FRICTION AND THE MECHANICS OF SKIDDING AUTOMOBILES

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•IT has been evident for many years that the skid resistance of automobile tires on pavement, particularly wet pavement, varies significantly with speed. Many researchers have published experimental evidence that shows that the coefficient of friction decreases with increasing speed (1, 2, 3, 4). In most cases this variation is approximately linear; in some cases it is quadratic. (Excellent, comprehensive review articles (5, 12) discuss various methods of measuring the coefficient of friction.) Despite the knowledge that friction varies with speed many people use a constant, average value for simplicity (6, 7, 8). Unfortunately, many people do not understand the nature of this simplification. As a result, some confusion and unnecessary testing have appeared in the literature. It is the intent of the following work to clear up some of the possible confusion and to illustrate that a more complete model of varying friction is possible without undue complication. Specifically it is shown that the differential equation of motion of a skidding automobile can still be integrated even when variable friction is included. An "exact" algebraic expression for the skidding distance is obtained in terms of the initial speed, weight distribution, and friction characteristics of the vehicle.

This exact expression is examined from three points of view. First, it is used to explain how the improper use of a constant, average friction value can lead to biased results. Second, it is shown that actual, variable friction curves can be found by using curve-fitting techniques with rather simple experimental data. Third, the exact expression is used from the point of view of accident investigation to show that the initial speed of a skidding automobile under very general vehicle-tire-road conditions can be read from a single graph.

DERIVATION OF EQUATIONS

Figure 1 shows a free body diagram of a vehicle, with wheels locked, skidding up a positive grade of angle θ. As shown in the figure, W is the total vehicle weight, N_1 is the total force between both rear tires and the pavement, N_2 is the total force between both front tires and the pavement, f_1 and f_2 are the total rear and front frictional forces respectively, D is the aerodynamic drag force, and ℓ_1 and ℓ_2 are the distances between the wheels and the center of gravity of the vehicle. From Newton's Law for assumed planar motion, the equation of motion in the x direction is

$$m(dv/dt) = -mg \sin \theta - f_1 - f_2 - D \tag{1}$$

where $m = W/g$, g is the acceleration due to gravity, and $v = dx/dt$ is the speed. For this type of analysis the vehicle can be treated as a rigid body; consequently, it is in equilibrium in the y direction. This gives

$$-W \cos \theta + N_1 + N_2 = 0 \tag{2}$$

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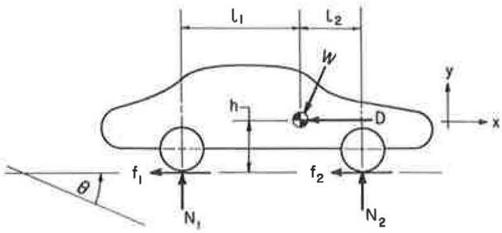

Figure 1. Free body diagram of a skidding vehicle.

Similarly because the vehicle is not rotating,

$$N_2(\ell_1 + \ell_2) - W \cos \theta \, \ell_1 + (W \sin \theta + D) h = 0 \tag{3}$$

The following assumptions are made:

1. The coefficients of friction are velocity dependent such that $f_1 = N_1\mu_1(v)$ and $f_2 = N_2\mu_2(v)$, where $\mu_1 = \mu_{10} - k_1 v - h_1 v^2$ and $\mu_2 = \mu_{20} - k_2 v - h_2 v^2$.
2. The center of gravity is low, i.e., $(W \cos \theta)\, \beta >> (W \sin \theta + D)\, \gamma \approx 0$ and $(W \cos \theta)\, \alpha >> (W \sin \theta + D)\, \gamma \approx 0$, where $\alpha = \ell_1/(\ell_1 + \ell_2)$, $\beta = \ell_2/(\ell_1+\ell_2)$, and $\gamma = h/(\ell_1+\ell_2)$.
3. The aerodynamic drag is proportional to the square of the velocity, i.e., $D = c_o v^2 = cWv^2$.

These assumptions can be used to reduce Eqs. 1, 2, and 3 to

$$dv/dt = - g \sin \theta - g \cos \theta \, [\beta\mu_1(v) + \alpha\mu_2(v)] - gD^* \tag{4}$$

where $D^* = D/W$, the drag force per unit weight. Eq. 4 is the equation of motion of a vehicle skidding in a straight line. Substitution of new variables simplifies Eq. 4. That is,

$$-(1/g)(dv/dt) = A v^2 + Bv + C \tag{5}$$

where

$A = c - (h_1 \beta + h_2 \alpha) \cos \theta$;
$B = - (k_1 \beta + k_2 \alpha) \cos \theta$; and
$C = \sin \theta + (\mu_{10} \beta + \mu_{20}\alpha) \cos \theta$.

Eq. 5 can be rearranged to a form convenient for integration.

$$\int_{v_o}^{v_f} v\,dv/(Av^2 + Bv + C) = - \int_{0}^{d} g\,dx \tag{6}$$

For $v_f = 0$ this gives

$$\begin{aligned} -gd = {} & (1/2A) \, \ell n C - \left\{B/[2A(B^2 - 4AC)^{1/2}]\right\} \\ & \ell n \left\{[B - (B^2 - 4AC)^{1/2}]/[B + (B^2 - 4AC)^{1/2}]\right\} \\ & - (1/2A) \, \ell n (Av_o^2 + Bv_o + C) + \left\{B/[2A(B^2 - 4AC)^{1/2}]\right\} \\ & \ell n \left\{[2Av_o + B - (B^2 - 4AC)^{1/2}]/[2Av_o + B + (B^2 - 4AC)^{1/2}]\right\} \end{aligned} \tag{7}$$

Here v_o is the velocity at the initiation of skidding, v_f is the final velocity, and d is the distance over which the car comes to rest. If the friction characteristics are known as well as the vehicle weight distribution, the aerodynamic drag, the grade, and the initial speed, Eq. 7 will give the skid distance for a complete stop. Some special cases of interest are when the coefficient of friction depends linearly on the velocity ($h_1 = h_2 = 0$) and when the coefficient of friction is a constant ($h_1 = h_2 = k_1 = k_2 = 0$). These cases are respectively as follows:

1. A = 0 (aerodynamic drag also neglected).

$$\ell n[(B/C)\, v_o + 1] - (B/C)\, v_o = - (gdB^2/C) \tag{8}$$

2. A = B = 0 (aerodynamic drag also neglected).

$$v_o = (2Cgd)^{1/2} \tag{9}$$

For level ground, $\theta = 0$. When the front and rear vehicle weights are equal, Eq. 9 further reduces to the "standard" stopping distance formula

$$d = v_o^2/2fg \tag{9a}$$

where f is an "average" coefficient of friction, which is called a "friction factor" in the sequel to distinguish it from the coefficients of friction, μ_1 and μ_2.

USE OF THE STOPPING-DISTANCE FORMULA

Two common methods of measuring the friction properties of tires on particular pavements are the stopping-distance method and the skid-trailer method. In the former, the vehicle is brought up to a given speed, the brakes are locked, and the vehicle skids to a stop. The skid distance is measured, and Eq. 9a is used to calculate a friction factor. In the latter method (12), a special trailer is pulled over a pavement at a constant speed with the wheels locked. The wheel torque can be measured, and the coefficient of friction can be calculated. Both methods are generally used for various speeds, and friction-speed curves can be plotted.

One important distinction between these two methods is made here. The skid trailer measures an "instantaneous" value of friction for a given speed, whereas the stopping-distance method yields an "average" value over a range of speeds, namely the friction factor.

Figure 2 shows a hypothetical friction coefficient that decreases quadratically from 0 to 80 mph, namely,

$$\mu = 0.6 - (3.25 \times 10^{-3})v + (1.11 \times 10^{-5})v^2$$

For zero grade and no aerodynamic drag, Eq. 7 yields the skidding distance shown in Figure 3. If one takes corresponding values of initial speed and stopping distance shown in Figure 3 (which is what one gets from stopping-distance experiments) and computes various values of the friction factor, f, from Eq. 9a, the friction factor curve shown in Figure 2 results. Any friction value from this curve can be used with Eq. 9a to compute stopping distances. On the other hand, skid-trailer data are instantaneous coefficient

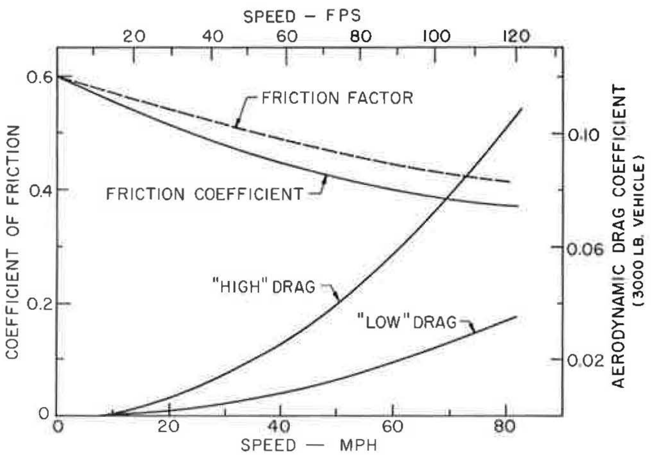

Figure 2. Friction and aerodynamic drag.

of friction values such as the friction coefficient curve shown in Figure 2. If these values of μ are used with Eq. 9a, a larger stopping distance is calculated than is obtained in practice because the friction coefficient is smaller than the friction factor. This should explain, at least in part, why 90 percent of 3,900 stopping-distance measurements (8) were smaller than the calculated stopping distances.

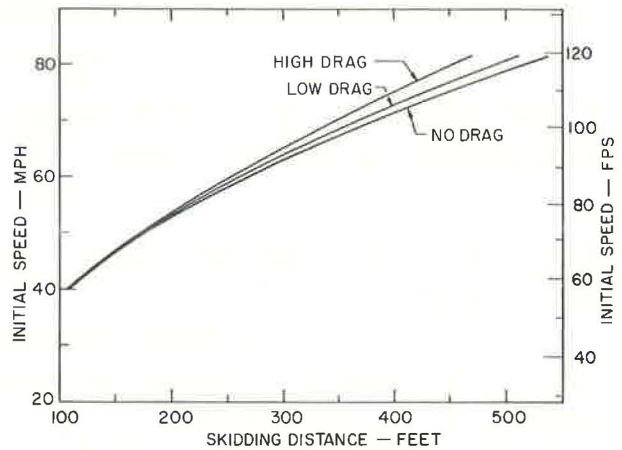

Figure 3. Skidding distance computed by using Eq. 7.

MEASUREMENT OF COEFFICIENTS OF FRICTION

This section discusses two topics: the importance of aerodynamic drag and how to obtain the instantaneous friction coefficient curve from stopping distance measurements.

For most practical purposes, the drag force is proportional to the square of the speed, i.e., $D = c_o v^2$. It depends essentially on the size and shape of the vehicle and generally ranges from 50 to 250 lb at a speed of 60 mph (9, 10). Quite obviously, in a low friction situation (say, on glare ice) a high drag force can be significant, particularly at high speeds. However, in typical test conditions (at speeds below 60 mph) drag is usually negligible. This is illustrated in Eq. 7 by solving for two values of drag: one designated as a "high" value, $c_o = 7.4 \times 10^{-3}$, and another designated as a "low" value, $c_o = 2.2 \times 10^{-2}$, also shown in Figure 2. (These are within the stated range.) The "typical" friction curve shown in Figure 2 is assumed. The results are shown in Figure 3 along with the zero drag case, $c_o = 0$. At speeds of less than 60 mph the percentage error in stopping distance with drag neglected is 8 percent or less. If this size of error is not tolerable or if the friction curve is considerably lower than the one shown in Figure 2, drag must be accounted for. Drag is neglected, however, in the remainder of this paper in order to simplify the results.

Generally, for safety reasons, tests to find the instantaneous friction curve from stopping-distance data involve speeds of less than 60 mph. Because in this range a linear relationship between friction and speed is usually found, and also to simplify the presentation, Eq. 8 is used. (Eq. 7 could be used with little additional difficulty.) Further, testing is generally done under controlled conditions, e.g., on level terrain ($\theta = 0$) and with identical tires, both front and rear. Thus $C = \mu_o$, the friction curve intercept, and $B = -k$, the slope of the friction curve. Equation 8 can then be rewritten as

$$d_i = -(10/a) \ln (1 - bv_i) - (10/a) bv_i \tag{10}$$

where $a = 10k^2 g/\mu_o$, $b = k/\mu_o$, and the subscript i indicates different experimental values of stopping distance for various initial speeds v. From the viewpoint of curve-fitting, Eq. 10 has two unknowns (a and b) appearing in nonlinear form. By using the classical method of differential corrections (11), now called a Newton-Raphson technique, we can minimize the sum of squares of deviations of d_i from the true stopping distance d.

$$Q = \sum_{i=1}^{n} (d_i - d)^2$$

with respect to a and b for all n experimental values. This furnishes a set of equations solvable for a and b.

Experimental data are required to illustrate the applicability of this technique. Because none was available to the author, some computer "experiments" were performed. First, a coefficient of friction curve was chosen (Fig. 4). For speeds of 20, 30, 40, 50, and 60 mph, two values of stopping distance were calculated from Eq. 8, each with a different, normally distributed "error" added. This was done by using a random number generating subroutine on an IBM 1130 digital computer. These fictitious experimental values are given in Table 1. The values of a and b were estimated by using the curve-fitting technique cited perviously. All numerical values are given in Table 1, and all results are shown in Figure 4. The exact stopping-distance curve from Eq. 8 and the curve fit from experimental data are essentially identical. The friction curves, exact and experimental, although not identical are very close. The maximum error in estimating μ (v) is 5.9 percent at v = 0. This example illustrates the usefulness and practicality of finding the friction curve by using nonlinear curve-fitting methods and data from stopping-distance experiments. One point of caution must be mentioned. When Eq. 8 is fitted for a and b, if the data indicate that the value of k is near zero, the natural logarithm in Eq. 8 must be expanded in a series to avoid differences of large numbers. This can be done automatically in the computer solution and presents no particular problem.

ACCIDENT STUDIES

A common situation occurs in accident investigation where a vehicle leaves measurable skid marks from a panic stop. When the car skids to a complete stop or when the speed at the end of the skid is known (or can be estimated), Eq. 7 can be used to furnish an expression for the initial speed v_o. For simplicity, it is assumed that the velocity at the end of the skid, v_f, is zero, although this is not necessary. For convenience, Eq. 8 will be used in the following examples.

In this situation, where everything is presumed known except v_o, Eq. 8 is best solved either by a computer using a root-finding method or by solving the equation for various initial velocities until the known stopping distance is found. In both cases, the generality and convenience of Eq. 8 is remarkable. Specifically, Eq. 8 takes into account the following:

1. Grade angle, θ;

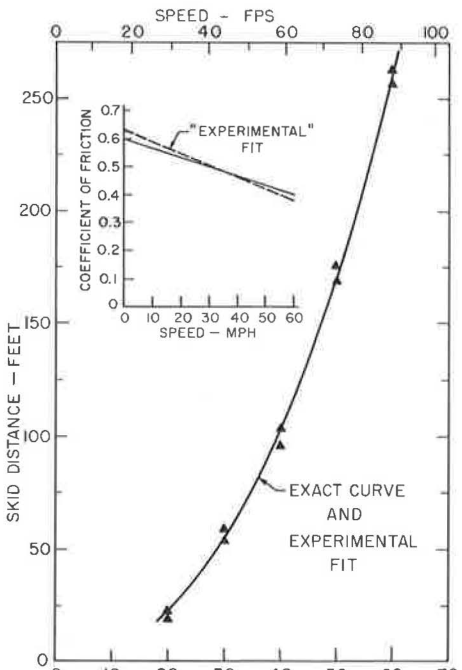

Figure 4. Least-squares curves from stopping-distance experiments.

TABLE 1

STOPPING-DISTANCE DATA

Actual Values, Eq. 8[a]		Values With Random Error[b]		Values From Fitted Curve[c]	
v (ft/sec)	d (ft)	v (ft/sec)	d (ft)	v (ft/sec)	d (ft)
29.3	24.1	29.3	19.8	29.3	23.1
		29.3	22.9		
44.0	56.5	44.0	59.3	44.0	54.7
		44.0	54.3		
58.7	105.1	58.7	103.7	58.7	102.6
		58.7	95.5		
73.3	172.1	73.3	175.2	73.3	170.0
		73.3	168.9		
88.0	260.5	88.0	257.0	88.0	260.7
		88.0	262.6		

[a] μ_o = 0.600 and k = 2.27×10^{-3}.
[b] μ_o = 0.600 and k = 2.27×10^{-3}. Error in the stopping distance is normally distributed with a mean of zero and a standard deviation of 5 ft.
[c] μ_o = 0.635 and k = 2.83×10^{-3}.

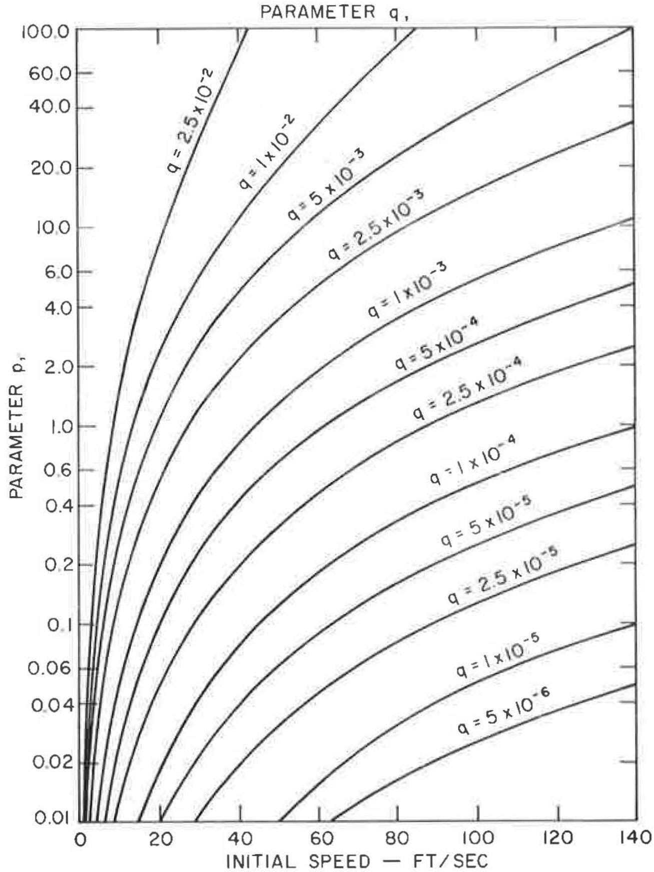

Figure 5. Initial speed from stopping distance.

2. Nonuniform weight distribution of the vehicle, α and β; and
3. Variable friction, μ_o and k, which can differ from the front to the rear of the vehicle.

Furthermore, all of these data are combined into only two parameters, q = -B/C and p = -gdB. In other words, once all of the friction data, weight distribution data, grade, and stopping distance are known, the initial speed depends only on p and q. Consequently, all of the information from Eq. 8 can be represented by a family of curves that gives the initial velocity v_o as a function of p and q. Figure 5 shows these curves for most frequently encountered values of p and q.

As an example of the use of the curves shown in Figure 5, suppose the following is known:

1. $\theta = 0.087$ radians (5 deg upgrade);
2. $\alpha = 0.55$ and $\beta = 0.45$ (55 percent of weight on front tires);
3. $\mu_{o1} = 0.6$, $\mu_{o2} = 0.5$, $k_1 = 0$, and $k_2 = 2 \times 10^{-3}$ (rear tires more effective than front tires); and
4. Stopping distance, d = 100 ft.

For these values, q = -B/C = 1.9×10^{-3} and p = -gdB = 3.523. From Figure 5, this gives an approximate initial speed of 62 ft/sec or 42 mph.

CONCLUSIONS

The major point demonstrated in this paper is that a rather general mathematical model of a skidding automobile can be constructed and solved with little difficulty. The solution, or simpler forms of the solution, can be useful in curve-fitting of experimental data from stopping-distance experiments. Further, the solution can also be used to obtain initial speeds in the case where the skid distance and friction characteristics are known. Finally, it was shown that the actual, instantaneous values of coefficient of friction should not be used with the standard stopping-distance formula, Eqs. 9 or 9a, but only with the more exact form, Eqs. 7 or 8. Conversely, "average" friction data should not be used with Eqs. 7 or 8, but only with Eqs. 9 or 9a.

NOTATION

The following symbols were used in the equations in this paper.

A = definition given following Eq. 5;
a = unknown coefficient (Eq. 10);
B = definition given following Eq. 5;
b = unknown coefficient (Eq. 10);
C = definition given following Eq. 5;
c = aerodynamic drag coefficient, lb-sec^2/ft^2;
c_o = aerodynamic drag coefficient, sec^2/ft^2;
D = aerodynamic drag force;
d = distance of skid;
f = friction factor (average coefficient over a variable speed skid);
f_1, f_2 = friction force between tires and pavement;
g = acceleration due to gravity;
h = perpendicular distance from pavement to vehicle, cg;
h_1, h_2 = coefficients in friction in expression;
k_1, k_2 = coefficients in friction in expression;
ℓ_1, ℓ_2 = length of vehicle;
m = mass of vehicle;
N_1, N_2 = normal force between tires and pavement;
p, q = parameters defined in preceding section;
v = velocity of vehicle, ft/sec;
v_f = velocity of vehicle at end of skid, ft/sec;
v_o = velocity of vehicle at initiation of skid, ft/sec;
W = total vehicle weight;
x = position coordinate of vehicle;
α = $\ell_1/(\ell_1 + \ell_2)$;
β = $\ell_2/(\ell_1 + \ell_2)$;
γ = $h/(\ell_1 + \ell_2)$;
θ = angle of grade (positive for upward motion); and
μ_1, μ_2 = coefficients of friction.

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