RATIONAL QUALITY ASSURANCE

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The paper describes aspects of a rational system for the application of statistical quality-control procedures in highway construction. Norms for the judgment of compliance with a double specification limit are defined in terms of parameters that will be meaningful to road engineers. These norms are further qualified in terms of a fundamental relation of the applicable coefficient of variation and the number of observations required for judgment purposes. Corresponding judgment norms are also presented for use when a product that fails to comply with specifications when first submitted is consequently resubmitted for acceptance. Information is also presented that is required for the practical application of the scheme. This includes coefficients of variation that are representative of current practice, desired frequency of sampling, and suggested lot sizes. Finally, the application of the method is illustrated by means of a proposed system logic and a practical example based on a double specification limit.

HIGHWAY engineers have for many years stressed the need for a rational approach in the judgment of the degree that highway construction processes comply with design specifications (1, 2, 3). Although that need has received some attention in the past, it is generally conceded that no comprehensive scheme is yet available for use in highway engineering (1, 4, 5). Past experience, however, has revealed the important factors that must be taken into account in the development of such a scheme. Some of these are as follows (6):

1. The scheme should be mathematically formulated (this requirement is satisfied only if the properties of the product that are subject to quality assurance are distributed in a reasonably random manner about a mean value);
2. The scheme should be adaptable to comply with the requirements of lower, higher, or double specification limits and should be applicable to both process and acceptance control;
3. The scheme should be based on variability requirements that are representative of existing practice and that can be adjusted from time to time by means of an information feedback service;
4. There should, if possible, be an incentive for the producer to improve the uniformity of the product and thereby to effect modified specification requirements and associated economic benefits to him;
5. There should be a rational means for deciding on the required number of tests, for they directly affect the determination of the judgment norms;
6. So that the same rejection risks apply throughout, provision must be made for the determination of the judgment norms that apply when a product is resubmitted after initially failing to comply with the specified requirements; and
7. The scheme should be relatively easy to apply in practice and should be adaptable to permit desired cost benefits or sophistication in quality-assurance techniques to be obtained.

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A quality-assurance scheme has recently been developed (6, 7) that substantially complies with these requirements; aspects of this work are described in this paper. Use is made of simple statistical theory associated with the normal and chi-square distributions.

QUANTIFICATION OF JUDGMENT NORMS

Quality assurance is the process used to determine whether the properties of a specified product representing particular design requirements have been satisfactorily met by the corresponding properties of the submitted or measured product. In practice the properties of the measured product cannot be directly compared with those of the specified product, and it is convenient to define a model product that effectively represents the specified product with which the measured product can be compared for judgment purposes. The specified product is quantified for both lower and double specification limits. In the latter case the standard and modified model products are also defined and mathematically formulated, and the use of that information for exercising rational quality assurance is demonstrated by means of a practical example.

Specified Product for Lower Specification Limit

A variable representing a product property that, if it satisfies design requirements, must comply with conditions for a lower specification limit can effectively be defined by a minimum value x_s, below which not more than ϕ percent of the individual values of the magnitude of the variable should fall, and by a maximum value represented by a standard deviation σ_s or coefficient of variation V_s. Because the distribution of the magnitude of the variable can be represented by a normal distribution (6, 8, 11, 12, 13) with a mean value $\bar{\bar{x}}$, the relation among the various parameters can be formulated as follows (Fig. 1, curve I):

$$\bar{\bar{x}} = x_s + t_\phi \sigma_s = x_s/(1 - t_\phi V_s)$$

where t_ϕ is the standard normal deviate for ϕ.

$\bar{\bar{x}}$ is furthermore the true mean value of a similar population of values that consist of the mean of n individual random values $\bar{x}_n$ instead of the single values x. In this case the standard deviation of the distribution of the $\bar{x}_n$ values about $\bar{\bar{x}}$ represented by σ_n is given as follows (Fig. 1, curve II):

$$\sigma_n = V_s \bar{\bar{x}}/\sqrt{n}$$

If, as in normal practice, $\bar{x}_n$ is taken as representative of the true mean value of the property, then it can be proved that the distribution of single values in this case has a standard deviation σ_3 (Fig. 1, curve III), which is given by

$$\sigma_3 = \sqrt{\sigma_1^2 + \sigma_2^2} = v_s \bar{\bar{x}} \sqrt{(n+1)/n} = \nu_s \bar{\bar{x}} \tag{1}$$

where $\nu_s = V_s \sqrt{(n + 1/n}$ represents the normalized coefficient of variation. It should be noted that $\nu_s = V_s$ for $n = \infty$. Henceforth in this paper the strictly correct ν instead of V will be used to indicate the applicable coefficient of variation. As a practical approximation, V can be substituted for ν in the relevant equations by assuming that $\sqrt{(n+1)/n} = 1$.

Specified Product for Double Specification Limit

The equations defining the specified products for lower and upper specification limits can be effectively combined to quantify the corresponding product for a double specification limit. These are as follows:

$$\text{Lower specification limit } \bar{\bar{x}} = x_s/(1 - t_\phi \nu_s) \tag{2}$$

$$\text{Upper specification limit } \bar{\bar{x}}' = x_s'/(1 + t_\phi \nu_s) \tag{3}$$

The following additional conditions must, however, be taken into account:

1. There should be a separation h between $\bar{\bar{x}}$ and $\bar{\bar{x}}'$ to allow for inherent variations in the mean of the measured product;
2. The allowable percentage defect ϕ must be the sum of the percentage defects at both x_s and x_s' that, as shown in Figure 2, are respectively $y\phi$ and $(1 - y)\ \phi$; and
3. The absolute mean value $X = (x_s + x_s')/2$ is the target value in double limit specifications.

An analysis of specification and test data from current practice revealed that the separation h effectively varies between $0.75\ \nu_s X$ and $1.25\ \nu_s X$. An average value of $h = \nu_s x$ has, therefore, been assumed for use in this paper.

If the acceptable approximation is made that $\nu_s X = \nu_s \bar{\bar{x}} = \nu_s \bar{\bar{x}}'$, it is evident from data shown in Figure 2 that $\bar{\bar{x}}' - \bar{\bar{x}} = h = \nu_s X \left[t_{(1-y)\phi} - t_{y\phi}\right]$ or

$$h/\nu_s X = \left[t_{(1-y)\phi} - t_{y\phi}\right] \tag{4}$$

The relation between y and ϕ is also shown in Figure 3.

The specified product for double specification limits can be formulated as follows:

$$\left.\begin{aligned} x_s &= X\left[1 - \nu_s(0.5 + t_{y\phi})\right] \\ x_s' &= X\left[1 + \nu_s(0.5 + t_{y\phi})\right] \end{aligned}\right. \tag{5}$$

Measured Product

The magnitude of a property of a measured product is characterized by the mean value $\bar{x}_n$ determined from a limited number of observation data n. The variability as characterized by the range R_n is determined from the same observation data. Those 2 quantities are then compared with the judgment norms established for the model product as the quality-assurance process is exercised.

Standard Model Product

The product with which a variability ν_s is associated is formulated to serve as a convenient link between the specified and measured products for quality-assurance purposes. One of the main motivations for this requirement is the desirability of using the average of multiple values of the variables n for evaluation and judgment purposes because of the associated increased accuracy. A suitable transformation from a specified to a model product has already been indicated by means of curves I and II shown in Figure 1. A similar transformation for a double specification limit is shown in Figure 4. The effect of using n values of the variable in the second case is suitably taken account of in the modified standard deviation $\nu_s X/\sqrt{n}$ that applies in this case.

Figure 4 shows that various judgment limits can now be defined for the magnitude of the variable. These are as follows:

1. x_a and x_a' are respectively the lower and upper acceptance limits below or above which not more than α_a percent of the population should fall, and it is intended that measured values of $\bar{x}_n$, which are greater than x_a or smaller than x_a', will represent completely acceptable products provided that the variability requirements have been satisfied;
2. x_r and x_r' are similarly the corresponding rejection limits respectively below or above which no more than α_r percent of the same population values should fall, and it is intended that measured values of $\bar{x}_n$, which are either smaller than x_r or greater than x_r', will be completely rejected; and
3. If the measured value of the magnitude falls within the ranges $x_r - x_a$ or $x_a' - x_r'$, the product will be conditionally accepted at reduced payment (6) provided that it complies with variability requirements.

Figure 1. Specified product for lower specification limit.

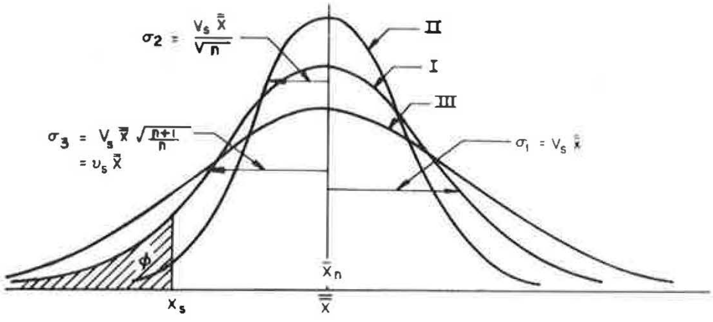


Figure 2. Specified product for double specification limits.

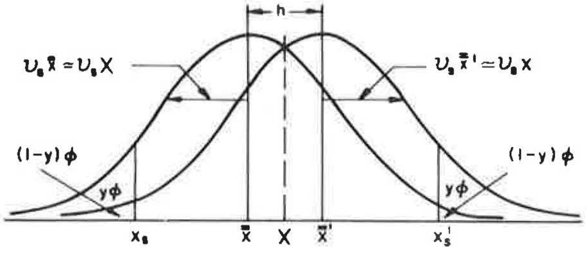


Figure 3. Factor y for different values of h and ϕ.

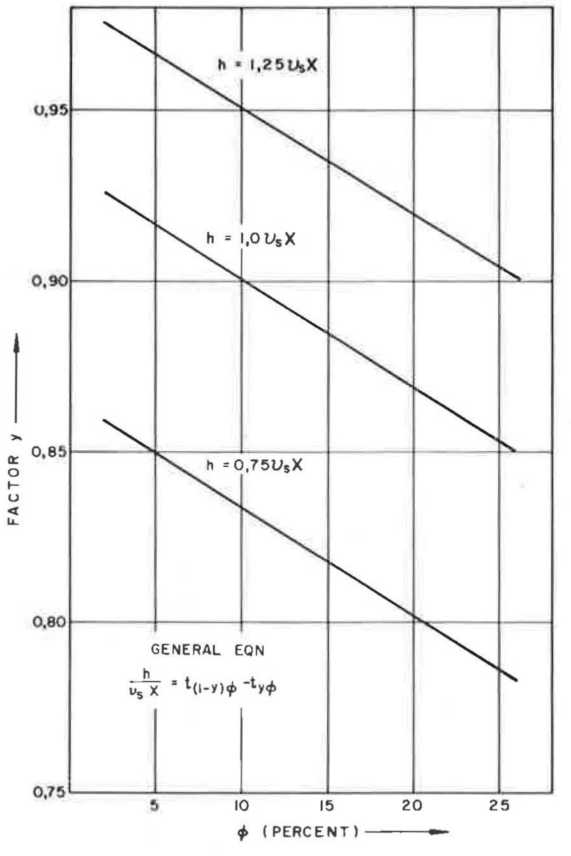
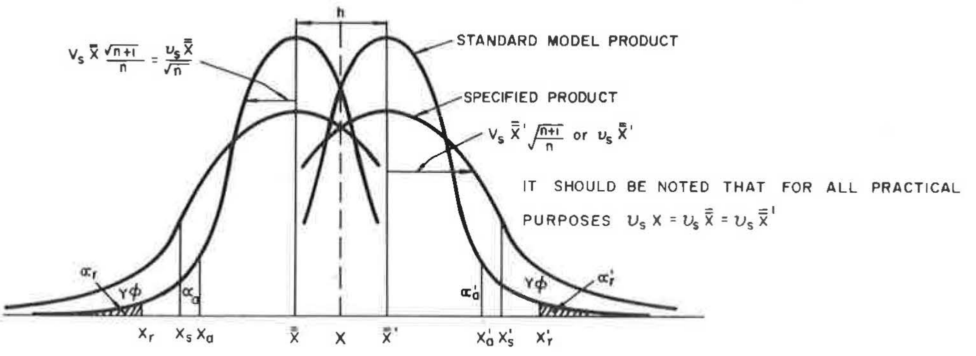


Figure 4. Specified and standard model products.



x_r should be small (assumed to be 0.1 percent) to ensure that rejected products have not been accepted. α_a, on the other hand, would normally be between 1 and 10 percent and can be optimized by taking into account certain economic factors (6, 12).

x_a', x_a', x_r, and x_r' can now be formulated as follows (Fig. 4):

$$x_a = X\left[1 - (t_{\alpha_a}/\sqrt{n})\,\nu_s\right] - 0.5h \qquad (6)$$
$$x_a' = 2X - x_a$$

$$x_r = X\left[1 - (t_{\alpha_r}/\sqrt{n})\,\nu_s\right] - 0.5h \qquad (7)$$
$$x_r' = 2X - x_r$$

Modified Model Product

The specified coefficient of variation ν_s is a maximum allowable (6) value that can be achieved by practically all producers. It is possible, on the other hand, that some producers can, as a matter of course, maintain a variability ν_p that is smaller than ν_s. In this case it is desirable that such a producer should be provided with some economic incentive. The modified model product is, therefore, defined with a variability ν_p, chosen by the producer and mandatory for quality-assurance purposes. The product can be formulated as follows (Fig. 5):

$$x_{ap} = X\{1 - (1/\sqrt{n})\left[t_{\alpha_r}\nu_s - \nu_p(t_{\alpha_a} - t_{\alpha_r})\right]\} - 0.5h$$
$$= x_r + X\left[(\nu_p/\sqrt{n})\,(t_{\alpha_r} - t_{\alpha_a})\right] \qquad (8)$$
$$x_{ap}' = 2X - x_{ap}$$

Whereas $\bar{\bar{x}}' - \bar{\bar{x}} = h = \nu_s X$ is required and just sufficient for the standard model product, a situation that can be economically exploited by the producer exists in the case of the modified model product. The required value of h for the latter product is $\nu_p X$, and the available latitude is $\bar{\bar{x}}_p' - \bar{\bar{x}}_p$. This implies that the acceptable product mean of the modified model product can be as low as $X - (\bar{\bar{x}}_p + 0.5\nu_p X) = X(1 - 0.5\,\nu_p) - \bar{\bar{x}}_p$ or as high as $X(1 + 0.5\nu_p) + \bar{\bar{x}}_p$.

The difference between the required mean for the standard and modified model products represents a potential saving to the producer who can maintain ν_p instead of ν_s. Alternatively, the consumer may wish to receive a product with a mean value of X and a coefficient of variation ν_p. In this case the producer would have to be compensated for the potential saving mentioned earlier. Either way there is consequently an economic incentive for a producer to strive for a more uniform product.

Modified Model Product Resubmitted

If a product has been rejected because $\bar{x}_n < x_r$ or $\bar{x}_n > x_r'$ and is resubmitted for judgment, whether or not it has been improved, there is only an α_r^2 percent risk that $\bar{x}_n$ will be either lower than x_r or higher than x_r' (1, 3). To maintain the same judgment standard throughout requires a determination of the corresponding judgment limits that must apply to the second submission of a product to meet this requirement.

Figure 6 shows how this condition applies to a double specification limit. The relevant judgment limits for the magnitude of the variable for the modified model product, when the pooled information from the 2n tests for both submissions is used, can readily be formulated as follows (6, 7):

$$^{(2)}x_{ap} = \{X\left[1 - (t_{\alpha_r}/\sqrt{n})\,(\nu_s - \nu_p)\right] - 0.5h\}\,(1 - k_1\nu_p) \qquad (9)$$

and

$$^{(2)}x_{ap}' = 2X - {}^{(2)}x_{ap} \qquad (10)$$

Figure 5. Standard model and modified model products.

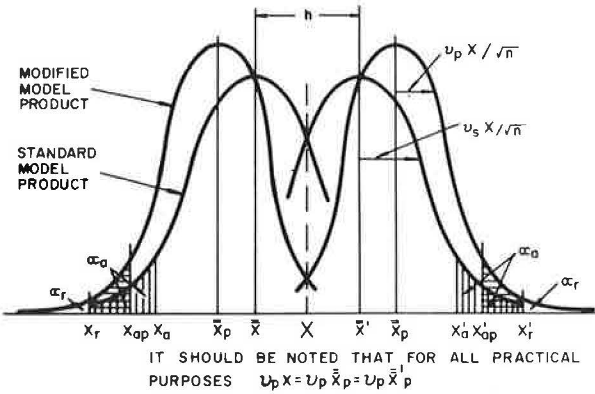


Figure 6. Control of magnitude for second submission.

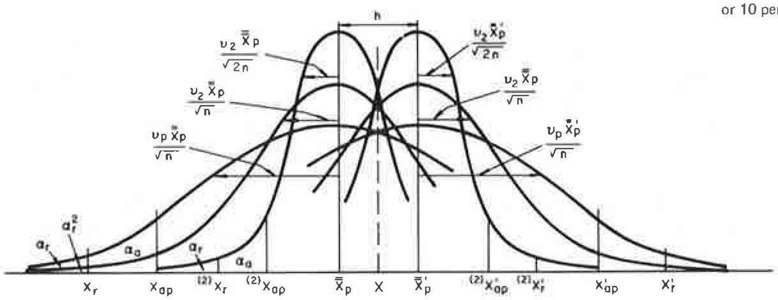


Figure 7. Relation between coefficient of variation ν_s and limit of accuracy p_r for various values of n.

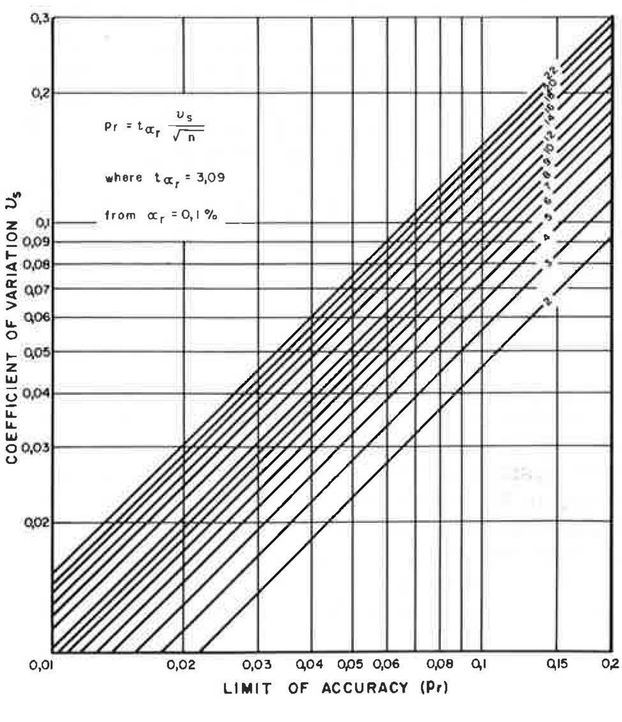


Table 1. Values of $k_{1,\alpha,n}$ for calculating rejection limit $^{(2)}x_r$ and acceptance limit $^{(2)}x_a$ for judgment of magnitude of product property resubmitted after rejection.

	$k_1 = t_{\alpha_r} t_{\alpha_a} / t_{\alpha_r}^2 \sqrt{2n}$ for α =			
n	0.1 percent	2.5 percent	5.0 percent	10 percent
2	1.004	0.637	0.535	0.417
3	0.820	0.520	0.437	0.340
4	0.710	0.451	0.378	0.295
5	0.635	0.403	0.338	0.264
6	0.580	0.368	0.309	0.241
7	0.537	0.341	0.286	0.223
8	0.502	0.318	0.267	0.208
9	0.474	0.300	0.252	0.196
10	0.449	0.285	0.239	0.186
12	0.410	0.260	0.218	0.170
14	0.380	0.241	0.202	0.158
16	0.355	0.225	0.189	0.147
18	0.335	0.212	0.178	0.139
20	0.318	0.201	0.169	0.132

Note: $^{(2)}x_r = \bar{\bar{x}}_p (1 - k_1 \upsilon_p)$ where $\alpha = \alpha_r$ = 0.1 percent; and $^{(2)}x_a = \bar{\bar{x}}_p (1 - k_1 \upsilon_p)$ where $\alpha = \alpha_a$ = 2.5, 5, or 10 percent.

where $k_1 = (t'_{\alpha_a} t'_{\alpha_r})/(t^2_{\alpha_r} \sqrt{2n})$.

Also

$$^{(2)}x_r = {}^{(2)}x_{ap} \tag{11}$$

as in Eq. 9 but with $\alpha_a = \alpha_r$, and

$$^{(2)}x_r' = 2X - {}^{(2)}x_r \tag{12}$$

Values for k_1 are given in Table 1.

The corresponding values for the standard model product can readily be determined from Eqs. 9 to 12 by substituting ν_s for ν_p.

The derivation of the judgment norms for the variability of the variable for double specification limits is identical to that for a lower specification limit and has already been published (13). This aspect will, therefore, not be dealt with here but is taken account of in the example illustrated in a following section.

DETERMINATION OF REQUIRED SAMPLE NUMBER

The required value of n for judgment purposes can be determined by at least 2 methods:

1. A method based on the relation given in Eq. 1 has been developed (8), in which both the cost of testing and the product cost are used, and has merit when reasonably reliable cost data are available from practice; and
2. A method that is perhaps more popular utilizes a relation that can be derived from Eq. 7 where, for a lower specification limit, $h = 0$ and $X = \bar{\bar{x}}$.

Furthermore, in the latter model, by putting $(\bar{\bar{x}} - x_r)/\bar{\bar{x}} = p_r$, where p_r is the limit of accuracy at the rejection limit for a standard model product, the relation between p_r and n can be formulated as follows:

$$p_r = t_{\alpha_r} (\nu_s/\sqrt{n}) \tag{13}$$

where t_{α_r} is the standard normal deviate for α_r. Figure 7 shows this relation for $\alpha_r' =$ 0.1 percent.

INFORMATION REQUIRED FOR APPLICATION OF QUALITY ASSURANCE

In addition to the availability of a rational method for the determination of judgment criteria, it is also essential to have various types of information available to ensure the effective use of quality assurance. This includes aspects such as the minimum yet practically achievable variability that should be specified; the percentage defect ϕ that should be allowed in the specification of properties of products; the economic lot size that should be used; and the various cost items related to testing, materials, construction, and maintenance. Although only limited systematic information is available with respect to most of these items, reasonably representative data have been established for the coefficients of variation representative of a number of product properties of importance in highway construction (6, 14).

Variability of Product Properties

Because variability is an important aspect of quality assurance, it is essential to use values that will ensure the best standard generally achievable by current practice. That was determined by establishing the distribution of coefficients of variation for various product properties from the analysis of extensive data from South African road practice. From these data, the median or V_{50} values of the coefficients of variation were determined as well as the ratio between this value and the 90 percentile value of the distribution or V_{90}. The average value for the ratio V_{90}/V_{50} was found to be 1.7.

More representative values of V_{50} were obtained by taking into account similar values determined from published information from practice in the United States (6). From these data, V_{90} values were again calculated by using the ratio of 1.7 given above. These V_{90} values have been chosen as the specified values V_s of the coefficient of variation to be used for quality-assurance purposes because they best comply with the qualifications stated above. This information is given in Table 2. It is intended that these data should be revised and updated from time to time as more reliable information becomes available. Useful information for the quantification of the variability for specifying grading for both bituminous surfacing and base course materials (6) is shown in Figure 8.

General Information for Quality Control

Although reliable values of certain parameters required in quality assurance are not yet available, approximate data obtained from a literature survey and an opinion survey of practicing engineers are given below as a guide.

1. The percentage defect ϕ varies between 10 and 25 percent, and the lower figures are associated with lower values of V_{90} and vice versa;
2. A value of $\alpha_r = 0.1$ percent is considered satisfactory for practical requirements although values of 0.2 percent or even higher may still be acceptable;
3. α_a should lie between 2.5 and 10 percent, and a preferable value for highway construction purposes is about 5 percent;
4. Depending on the applicable parameter, the limit of accuracy should preferably be below 15 percent and have a probable practical range of 6 to 12 percent; and
5. The rational determination of lot sizes is not yet possible, and currently accepted practice such as a day's work or estimated general lot sizes of about 4,000 m^2 should be used.

PRACTICAL APPLICATION OF QUALITY CONTROL

The effective utilization of quality assurance demands an integrated interaction among certain important functions such as the interest and activities of both producer and consumer as well as the nature and quantity of the available input information. Such a system is shown in Figure 9.

Consumer Discipline

Apart from the consideration, approval, and financing of the product property, the consumer shall be responsible for designing and specifying the desired product property as well as for the associated quality control to ensure that the delivered goods comply with the specified requirements, which at all times should be mutually acceptable to both consumer and producer and practically attainable.

Design Function—This function includes establishing and calculating norms required for control judgments and making them known to the producer discipline by means of the specifications. At the same time a simple scheme should be prepared for use by the application function for acceptance control. This function must also decide on details such as lot size; number of test samples; and test positions, procedures, methods, apparatus, and calibration. This function should constantly draw information from data storage that should be kept as up to date as possible. It is, therefore, essential that information gained by the producer should be fed into storage so that the design and specification function can recognize and readily allow for any new and improved techniques.

Application Function—It is the duty of this function to perform tests, make calculations, and execute judgments on product properties in accordance with the norms established by the design function.

Producer Discipline

This function includes storing and supplying performance and cost data and properly controling the process. The control of quality during the process of manufacture or construction can reduce costs by reducing rejections. The direct supply of test results by the producer to the design and application functions as well as to a central data

Table 2. Recommended V_{50} and V_{90} values for product properties.

Course	Property	V_{50} Values (percent) South Africa	United States	Recom-mended	V_{90} $\equiv V_e$
Wearing and level-ing	Binder content	3.4	5.68	4.9	8.3
	Marshall stability	16.7	13.07	15.5	27.9
	Marshall flow	11.8	15.5	14.3	24.3
	Marshall void content	20.8	20.68	20.7	35.2
	Thickness		11.84	11.8	20.0
Subbase and base	Percentage density (general)	2.7	3.57	3.3	5.6
	Percentage density (asphalt)	1.75	1.25	1.6	2.7
	Thickness	6.4	6.8	6.7	11.4
	Moisture content		14.8	14.8	25.2
	Cement content (stabilization)		13.6	13.6	23.2
Concrete pave-ment	Thickness				
	8 in.		3.6	3.6	6.1
	9 in.		3.2	3.2	5.4
	10 in.		2.6	2.6	4.4
	Strength, 28 days		14.5	14.5	24.6
	Air void content (plastic sheeting)		18.34	18.3	31.1
	Cone slump		31.5	31.5	53.5

Figure 8. Relation between V_{90} and cumulative percentage passing sieve size.

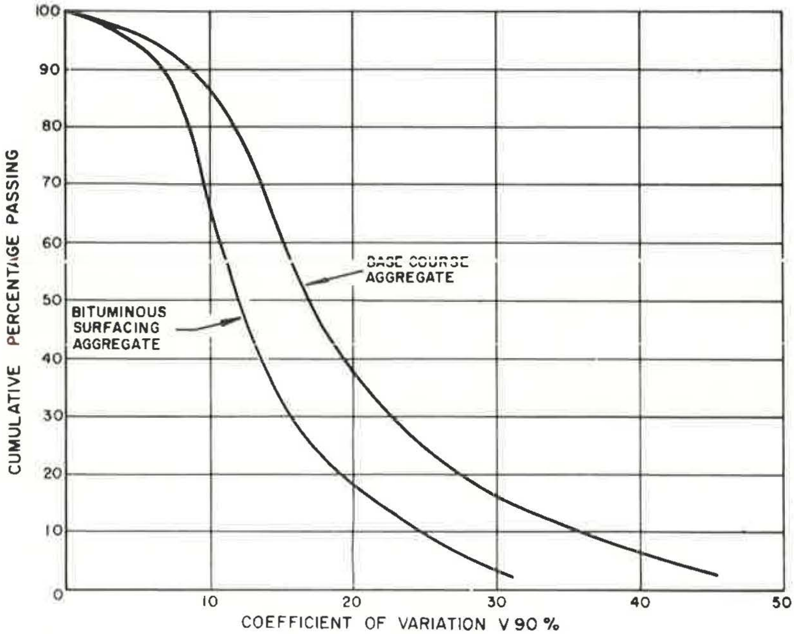
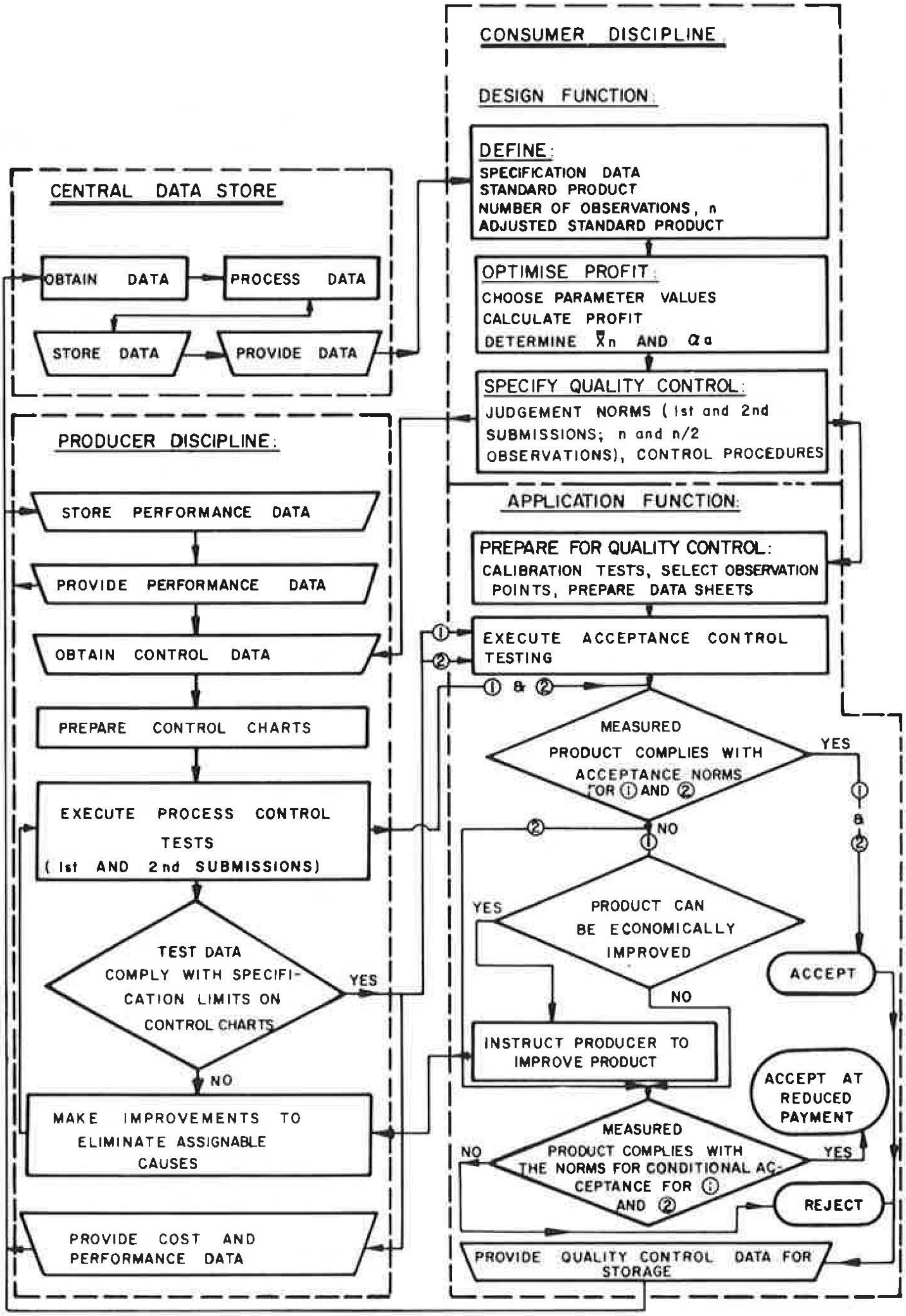


Figure 9. Quality control system for road construction.



NOTE (1), (2) RESPECTIVELY 1st AND 2nd SUBMISSIONS storage will contribute largely to continually improved judgment discipline and to design and specification techniques.

Central Data Storage

Some form of central data storage and processing of production costs and quality variations from which the latest parameters could be drawn would be of benefit to the producer discipline and would help to bring about improved cooperation between design and construction disciplines.

PRACTICAL EXAMPLE

The following example of a double limit specification illustrates the practical application of the method.

It is desired to control binder content in a pre-mix to an average of 5 percent by mass of total mix. The choice of parameters for specification requirements is as follows.

1. Average magnitude $X = 5$ percent $= 0.05$;
2. Coefficient of variation $= V_s = V_{90} = 0.083$ (Table 2);
3. Spread of higher and lower population means, i.e., $\bar{\bar{x}}' - \bar{\bar{x}} = h = \nu_s X$, say;
4. Values of ϕ, α_a, α_r, and $p_r = 15$, 5, 0.1, and 10 percent; and
5. Lot = 1 day's output, say.

The magnitude of the variable is calculated as follows:

1. Number of tests per lot—

$$\sqrt{n + 1/n} = p_r/t_{\alpha_r} \nu_s = 0.10/(3.10 \times 0.083)$$

from which $n = 7.4$ or taken as 8, $p_r = 0.096$ or 9.6 percent, and $\nu_s = \sqrt{[(n+1)/n]}\ V_s = 0.088$.

2. Specification limits—From the data shown in Figure 3, ($h = \nu X$ for $\phi = 15$ and $y = 0.883$), $y\phi = 13.25$ and $t_{y\phi} = 1.115$.

$$\bar{\bar{x}} = X(1 - 0.5\nu_s) = 0.05(1 - 0.5 \times 0.088) = 0.0478$$

$$\bar{\bar{x}}' = 2X - \bar{\bar{x}} = 0.10 - 0.0478 = 0.0522$$

$$x_s = X\ [1 - \nu_s(0.5 + t_{y\phi})] = 0.05\ [1 - 0.088(0.5 + 1.115)] = 0.0430 \text{ (if required)}$$

$$x_s' = 2X - x_s = 0.10 - 0.043 = 0.0570 \text{ (if required)}$$

This implies that not more than 15 percent of the values of binder content observations shall fall outside the limits of 4.33 and 5.67 percent by mass and that the product property variation shall not exceed $\nu_s = 8.8$ percent.

3. Rejection limits (first submission)—

$$x_r = X(1 - p_r) - 0.5h = 0.05 \times 0.904 - 0.5 \times 0.88 \times 0.05 = 0.0431$$

$$x_r' = 2X - x_r = 0.10 - 0.0431 = 0.0569$$

4. Acceptance limit (first submission)—

$$x_a = X\{1 - \nu_s\ [(t_{\alpha_a}/\sqrt{n}) + 0.5]\} = 0.05\{1 - 0.088\ [(1.645/\sqrt{8}) + 0.5]\} = 0.0455$$

$$x_a' = 2X - x_a = 1.00 - 0.0455 = 0.0545$$

5. Rejection limits (second submission)—

$$^{(2)}x_r = X(1 - 0.5\nu_s)\ (1 - k_1\nu_s)$$

Table 3. Factor f_3 values for varying percentages of ϕ and α required for calculating control limits for range variability.

	α (percent)					α (percent)				
	0.1	2.5	5.0	10	50	0.1	2.5	5.0	10	50
n	ϕ = 10 percent					ϕ = 15 percent				
2	2.827	1.927	1.684	1.416	0.578	3.216	2.194	1.916	1.612	0.657
3	3.335	2.425	2.181	1.924	1.048	3.695	2.686	2.417	2.132	1.161
4	3.679	2.758	2.515	2.258	1.372	3.991	2.993	2.728	2.450	1.488
5	3.930	3.011	2.768	2.503	1.621	4.244	3.251	2.989	2.703	1.750
6	4.135	3.209	2.965	2.700	1.817	4.429	3.436	3.176	2.892	1.947
7	4.302	3.372	3.131	2.868	1.990	4.588	3.596	3.339	3.058	2.122
8	4.442	3.517	3.274	3.007	2.129	4.725	3.743	3.483	3.199	2.265
9	4.565	3.638	3.397	3.126	2.259	4.838	3.854	3.599	3.312	2.394
10	4.674	3.751	3.500	3.233	2.364	4.939	3.961	3.698	3.417	2.498
12	4.860	3.926	3.687	3.415	2.561	5.111	4.128	3.877	3.592	2.694
14	5.014	4.082	3.840	3.572	2.722	5.260	4.284	4.028	3.748	2.855
16	5.150	4.215	3.977	3.706	2.854	5.378	4.400	4.154	3.871	2.980
18	5.261	4.322	4.084	3.819	2.974	5.495	4.515	4.266	3.989	3.107
20	5.357	4.431	4.187	3.920	3.084	5.577	4.611	4.359	4.080	3.210
	ϕ = 20 percent					ϕ = 25 percent				
2	3.629	2.473	2.162	1.818	0.741	4.043	2.755	2.408	2.026	0.826
3	3.988	2.900	2.609	2.302	1.253	4.297	3.124	2.811	2.480	1.350
4	4.269	3.200	2.918	2.621	1.592	4.538	3.403	3.102	2.786	1.692
5	4.479	3.431	3.155	2.852	1.847	4.723	3.620	3.327	3.008	1.948
6	4.655	3.610	3.338	3.040	2.046	4.882	3.789	3.501	3.188	2.146
7	4.798	3.758	3.492	3.199	2.219	5.012	3.929	3.648	3.342	2.318
8	4.918	3.895	3.625	3.329	2.358	5.122	3.957	3.776	3.468	2.456
9	5.025	4.004	3.739	3.441	2.487	5.220	4.160	3.884	3.575	2.584
10	5.119	4.105	3.833	3.541	2.589	5.307	4.258	3.974	3.671	2.685
12	5.281	4.266	4.006	3.711	2.783	5.457	4.408	4.140	3.835	2.876
14	5.415	4.410	4.147	3.858	2.940	5.582	4.546	4.275	3.977	3.030
16	5.535	4.528	4.274	3.984	3.067	5.694	4.662	4.398	4.098	3.155
18	5.631	4.630	4.372	4.088	3.184	5.784	4.755	4.491	4.199	3.270
20	5.715	4.728	4.467	4.182	3.290	5.862	4.850	4.582	4.289	3.375

Note: $\bar{\bar{R}}$, R_a, or $R_r = f_1 R_{\alpha,n}\sigma_s = f_3\sigma_s$, where α = 0.1 and 50 percent respectively for R_r and $\bar{\bar{R}}$ and $\alpha = \alpha_a$ = 2.5, 5.0, and 10 percent for R_a.

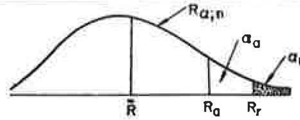


Table 4. Factor k_3 values for varying percentages of ϕ and α required for calculating acceptance limit R''_a and rejection limit R''_r at resubmission for range variability.

	α (percent)				α (percent)			
	0.1	2.5	5.0	10	0.1	2.5	5.0	10
n	ϕ = 10 percent				ϕ = 15 percent			
2	2.026	1.518	1.384	1.233	2.339	1.753	1.599	1.427
3	2.502	1.942	1.795	1.629	2.769	2.148	1.986	1.803
4	2.839	2.249	2.093	1.916	3.086	2.445	2.275	2.084
5	3.100	2.487	2.321	2.145	3.336	2.676	2.497	2.307
6	3.314	2.677	2.514	2.335	3.540	2.861	2.686	2.494
7	3.494	2.844	2.676	2.490	3.715	3.024	2.844	2.646
8	3.654	2.990	2.822	2.629	3.866	3.164	2.986	2.782
9	3.788	3.114	2.941	2.750	3.996	3.285	3.102	2.901
10	3.907	3.231	3.054	2.859	4.110	3.399	3.212	3.007
12	4.111	3.424	3.246	3.051	4.306	3.585	3.400	3.195
14	4.287	3.592	3.410	3.216	4.475	3.749	3.560	3.356
16	4.441	3.736	3.550	3.350	4.622	3.889	3.695	3.488
18	4.575	3.863	3.674	3.471	4.752	4.013	3.816	3.605
20	4.694	3.970	3.791	3.583	4.865	4.115	3.929	3.714
	ϕ = 20 percent				ϕ = 25 percent			
2	2.600	1.949	1.777	1.587	2.896	2.171	1.980	1.767
3	2.993	2.322	2.146	1.949	3.224	2.502	2.312	2.100
4	3.295	2.610	2.456	2.225	3.502	2.774	2.581	2.365
5	3.533	2.834	2.654	2.444	3.726	2.989	2.790	2.578
6	3.730	3.014	2.829	2.628	3.912	3.161	2.968	2.756
7	3.897	3.173	2.984	2.777	4.072	3.315	3.118	2.901
8	4.044	3.310	3.124	2.911	4.213	3.448	3.254	3.032
9	4.170	3.428	3.237	3.027	4.332	3.561	3.363	3.145
10	4.280	3.538	3.344	3.131	3.437	3.669	3.468	3.247
12	4.468	3.720	3.528	3.315	4.617	3.844	3.646	3.426
14	4.631	3.880	3.684	3.473	4.772	3.999	3.797	3.579
16	4.773	4.015	3.815	3.600	4.910	4.131	3.925	3.704
18	4.897	4.136	3.933	3.715	5.030	4.248	4.039	3.816
20	5.007	4.235	4.044	3.824	5.136	4.344	4.148	3.922

Note: $R''_a = k_3 \upsilon_s \bar{\bar{x}}$ ($\alpha = \alpha_a$ = 2.5, 5.0, and 10 percent), and $R''_r = k_3 \upsilon_s \bar{\bar{x}}$ ($\alpha = \alpha_r$ = 0.1 percent), where $k_3 = R_{\alpha,(2n-1)} \sqrt{\left[(n-1)\chi^2_{\alpha_r,(n-1)}\right] / \left[\chi^2_{\phi,(n-1)}\, \chi^2_{\alpha_r^2,(n-1)}\right]}$.

For $R_{\alpha,(2n-1)}$, values are given in standard tables for the distribution of the range for corresponding values of α and (2n - 1).

Figure 10. Data for acceptance control.

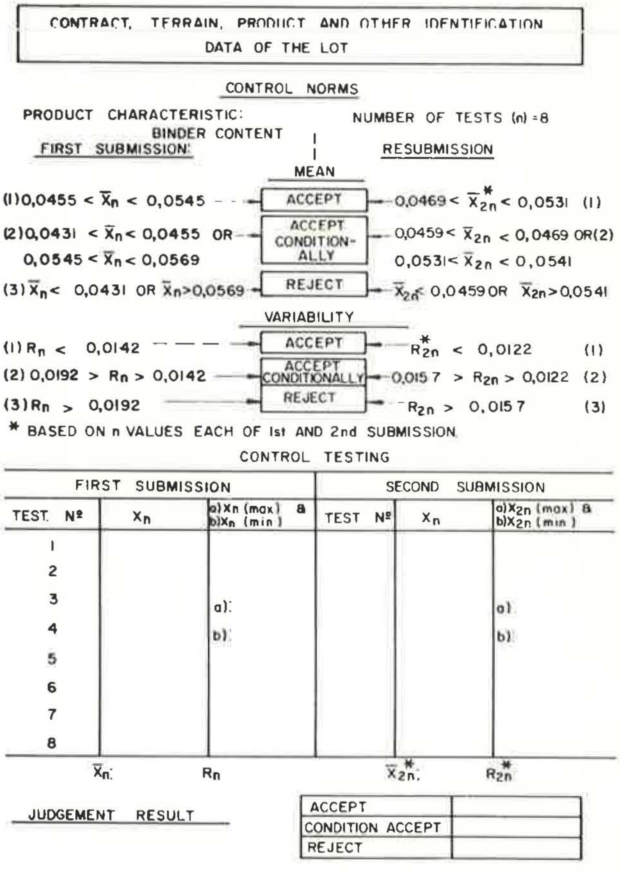
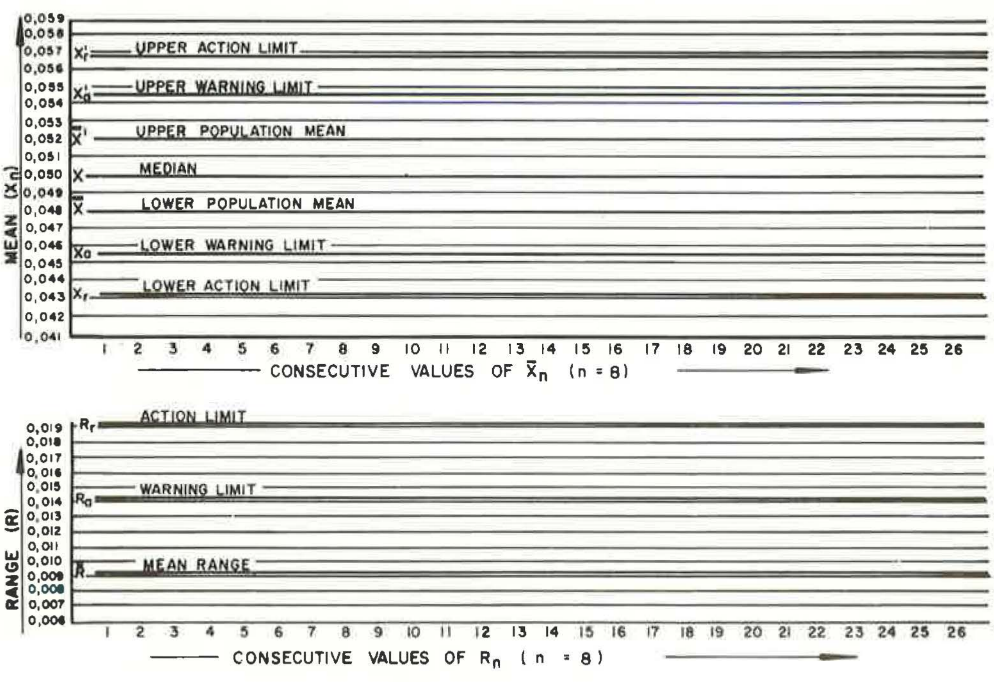


Figure 11. Control charts for magnitude and variability of binder content.



$$^{(2)}x_r = 2X - {}^{(2)}x_r$$

From data given in Table 1 ($\alpha_r = 0.1$ percent and $n = 8$), $k_2 = 0.502$; therefore,

$$^{(2)}x_r = 0.05\ [1 - (0.5 \times 0.088)]\ [1 - (0.502 \times 0.088)] = 0.0459$$

$$^{(2)}x_r = 0.100 - 0.0459 = 0.0541$$

6. Acceptance limits (second submission)—

$$^{(2)}x_a = X(1 - 0.5\nu_s)\ (1 - k_1\nu_s)$$

$$^{(2)}x_a = 2X - {}^{(2)}x_a$$

From data given in Table 1 ($\alpha_a = 5$ percent and $n = 8$), $k_1 = 0.267$; therefore,

$$^{(2)}x_a = 0.05\ [1 - (0.5 \times 0.088)]\ [1 - (0.267 \times 0.083)] = 0.0469$$

$$^{(2)}x_a = 0.10 - 0.0969 = 0.0531$$

For the variability of the variable, the range is selected to represent variability rather than the standard deviation for practical reasons. The general expression for range is given by $R_\alpha = f_3\bar{\bar{X}}_p\nu_p$ (where f_3 may be interpolated from data given in Table 3) for the mean range $\bar{\bar{R}}_p$, the acceptance limit R_{ap}, and the rejection limit R_r (6, 7, 13). ${}^{(2)}R_\alpha = k_3\bar{\bar{X}}_p\nu_p$ (where k may be interpolated from data given in Table 4) for ${}^{(2)}R_{ap}$ and ${}^{(2)}R_r$. For the example, the value of $\bar{\bar{R}}_p$ can be calculated as follows: $\bar{\bar{R}}_p = f_3\bar{\bar{X}}_p\nu_p$, where $\nu_p = 0.088$, f_3 is the value for $\phi = 13.25$ and $\alpha = 50$, and $\bar{\bar{X}}_p$ is the mean of the product value as specified and $\alpha = 0.05$. Therefore,

$$\bar{\bar{R}}_p = 2.218 \times 0.05 \times 0.083 = 0.00920$$

Similarly, for the first submission,

$$R_{ap} = 0.0142\ (f_3 = 3.410 \text{ for } \alpha = 5 \text{ percent and } n = 8)$$

$$R_r = 0.0192\ (f_3 = 4.626 \text{ for } \alpha = 0.1 \text{ percent and } n = 8)$$

and for the second submission,

$$^{(2)}R_{ap} = 0.0122\ (\text{for } k_3 = 2.929 \text{ for } \alpha = 5 \text{ percent and } n = 8)$$

$$^{(2)}R_r = 0.0157\ (\text{for } k_3 = 3.792 \text{ for } \alpha = 0.1 \text{ percent and } n = 8)$$

From the data given above, an acceptance control sheet (Fig. 10) may now be prepared by the design function for use by the application function. From time to time revision of this sheet may be called for to allow for the modified product coefficient of variation ν_p in place of ν_p if the quality of the product property merits this action. Control charts as shown in Figure 11 can be used to plot information required to exercise process control. Although in this example $n = 8$ has been used for convenience, a value of n used for process control is normally lower than that used for acceptance control.

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DISCUSSION

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The authors of this paper are to be congratulated on exploring in such detail the acceptance and rejection limits for both first and second submission of a product. In discussing the number of observations or tests, the authors mention a method based on optimization of cost and then go on to develop an alternative or popular method based on a standard normal deviation. I would have much preferred that they pursue the method of economic optimization of cost because this is usually the primary objective of quality assurance plans. There are probably instances where safety and legal requirements are of overriding importance, but in most instances a quality assurance plan is to protect the buyer from the economic consequences of a poor quality product. Therefore, the problem is best approached by balancing the cost of the quality assurance program against the savings that will be realized through the level of quality assured by this program.

Intrinsic to the problem of economic optimization in rational quality assurance is to decide whether to use acceptance sampling alone or a combination of quality control and acceptance sampling. The amount of acceptance sampling necessary for a certain level of quality assurance is related to the information coming from the quality control program. Where the buyer is intimately familiar with the quality control program, he often can judge the level of quality assurance with little or no acceptance testing. The buyer pays for both the quality control program and the acceptance sampling plan, and he should not overlook the benefits that can come from the proper use of both.

In my opinion, the quality of some products of the highway industry can best be ensured through the buyer's participation in the quality control program. There has been a drive in recent years to remove the buyer from the quality control of all highway products to allow the seller to make full use of his ingenuity in improving the product and reducing the costs. There are instances where the buyer has contributed to higher costs by being too restrictive, but there are other areas where the best approach to quality assurance involves the buyer in the quality control of the product.

One of these is in the acceptance of the finished roadway. The important thing is to build it right in the first place. A road that is poorly constructed can seldom be successfully corrected afterwards. Penalties (which in the long run are paid by the buyer) will not correct fundamental errors in construction. Therefore, the major effort of all concerned should be the proper control of quality in the first place. Few highway engineers have any confidence in their ability to judge the useful life of a road by merely viewing the finished pavement. Most think that it is necessary for them to be involved in the quality control program to have assurance of the quality of the finished roadway.

The buyer should approach penalties with the realization that in the long run he will pay them as he will all the costs of the products he buys. This does not mean that a system of penalties may not be a good investment for a buyer, but he should look at what he is buying with his money and determine whether he is getting a proper return for his money.

I would much prefer that the authors use standard deviations rather than coefficients of variation in describing the variations of the various properties described in their paper. This is a personal observation, and I am not sure that all engineers would share my views on the greater simplicity of the use of standard deviations.

I enjoyed reading this paper and would be interested in reports on the application of this approach to acceptance and rejection of road materials and construction.

AUTHORS' CLOSURE

The authors wish to thank Mr. Davis for his constructive comments.

We agree that the determination of the optimum sample size based on economic considerations is desirable. This approach, together with other aspects concerning the choice of other parameters such as α_a to ensure maximum economic gain, has in fact been developed and published elsewhere (6, 15).

Either the standard deviation σ or the coefficient of variation V can be used to describe variability, and the theory presented in the paper is, with the proper adaptation, applicable to both cases. In the first case σ is independent of the mean $\bar{\bar{x}}$, while in the second case σ must be proportional to $\bar{\bar{x}}$. According to information analyzed by the authors as well as independently substantiated (16), the second case is more applicable to practical conditions, and V instead of σ was, therefore, chosen to represent variability.

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