HYDRAULICS OF RIGID BOUNDARY BASINS

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The mechanics of flow in a particular type of energy dissipator is investigated from a basic point of view. The dissipator uses artificial roughness elements to induce a hydraulic jump. A comprehensive test program was conducted to determine the energy and momentum coefficients and the drag coefficients necessary to analyze the basins analytically. The method outlined is general and allows the designer flexibility in his choice of dimensions. The test program included studies of flow from both circular and rectangular culvert outfalls. Discharges ranging from 6.75 to 23.5 ft^3/sec were investigated in a 1.45-ft diameter circular pipe and a 1.25- by 1.25-ft rectangular box.

•THIS PAPER presents a design procedure for a particular type of energy-dissipating basin at culvert outfalls. The basin features a simple geometrical design, readily adaptable to field construction methods. It utilizes roughness elements to induce a hydraulic jump that enhances the dissipation of energy. The necessary coefficients have been developed in an experimental program so that the designer is equipped to analyze a proposed basin by using fundamental hydraulic principles.

When tailwater submerges a culvert outlet section, the jet of water emerging from the conduit has the characteristics of a submerged jet. When low tailwater occurs and the conduit walls terminate abruptly but the floor continues at the same slope, the efflux has the characteristics of flow at an abrupt expansion. Most culverts function somewhere between these two extremes. This report is concerned with the case where the inverts of culverts are set so that flow at the outfall has the characteristics of flow at an abrupt expansion; i.e., the inverts are set sufficiently high so that the flow will plunge and spread in a predictable manner.

The basin investigated is shown in plan and section in Figure 1. It features an optional width and roughness elements of selected height and spacing.

The basic equations used in the analysis are the continuity equation and momentum equation with an appropriate drag term. The procedure requires the use of the following design aids developed during this study: momentum equation correction coefficients for nonhydrostatic pressure and nonuniform distribution of velocity at the outfall section; dimensionless water surface contours and relative velocities for the rapidly varied flow region downstream of the outfall; and suitable drag coefficients for a particular size and grouping of roughness elements placed on the floor of the basin.

In the interest of brevity, only one each of the design aids and only a brief description of the experimental programs required to develop the design aids are presented. The number of references cited is also limited by length considerations. The notation used is given at the end of the paper.

METHOD OF ANALYSIS

Basic Equations

With reference to Figure 1, the momentum equation written in the direction of flow for the control volume between station 0.0 and station B is

$$\beta_2 \rho V_o Q + \beta_1 \gamma (y_o^2/2) W_o = F_T + F_R + \beta_4 \rho V_2 Q + \beta_3 \gamma (y_2^2/2) W_2 \tag{1}$$

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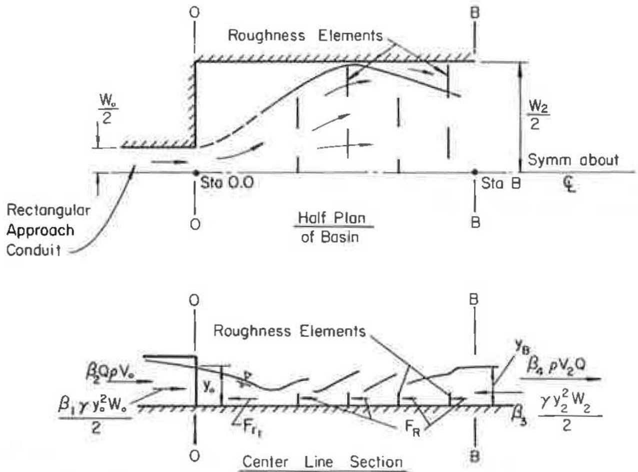

Figure 1. Energy-dissipating basin.

The drag force F_R is defined as

$$F_R = C_D A_F N \rho(V_a^2/2) \tag{2}$$

V_a is the approach velocity at the first row of roughness elements, defined as the average velocity 2 pipe diameters downstream of the outlet. F_T, the shear force exerted by the floor on the flow in the area upstream of the roughness elements and downstream of the outlet, is small and henceforth is included in the F_R term.

Making use of the continuity equation, we obtain

$$y_2 = Q/V_2W_2 \tag{3}$$

Inserting the value of y_2 and F_R (Eq. 2) into Eq. 1 and assuming $\beta_3 = \beta_4 = 1$ yield the following relationship

$$\beta_2\rho V_o Q + \beta_1\gamma(y_o^2/2) W_o = C_D A_F N \rho(V_a^2/2) + \rho Q V_2 + (\gamma Q^2/2V_2^2W_2) \tag{4}$$

This is the design equation. For a given discharge, depth of flow at the outfall section, approach pipe width, β_1, β_2, a particular set of roughness elements, C_D, and V_a, and estimate of V_2 the exit velocity from the basin is readily obtained.

The following sections describe the experimental program conducted for the purpose of developing design aids that provide suitable values of β_1, β_2, V_a, and C_D.

ENERGY AND MOMENTUM CORRECTION FACTORS

Theoretical Development

At any cross section, the amount of energy per pound of water at any point is

$$H = \alpha_1[(P/\gamma) + y] + \alpha_2(V^2/2g) \tag{5}$$

Where nonuniform steady-flow conditions prevail, it is convenient to evaluate the power of the flow at a section by multiplying the quantity of energy per pound of water by the number of pounds of water per second that pass through the incremental area surrounding the point; i.e.,

$$\Delta P_T = \{[(P/\gamma) + y] + (V^2/2g)\} \gamma\Delta Q \tag{6}$$

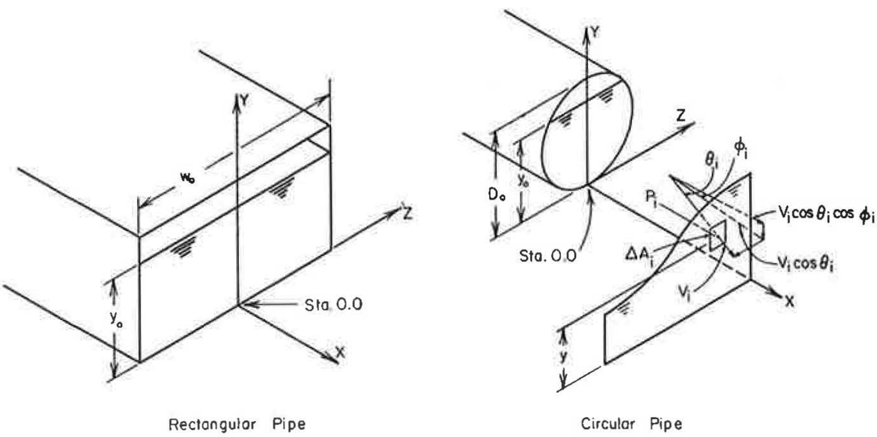

Figure 2. Outfall definition.

Referring to Figure 2, we have

$$\Delta Q_i = V_i \cos\theta_i \cos\phi_i \Delta A_i \tag{7}$$

At any cross section, the total power available is

$$P_T = \sum_i \{[(P/\gamma) + y]_i + (V_i^2/2g)\} \gamma V_i \cos\theta_i \cos\phi_i \Delta A_i \tag{8}$$

where the sum is taken over the entire section in question.

The specific energy equation that is the most convenient is made up of gross flow quantities.

$$H = \alpha_1 y + \alpha_2 [(Q/A)^2/2g] \tag{9}$$

This is converted to power by multiplying H by γQ.

$$P_T = HQ\gamma = \{\alpha_1 y + \alpha_2 [(Q/A)^2/2g]\} \gamma Q \tag{10}$$

Equating Eqs. 8 and 10 yields

$$\sum_i \{[(P/\gamma) + y]_i + (V_i^2/2g)\} (\gamma V_i \cos\theta_i \cos\phi_i) \Delta A_i = \{\alpha_1 y + \alpha_2 [(Q/A)^2/2g]\} \gamma Q \tag{11}$$

Canceling out γ, equating the like terms from each side, and solving for α_1 and α_2, we obtain

$$\alpha_1 = \frac{\sum_i \{[(P/\gamma) + y]_i (V_i \cos\theta_i \cos\phi_i \Delta A_i)\}}{yQ} \tag{12}$$

and

$$\alpha_2 = \frac{\sum_i (V_i^3 \cos\theta_i \cos\phi_i \Delta A_i)}{Q^3/A^2} \tag{13}$$

Utilizing the impulse and momentum principle and similar reasoning, we can show in differential form that the external force and momentum flux at any cross section in the x-direction are

$$F = \sum_i P_i \, \Delta A_i + \sum_i \rho V_i^2 \cos^2\theta_i \cos^2\phi_i \, \Delta A_i \tag{14}$$

The convenient expression for momentum and pressure force in terms of gross flow quantities is

$$F = \beta_1 (\gamma y^2 W/2) + \beta_2 Q \rho V \tag{15}$$

Equating Eqs. 14 and 15 and sorting out similar terms easily shows that

$$\beta_1 = \sum_i P_i \Delta A_i / [\gamma (yA/2)] \tag{16}$$

and

$$\beta_2 = \frac{\sum_i (V_i^2 \cos^2\theta_i \cos^2\phi_i \, \Delta A_i)}{Q^2/A} \tag{17}$$

where A = wetted area at the outfall section for either circular or rectangular conduits.

Equations 8 and 14 are general. There are no limiting assumptions; i. e., if the quantities can be measured precisely and if the incremental areas are taken small enough so that the summation is a good approximation of the integral, the quantities are correct for that particular cross section.

The procedure used to evaluate these quantities was to divide each cross section into a grid; measure the velocity, total head, and elevation at the centroid of each incremental area; deduce the pressure at the centroid by subtracting the sum of the velocity head and elevation head from the measured total head; and perform the various summations. Yaw and pitch probes were used in combination to obtain the yaw (horizontal) angle and pitch (vertical) angle of the velocity vector simultaneously with the measurements of total head and velocity magnitude at each grid point. The measured data were used to evaluate Eqs. 12, 13, 16, and 17. (Basic data are not presented in this paper but can be obtained at cost from Colorado State University.)

Sufficient data were also gathered at downstream sections for the purpose of plotting dimensionless water surface contours and relative velocities.

Test Facility

A rectangular basin with a horizontal aluminum floor 10 ft wide by 14 ft long with 1-ft vertical walls was positioned symmetrically downstream of a 20-ft length of approach pipe. The entire assembly was constructed within a large (185 ft long by 20 ft wide by 8 ft deep) outdoor flume. Data were collected for 2 approach pipes: a 1.45-ft diameter circular pipe and a 1.25- by 1.25-ft rectangular box. Both culverts had smooth walls. The pipe invert was horizontal and was carefully matched to the top surface of the basin floor. A rectangular, sharp-crested weir at the lower end of the large flume was used to check the discharges that were obtained by integration of experimental data. Tailwater effects from the weir were avoided by installing the floor of the test basin 2 ft above the concrete floor of the large flume. A variable-height dam for the purpose of tailwater control was constructed 35 ft downstream of the pipe outlet. The crest of the dam was maintained at the elevation of the top surface of the basin floor for all runs.

The measuring probes and supporting equipment were mounted on a large instrument carriage spanning the large flume.

Test Program and Range of Parameters

Seven discharges varying from 9.77 to 23.5 ft^3/sec were examined for the 1.45-ft circular approach pipe. The relative depth ratio y_o/D_o ranged from 0.75 to 1.00. The parameter $Q/D_o^{5/2}$ varied from 3.87 to 9.28. This encompasses the usual range of these parameters in highway culvert operation. Velocity data were taken at stations 0.0 and 2.9, and water surface contours were obtained at stations 0.0, 1.45, 2.9, and 4.35. In this paper, the station number indicates the distance downstream from the outfall section.

For the rectangular approach pipe, 6 discharges varying from 6.75 to 21.3 ft^3/sec were examined. The relative depth ratio y_o/W_o (depth of flow divided by pipe width) ranged from 0.61 to 0.94, and the Froude number $V_o/\sqrt{gy_o}$ varied from 1.44 to 2.35, the usual range of culvert operation. Velocity and water surface data were collected at stations 0.0, 2.5, 5.0, and 10.0.

Energy and momentum coefficients for circular and rectangular outfall sections are shown in Figures 3 and 4. One typical plot of dimensionless water surface contours and relative velocities is shown in Figure 5. Measurement apparatus, measuring procedures, analysis of data, and additional plots for the range of Froude numbers and relative depths mentioned earlier are included in other reports (5, 6, 7).

COEFFICIENTS OF DRAG

Problem Analysis

It has been shown by previous studies (2, 3, 4) that, for both supercritical and subcritical flow, an important correlating parameter with respect to the drag exerted by a roughness element on the flow is the relative depth, y/a, the ratio of the depth of flow striking the element to the height of the element. In the energy-dissipating basin where the water is diverted upward by the element, it is obvious that, up to a limiting point at least, the deeper the flow over the element is, the larger the quantity of water disturbed by the element will be and, consequently, the larger the apparent coefficient of drag will be.

The depth of flow at a point 2 pipe diameters downstream of the outlet (the approximate location of the first row of elements) was chosen as the scaling length y. For design purposes, this length is readily obtained from an appropriate plot of dimensionless water surface contours (Fig. 5 shows an example). Because the width of the expanding jet is not controlled by the walls at this point (station 2 D_o), this height is significant for a basin of any width when only the first 2 rows of elements are considered. This is not the case for the remaining rows of elements; i. e., the wider the basin is, the shallower the flow for a given discharge will be. For this reason, an additional correlating factor W_2/W_o, the basin width divided by the conduit width, is necessary.

The longitudinal spacing of the elements, J, is significant. Because of the complexity of the flow, it does not appear practical to include this factor as a density term. Instead, the ratio J/a is included as a geometric ratio and accompanies each design curve.

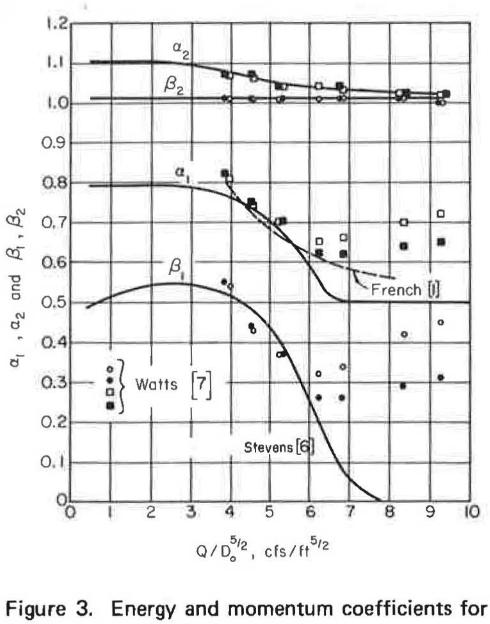

Figure 3. Energy and momentum coefficients for circular approach pipe.

The lateral spacing of the element, 2M, is not considered critical. The important point is that the elements in each row occupy half the width of the channel and that the elements be staggered in successive rows. This ensures that there will be no smooth longitudinal corridors through the basin. In order that the elements will serrate the flow and not act as a long sill, it is recommended that the ratio M/a be restricted to a range of 2 to 8.

Experimental Procedure for Obtaining Coefficients of Drag

Equation 4 with slight modification was used to evaluate C_D. The procedure was as follows:

1. A basin of known dimensions and pattern of roughness elements was subjected to a specific discharge;

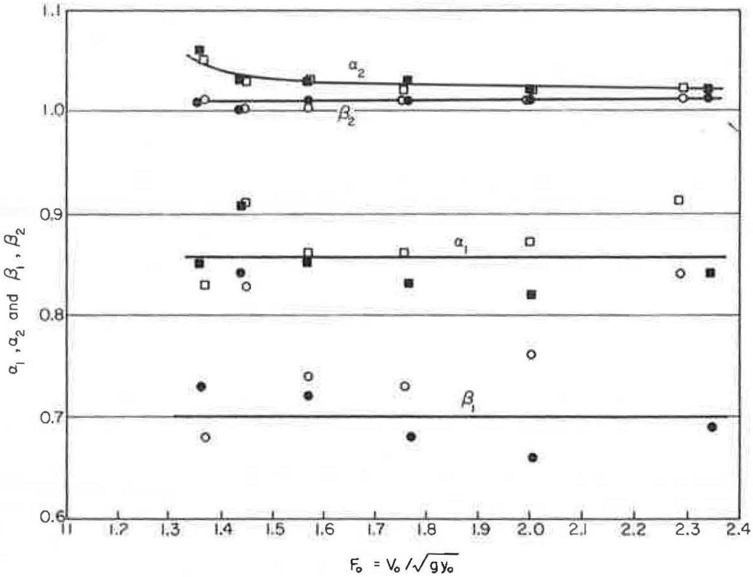

Figure 4. Energy and momentum coefficients for rectangular approach pipe.

2. At a section downstream of the last row of roughness elements, the yaw probe was used to measure the flow quantities, velocity, and pressure;

3. Equation 14 and the measured quantities from step 2 were used to evaluate the quantity $\sum_i P_i \Delta A_i + \sum_i \rho V_i^2 \cos^2\theta_i \cos^2\phi_i \Delta A_i$, which is equal to $\beta_4 \rho V_2 Q + (\beta_3 \gamma Q^2/2V_2^2 W_2)$;

4. The terms $\beta_2 \rho V_o Q + \beta_1 \gamma (y_o^2/2) W_o$ were evaluated (the necessary information was available from the study of flow properties at station 0.0, previously described);

5. The quantity obtained in step 3 was subtracted from the quantity obtained in step 4, and the remaining quantity is the drag force exerted by the group of elements on the flow, F_R;

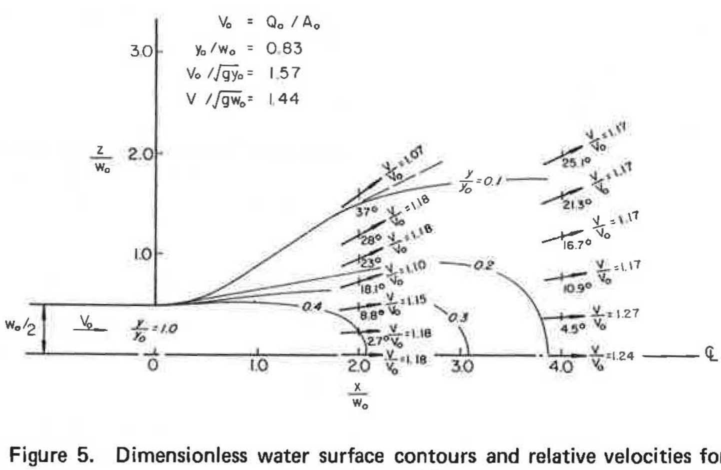

Figure 5. Dimensionless water surface contours and relative velocities for rectangular outfalls.

6. When F_R was known, Eq. 2 was solved for C_D; and
7. C_D was plotted as a function of Y/a, J/a, and W_2/W_o for a particular basin configuration.

Test Program for Evaluating Coefficients of Drag

Fifty-four runs were made to evaluate C_D. For the primary tests, 12 basin and element arrangements were examined. Each basin was subjected to 2 discharges. The lower discharge was approximately the design discharge (based on Wyoming State Highway Department specifications) for the approach pipe. The higher discharge was approximately 50 percent larger.

Two heights of elements were used for each discharge: a = $1\frac{1}{4}$ in. and a = $2\frac{1}{4}$ in. A variation of relative depth, y/a, from 1.1 to 2.7 resulted from the combination of 2 discharges and 2 element heights.

One pattern of longitudinal and lateral spacing was used for all runs. With 2 element heights, a two-fold variation of J/a, 6.0 and 12.0, was obtained.

For the 1.25-ft rectangular approach pipe, 2 basin widths, W_2 = 5 ft and W_2 = 10 ft, were tested. One width of basin, W_2 = 10 ft, was used with the 1.45-ft diameter circular approach pipe.

In addition to the primary runs described, 6 special runs were made. The circular approach pipe and 10-ft wide basin were used with two patterns of 4- by 1-in. elements. The significant difference between these basins and those used for the primary runs was the size of the elements. The 4-in. elements were spaced on 18-in. centers laterally; thus, large gaps existed between the elements. As expected, high-speed cores of water were measured downstream of the field of elements. The coefficient of drag deduced for the small, widely spaced elements was somewhat larger than comparable coefficients of drag for the elements 9 in. long. However, because of the probability of high-speed cores of water downstream of the basin, elements spaced laterally at more than twice their length are not recommended.

The 10- by 14-ft basin previously described, with a horizontal aluminum floor, tapped and threaded to accommodate roughness elements anchor bolts, was used for all experiments. False walls were installed for the 5-ft wide basins.

Data from 12 of the runs are shown in Figure 6. Similar figures for other combinations of roughness elements and for basins downstream of circular conduits are presented in other reports (5, 6, 7).

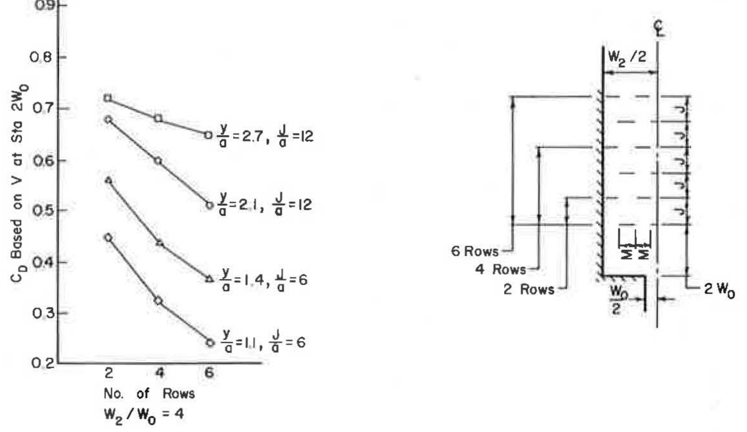

Figure 6. Coefficients of drag for roughness elements in rectangular approach pipe.

BASIN ANALYSIS

The design procedure and use of design aids are most readily explained by the solution of practical problem. A 6- by 6-ft culvert is used as an example, where $Q = 420$ ft^3/sec, $W_o = 6$ ft, and $y_o = 4$ ft. The designer's choice is $W_2/W_o = 4$, $y/a = 1.1$, and 6 rows of elements. Working with one-half of the basin, we have $W_2/W_o = 4$; $W_2 = W_o^4 = (6)(4) = 24$ ft; $Q/2 = 210$ ft^3/sec; $y_o = 4$ ft; $W_o/2 = 3$ ft; $V_o = (Q/2)/\text{area} = 210/[(W_o/2)(y_o)] = 210/[(3)(4)] = 17.5$ ft^3/sec; and $F_o(V_o/\sqrt{gy_o}) = 17.5/\sqrt{(32.2)(4)} = 1.54$.

As shown in Figure 4, $\beta_1 = 0.71$ and $\beta_2 = 1.01$.

The estimate from Figure 5 is $y/y_o = 0.21$; and $y = y_o(0.21) = (4)(0.21) = 0.84$ ft at $x/W_o = 2$, $V_a/V_o = 1.18$, and $V_a = (1.18)\,V_o = (1.18)(17.5) = 20.6$ ft^3/sec.

The height of element a is obtained by using $y = 0.84$, and $y/a = 1.1$ (designer's choice); therefore, $a = 0.76$ ft or use 0.75 ft.

The length of element m, as shown in Figure 6, is $(W_2/2)/3\frac{1}{2}$ spaces = 3.43 ft. Area of element a = (M)(a) = (3.43) (0.75) = 2.57 sq ft.

The longitudinal spacing of element J, as shown in Figure 6 for $y/a = 1.1$, is $J/a = 6.0$ or $J = (6.0)(0.75) = 4$ ft.

The number of elements N shown in Figure 6 is 10.5.

$C_D = 0.23$ for 6 rows of elements and $y/a = 1.1$ (Fig. 6).

Velocity at outfall of basin V_2 is estimated by using design Eq. 4.

$$\beta_1\gamma(y_o^2/2)(W_o/2) + \beta_2\rho V_o(Q/2) = C_D N A_F \rho\,(V_a^2/2) + \rho(Q/2)V_2 + [\gamma(Q/2)^2]/(2V_2^2 W_2/2)$$

$\gamma = 62.4$ lb/ft^3; $C_D = 0.23$; $\beta_3 = 1$; $V_o = 17.5$ ft^3/sec; $\rho = 1.94$ $lb\text{-}sec^2/ft^4$; $\beta_1 = 0.71$; $\beta_4 = 1$; $y_o = 4$ ft; $\beta_2 = 1.01$; $w_o = 6$ ft; $V_a = 20.6$ ft^3/sec; $W_2 = 24$ ft; and $Q = 420$ ft^3/sec. All values in the equation have been determined except V_2, the unknown. Substituting known values into the equation,

$$(0.71)(62.4)(4^2/2)(6/2) + (1.01)(1.94)(17.5)(210) = (0.23)(10.5)(2.57)(1.94)(20.6^2/2)$$
$$+ (1.94)(210)\,V_2 + (62.4/2)\,\{210^2/[V_2^2\,(24/2)]\}$$

and solving for V_2, we obtain

$$407\,V_2 + (114{,}600/V_2^2) = 5{,}710$$

These are three possible values of V_2: one value is negative and meaningless, and the other two are significant. The lower value is associated with subcritical flow, and the higher value is the conjugate velocity. When the preceding equation is solved, V_2 subcritical = 5.9 ft^3/sec and V_2 supercritical = 12.1 ft^3/sec. The depths of flow at the outfall corresponding to these velocities are $y_2 = (Q/2)/[(W_2/2)\,V_2]$; y_2 subcritical = $210/[(12)(5.9)] = 3.0$ ft; and y_2 supercritical = $210/[(12)(12.1)] = 1.4$ ft.

If tailwater is less than 1.4 ft, flow will be supercritical and the outfall velocity will be about 12.1 ft^3/sec. If the tailwater is 3.0 ft or higher (it is difficult to imagine a natural channel carrying 420 ft^3/sec at a depth less than this), the exit velocity will be about 5.9 ft^3/sec or less.

If the exit velocity and depths are satisfactory, the basin dimensions are as follows: length = $2W_o + 5J + 1J$ (add J downstream of last row of elements) = (2)(6) + (5)(4) + 4 = 36 ft; width = $(4)(W_o) = (4)(6) = 24$ ft; height of basin walls = y_2 subcritical + freeboard = 3.0 + 1.5 = 4.5 ft; size of element: 0.75×3.43; number required: $2 \times 10.5 = 21$; longitudinal spacing of elements J = 4 ft; and lateral spacing of elements 2M = 6.8 ft.

If V_2 deduced from Eq. 4 is close to critical velocity (this was not the case in the example solved in the preceding) and the tailwater depth downstream of the basin is, coincidentally, near critical depth, an unstable water surface (such as standing waves) is probable. If tailwater depth is near critical, the basin should be redesigned in such a way as to ensure adequate depth. Widening the basin or lowering the downstream portion of the basin are 2 effective means of attaining a suitable depth.

Further explanation of the design procedure and the application to other types of energy basins and a more complete description of the experiments used to develop the design aids are given in other reports (5, 6, 7).

CONCLUSIONS

A method of design for artificially roughened energy basins at culvert outfalls is presented. The momentum and continuity equation used in conjunction with experimentally derived design aids can be used to predict approximate exit velocity from the basin. The procedure is general and is readily applicable to other energy-dissipating structures.

NOTATION

The following notation is used in this paper:

a = height of roughness element, ft;
A = area of wetted cross section, ft^2;
A_F = frontal area of a roughness element, ft^2;
C_D = drag coefficient of roughness element, dimensionless;
D_o = diameter of circular approach pipe, ft;
F = force, lb;
F_R = drag force exerted by roughness elements on the flow, lb;
F_T = shear force exerted by the floor on the flow, lb;
g = acceleration of gravity, ft/sec^2;
H = total energy, ft-lb/lb;
J = longitudinal spacing of roughness elements, ft;
M = width of roughness element, ft;
N = number of elements;
P = pressure intensity at a point, lb/ft^2;
P_T = power, ft-lb/sec;
Q = discharge, ft^3/sec;
V = velocity, ft/sec;
V_a = approach velocity at the first row of roughness elements defined as the average velocity 2-pipe diameters downstream of the outlet, ft/sec;
V_o = average velocity at outfall section, ft/sec;
V_2 = average velocity at section B, ft/sec;
W = width of section, ft;
W_o = width of outfall, ft;
W_2 = width of channel at section B, ft;
x = longitudinal coordinate measured from the outlet section, ft;
y = vertical distance above a datum or depth of flow, ft;
y_o = depth of flow at outfall section, ft;
y_2 = average depth of flow at section B, ft;
z = lateral coordinate measured from the longitudinal centerline, ft;
α_1 = corrective coefficient (energy equation) for nonhydrostatic distribution of pressure, dimensionless;
α_2 = corrective coefficient (energy equation) for nonuniform distribution of velocity, dimensionless;
β_1 = corrective coefficient (momentum equation) for nonhydrostatic distribution of pressure at section 0, dimensionless;
β_2 = corrective coefficient (momentum equation) for nonuniform distribution of velocity at section 0, dimensionless;
β_3 = corrective coefficient (momentum equation) for nonhydrostatic distribution of pressure at section B, dimensionless;
β_4 = corrective coefficient (momentum equation) for nonuniform distribution of velocity at section B, dimensionless;
γ = specific weight of fluid, lb/ft^3; and
ρ = mass density of fluid, lb-sec^2/ft^4.

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