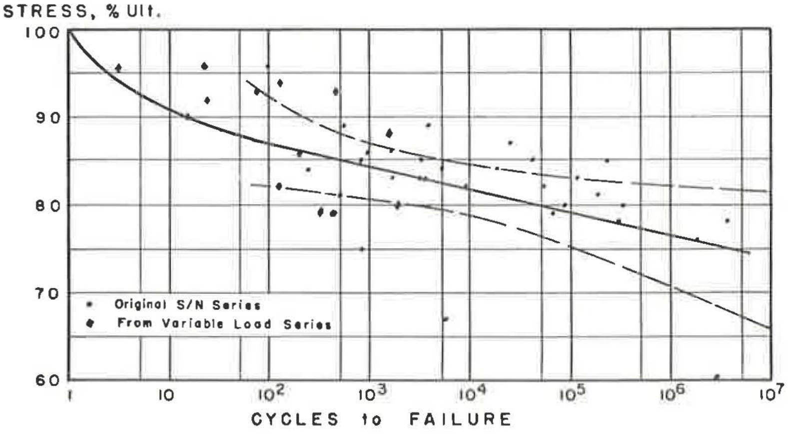

Figure 4. S/N diagram for monotonic fatigue tests.

The S/N line that was developed for this study is shown in Figure 4. It may be noted that 11 points represent specimens from the variable load fatigue series, which failed prior to changing of the cyclic loads. The linear portion of the S/N line was established by neglecting the specimens that failed at less than 70 cycles, and the computer was used to calculate a regression line through the remaining points. The equation for this portion of the line is as follows:

$$\text{Stress, percent ultimate} = 92.6 - \underset{(\pm 1.914)}{2.70} (\text{log cycles})$$

The 99 percent confidence limits for the slope of the line is shown in parentheses beneath that term in the equation. The 99 percent confidence band for the linear portion of the S/N line is also shown in Figure 4. It has been stated that the Miner theory postulates that fatigue damage occurs linearly with an increase in cycles of load. The data given in Table 5 and the S/N line were used to calculate the apparent damage, d, for each of the specimens. These values are also given in Table 5. Although it is difficult to evaluate the apparent damage, the very wide range is interesting. It may be noted that the apparent damage at failure ranges from 0.00 to 384.34.

The S/N relationship that has been established is the best result of several methods of analysis. There is obviously considerable scatter of the data points plotted on the diagram. This scatter is due primarily to errors in accurately predicting the strength of the individual test specimens. However, this was caused by the basic variations in strength within and between the specimens cast from each batch of concrete.

VARIABLE LOAD FATIGUE TEST DATA AND ANALYSIS

In the third phase of this study, 28 of the 64-in. beams were subjected to two levels of repetitive loads to develop some information on the cumulative damage characteristics of plain concrete. Although the number of load variations was limited, the following variables were included: the stress and the number of cycles at the first level, the direction of change to the second stress level, and the magnitude of change in stresses.

Because the number of tests that could be run was limited, the testing plan was designed to provide a range of conditions that could be used to evaluate the Miner hypothesis. This theory is based on the assumption that fatigue damage will occur in proportion to the work done on the specimens and that the resulting damage may be expressed by the ratio of the number of cycles of stress at a particular level to the corresponding number of cycles to cause failure at that same level (2). In other words, a specimen subjected to n_1 cycles of load at a constant stress level S_1, which would cause the specimen to fail after N_1 cycles, will use up 100 (n_1/N_1) percent of its fatigue life. Additional repetitions of load at different stress levels will cause damage at the same proportional rate.

If the Miner theory is appropriate, the damage created under the two levels of stress could be accumulated to yield a set of values that could be compared against the S/N diagram. If the resulting points fell within the scatter of the S/N points, it could be said that damage occurred on a linear basis.

Accordingly, the data from this phase of testing were evaluated as follows: The S/N line and the normalized maximum stresses were used to calculate the number of cycles at the second stress level necessary to cause the damage, d, theoretically created at the first level and yielded n_1'. This value was added to the actual number of cycles that were applied at the second stress level, and an equivalent total number of cycles was obtained at the second stress level, n_2^*. These calculations were made with the computer. In equation form the preceding may be expressed as follows:

$$n_1/N_1 = d_1$$

$$d_1\ (N_2) = n_1'$$

$$n_1' + n_2 = n_2^*$$

TABLE 6

DATA FROM VARIABLE FATIGUE TESTS WITH LOADS LOWERED

Beam	Maximum Stress S_1 (percent)	First Level Cycles n_1	Damage d_1 (n_1/N_1)	Equivalent First Level Cycles n_1'	Maximum Stress S_2 (percent)	Second Level Cycles n_2	Damage d_2 (n_2/N_2)	Stress Change (percent)	Total Cycles $n_2^\#$	Total Damage d^*
78	85	1,000	1.48	922,677	77	42,954	0.07	7	965,631	1.55
67	86	1,000	3.47	922,677	78	633,634	2.38	8	1,556,311	5.85
66	86	2,000	6.94	1,845,353	78	1,043,742	3.93	8	2,889,095	10.87
72	86	2,000	6.94	1,845,353	78	405,135	1.52	8	2,250,488	8.46
92	92	100	16.67	26,453	84	1,363	0.86	8	27,816	17.53
119	93	50	11.49	3,310	86	1,774	6.16	7	5,084	17.65
91	93	100	22.99	15,542	85	12,786	18.91	8	28,328	41.90
124	97	50	29.94	666	89	41,763	1,754.75	8	42,429	1,784.69

The test data and the results of the preceding calculations are given in Tables 6 and 7 for the tests in which the stress level was lowered and raised respectively. The theoretical damage created at the 2 stress levels has been indicated. The extreme values are due to scatter of the points about the S/N line, which emphasizes the error that could be created by not recognizing the effect of variation in strength of concrete on its fatigue life.

The preceding information for each specimen was used to plot points that are shown in Figure 5 and that represent the equivalent total number of cycles at the second stress level versus that stress level. The equation for the regression line through the data is as follows:

$$\text{Stress, percent ultimate} = 92.7 - \underset{(\pm 1.132)}{1.88}\,(\log \text{cycles})$$

The 99 percent confidence limits for the slope of the line are shown beneath that term in the equation. The 99 percent confidence band for the line is also shown in Figure 5. The results of these tests were evaluated by comparing this regression line with the S/N line and its 99 percent confidence band. This comparison is shown in Figure 6. The line representing the variable load fatigue test data falls within the 99 percent band for most of its length.

TABLE 7

DATA FROM VARIABLE FATIGUE TESTS WITH LOADS RAISED

Beam	Maximum Stress S_1 (percent)	First Level Cycles n_1	Damage d_1 (n_1/N_1)	Equivalent First Level Cycles n_1'	Maximum Stress S_2 (percent)	Second Level Cycles n_2	Damage d_2 (n_2/N_2)	Stress Change (percent)	Total Cycles n_2^*	Total Damage d^*
53	69	12,000	0.00	100	84	6,262	3.95	15	6,362	3.95
33	69	24,000	0.00	0	85	4,905	7.26	16	4,905	7.26
50	69	24,000	0.00	0	85	3,460	5.12	16	3,460	5.12
120	69	96,000	0.00	0	85	32,080	47.45	16	32,080	47.45
77	70	48,000	0.00	0	85	172,571	255.26	15	172,571	255.26
41	70	48,000	0.00	0	86	167	0.58	16	167	0.58
34	71	12,000	0.00	0	87	905	7.38	16	905	7.38
123	71	24,000	0.00	0	87	3,735	30.45	16	3,735	30.45
43	76	3,000	0.00	8	83	43,932	11.79	7	43,940	11.79
83	76	6,000	0.00	15	83	571	0.15	7	586	0.15
86	76	6,000	0.00	7	84	2,833	1.78	8	2,840	1.78
68	77	12,000	0.02	13	85	19,625	29.03	8	19,638	29.05
37	78	3,000	0.01	3	86	2,776	9.64	8	2,779	9.65
59	78	12,000	0.04	13	86	34,547	119.96	8	34,560	120.00
80	78	12,000	0.04	13	86	387,995	1,347.30	8	388,008	1,347.34
38	79	3,000	0.03	3	87	521	4.24	8	524	4.27
46	79	3,000	0.03	3	87	18,909	154.15	8	18,912	154.18
69	79	6,000	0.05	7	87	7,015	57.19	8	7,022	57.24
40	80	3,000	0.06	3	88	1,943	37.19	8	1,946	37.25
70	80	6,000	0.12	3	89	1,737	72.98	9	1,740	73.10

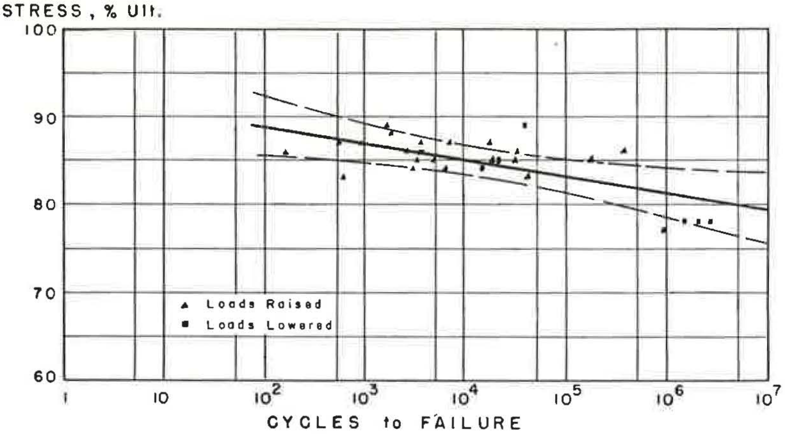

Figure 5. Effect of 2 levels of fatigue loading on fatigue life.

Whether these 2 lines are significantly different was determined by considering 2 statistical factors. The appropriate terms and their confidence limits in the preceding equation show that the slopes of the 2 lines are not statistically different and that their intercepts at the vertical axis are almost identical. Also, the standard error of fit for the variable load data points to the S/N line has been compared against the corresponding value for the S/N points with respect to the S/N line. The F-ratio for this comparison was computed to be 1.63. Therefore, the variable load test data are not significantly different from the S/N test data. Therefore, the conclusion was that there is no evidence of a significant difference in the 2 lines shown in Figure 6.

PREVIOUS PLAIN CONCRETE FATIGUE RESEARCH

As mentioned previously, there has been considerable research on the fatigue characteristics of plain concrete. How do the results of this study relate to earlier studies? Rather than to examine those studies in detail, it is perhaps more appropriate to comment on one of the main findings of this study and to relate this finding to one of the previous studies. That is, it has been found that the accuracy of the S/N relationship is

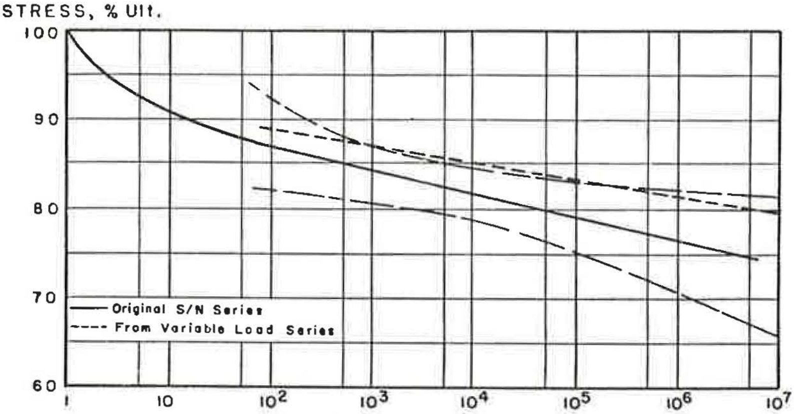

Figure 6. Comparison of cumulative damage line with S/N line.

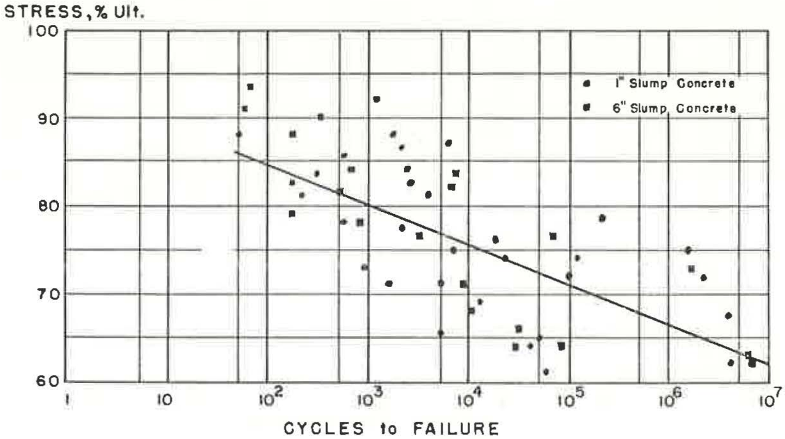

Figure 7. S/N diagram developed by Murdock and Kesler (4).

dependent on the accuracy of the normalizing static strength prediction. As the accuracy diminishes, the level of scatter of the data points increases, and the position and slope of the S/N line are changed.

The S/N diagram reported in a related fatigue study conducted by Murdock and Kesler (4) is shown in Figure 7. For these tests of beams, the normalizing value was the average flexural strength of the halves of the broken beams. The variation in the indicated percentage of ultimate strength for the specimens, which failed at the same number of cycles, is about twice as large as that shown in Figure 4.

Therefore, it is concluded that many of the previously reported S/N relationships for concrete may leave something to be desired but that their use for comparative purposes within a study is probably reasonable. However, the true accuracy of those S/N relationships may be questionable.

CONCLUSIONS

1. The accuracy of the S/N line representing the fatigue strength of concrete is very dependent on the accuracy of the prediction of the static strength that is used to normalize the data.
2. The Miner hypothesis represents the cumulative damage of plain concrete due to variations in fatigue loading in a reasonable manner.
3. Concrete should not be stressed to a level where the effect of fatigue is a real possibility. Because of the local, or gross, variations in the strength of concrete, fatigue effects may become critical at about 70 percent of the static ultimate strength.

ACKNOWLEDGMENTS

The study reported here is a portion of a major investigation that is being conducted by the Office of Research of the Federal Highway Administration (5) on the life expectancy of highway bridges.

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