NUMERATOR-DENOMINATOR ISSUE IN THE CALCULATION OF BENEFIT-COST RATIOS

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The application of the benefit-cost ratio method to the evaluation of alternative highway designs and programs is of substantial interest. Several important reference works in this area point out that the magnitude of the ratio will be affected by the category to which a specific consequence is assigned, that is, whether an economic gain will be considered as a benefit (added to the numerator) or as a negative cost (subtracted from the denominator). The writers of these references proceed to justify the specific classification of certain consequences such as roadway maintenance costs and user costs. However, inasmuch as the only relevant issue is whether the ratio exceeds unity, the numerator-versus-denominator issue is without interest. A ratio cannot be altered from greater than unity to less than unity merely by adding (or subtracting) a constant from both numerator and denominator.

• SOME AUTHORS are critical of the benefit-cost ratio method on the grounds that the magnitude of the ratio is dependent on whether a particular economic consequence is considered in the numerator as a benefit or in the denominator as a "negative cost." (Alternatively, one may choose between inclusion in the denominator as a cost and inclusion in the numerator as a "negative benefit.")

This question occurs frequently in problems relating to highway construction and design. In particular, consider three major consequences of new highway construction or improvement: (a) capital costs, (b) benefits accruing to users of the facility, and (c) highway maintenance expenses. The issue raised at this point is whether maintenance expenses should be deducted from road-user benefits numerator) or, conversely, added to capital costs (denominator), inasmuch as each strategy will result in a different benefit-cost ratio (except in the case where the benefit-cost ratio equals unity).

A simple numerical example will serve to illustrate this point. Let X = road-user benefits = 15, Y = facility capital costs = 8, and Z = facility maintenance costs = 5. (All economic consequences are stated in terms of their equivalent uniform annual amounts over the life of the project. Alternatively, they could have been stated in terms of equivalent net present value. Either convention is appropriate to the following discussion.)

Now, in the event that maintenance costs are subtracted first from benefits

$$B:C = \frac{X - Z}{Y} = \frac{15 - 5}{8} = 1.25$$

If maintenance costs are considered in the denominator,

$$B:C = \frac{X}{Y + Z} = \frac{15}{8 + 5} = 1.15$$

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The critics claim that the resulting ambiguity makes it difficult, if not impossible, to compare two projects by using the benefit-cost ratio method. For example, suppose that we are considering an alternative (Project II) with a benefit-cost ratio of 1.20. Which, then, is preferable? Project I with B:C = 1.25 (or 1.15) or Project II with B:C = 1.20?

In response to this apparent problem, several writers have attempted to specify precisely where annual expenses ought to be placed. The "Red Book" of the American Association of State Highway Officials, for example, suggests that only road-user costs (and benefits) should appear in the numerator; all other economic consequences of the proposed investment should appear in the denominator (1, p. 14). On the other hand, it has been argued that, "...in terms of economics, economy and cost accounting, it is much more logical to put the repetitive annual cash flows in the numerator and the capital investments in the denominator" (4, p. 149. Although these views are not necessarily incorrect, they are at best much ado about very little; and, at worst, they reflect a serious misunderstanding of the application of the benefit-cost ratio method.

There is only one characteristic of the benefit-cost ratio that is relevant to the decision-making process: Is the ratio greater than unity? Otherwise, the absolute value of the ratio is irrelevant. This comment holds for both positive and negative values of the denominator. That is, the decision rules are:

For denominator > 0, accept if B:C > 1.0; reject otherwise.
For denominator < 0, reject if B:C > 1.0; accept otherwise.

In both instances the critical value of the ratio is 1.0. It is the comparison with this benchmark that leads to the decision.

Returning to our example, let us suppose that the benefit-cost ratio of Project II resulted from the following estimates: X(II) = 24, Y(II) = 20, and Z(II) = 0.

Now, let us determine the preferable alternative, I or II, considering maintenance costs in the numerator (as a negative benefit) or in the denominator. In the first, we note that B:C = 1.25, which, because it is greater than unity, leads us to conclude that Project I is preferable to "doing nothing," i.e., investing elsewhere. But is II preferable to I? To answer this question we note that incremental benefits = 24 - (15 - 5) = 14, incremental costs = 20 - 8 = 12, and ΔB:C = 14/12 = 1.17; thus Project II is preferred. Solving under the assumption that maintenance costs should be included in the denominator, we have incremental benefits = 24 - 15 = 9, incremental costs = 20 - (8 + 5) = 7, and ΔB:C = 9/7 = 1.29; as before, Project II is preferred because the incremental benefit-cost ratio exceeds unity.

To prove that this conclusion holds in all cases, we need only note that our decision rule (for positive denominator) is simply to accept the incremental investment if the resulting incremental benefit-cost ratio exceeds unity; otherwise, reject it. Stated in prior notation, the rules are:

$$\text{If B:C} = \frac{X - Z}{Y} > 1.0, \text{ accept; otherwise, reject.}$$

The alternative formulation is

$$\text{If B:C} = \frac{X}{Y + Z} > 1.0, \text{ accept; otherwise, reject.}$$

These inequalities clearly will lead to identical results; that is,

$$\text{If } \frac{X - Z}{Y} > 1.0, \text{ then } X > Y + Z, \text{ and } \frac{X}{Y + Z} > 1.0$$

This result arises from the fact that the direction of an inequality cannot be reversed merely by subtracting a constant from both sides of the inequality.

Another defense of the "annual expenses in the denominator" convention arises from the assertion that the benefit-cost ratio represents (or ought to represent) a measure of the profitability of the investment. By way of illustration, Winfrey (4) provides the following example with X = 100, Y = 1, and Z = 20. Thus,

$$B:C = \frac{X - Z}{Y} = \frac{100 - 20}{1} = 80.0$$

Alternatively,

$$B:C = \frac{X}{Y + Z} = \frac{100}{1 + 20} = 4.8$$

"The ratio of 4.8 really has no meaning," Winfrey writes, "because essentially it means that the gross profits were 4.8 times the annual operating expense, and the return on invested capital is not calculated. But the ratio of 80.0 does reflect the size of the net return on the invested capital" (4, p. 150).

The difficulty here is that the benefit-cost ratio is not a measure of return on invested capital. It is not meant to be an index of profitability, merely acceptability. In this illustration, both results lead to the conclusion that the proposed investment is attractive. But nothing more! It does not tell us, for example, that a project with B:C = 80 is preferable to an alternative with B:C = 5.

This principle may be illustrated by another example. Consider these alternatives in competition with another. Designate the two alternatives R and S respectively. Assume the following data for S: X = 120, Y = 20, and Z = 4. Thus,

$$B:C = \frac{X - Z}{Y} = \frac{120 - 4}{20} = 5.8$$

Alternatively,

$$B:C = \frac{X}{Y + Z} = \frac{120}{20 + 4} = 5.0$$

Summarizing the results of the two sets of calculations gives us the following benefit-cost ratios:

Maintenance Costs	Alternative R	Alternative S
Numerator	80.0	5.8
Denominator	4.8	5.0

The only conclusions that may be drawn from these data are that (a) alternative R is preferable to the base condition because its B:C > 1.0, and (b) alternative S is preferable to the base condition for the same reason. We do not conclude that R is "very good" nor that R is not preferable to S. Indeed, the choice between R and S awaits the following analysis of the differences between alternatives: with maintenance costs in numerator

$$\Delta B:\Delta C = \frac{116 - 80}{20 - 1} = 1.9$$

With maintenance costs in denominator

$$\Delta B:\Delta C = \frac{120 - 100}{24 - 21} = 6.7$$

The incremental benefit-cost ratios (in both instances) exceed unity, indicating that the incremental investment in S over R is justified. Again, other information about the size of the ratio is irrelevant to the decision problem.

In summary, then, the numerator or denominator problem is an empty issue. As a practical matter, the decision-maker should be concerned only with whether the benefit-cost ratio exceeds unity. The absolute value of the ratio, although it can be affected by the choice of numerator or denominator for certain income-expense items, is not relevant to the choice between alternatives.

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DISCUSSION

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The paper by Fleischer calls attention to two misunderstood or misapplied principles of economic analysis of alternative investment schemes, whether mutually exclusive or independent proposals. One of the principles of the analysis for economy is that it is the difference between proposals that is significant and not the magnitude of the cash flows. The second point often misunderstood is the principle that, when using the rate of return or benefit-cost ratio method, the procedure must be by pairs of alternatives (all possible pairs within the specific alternative projects being considered) by the principle of differences. This principle of comparison of alternatives by their differences is the only procedure that will identify the one alternative that will maximize the total net income, or net benefits.

This paper arrives at the right answer to the numerator-denominator controversy when making a choice from a pair of alternatives or from a series of pairs of alternatives, whether mutually exclusive or independent. If income is to be maximized, the factor to watch is simply that the benefit-cost ratio for a pair of alternatives is 1.0 or more or, for the rate of return method, that the calculated rate of return is at least the minimum attractive rate of return sought. In the book by Winfrey, cited by Fleischer, this principle is adhered to in the table on page 136 but, as stated by Fleischer, forgotten on page 148 in the discussion of the numerator and denominator locations for the annual expense factor.

If we accept the truth that, whether the annual expense factor is in the numerator or denominator, either procedure will indicate the same alternative as having the greater economy, then the magnitude of the benefit-cost ratio is irrelevant, except that it is less than or greater than 1.0.

But there are applications of the B/C ratio method of analysis in which the magnitude of the ratio may be of significance to the decision-maker. Consider the following three applications.

In choosing one alternative out of two or more mutually exclusive alternatives, the decision-maker would like to know whether the benefit-cost ratio between a pair of alternatives was 0.9 or 1.1. These near-to-one ratios for all practical purposes are equivalent to 1.0. The degrees of precision and of certainty in the whole process of economic analysis are rough, so that the final answer should not be used to precise magnitudes. The size of the ratio, when considered against external consequences, may be important to the decision-maker.

A second application where the benefit-cost ratio magnitude may be important is in determining the sensitivity of the discount rate (or other factor) in affecting the ratio. The comparative sensitivity of a 7 percent discount factor and a 10 or 12 percent discount factor cannot be determined by whether the benefit-cost ratio is more or less than 1.0. The analyst or the decision-maker needs to know the relative magnitude of the ratios obtained by using, say, a 7 percent rate and when using a 10 or 12 percent rate. Table 1 gives data that illustrate that the benefit-cost ratio is sensitive in the numerator position but not in the denominator position. This fact should be known to the analyst and to the decision-maker.

A third application may arise where the magnitude of the ratio is important. Consider application of the benefit-cost ratio method to a single proposal under study. Let X = 100, Y = 5, and Z = 20. The calculated benefit-cost ratios are as follows:

1. X in numerator = (100 - 20)/5 = 16;
2. X in denominator = 100/(5 + 20) = 4.

Table 1. Sensitivity of benefit-cost ratio.

		Present Worth for 20 Years		
Item	Amount ($)	7 Percent	10 Percent	12 Percent
X[a]	100	1,059	851	747
Y[b]	120	120	120	120
Z[c]	30	318	255	224
Calculated benefit-cost ratio				
Z in numerator		6.2	5.0	4.4
Z in denominator		2.4	2.3	2.2

[a]X = annual user benefits. [b]Y = initial capital costs.
[c]Z = annual maintenance.

The fact that these two calculated benefit-cost ratios are greater than one is not sufficient information to the decision-maker. It is important for him to know that the answer is either 16.0 or 4.0 before he can judge the effectiveness of the proposed investment. And if that person is concerned about the rate of return on his investment he should be guided by the ratio 16.0, the numerator result.

Some highway engineers may calculate the benefit-cost ratio of a proposed improvement to a highway facility as compared to the existing facility for many independent projects and then use the benefit-cost ratios as an index of priority of construction. This procedure is in theoretical error because each of the independently proposed projects is not compared with each other by the differential procedure.

What are the implications of this practice? What procedure do you recommend for priority selection when there may be several hundred or several thousand projects to compare with each other? At times this priority study is made for just a general guide to what projects or what type of improvements are likely to afford the greater benefits. Actual construction priorities are determined with more refined calculations.

Fleischer is to be commended for giving us this simple and correct explanation to the long-existing numerator-denominator controversy when selecting from a group of mutually exclusive alternatives and in priority selections.

AUTHOR'S CLOSURE

I note with interest Winfrey's three exceptions to the rule that the magnitude of benefit-cost ratio is irrelevant, other than the information that the ratio is less than (or greater than) unity. I concur with the first observation. Clearly, project benefit-cost ratios close to unity suggest that the accept or reject decision may be sensitive to the estimates associated with one or more of the inputs. In this case, sensitivity analyses may be in order.

With regard to the second exception, it is not clear to me that the numerical example supports the conclusion that "the benefit-cost ratio is sensitive in the numerator position but not in the denominator position." In both instances—annual maintenance costs in the numerator or the denominator—the benefit-cost ratios are well in excess of unity. Sensitivity has to do with changes in decisions as the result of changes in input values. The fact that the ratio is reduced from 6.2 to 4.4 as the discount rate increases from 7 to 12 percent is of no particular relevance.

The third exception provided by Winfrey also is not convincing. With regard to the decision, a benefit-cost ratio of 16.0 is equivalent to one of 4.0 inasmuch as both lead to the same result, i.e., accept the proposed investment. Neither ratio indicates the rate of return of this project. In fact, this example is instructive in that it illustrates that a single proposal can result in two entirely different benefit-cost ratios. Yet these ratios are (necessarily) consistent in that they lead to identical decisions.