AN EXPERIMENTAL INVESTIGATION OF HIGHWAY SURVEYING AND MAPPING CONTROL EXTENSION BY TRILATERATION AND CONVENTIONAL TRAVERSE

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A chain of 16 quadrilaterals approximately 6 miles in length along I-71 was used to evaluate ground control extension by trilateration and conventional traverse. Distances were measured with a Geodimeter, and angles were measured with a Wild T2 theodolite. A FORTRAN computer program was developed to adjust the trilateration network. The method is based on a previously developed and published method of trilateration adjustments. The traverse adjustment was accomplished by using the least squares method. Results of the experiments show that trilateration is more accurate than conventional traverse and is at least as economical. The authors recommended that a trilateration chain of quadrilaterals be used where a double centerline or double traverse is needed for highway surveying or mapping control. The methodology and computer program have permitted this method to be used wherever economically advantageous.

•THIS is the abridged version of the fourth report of a trilateration study. The first paper (2) established the basic ideas of a trilateration scheme and its adjustment by use of the area equation for fundamental figures. The second paper (3) dealt mainly with the geodetic scheme of trilateration and its adjustment. The third paper (4) explained easy application of the area adjustment method for engineers and surveyors.

This paper reports a trilateration experiment using conventional traverse in highway mapping control. The experiment was done intermittently from the summer of 1965 through the winter of 1967 on a spare-time basis by the field survey party of the aerial engineering section of the Ohio Department of Highways. The computer programs were developed from the summer of 1967 through early 1969.

THE SCHEMES OF THE EXPERIMENT

The experimental network is located primarily along I-71 in Delaware County, Ohio, about 17 miles north of Columbus. The network spans between two U. S. Coast and Geodetic Survey (CGS) first-order triangulation monuments, Shannahan (established 1928, abbreviated S) at the south end and Galena (established 1933, abbreviated G) at the north end. The geodetic positions, the state plane coordinates, and other related data for these two points are listed in the Horizontal Control Data published by the CGS.

The two points were not directly connected and observed. S belongs to the north-south triangulation chain, and G belongs to the east-west triangulation chain. Direct measurement by Geodimeter of the distance between the two triangulation monuments was attempted but found to be impractical because too much work would be involved in establishing towers to overcome the obstacles along the line of sight.

According to the CGS (11), the scale factors at latitude 40 deg 10 min are $1/16{,}200$ too great for north zone plane coordinates and $1/34{,}400$ too great for south zone plane coordinates, and at latitude 40 deg 11 min are $1/19{,}000$ too great for north zone plane coordinates and $1/32{,}000$ too great for south zone plane coordinates. Therefore, the south zone

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plane coordinates for the state of Ohio were used in all computations because they were comparatively more accurate.

Reference mark 2 (abbreviated A) of station S was used as one vertex of the first quadrilateral in the network. This point serves not only as a point in the network but also as the azimuth mark from S in order to use the original grid azimuth value in the preliminary orientation of the network. The distance between X and A was also measured with the Geodimeter to check the accuracy of the original distance value obtained by CGS in 1928. The results are as follows:

Item	Year	Distance (ft)
CGS, original	1928	691.272
Geodimeter	1967	690.244
Geodimeter, reduced (sea level)	1969	690.212
Adjusted by trilateration	1969	690.214

To fit the usual highway control situation required that the network consist primarily of a chain of quadrilaterals with braced diagonals for trilateration investigation. A comparison was made with traverses formed by the external sides of the quadrilaterals. The basic geometric features and related field work of the two kinds of experiments are shown in Figures 1 and 2.

FIELD OBSERVATION AND REDUCTION OF GEODIMETER DISTANCES AND THEODOLITE ANGLES

Field observations of Geodimeter distances and theodolite angles for trilateration and traverse experimental networks were done in the usual manner as it has been practiced by the field crews of the aerial engineering section in recent years.

The reduced distances at sea level were used for trilateration and traverse computations. In the trilateration and traverse adjustment, a weight was assigned to each averaged distance according to the number of reduced sea level distances used for the average. (A weighting scheme according to the standard error of the measured distances was not used because maximum spread, and not standard error, was the only available value.)

All the horizontal angles have three sets of measurements. Therefore, the weights of the horizontal angles are assumed to be equal.

ADJUSTMENT OF THE TRILATERATION CHAIN

The method of adjustment of plane trilateration in fundamental figures by area equations developed in the first report of this series was used to adjust the quadrilateral chain of the experimental trilateration. The basic theory and equations have been treated thoroughly and published in previous reports. The practical application of the method was facilitated by use of an electronic computer in solving a large number of simultaneous equations. The FORTRAN IV computer program will be made available in a separate report (6). This report will deal only with data that have been used, the procedures followed, and results obtained.

As shown in Figure 1, there are 16 quadrilaterals in the network. The areas of triangles were computed by using the distance values of each side of the four triangles of each quadrilateral. The discrepancies between the area sums of each pair of opposite triangles in all of the 16 quadrilaterals and their relative ratios are given in Table 1. The three largest area errors are noted.

The purpose of the trilateration adjustment is to apply corrections to each side of the quadrilaterals so as to eliminate the area errors while keeping the sum of the squares of the corrections at a minimum.

In the computer program for trilateration adjustment, the input data were the 81 averaged sea level distances from the Geodimeter measurements and their proper weights. After the processing, the residual or the correction, the corrected value,

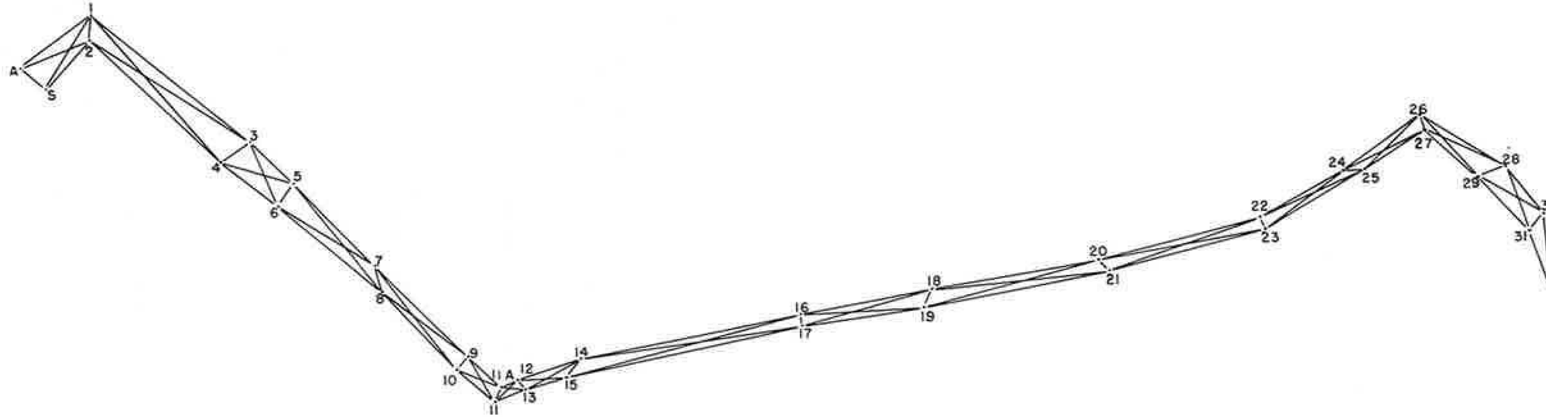

Figure 1. Geometric features and field work involved in trilateration network.

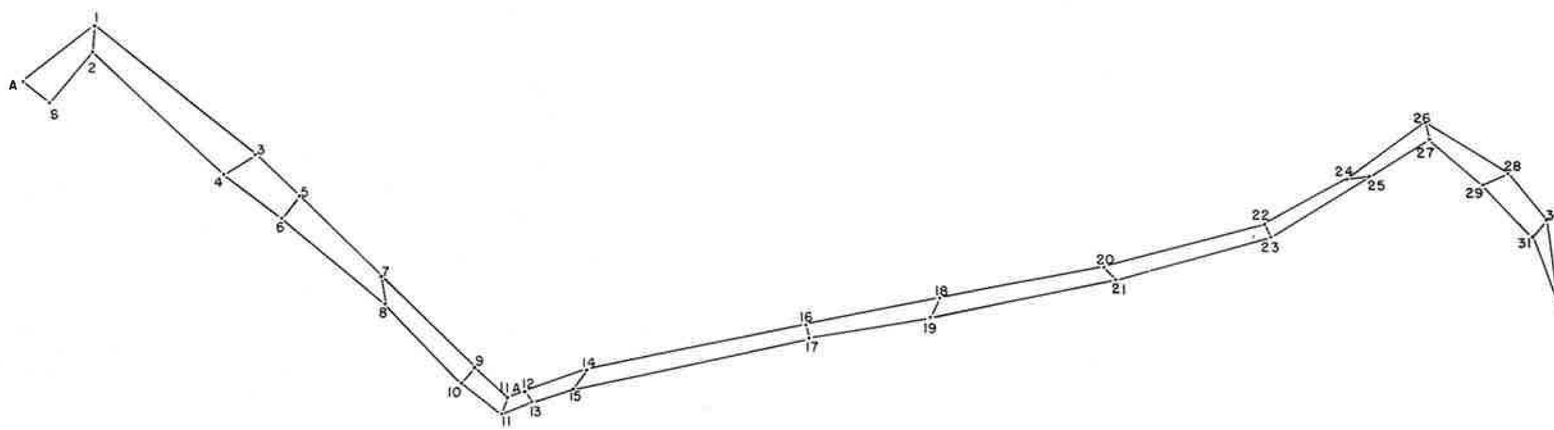

Figure 2. Geometric features and field work involved in traverse network.

TABLE 1

AREA ERRORS OF THE QUADRILATERALS OF EXPERIMENTAL TRILATERATION

Quadrilateral	Side-Length Ratio[a]	Area Sum (sq ft)	Area Error (sq ft)	Relative Error (1 in)
1	4.345	+645,178.209045 -645,200.004112	-21.795067	29,603
2	11.446	+1,728,819.656384 -1,728,821.294457	-1.638073	1,055,399
3	5.392	+536,354.382108 -536,379.650455	-25.268374	21,227
4	14.168	+608,719.033576 -608,664.446298	54.587278	11,151
5	8.483	+423,280.549834 -423,258.345555	12.204379	34,682
6	3.816	+448,860.854501 -448,876.703583	-15.839082	28.339
7	3.466	+240,672.253517 -240,649.371756	22.881761	10,518
8	3.616	+242,982.566471 -242,993.874676	-11.308205	21,488
9	21.655	+1,296,629.342527 -1,296,588.007834	41.334693	31,369
10	10.619	+517,974.509942 -517,822.283861	152.226081	3,402[b]
11	12.333	+1,021,591.253596 -1,021,555.186423	36.167173	28,246
12	13.136	+874,354.161284 -874,510.336224	-156.174940	5,599[b]
13	10.065	+505,890.469942 -505,515.604169	374.865773	1,349[b]
14	4.357	+455,782.294891 -455,762.123519	20.171372	22,595
15	4.122	+933,583.158365 -933,598.483824	-15.325460	60,918
16	3.808	+883,385.387404 -883,412.136686	-26.749282	33,025

[a]Side-length ratio = longest side/shortest side.
[b]Largest errors.

and the relative error of each distance are obtained. The accuracy of the trilateration will be discussed later.

In the adjustment of the experimental trilateration, orientation and scaling did not enter into the problem because the accuracy of the known azimuth angles and the distance between the two known triangulation stations were not necessarily better than the Geodimeter measurements. Because the rigorous simultaneous adjustment of distances, directions, and coordinates would mask the error contribution of the Geodimeter distances (which is the primary concern in this experiment), the authors did not use the rigorous method as used in geodetic adjustment.

ORIENTATION AND COMPUTATION OF THE UNADJUSTED AND THE ADJUSTED TRILATERATION

As stated earlier, the coordinates of a first-order triangulation point at each end of the trilateration network were known, and an azimuth mark at one end was also used as one point of the quadrilaterals. Thus, the trilateration was computed and oriented between these two known points.

The plane coordinates and orientation of the azimuths were computed by using the trilateration distances and the angles computed from the distances. The computer program used was the M.I.T. Integrated Civil Engineering System COGO (8). Two tests have been made with the computation. One test used the adjusted lengths of the trilateration; the other test used the observed distances of the original unadjusted trilateration. In both cases, computations were carried out through two different simple triangle chains of the trilateration scheme to the ending point G from the starting point S by using the preliminary grid azimuth from S to A, 90 deg 57 min 58 sec from south. By keeping the grid azimuth from S to G, 235 deg 04 min 49.31 sec from south, which was computed from the given coordinates as fixed, we found the azimuth from S to A by trial and error as follows: northward through left chain of adjusted trilateration, 90 deg 57 min 04.52 sec; northward through right chain of adjusted trilateration, 90 deg 57 min 04.06 sec; northward through left chain of unadjusted trilateration, 90 deg 59 min 08.81 sec; and northward through right chain of unadjusted trilateration, 90 deg 56 min 56.74 sec.

It can be seen that the discrepancy is much less between the adjusted trilateration values than between the unadjusted trilateration values. Similar results were obtained with the computation of the grid distance from S to G by using the computed coordinates: northward through left chain of adjusted trilateration, 31,722.715 ft; northward through right chain of adjusted trilateration, 31,722.714 ft; northward through left chain of unadjusted trilateration, 31,721.122 ft; and northward through right chain of unadjusted trilateration, 31,724.047 ft.

The geodetic and grid distances from S to G as computed from the coordinates given by CGS are 31,721.473 ft and 31,722.127 respectively. The difference in the computations of the grid distance through either chain of the adjusted trilateration is only 0.001 ft, or 0.588 and 0.587 ft from the CGS grid distance. Therefore, use of the adjusted trilateration should be standard practice in orientation and computation of trilateration coordinates.

Because the trilateration was not adjusted by considering the orientation and scaling errors simultaneously, there was a discrepancy between the final coordinates of the adjusted trilateration at G and the coordinates of G given by CGS. To simplify the adjustment of this discrepancy, we computed two simple traverses, which can also be designated as west chain of legs and east chain of legs, for each simple triangle chain of adjusted and unadjusted trilaterations and adjusted them by COGO.

The computed coordinates of the points of both the adjusted and unadjusted trilateration of computation with chain of triangles and chain of legs along with the differences between these coordinates and the sums of squares of these differences are omitted in this account. The magnitudes of the sums of the squares of the differences of the coordinates are given in Table 2. From the magnitudes of the sums of the squares of the differences of the west from east coordinates of the adjusted chain of triangles and of the adjusted chain of legs, it can be seen that there is no appreciable difference between computing either from the west or the east chain of triangles or chain of legs of the adjusted trilateration.

The magnitude of the sums of squares of the differences of coordinates computed from the west chain of triangles and the west chain of legs for adjusted and for unadjusted trilateration and from the east chain of triangles and the east chain of legs for adjusted and for unadjusted trilateration is of the same order, but the values are larger in the computations obtained from the unadjusted trilateration. Therefore, the differences of computation between chain of triangles and chain of legs are not significant, but the differences between that of adjusted and unadjusted trilaterations are significant. The latter statement is strongly supported by the sums of squares of the differences of coordinates of the computations of the unadjusted trilateration either in chain of triangles or in chain of legs and the differences between the adjusted and the unadjusted trilaterations in the west chain of triangle and chain of legs. The small differences of the east chain of triangles and east chain of legs between the adjusted and unadjusted trilaterations may indicate that the computation from the unadjusted trilateration is irregular.

Finally, in order to see the change in distances after the final coordinates have

TABLE 2

SUM OF THE SQUARES OF THE DIFFERENCES OF THE COORDINATES

Trilateration	Sum of x^2	Sum of y^2
Adjusted trilateration		
West triangles and legs	5.150	3.477
East triangles and legs	5.125	2.808
West and east legs	0.0003	0.0003
West and east triangles	0.0002	
Unadjusted trilateration		
West triangles and legs	13.304	15.927
East triangles and legs	55.383	37.826
West and east legs	1,167.999	6,075.095
West and east triangles	952.672	4,857.954
Adjusted and unadjusted trilateration		
West triangles	957.894	4,762.219
West legs	1,086.693	5,441.824
East triangles	10.597	3.572
East legs	5.379	21.194

been computed through adjustment of orientation and scaling of the adjusted trilateration, we calculated the grid distances for corresponding Geodimeter distances of the trilateration quadrilaterals from the coordinates by COGO. The absolute change from the observed distances and their relative changes are also computed but are not shown here.

ACCURACY OF THE TRILATERATION IN THE EXPERIMENT

The accuracy of the trilateration depends on the accuracy not only of the individual distance measurement but also of the network distance measurement. Both aspects are of primary interest in this section. Let us first examine the accuracy of the individual distance measurements. For two measurements, it can be shown that the standard deviation is one-half of the "spread." Therefore, standard error and spread are directly related for two measurements.

For the Geodimeter measurements, a distance was observed with three different frequencies. The maximum spread for each pair of frequencies may be used as an indication of the accuracy of the Geodimeter distance. For easy comparison, the relative error, i. e., the maximum spread divided by the average distance and expressed as a unit fraction of the distance, was selectively computed for the short distances with the following significant maximum spread:

Station	Distance (ft)	Maximum Spread (ft)	Relative Error (1 in)
5-6	278.6631	0.1147	2,430
13-12	323.7927	0.1296	2,498
23-22	267.8335	0.2093	1,280
21-20	330.1614	0.1982	1,666

It seems that the errors of these four distances are relatively quite large. The statistics for the errors of all the distances are as follows:

Class of Relative Error (1 in)	Number of Geodimeter Distances
1,280	
	8
5,000	
	25
10,000	
	62
50,000	
	9
100,000	
	4
430,000	
Total	108

From statistics, we conclude that approximately 60 percent of the Geodimeter distances have accuracies in the range of $^1/_{10,000}$ to $^1/_{50,000}$.

The accuracy of the trilateration network as a whole can be examined in several ways.

1. The absolute and relative area errors of the quadrilaterals have the same pattern in magnitude (Table 1). Therefore, the area errors are independent of the area size. Quadrilaterals 13, 12, and 10 have, in that order, the largest absolute and relative area errors and, especially, quadrilaterals 12 and 13, where the distances 20-21 and 22-23, with the largest relative distance errors, lay. Therefore, the accuracy of the trilateration network depends on the accuracy of the individual distances. Also, there seems to be no apparent relationship between the area errors and the side-length ratio. Therefore, the area errors are also independent of the area shape.

2. The corrections of the distances in the trilateration adjustment is another indication of the accuracy of the network. The three largest absolute corrections of the distances are -0.0351, -0.0265, and -0.0262, and the three largest relative adjustments are only $1/_{25,141}$, $1/_{31,877}$, and $1/_{36,009}$. The standard deviation for the distance of unit weight in the trilateration network as computed from the corrections is +0.0146 ft.

3. The grid distance from S to G as computed from the state plane coordinates is 31,722.127 ft. The discrepancies between the state plane coordinate distance and the trilateration distance of S to G (given earlier) and their relative errors are listed in the following:

Network	Absolute Discrepancy (ft)	Relative Discrepancy (ft)
Adjusted trilateration		
Left chain	0.588	54,041
Right chain	0.587	54,041
Unadjusted trilateration		
Left chain	1.005	31,563
Right chain	1.920	16,523

This table shows that the experimental trilateration distances are in excellent agreement with the values computed from the CGS data. There is some uncertainty as to which of these two distances is the more accurate inasmuch as a direct measurement between the two points could not be made.

4. The errors of closure of the chain of legs between the CGS triangulation points may be used as another indication of the accuracy of the trilateration. These are given in Table 3.

5. From the sums of squares of the differences of the coordinates computed from the adjusted and unadjusted triangulation through left or right chain of triangles and chain of legs, the accuracy of the trilateration coordinates can be computed as given in Table 4. Clearly, the coordinates computed from the adjusted trilateration have a higher degree of accuracy than do those computed from the unadjusted trilateration.

6. The three largest absolute changes of the grid distances from the observed distances are -0.083, +0.080, and -0.072 ft. Eight of the 83 relative changes are greater than $1/_{10,000}$. The root-mean-square change of the grid distance is ±0.0326 ft.

ADJUSTMENT AND COMPUTATION OF THE TRAVERSE COORDINATES

The M.I.T. COGO program was used in the adjustment and computation of the experimental traverses. In the program, there are four methods for traverse adjustment: compass rule, transit rule, Grandall's method, and method of least squares. The method of least squares has been used throughout because it is comparable to the method used for adjustment of the trilateration.

In the method of least squares, a weighting scheme for angles and distances can be used. The preliminary angular closure error is adjusted first. The errors of closure in the latitudes and departures are then adjusted. The angular errors, linear errors, and relative errors for all the quadrilaterals in the network are given in Table 5. No relative errors are larger than $1/_{10,000}$.

TABLE 3

ERRORS OF CLOSURE

Network	Error of Closure (ft)	Relative Accuracy (1 in)
Adjusted trilateration		
Left legs of left chain	0.588	66,927
Right legs of left chain	0.587	64,050
Left legs of right chain	0.587	64,088
Right legs of right chain	0.587	66,999
Unadjusted trilateration		
Left legs of left chain	1.005	39,141
Right legs of left chain	1.760	21,381
Left legs of right chain	1.920	19,598
Right legs of right chain	1.920	20,488

TABLE 4

ACCURACY IN TERMS OF THE ROOT-MEAN-SQUARE DIFFERENCES OF THE COMPUTED GRID COORDINATES OF EXPERIMENTAL TRILATERATION AND TRAVERSES

Trilateration and Traverse	x (ft)	y (ft)	Linear Resultant (ft)
Adjusted trilateration			
West and east triangles	0.002	0.003	0.003
West and east legs	0.003	0.003	0.004
West triangles and legs	0.389	0.320	0.504
East triangles and legs	0.388	0.287	0.483
Unadjusted trilateration			
West and east triangles	5.293	11.956	13.075
West and east legs	5.861	13.367	14.595
West triangles and legs	0.626	0.684	0.927
East triangles and legs	0.276	1.056	1.656
Unadjusted and adjusted trilateration			
West triangles	5.307	11.835	13.021
West legs	5.653	12.696	13.898
East triangles	0.558	0.324	0.645
East legs	0.398	0.790	0.891
Traverses			
Quadrilateral chain and long loop	1.034	0.546	1.169
Quadrilateral chain and simple traverse	0.449	0.865	0.975
Long loop polygon and simple traverse	1.194	1.182	1.680

The experimental traverse in the network consists of 16 quadrilaterals and one triangle as in trilateration but without the braced diagonals in the quadrilaterals. After the field work had been completed over a 2-year span, it was discovered during computation that there were discrepancies between the distance and angular measurements of quadrilaterals 12 and 13, which could not be checked with each other. Therefore, quadrilaterals 12 and 13 had to be combined into one polygon. Also, in quadrilateral 16

TABLE 5

ERRORS OF CLOSURE OF QUADRILATERALS AND POLYGONS OF EXPERIMENTAL TRAVERSES

Quadrilateral or Polygon	Angular Error (min)	Perimeter (ft)	Linear Error	Relative Error (1 in)
1	5 x 0.82	3,812.855	0.039	97,579
2	4 x 5.17	8,493.345	0.101	83,943
3	4 x 6.61	2,912.258	0.250	11,639
4	4 x 9.38	6,387.051	0.173	36,830
5	4 x 7.00	3,921.232	0.237	16,548
6	4 x 15.01	2,801.919	0.092	30,313
7	4 x 9.00	2,052.097	0.047	43,775
8	4 x 2.37	2,249.062	0.075	29,849
9	4 x 5.21	10,531.266	0.294	10,531
10	4 x 11.62	5,215.380	0.068	76,675
11	4 x 4.54	8,014.118	0.266	30,119
12-13	6 x 4.14	12,064.549	0.115	104,910
14	4 x 3.23	4,015.470	0.055	73,035
15	4 x 0.71	4,353.260	0.120	36,359
16	—	3,861.025	0.111	24,695
Triangle	—	2,592.559	0.115	22,455
Long loop polygon	33 x 4.10	71,700.172	2.102	34,105
Quadrilateral chain	—	3,861.025	0.108	35,633
Triangle	—	2,592.559	0.115	22,487

two angular measurements were missing. Therefore, quadrilateral 16 does not have an angular error of closure. These two exceptions are given in Table 5.

The traverse chain of quadrilaterals was also oriented and computed following the same procedure as for trilateration except that only one route of computation was possible for the adjusted traverse chain.

The azimuth of Shannahan to its reference mark 2 (S to A) was found to be 90 deg 57 min 57.01 sec from south. The grid distance from S to G computed from the coordinates of the adjusted quadrilateral traverse was 31,720.127 ft computed from CGS data.

The computed coordinates of the points of the adjusted quadrilateral traverse chain are omitted here. In order to compare the distances obtained by the adjusted traverse chain with those original Geodimeter distances obtained in the field, i. e., the unadjusted traverse or trilateration distances, we obtained the grid distances computed from the adjusted traverse.

No attempt was made to orient and compute the unadjusted traverse chain of quadrilaterals for experimental purposes, as was done with the trilateration, because it is not conventional practice. However, the orientation and computation of the long loop traverse, which is the overall polygon of the traverse chain without any intermediate connections, were tested. The azimuth from S to A in this long loop traverse is 90 deg 57 min 51.60 sec, and the grid distance from S to G is 31,720.889 ft. Both of these values have the same order of accuracy as the traverse chain of quadrilaterals.

The final discrepancies of the coordinates of G in the traverse chain of quadrilaterals with the coordinates given by CGS are also adjusted through two chains of legs or two simple traverses as was done for the trilateration. The coordinates of the traverse points from both the long loop and the simple traverse computations are computed but are not shown here. The grid distances computed from these adjusted traverses are also omitted in this paper.

The differences of the coordinates between any two of the three kinds of traverses—the quadrilateral chain, the long loop polygon, and the simple traverse—and the sums of squares of those differences were also computed. The magnitude of the sums of the squares of the differences of the coordinates between the long loop polygon and the simple traverse ($x = 48.448$ and $y = 47.506$) is the largest among the three pairs of sums. This suggests that the traverse chain of quadrilaterals was best for the computation of coordinates among the traverses tested.

Another way to compare the different traverses in adjustment and computation is to examine the changes of the computed distances from the field-observed distances. The sums of squared of the changes of the computed distances from the quadrilateral chain, the long loop polygon, and the simple traverse are respectively 0.439, 59.831, and 53.586. Therefore, the quadrilateral chain is undoubtedly the one with least alteration to the original field measurements.

ACCURACY OF THE TRAVERSES

The accuracy of the traverse depends not only on the Geodimeter distances but also on the horizontal angle measurements. The statistics of the relative errors of the observed Geodimeter distances for traverse use only are as follows:

Class of Relative Error (1 in)	Number of Geodimeter Distances
1,280	6
5,000	12
10,000	28
50,000	5
100,000	2
Total	53

From these statistics, it may be seen that approximately one-third of the traverse distance measurements have an accuracy greater than $1/10{,}000$.

The statistics of the maximum spread of the horizontal angle measurements are as follows:

Maximum Angular Spread (sec)	Number of Angles
1	9
5	31
10	10
15	9
20	4
26	
Total	63

From this list, it is seen that the most frequent angular error is in the range from 5 to 10 sec.

The accuracy of the horizontal angles can also be judged by the total angular error of closure of the traverse of the mean correction to each traverse angle as given in Table 3. The maximum mean correction is 15.01 sec, and the minimum is 0.71 sec. The most frequent mean correction is in the range from 5 to 10 sec.

The combined effect of the observed errors of traverse distance and horizontal angle measurements can be expressed by the linear error of closure or the relative error of the traverses as given in Table 5. The absolute linear error of the long loop traverse is a large value, 2.102 ft, but its relative error is only $^1/_{34,105}$. Generally, the absolute linear error of closure and the relative error of the quadrilateral traverses are very small. No errors of quadrilateral traverses have exceeded 0.3 ft or $^1/_{10,000}$.

The accuracy of the individual coordinates of the traverses can be examined by the differences between the computed grid coordinates of the experimental traverses. The accuracy of the coordinates of the individual traverses as a whole may be expressed in terms of the root-mean-square difference (Table 4).

The accuracy of the final adjusted traverse of quadrilateral chain may also be represented by the changes of the horizontal angles and the grid distances computed from the final coordinates. For the traverse of quadrilateral chain, the maximum change in distance is 0.625 ft or $^1/_{1,805}$ relative to its distance value. The root-mean-square change of the grid distances is ±0.0768 ft.

COMPARISON AND EVALUATION OF THE TRILATERATION AND THE TRAVERSE EXPERIMENTS

As stated earlier, the scheme of the experiments is primarily to fit the highway surveying and mapping control situation. The network consists of a chain of quadrilaterals with braced diagonals for the trilateration experiment and without diagonals for the traverse experiment. The basic geometric features and field work involved in the two kinds of experiments are shown in Figures 1 and 2.

First, to compare and evaluate these require that the actual field and office work be analyzed in terms of the personnel employed and time expended. According to the field notes and time sheets submitted by the field and office personnel who worked on this trilateration research project, a total of 246 hours was spent on reconnaissance and layout of the network, 484 hours on horizontal and vertical angle measurements, 372 hours on Geodimeter measurements, and 208 hours on checking and reducing field notes, including theodolite angles, hand computation of Geodimeter distances, angular closure of traverse, and preliminary computation of triangle areas. The record of hours includes the time spent resetting missing station marks and remeasurement of the angles and distances.

By estimation, about one-quarter of the time spent on angular measurements was for vertical angles used in Geodimeter distance reduction. Of the 83 Geodimeter distances of the trilateration network, 51 were also used in the traverse experiment. In regard to the network reconnaissance, the chain of quadrilaterals without diagonals in traversing required one-third less time than that required for the chain of quadrilaterals with diagonals in trilateration.

The time spent checking and reducing field notes is not easily allocated between traversing and trilateration. However, approximately one-third of this time was assigned to traversing and two-thirds to trilateration, based on the degree of difficulty of the work. Therefore, the overall time required for experimental traverse and trilateration excluding the network adjustment and coordinates computation was 831 hours for trilateration and 863 hours for traverse.

Computer time records for the network adjustment and computation of coordinates were not available. However, if the trilateration adjustment can be incorporated into the COGO program, there would be little difference in the computer time required. Therefore, it can be concluded that trilateration for highway control is no more time-consuming than traversing, and with additional field experience on trilateration it may require considerably less time than traversing.

A bottleneck for trilateration field work is that the vertical angles must be observed separately by theodolite. If the Geodimeter or any other electronic distance-measuring instrument had a vertical-angle-measuring device attached to it, an appreciable amount of field time could be saved in distance measuring.

Second, the accuracy achieved by either trilateration or traversing should be analyzed. For the same number of control points and the same size of field crew, the trilateration establishes more reliable positions than does traversing. There are several facts displayed in the experiment that can be summarized to support this conclusion, including the following:

1. From an analysis of the observed values, the angular measurements by theodolite were subject to personal errors in pointing or sighting and in reading operations, while the distance measurements by Geodimeter were independent of personal errors.
2. Both Geodimeter and theodolite measurements are subject to errors for short distances. A quadrilateral traverse has four angles to be measured, and each angle has one short line of sight. But for the trilateration quadrilateral, with two long diagonals, only two of the six distances to be measured are short.
3. The discrepancies between the distances from S to G computed from the CGS-listed coordinates and from either the experimental trilateration or the experimental traverse were 0.588 ft for trilateration and 1.564 ft for traversing.
4. As given in Table 4, the accuracies in terms of the root-mean-square differences of the computed grid coordinates of the adjusted trilateration are definitely higher than those of the traverses.
5. The accuracy in terms of the sum of the squares of the changes in the distances computed from the adjusted trilateration from the field Geodimeter distances, 0.088, is much less than the sum of the squares of the changes of the distances computed from the adjusted traverse of the quadrilateral chain from the field Geodimeter distances, 0.489.

RECOMMENDATIONS FOR APPLICATION OF THE TRILATERATION TO THE CONTROL OF THE HIGHWAY SURVEYING AND MAPPING

In the preceding sections, the experimental trilateration and traverse have been investigated in detail. It was concluded that the trilateration is more accurate than the traverse and is at least as economical.

For highway surveying and mapping, as well as for other surveys such as railways and waterways, the area under consideration is usually a long narrow band. If a simple open traverse is not accepted as safely accurate enough for the surveying or mapping control, a double simple chain closed traverse such as the double centerline in the location survey of the Interstate highway, may be adopted. The latter traverse is the same as the long loop extension so named in our experiment. Obviously, the long loop extension without middle connecting or check lines is still not safely accurate as shown in the experiment. As the middle connections or check lines are increased, the maximum connection will be reached as the traverse that consists of a chain of quadrilaterals without diagonals as in the experiment. This is the traverse chain of quadrilaterals with all distances and angles measured.

A simple chain of triangles may also be called a simple chain trilateration if all sides of the triangles are observed in the field. A simple chain trilateration is enough to fix the points as for a simple open traverse. However, a simple chain trilateration run between two known control points has no self-check for each triangle.

A trilateration chain of quadrilaterals is a traverse chain of quadrilateral polygons braced with double diagonals or a chain of overlapping triangles. It has a self-check for each quadrilateral, which was termed the fundamental figure of adjustment in the first report. The quadrilateral chain of trilateration should be run between two known points. However, if only one point and one azimuth are known, these are enough to fix the orientation and position of the trilateration with respect to the existing control system, but not enough to give a check.

A trilateration chain of quadrilaterals has the same number of points as the traverse chain of quadrilateral polygons or as the long loop extension or as the closed double simple chain of legs as mentioned before, and is also more accurate and at least as economical. Therefore, it is recommended that the trilateration chain of quadrilaterals be used where a double centerline or double traverse is needed for highway surveying and mapping control.

To apply the scheme of trilateration to mapping and surveying control of highway engineering, we recommend the following specifications and procedures:

1. A regular route survey control by trilateration chain in highway engineering should start from at least one first- or second-order CGS triangulation monument with known azimuth marks (or should both start and end at such a monument?).
2. In all cases, a chain of quadrilaterals with braced double diagonals should be used except that a simple chain of triangles may be used if two known monuments at the termini are available.
3. The line to the azimuth mark may be used as one line of the trilateration. In exceptional cases, the orientation of the trilateration may be determined or checked by trilateration astronomical observations.
4. For independent surveys or small projects such as bridge sites, known monuments may not be available in the vicinity, and orientation and positioning with state plane coordinates may not be practical. One or several quadrilateral networks may be used as an independent network of trilateration.
5. The shape of the triangles in all of the trilateration networks may be acute with the ratio of the longest side to the shortest side as large as 30:1. The length of the shortest side should never be less than 200 ft.
6. All of the distances in the trilateration can be measured with electronic or optical distance-measuring instruments.
7. The difference of the sums of the two pairs of opposite triangles in a quadrilateral is an indicator of the accuracy of the field work. It can be used as a check for blunders. This area discrepancy should not exceed 100 sq ft at any time.
8. The area errors of the trilateration network of quadrilaterals may be adjusted by the method of area equations as described in the first three reports. A single quadrilateral trilateration may be adjusted by use of a desk calculator with the steps prescribed in the third report. A computer program will soon be available for any sophisticated network.
9. The adjustment and computation of the coordinates of the points of the trilateration may use any computer program available for these purposes.

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