

DYNAMIC PROPERTIES OF CEMENT-TREATED SOILS

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An experimental study of the dynamic properties of cement-treated soils is described. The dynamic shear modulus and damping characteristics of 2 soils, a uniform sand and a silty clay, treated with Type 1 portland cement are determined by the resonant column technique. Test variables studied are cement content, confining pressure, shear-strain amplitude, and moisture content. The dynamic shear modulus and damping of both uniform sand and silty clay can be greatly increased by adding a small amount of cement. The effect of confining pressure, which increases shear modulus and damping, is more pronounced in cement-treated cohesionless soils than in cohesive soils. Higher shear-strain amplitude reduces the dynamic shear modulus and increases the damping; the cement-treated soils are subjected to a higher rate of change than are the untreated soils. The moisture content of soils has a significant effect on the dynamic properties of cement-treated cohesive soils but has no appreciable influence on cement-treated cohesionless soils. This investigation shows that the use of cement to stabilize sand and clay subjected to dynamic loading is very effective in increasing the rigidity of the soils and in reducing the deformation.

•A PROPERLY designed highway or airport pavement must meet the general requirements for all foundations. The loads imposed by traffic have to be transferred to soil layers capable of supporting them without failing in shear (bearing capacity), and the deformations of the soil layers should be such that the pavement should not suffer excessive permanent settlement. In the design and analysis of pavement-soil systems, therefore, the most critical quantities considered are the bearing capacity and deformation characteristics of the underlying soil, and those are, in turn, dependent on the material constants of the soil (such as modulus of elasticity, shear modulus, Poisson's ratio, and coefficient of subgrade reaction). The success with which the road or runway is built then depends on the accuracy with which the material properties of soil are determined.

To date, roads and runways are largely designed on the basis of knowledge of the performance of other roads built under similar traffic and subgrade conditions and of soil tests under static loading such as unconfined compression, plate loading, density, and CBR essentially dynamic nature. The use of static loading, however, does not represent the actual loading due to traffic. Because the applied load is alternating and repetitive in nature, the material properties under such a load must be investigated in terms of dynamic response and behavior of pavement-soil systems. Such a dynamic testing, in a reversed process, will also provide a rapid, nondestructive testing technique for assessing the performance of a road.

The past research indicates that theoretical values of the stresses and deformations generated by moving vehicles in multilayered elastic systems depend on the relative values of the dynamic elasticity of the layers forming the road and their thicknesses. It is, therefore, essential to obtain information concerning the dynamic material properties of the common road-making materials under repeated loading of the form applied by traffic.

For soils possessing low strength and high deformation characteristics, chemical stabilization techniques have been successfully employed in the past for road and runway construction. Studies of these stabilized soils under static loading have been extensive.

However, very little is known about the response and behavior of these soils subjected to dynamic loading.

This paper describes the first part of a two-part experimental investigation conducted to determine the dynamic properties of 2 chemically stabilized soils: a sand and a silty clay treated with cement. In the second part the 2 soils are stabilized with lime-fly ash. The experimental results on the second part will be reported in a later publication.

DETERMINATION OF DYNAMIC PROPERTIES OF SOILS

There are a number of experimental methods to determine the dynamic properties of soils. These methods may be divided into 2 general categories: (a) those in which tests are performed on a small specimen of soil usually in a triaxial compression in the laboratory (resonant column method, amplitude ratio method, and repeated-loading method) and (b) those in which the testing on soils is considered to be in situ (seismic or pulse method and elastic half-space method). A detailed discussion of these various methods (except for repeated-loading test) and a comparison of the test results obtained thereby were given by Chae (2) and will not be repeated here. A detailed description of repeated-loading technique was given by Seed and Fead (18).

Of these methods, the resonant column method has been most widely used by many researchers since Iida's original work (14) on wave propagation in sand columns. Most of these studies, however, have been confined to evaluation of cohesionless soils (4, 5, 6, 7, 11, 14, 15, 17). Few studies have been made on the dynamic behavior of clays (9, 12, 13, 20). It is believed that no work has ever been reported on chemically stabilized soils except those under repeated loading. A comprehensive test program and results were published by Mitchell et al. (16) using the repeated-loading technique.

For untreated soils it has generally been found that the most influential test parameters for sands are confining pressure and shear-strain amplitude, especially in small amplitude vibrations. Study of cohesive soils is much more complicated because of the additional variables involved, such as consolidation and other secondary effects. Specifically, a functional equation, proposed by Hardin and Black (9), for the dynamic shear modulus includes the following parameters: isotropic component of ambient effective stress, void ratio, ambient stress and vibration history, degree of saturation, deviatoric ambient stress, grain characteristics, amplitude of vibration, secondary effects that are functions of time and load increment, soil structure, and temperature.

For chemically stabilized soils the additional major parameters are cement or chemical agent content and moisture content.

EXPERIMENTAL INVESTIGATION

Materials

The materials used for this study were a uniform sand and a fine silty clay. These 2 soils were mechanically separated from the same parent material, a glacial outwash locally known as Dunellen soil, so that a comparison could be made of the dynamic response of coarse-grained soil and fine-grained soil from the same parent material. The grain size distribution curves for both soils are shown in Figure 1. The sand particles ranged from 2 to 0.074 mm in diameter and had a uniformity coefficient of 3.43 and a specific gravity of 2.63. The clay had a specific gravity of 2.65, liquid limit of 30.1 percent, and plasticity index of 9.8 percent. According to AASHTO classification, the sand is classified as A-3 and clay as A-6. Type 1 portland cement was used throughout the investigation.

Specimen Preparation

Tests were performed on remolded specimens prepared by a modified Harvard miniature compactor. A hammer, 0.75 in. in diameter and weighing 0.82 lb, was dropped 6 in. to compact the specimen in 5 layers with 10 drops on each layer. The size of the

specimen after extrusion was 1.31 in. in diameter and 2.93 in. in height. Based on strength criteria, cement contents for both sand and clay were chosen at 2, 4, and 6 percent of the dry weight of soil. A soil-cement mixture was dry-mixed, and a predetermined amount of distilled water was added to bring the mixture to a desired moisture content. The mixture was mixed thoroughly, and the specimen was molded immediately. The moisture-density curves for the sand and the silty clay are shown in Figure 2. With changes in cement content, for the sand the optimum moisture content does not vary significantly but does vary markedly for the silty clay.

Immediately following the extrusion, each specimen was wrapped in plastic and placed in a glass container to prevent moisture change and absorption of CO_2 . The specimens were then stored in a water bath where 70 F temperature and 95 percent relative humidity were maintained at all times. The cement-treated soils were cured for 28 days before testing. The cement-treated sand specimens were first cured in the compaction mold and were extruded after 2 weeks of curing.

Test Setup and Procedure

The dynamic shear modulus and damping were determined by means of the resonant column technique in torsion. Figure 3 shows a schematic diagram of the instrumentation sequences in the overall test setup. Figure 4 shows a view of the overall setup in the laboratory. The apparatus consists essentially of a driving mechanism (oscillator) on a specimen contained in a triaxial chamber and auxiliary equipment for excitation and readout. The torsional oscillator used was developed by Hardin. A detailed description of the oscillator and the theory of vibration for the specimen-apparatus model may be found in an article by Hardin and Music (10). Torque is applied to the center ring of this mechanism by applying sinusoidally varying voltage to the coils, producing sinusoidally varying forces between the coils and magnets. For balancing the weight of the apparatus on the specimen, a loading truss and lever-fulcrum are used outside the cell. Because the specimen is fixed at the bottom, a free-fixed end condition results in the vibration.

After the equipment was connected as shown in Figures 3 and 4 the input current was set at a desired level and kept constant. The frequency was varied over a large range to find the approximate position of the resonant frequency. When the approximate position had been ascertained, the frequency was varied in much smaller increments for precise measurement of the resonant frequency. The peak-to-peak value of acceleration and the phase angle between the input force and acceleration were recorded at each frequency. The dynamic response data so obtained, together with the apparatus-specimen parameters, were fed into a computer based on the theory of the apparatus-specimen model developed by Hardin (10). The dynamic shear modulus was determined by the process of iteration. Damping characteristics in terms of logarithmic decrement were obtained from the frequency-displacement amplitude response curves.

The independent test variables were moisture contents and cement contents for the specimens and confining pressures and shear-strain amplitudes for the testing apparatus. The specimens were tested at confining pressures of 3, 10, 20, and 35 psi. At a given confining pressure application, the dynamic shear modulus and damping were determined in sequence at the following 5 preselected shear-strain amplitudes: 1.4×10^{-5} , 2.3×10^{-5} , 3.7×10^{-5} , 7.5×10^{-5} , and 1.4×10^{-4} . The strain amplitudes used were limited to these values because of restrictions imposed by the apparatus and the specimen. These strain amplitudes may be small for traffic loading on real pavements, except perhaps that in the underlying subgrade layer; nevertheless, the test results obtained will be useful for they may be correlated with those expected in a real situation.

Immediately after the resonant column test, the specimens were tested in a triaxial apparatus for the evaluation of static properties of the soils. The confining pressure was chosen at 20 psi and the strain rate at 0.02 in./min. The stress-strain relationship for each specimen was then plotted by computer.

Figure 1. Grain size distribution.

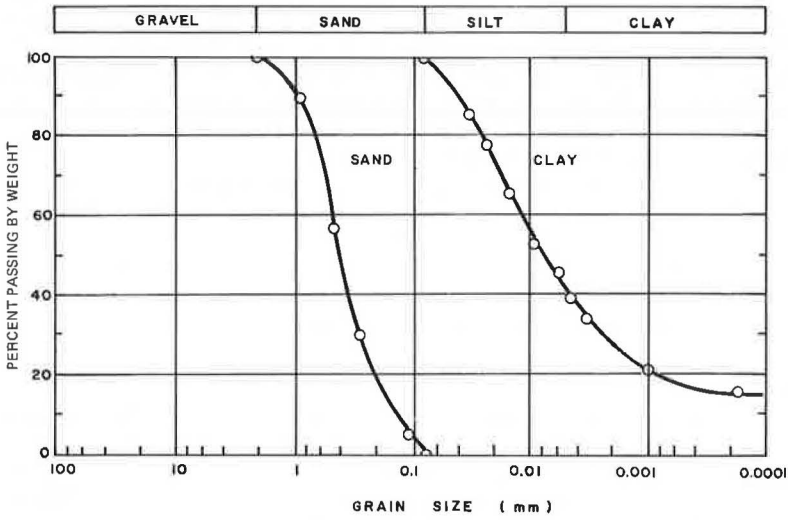


Figure 2. Density versus moisture content for sand and clay.

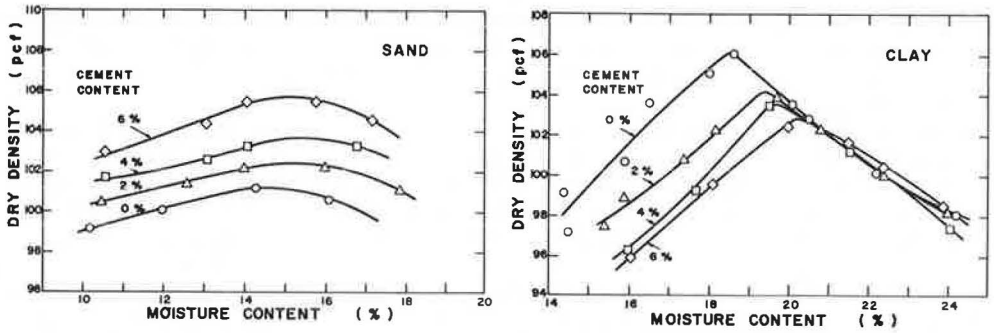
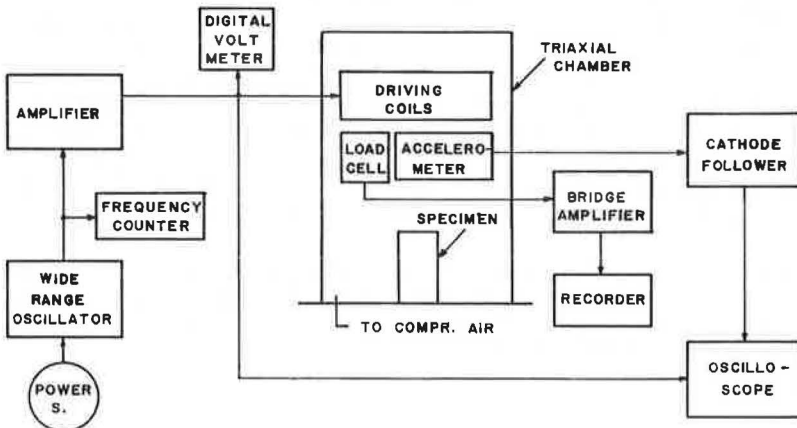


Figure 3. Schematic of overall test setup.



ANALYSIS OF TEST RESULTS

Many variables are involved in a single test, and their interrelations must be considered in the analysis of test results. The variables may include confining pressure, shear-strain amplitude, cement content, moisture content, density, void ratio, and degree of saturation. Consequently, when the effect of one variable on the dynamic properties of a material is evaluated alone, the other variables must be kept constant and the interrelations among these variables should be known.

DYNAMIC SHEAR MODULUS

Effect of Confining Pressure

Variations of dynamic shear modulus with confining pressure for sand and clay are shown in Figures 5 and 6. Each line shown in these figures represents the response of a specific specimen at the optimum moisture content. Only the results at a specific shear-strain amplitude (1.4×10^{-5}) are given. Similar results were obtained at various levels of strain amplitude. In general, dynamic shear modulus increases with increasing confining pressure and can be plotted as a straight line in a log-log scale. The increase in shear modulus with increased confining pressure becomes more pronounced for sand with greater cement content as indicated by the slope of the shear modulus-confining pressure curves. The slope increases from 0.50 for untreated sand to 0.86 for 6 percent cement-treated sand. On the other hand, the rate of increase of shear modulus for the silty clay samples remains essentially the same at 0.45 irrespective of cement content. The results obtained in this study indicating that dynamic shear modulus for both untreated sand and clay varies with approximately 0.5 power of confining pressure confirm the previous test results obtained by many investigators (4, 5, 7, 9, 12, 13). An empirical equation for the dynamic shear modulus of dry sand was proposed by Hardin (7). For small strain amplitudes,

$$G = \{ [2,630(2.17 - e)^2] / (1 + e) \} \bar{\sigma}^{0.5} \quad (1)$$

in which G is the dynamic shear modulus in psi, e is the void ratio, and $\bar{\sigma}$ is the effective confining pressure in psi. For the purpose of comparison, Hardin's equation using $e = 0.633$ is shown in Figure 5. There is a reasonably good agreement between Hardin's equation and the results obtained in this study for untreated soils. The effect of moisture appears to be negligible, as will be explained in a later section. On the basis of this agreement an effort has been made to find a general mathematical expression to accommodate cement content as a variable. The straight-line relationship of a log-log plot gives a general form of $G = a \times \bar{\sigma}^b$ in which a is the unit intercept and b is the slope of the line. By obtaining the relation between cement content, a and b , one can express the modified equation as

$$G^* = (G - 0.343 \times C \times \bar{\sigma}^{0.5}) \bar{\sigma}^{0.06C} \quad (2)$$

where G^* and G are the dynamic shear moduli in psi of cement-treated and untreated soils respectively C is the cement content in percent, and $\bar{\sigma}$ is the confining pressure in psi.

Based on the study of normally consolidated clay with low surface activity, Hardin and Black (9) also proposed a dynamic shear modulus equation for clay as follows:

$$G = \{ [1,230 (2.973 - e)^2] / (1 + e) \} \bar{\sigma}^{0.5} \quad (3)$$

This equation (with $e = 0.561$) is shown in Figure 6. By the same procedure, the modified dynamic shear modulus equation is determined for cement-treated clays as follows:

Figure 4. Test setup equipment.



Figure 5. Variation of dynamic shear modulus with confining pressure and cement content for sand.

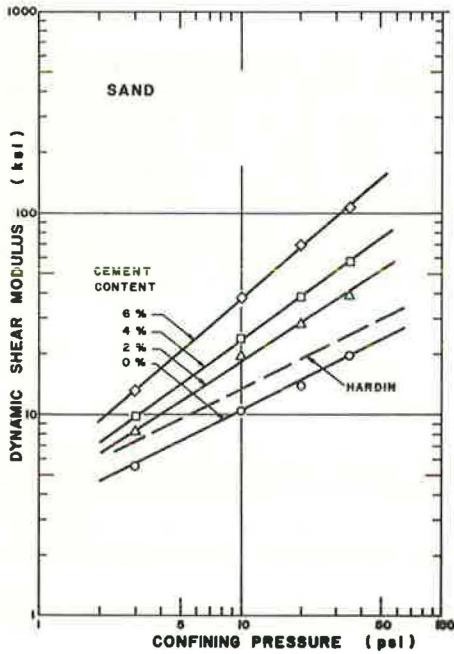
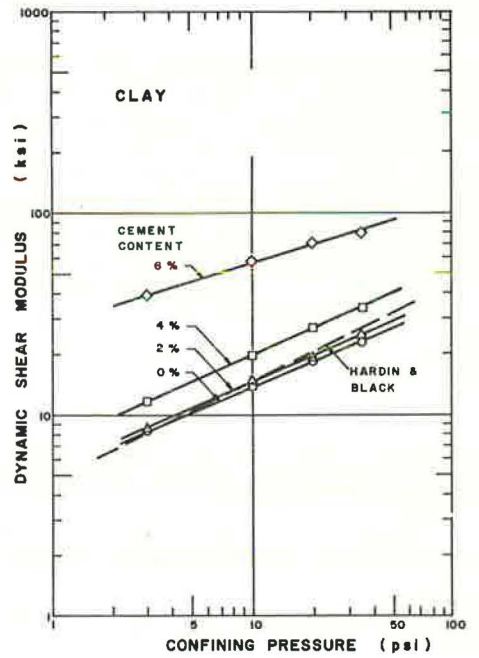


Figure 6. Variation of dynamic shear modulus with confining pressure and cement content for clay.



$$G^* = G + C [1,269 - 2,784C + 1,826C^2 - 440C^3 + 37C^4] \bar{\sigma}^{0.45} \quad (4)$$

In using these 2 modified equations, one should take care to note the comparable features such as chemical compositions, soil structure, and strain amplitude. As will be seen later on the shear modulus-strain amplitude curves, the slope for both cement-treated and untreated soils change significantly at higher amplitude.

The slope of dynamic shear modulus-confining pressure curves for sand increases with an increasing strain amplitude imposed. The same was observed for untreated soils, although to lesser degrees, by Silver and Seed (19) using a dynamic simple shear device.

Effect of Strain Amplitude

The dynamic shear modulus as a function of shear-strain amplitude for various confining pressures and cement contents is shown in Figures 7 and 8. The dynamic shear moduli of both sand and clay decrease with increasing strain amplitude, and the rate of decrease is much more pronounced with higher cement content and confining pressure.

Within the range of shear strain studied, the average decrease of dynamic shear modulus is about 55 percent for a 6 percent cement-treated sand specimen but only 25 percent for an untreated sand specimen. Relatively speaking, the effect of cement content on dynamic shear modulus within the strain amplitude studied is small for clay soils. With the largest strain imposed, the shear modulus is reduced by about 20 percent for untreated clay and 30 percent for 6 percent cement-treated clay.

The maximum values of the dynamic shear modulus for both cement-treated and untreated clays usually occur at very small shear-strain amplitude in the 0.5×10^{-5} range. Cohesionless soils, on the other hand, exhibit different behavior in that the maximum dynamic shear moduli for cement-treated sands occur near the zero-strain amplitude. This is especially true for soils at higher cement-treatment levels.

Effect of Moisture Content

Figure 9 shows that the dynamic shear modulus of sand does not vary significantly with moisture content for a given cement content and strain level. The presence of water does not appear to reduce the velocity of wave propagation significantly in either untreated or treated sands. Similar results were obtained by Barkan (1) on Young's modulus of sand with moistures ranging between 0 and 10 percent. Hardin and Richart (11) observed that the presence of moisture in sand reduced the shear modulus slightly when the moisture is less than about 1.4 percent, but higher moisture contents do not affect the value of dynamic shear modulus.

The results for shear modulus-moisture relations for cohesive soils are quite different as shown in Figure 10. For a given confining pressure and cement content, the maximum value of dynamic shear modulus occurs near the optimum moisture but drops sharply with a further increase in moisture. For clay specimens with moisture content less than the optimum the dynamic shear modulus remains close to its maximum value. This is true for both cement-treated and untreated clay specimens within the moisture range studied and regardless of the shear-strain amplitude level. A similar observation was noted by Barkan (1) on Young's modulus of clay samples with moisture ranging from 0 to 30 percent. In studying compacted kaolinite at low stresses, deGraft-Johnson (3) also found similar results.

Effect of Cement Content

The effects of cement content have been discussed along with the other parameters in previous sections. Figures 11 and 12 show the relation between the shear modulus and cement content at the given strain amplitude of 1.4×10^{-5} and optimum moisture conditions. Cement content strongly influences the shear modulus for both sand and clay. On a semilog plot, a straight line is obtained for sand over the range of cement content tested. For clay, however, the rate of increase becomes greater as the cement content is increased. Cement stabilization can be used very effectively in weak soils

Figure 7. Variation of dynamic shear modulus with shear-strain amplitude for sand.

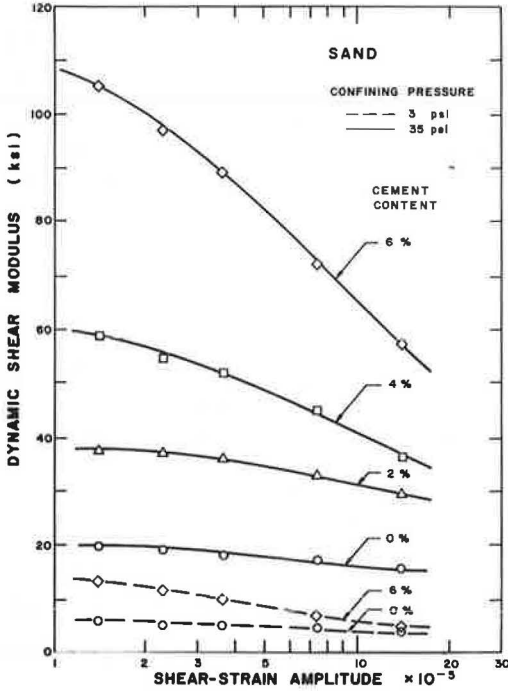


Figure 8. Variation of dynamic shear modulus with shear-strain amplitude for clay.

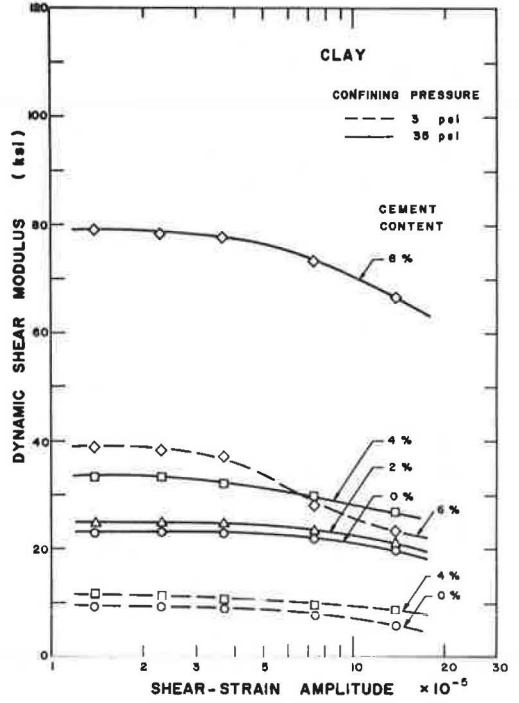


Figure 9. Variation of dynamic shear modulus with moisture content for sand.

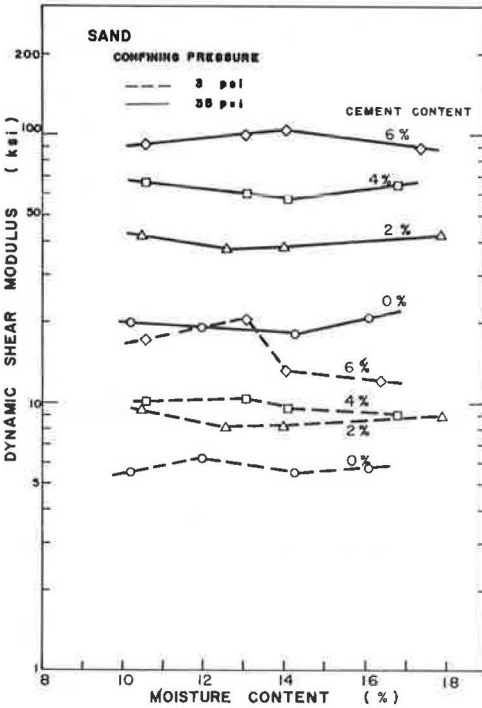


Figure 10. Variation of dynamic shear modulus with moisture content for clay.

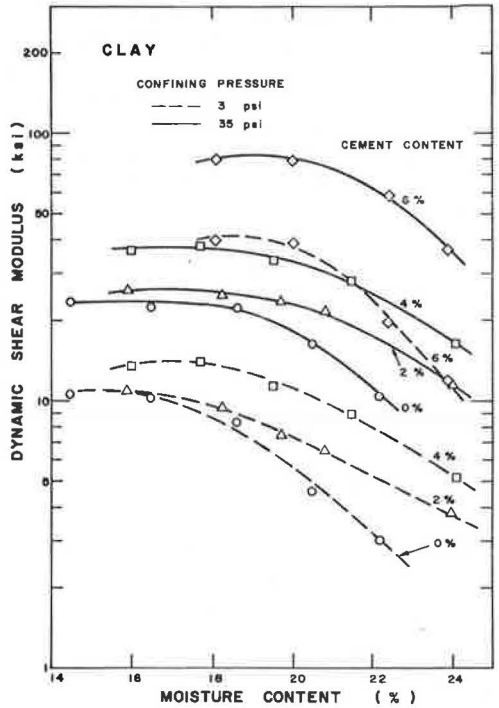


Figure 11. Effect of cement content on the dynamic shear modulus of sand.

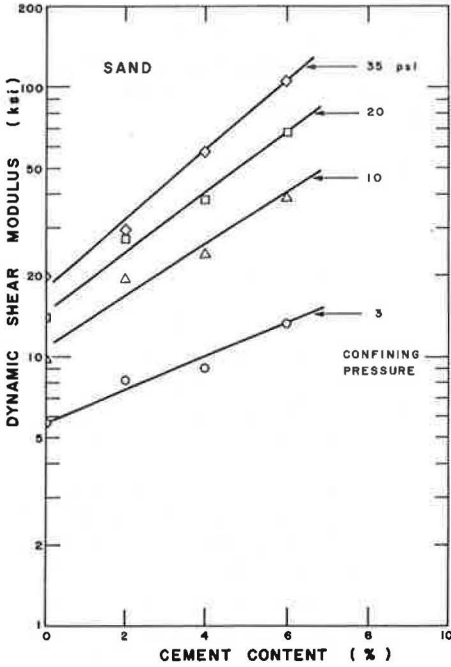


Figure 12. Effect of cement content on the dynamic shear modulus of clay.

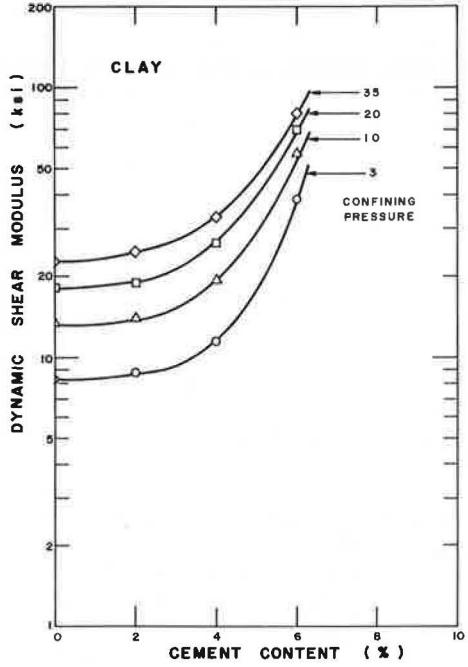
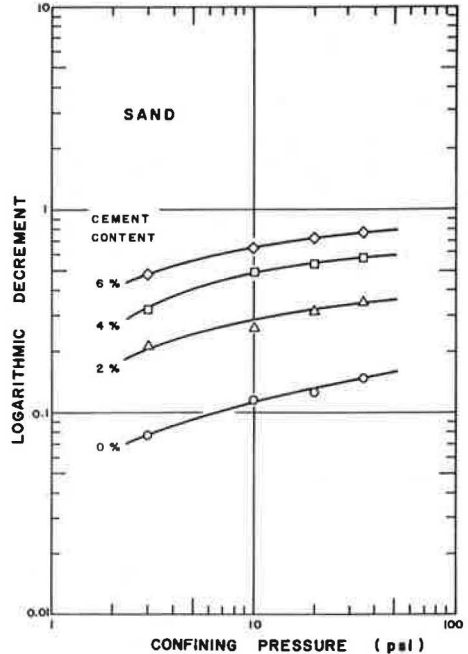


Table 1. Effect of cement content on static and dynamic properties.

Soil	Cement Content (percent)	Static Modulus of Elasticity (psi)	Dynamic Shear Modulus (psi)	Logarithmic Decrement
Sand	0	645	13,684	0.1264
	2	2,050	27,441	0.3174
	4	2,360	38,456	0.5369
	6	6,870	67,856	0.7192
Clay	0	790	18,114	0.2210
	2	800	18,700	0.2220
	4	2,130	26,633	0.2422
	6	8,600	70,447	0.3612

Note: Confining pressure = 20 psi.

Figure 13. Variation of logarithmic decrement with confining pressure and cement content for sand.



because of increased rigidity of soils subjected to dynamic loading. Table 1 gives typical results obtained for static and dynamic properties of the sand and silty clay tested under the same confining pressure of 20 psi. The static modulus of elasticity was taken to be secant modulus at one-third of the ultimate load. Both the static and dynamic moduli increase with an increasing cement content as expected. However, because of the many test parameters involved in both static and dynamic tests, quantitative correlations between dynamic and static response are difficult.

DAMPING CHARACTERISTICS

Effect of Confining Pressure

The test results for logarithmic decrement and confining pressure relations for sand specimens treated at different cement contents are shown in Figure 13 for small strain amplitude of 1.4×10^{-5} . In general, damping increases with confining pressure, and the greatest increase occurs at low confining pressures. The curves are mostly parallel to each other, indicating that the rate of change in damping for sand specimens is independent of cement content. For the silty clay no consistent trend was observed because of the very strong influence of moisture content on damping as will be explained later.

Damping characteristics of granular soils have been measured by several investigators (6, 8) using a vibrational decay technique in a resonant test. Hardin (8) showed that logarithmic decrement decreases with 0.5 power of confining pressure. Damping in clays was studied by Humphries (12), who found that logarithmic decrement of clay increased with 0.3 to 0.7 power of confining pressure. Humphries also compared the results of logarithmic decrement as determined by the amplitude-frequency method and decay method and found that when logarithmic decrement was small both methods gave the same value, but the logarithmic decrement as determined from the decay method yielded a higher value when the logarithmic decrement was large.

Effect of Shear-Strain Amplitude

Figures 14 and 15 show the relations among logarithmic decrement, strain amplitude, cement content, and confining pressure. For all soils, logarithmic decrement increases with shear-strain amplitude, and the greater increase is at larger amplitude. A greater increase in damping is also observed in soils with higher cement content and larger confining pressure as previously stated.

Effect of Moisture Content

Figure 16 shows that the damping characteristics of sand is independent of the moisture as was the case for shear modulus. Damping capacity of sand can be improved by the addition of cement regardless of its moisture condition.

The effect of moisture on damping in cohesive soils is shown in Figure 17. Under low confining pressure damping is reduced as moisture is increased, but under high confining pressure the same trend is not observed. For specimens having moisture content higher than the optimum, an increase in confining pressure increases damping; for specimens having moisture content lower than the optimum, an increase in confining pressure decreases damping. This is true for both untreated and treated clay specimens.

Effect of Cement Content

The effect of cement content on damping has been discussed in conjunction with the effects of the other parameters previously. In general, an increase in cement content increases damping and dissipates larger wave energy as confining pressure is increased. This is an advantage in practical design of roadways because of reduction in amplitude of vibration. Table 1 gives the effect of cement content on attenuation of wave energy. At a constant confining pressure of 20 psi and a 6 percent cement-treatment level, the increase of logarithmic decrement is 5.7 times for sand and only 1.7 for clay, although the increase of static strength is much higher for clays.

Figure 14. Effect of shear-strain amplitude and cement content on logarithmic decrement of sand.

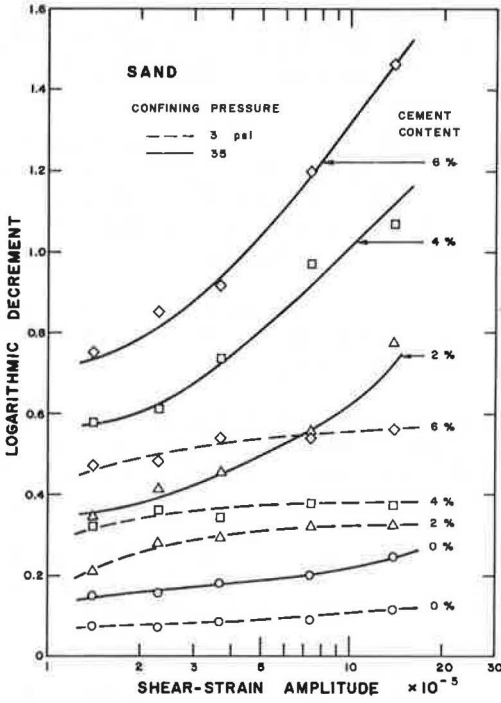


Figure 15. Effect of shear-strain amplitude and cement content on logarithmic decrement of clay.

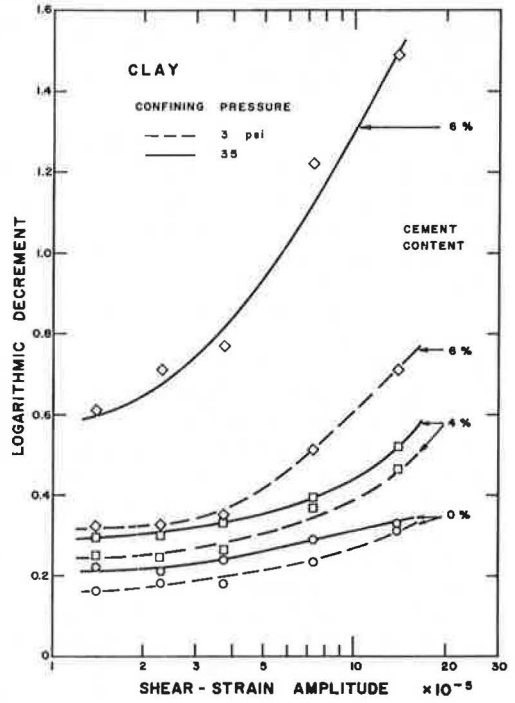


Figure 16. Variation of logarithmic decrement with moisture content for sand.

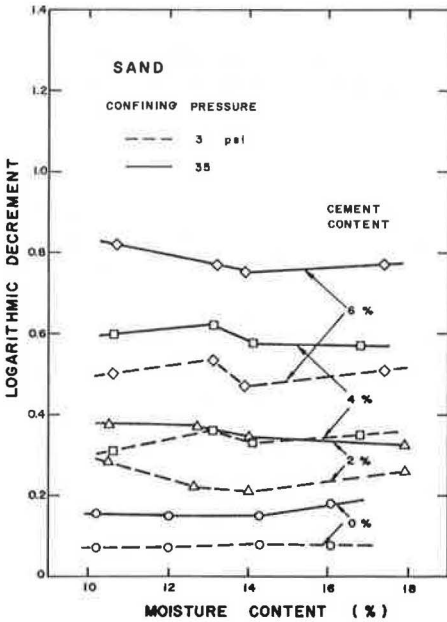
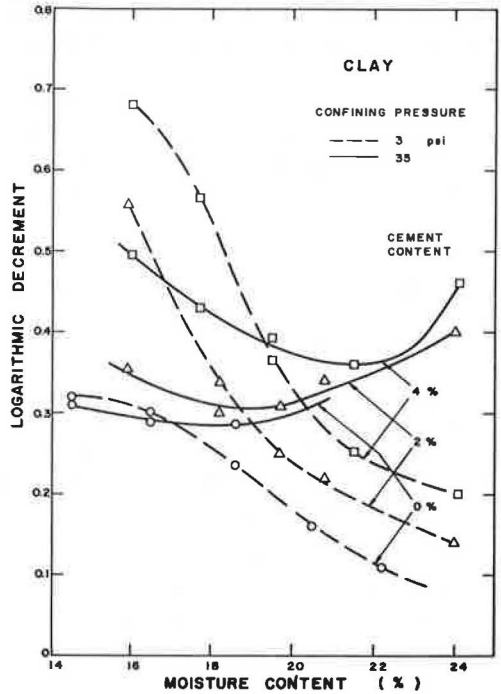


Figure 17. Variation of logarithmic decrement with moisture content for clay.



CONCLUSIONS

The use of cement to stabilize sand and clay subjected to dynamic loads is very effective in increasing the rigidity of the soil and in reducing the deformation. When resonance is a problem in foundation design, this condition may be avoided successfully by cement stabilization that produces changes in the elastic properties of the soil. The essential conclusions to be derived from this experimental study are summarized as follows:

1. Dynamic shear moduli for both sand and clay increase with increasing confining pressure. Cement treatment of both soils results in a significant increase in dynamic shear modulus, and the effect is more pronounced at a higher confining pressure for sand. Modified equations for dynamic shear moduli for cement-treated sand and clay at small shear-strain amplitudes are derived as functions of confining pressure, void ratio, and cement content.
2. Dynamic shear moduli for both materials decreased with increasing shear-strain amplitudes, and the effect is most pronounced at a small strain amplitude within the range studied. Cement-treatment increases the rates of change of the dynamic shear moduli at various strain amplitudes for both soils.
3. At a given confining pressure, cement content, and strain amplitude the dynamic shear modulus does not vary significantly with moisture for cohesionless soils. For clay soils, however, it decreases rapidly when the moisture of the specimen is increased beyond the optimum moisture content; the maximum value occurs near the dry side of the optimum.
4. Logarithmic decrement of sand does not vary with moisture content of the specimen. Higher confining pressure and cement content will result in higher damping. Logarithmic decrement in clay, both untreated and treated, depends on the relative amount of moisture with respect to the optimum.
5. Logarithmic decrement increases with increasing shear-strain amplitude for both materials. Within the range of the strain amplitude studied, damping increases at a faster rate in cement-treated clay specimens than in untreated ones. However, the rate of change remains about the same for sand.
6. The practical aspect of this study is the effect of cement treatment of soils on the increase of both dynamic shear modulus and logarithmic decrement. The combined effect of these properties can be utilized to a great advantage in designing cement-stabilized soil-foundation system subjected to dynamic loading.

ACKNOWLEDGMENTS

The authors are grateful to the Rutgers Research Council for supporting this research. Computer time was made available through the facilities of the Rutgers University Computing Center.

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