ESTIMATION OF SPEED FROM PRESENCE DETECTORS

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Presence detectors are currently in use in surveillance and ramp control projects in several cities in the United States. The data obtained from those detectors are usually processed to produce traffic volumes and occupancies. By making certain assumptions, it is possible to obtain estimates of the average (space mean) speed of a group of vehicles passing over a single sensor. The simplest estimators (of the form length divided by detector on time) are biased. The purpose of this paper is to develop unbiased, minimum mean square error estimates of speed. These new speed estimators differ from the simplest estimators in that on-times associated with the passage of individual vehicles are processed separately. Although individual speeds cannot be determined very accurately, the estimators developed here are capable of determining the average (space mean) speed of, say, 20 vehicles to within 4 mph. A freeway traffic simulation is employed to illustrate the superiority of the estimators derived here.

The surveillance of traffic on freeways in several cities (Los Angeles, Chicago, Houston, and others) is based on measurements acquired from presence detectors. These devices consist of a sensor (either a magnetic loop or a sonic instrument) and a detector that converts the changes of a signal produced when vehicles pass through the influence zone of the sensor into an on-off type of signal. The presence detector measurements are typically converted into estimates of volume (vehicles/hour), occupancy (fraction of time the sensor is activated, usually expressed in percent), and density (vehicles/lane/hour). It is the purpose of this paper to examine the use of presence-detector measurements for the unbiased estimation of individual and space mean vehicle speeds.

If the on-time measured by the presence detector for the passage of an individual vehicle is \( t \), the speed of that vehicle can be determined from

\[
V = \frac{L}{t}
\]  

(1)

if \( L \), the effective length of the vehicle and the sensor combined, is available. This effective length, however, varies from vehicle to vehicle (and for a fixed vehicle, possibly from sensor to sensor) so that at best one only has available a distribution of effective lengths. This distribution is to be identified in experiments under way for the specific hardware used on Los Angeles freeways. A description of the experiments is contained in the Appendix. For our purposes, we will assume that the mean length \( \mu_L \) and standard derivation \( \sigma_L \) are known.

In addition to the uncertainty in the effective length, the presence signal is usually sampled, for example, every \( T \) seconds, so that the on-time is quantized in the form \( t = nT \) where \( n \) is an integer. This, of course, leads to additional errors because it is possible for this quantized measurement of time to be in error by nearly a full sampling

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interval. If a very high sampling rate is used, a variation of one in the number of presence pulses is insignificant. However, the current trend is toward lower sampling rates because of the cost of transmitting and processing the large amounts of data generated by high sampling rates. Significant errors in the measurement of time will exist with sampling rates on the order of 15 to 20 samples per second. In the next section, the statistics of the number of presence pulses are determined.

Because an individual vehicle speed cannot be precisely measured by a presence detector, one can at most ask for an unbiased estimate. An unbiased estimate, in the instance of estimating vehicle speed, is one that has the property that a collection of vehicles passing over the sensor would generate a set of estimates whose average would tend to the average of the true speeds as the size of the collection is increased. The speed estimator \( \hat{v} = \frac{m_t}{nT} \) is not unbiased; the unbiased estimator is determined in a later section.

In traffic surveillance, the average speed of the vehicles on the road is usually of more interest than a collection of estimates of individual speeds. This space mean speed, under conditions of space and time homogeneity of traffic conditions (1), can be determined from the speeds of vehicles as they pass a fixed point on a road from the relation

\[
v_s = \frac{N}{\left[ \sum_{i=1}^{N} \left( 1/v_i \right) \right]}
\]

(2)

where \( v_i, i = 1, \ldots, N \) are the sequence of individual speeds.

If one uses the estimate

\[
\hat{v}_i = \frac{m_t}{n_t T}
\]

(3)

for the \( i \)th vehicle (which generates \( n_t \) presence pulses), one is naturally led to estimate space mean speed by

\[
\hat{v}_s = \frac{Nm_t}{T \sum_{i=1}^{N} n_t}
\]

If these measurements are taken during the time period MT, occupancy is defined by

\[
\theta = \left( \frac{\sum_{i=1}^{N} n_t}{M} \right)
\]

(4)

With this notation, the space mean speed estimate becomes

\[
\hat{v}_s = \left( \frac{N}{MT} \right) \left( \frac{m_t}{\theta} \right)
\]

(5)

Note that \( N/MT \) is the volume during this period of time. This estimate of space mean speed is also biased; the unbiased estimator is determined in a later section.

In the last section, the various estimators are compared by using data obtained from a car-following simulation of freeway traffic.

The Appendix contains a description of 2 experiments, which had 2 primary results. First, values for \( m_t \) and \( \sigma_t \) for the magnetic loop sensors and associated detectors in use on Los Angeles freeways were determined. Second, it was determined that, as long as vehicles stay within 12-ft lanes, centered magnetic loops, 6 by 6 ft, produce presence signals that vary by less than 2 percent for any fixed speed.

DISTRIBUTION OF NUMBER OF PRESENCE PULSES

The number of presence pulses that will be generated by a vehicle passing over the sensor is given by

\[
n = \left[ \left( \frac{L_f}{v} + \epsilon \right) \right]
\]

(6)
where $L$ is the combined vehicle and sensor effective length, $f$ is the frequency of sampling, $v$ is the vehicle speed, $e$ is a random variable uniformly distributed on $(0, 1/f)$ that represents the random arrival time, and $[x]$ is the greatest integer less than or equal to $x$. With $m$ defined by

$$m = Lf/v$$

it is readily seen that the passage of this vehicle will generate either $m$ or $m + 1$ presence pulses. Noting the uniform distribution for $e$, one finds that

$$p(n = m | L, v) = m + 1 - (Lf/v)$$

and

$$p(n = m + 1 | L, v) = (Lf/v) - m$$

We shall employ the notation $p(n | V)$ to denote this probability, with the dependence on $L$ suppressed in the notation. Defining the set

$$A_n = \{v: \lfloor fL/(n+1) \rfloor < v \leq \lfloor fL/n \rfloor \}$$

one also has

$$p(n | v) = \begin{cases} 
n + 1 - (Lf/v) & \text{for } v \in A_n \\
(Lf/v) - n + 1 & \text{for } v \in A_{n-1} \\
0 & \text{for } v \not\in A_n \cup A_{n-1} 
\end{cases}$$

**ESTIMATION OF INDIVIDUAL VEHICLE SPEEDS**

Suppose that the measurements $y_1, y_2, \ldots, y_N$ are available to estimate the random variable $x$, e.g., by means of

$$\hat{x} = g(y_1, y_2, \ldots, y_N)$$

The mean square error for this estimator is given by

$$E \{[g(y_1, y_2, \ldots, y_N) - x]^2\}$$

($E \{ \}$ denotes the expected value.) It is easily shown (2) that the minimum mean square error estimator is given by the conditional expectation

$$\hat{x} = E(x | y_1, y_2, \ldots, y_N)$$

Moreover, this estimator is unbiased.

In this section, we determine the unbiased speed estimator

$$\hat{v}(n) = E(v | n)$$

This estimator is readily obtained if the conditional probability density function $p(v | n)$ is known. From Bayes' formula

$$p(v | n) = p(n, v) / p(n)$$

and the 2 relations

$$p(n, v) = p(n | v) p(v)$$

$$p(n) = \int p(n, v) dv$$
it can be seen that \( p(v|n) \) can be found if \( p(v) \) is specified, for \( p(n|v) \) was determined in the previous section.

Noting that \( p(n|v) \) for \( v \in A_n \cup A_{n-1} \), we see that only a portion of the probability density \( p(v) \) is required. In fact, over the limited domain \( A_n \cup A_{n-1} \), we shall take \( p(v) \) to be constant, i.e.,

\[
p(v) = k \quad \text{for } v \in A_n \cup A_{n-1}
\]

Then from Eqs. 11, 17, 18, and 19, one finds

\[
p(n) = kfL \log \left[ \left( \frac{n^2}{(n^2 - 1)} \right) \right]
\]

And then, using Eq. 16 we have

\[
p(v|n) = \begin{cases}
\frac{1}{fL} \log \left[ \left( \frac{n^2}{(n^2 - 1)} \right) \right] \left( n + 1 - \frac{L}{v} \right) & \text{for } v \in A_n \\
\frac{1}{fL} \log \left[ \left( \frac{n^2}{(n^2 - 1)} \right) \right] \left( \frac{L}{v} - n + 1 \right) & \text{for } v \in A_{n-1} \\
0 & \text{for } v \notin A_n \cup A_{n-1}
\end{cases}
\]

(Note that \( k \) does not appear in this result.)

From Eq. 21, it is an easy matter to determine the desired estimator by integration,

\[
\hat{v}(n) = E(v|n) = \int vp(v|n)dv
\]

\[
= fL/n \left\{ 1/\left( n^2 - 1 \right) \log \left[ \left( n^2/(n^2 - 1) \right) \right] \right\}
\]

Note that \( fL/n \) represents the standard speed estimator (Eq. 3); to identify the effect of the term in brackets, it can be expanded in a series to yield

\[
\hat{v}(n) = fL/n \left[ 1 + \left( \frac{1}{n^2} \right) + O\left( \frac{1}{n^4} \right) \right]
\]

where \( O(x) \) denotes a term with the property that \( O(x)/x \) tends to a finite limit as \( x \) tends to zero.

Throughout these calculations \( L \) was considered to be fixed. Averaging over the population of lengths, \( L \) is merely replaced by the mean length \( m_L \) so that there is no bias introduced by the variability of lengths about the mean.

The minimum mean square estimate of the time mean speed of a sequence of \( N \) vehicles that produces the sequence \( n_1, n_2, \ldots, n_N \) of presence pulses can be expressed as

\[
\hat{v}_t = \frac{1}{N} \sum_{i=1}^{N} E(v_i|n_1, n_2, \ldots, n_N)
\]

Because most of the information concerning the speed of the \( i \)th vehicle is contained in \( n_i \), we make the approximation

\[
E(v_i|n_1, n_2, \ldots, n_N) \approx E(v_i|n_i)
\]

so that the time mean speed estimator becomes

\[
\hat{v}_t = \frac{1}{N} \sum_{i=1}^{N} E(v_i|n_i)
\]

ESTIMATION OF SPACE MEAN SPEED

Space mean speed, under conditions of time and space homogeneity of traffic conditions (1), can be determined from the sequence of speeds measured at a fixed point in space during a period of time by the formula
\[ v_s = N/\left( \sum_{i=1}^{N} 1/v_i \right) \]  

(27)

where \( v_i, i = 1, \ldots, N \) are the speeds of the \( N \) vehicles. In fact, it is an easier matter to determine an estimate of \( 1/v_s \). To correct the 2 estimates, we define

\[ u_s = 1/v_s = 1/N \sum_{i=1}^{N} u_i \]  

(28)

where

\[ u_i = 1/v_i \]  

(29)

Suppose \( \hat{u}_s \) is the estimate of \( u_s \) based on presence pulse data and \( \sigma^2_{u_s} \) is the variance of this estimate; i.e.,

\[ \hat{u}_s = \text{E}(u_n | n_1, n_2, \ldots, n_n) \]  

(30)

\[ \sigma^2_{u_s} = \text{E}[(u_n - \hat{u}_s)^2 | n_1, n_2, \ldots, n_n] \]  

(31)

If \( \sigma_{u_s}/\hat{u}_s < 1 \), it is readily shown that

\[ \text{E}(v_s | n_1, n_2, \ldots, n_n) = 1/\hat{u}_s \left[ 1 + \left( \sigma^2_{u_s}/(\hat{u}_s)^2 \right) + 0(1/\hat{u}_s^5) \right] \]  

(32)

We shall use the first 2 terms in the parentheses for our space mean speed estimator.

It remains to determine \( \hat{u}_s \) and \( \sigma^2_{u_s} \). As for the time mean speed estimator, we shall make the approximation

\[ \text{E}(u_n | n_1, n_2, \ldots, n_n) = \text{E}(u_i | n_i) \]  

(33)

so that we shall take

\[ \hat{u}_s(n_1, n_2, \ldots, n_n) = 1/N \sum_{i=1}^{N} \text{E}(u_i | n_i) \]  

(34)

Similarly,

\[ \sigma^2_{u_s} \approx 1/N^2 \sum_{i=1}^{N} \sigma^2_{u_i} \]  

(35)

where

\[ \sigma^2_{u_i} \approx \text{E} \left\{ (u_i - \text{E}(u_i | n_i))^2 \right\} n_i \]  

(36)

Our computations can be completed once \( \text{E}(u_i | n_i) \) and \( \sigma^2_{u_i} \) are determined.

Now (suppressing the subscript notation),

\[ \text{E}(u | n) = \int (1/v) p(v | n) dv \]

which is readily determined to be

\[ \text{E}(u | n) = (n/IL) \left( \left\{ \log \left[ (n + 1)/(n - 1) \right] \right\} / \left\{ n \log \left[ n^2/(n^2 - 1) \right] \right\} - 1 \right) \]  

(37)

\[ = (n/IL) \left[ 1 - 1/3n^2 + 0(1/n^5) \right] \]  

(38)

This has been obtained under the same assumptions as employed in the previous section [e.g., \( p(v) \) is constant].
Similarly, one finds

$$E(u^2|n) = (1/\bar{L})^2 \{1/\log \left[ n^2/(n^2 - 1) \right] \}$$  \hspace{1cm} (39)$$

$$= (n/\bar{L})^2 \left[ 1 + (1/2n^2) + 0 (1/n^4) \right]$$  \hspace{1cm} (40)$$

so that

$$\sigma^2_s = (n/\bar{L})^2 \left[ (7/6n^2) + 0 (1/n^4) \right]$$  \hspace{1cm} (41)$$

The space mean speed estimator is then defined by Eqs. 32, 34, 35, 38, and 41. The assumption that $\sigma_s/\bar{u}_s << 1$ is justified for even moderate values of $n_1$. 

Up to this point $\bar{L}$ has been fixed. To account for the variability in $\bar{L}$ requires that an expectation with respect to $\bar{L}$ be taken. In contrast to the situation with the time mean speed, $\bar{L}$ cannot be simply replaced by $m_\bar{L}$; in fact, we require $E(1/\bar{L})$ as $\bar{L}$ appears in the denominator of $E(u_1|n_1)$. Using the same reasoning that led to Eq. 32, one has

$$E(1/\bar{L}) = 1/m_\bar{L} \left[ 1 + \sigma^2_s/m_\bar{L}^2 + 0 (1/m_\bar{L}^4) \right]$$  \hspace{1cm} (42)$$

if one assumes $\sigma_s/m_\bar{L} << 1$. The first 2 terms of those within the brackets will be employed. This correction need not be made in calculating $\sigma_s$ because the net correction is of the same size as terms that are already being ignored; i.e., $\bar{L}$ can be replaced by $m_\bar{L}$.

In the preceding discussion, the speed distribution was taken to be uniform in the limited range of interest. There are instances, to be discussed in the next section, in which a ramp function more closely approximates the situation. If the distribution of speeds over the range of interest is assumed to be

$$p(v) = \begin{cases} k(v_b - v) & v < v_b \\ 0 & v > v_b \end{cases}$$  \hspace{1cm} (43)$$

one finds in a similar manner

$$E(u|n) = (n/\bar{L}) \left\{ (1 + m/n) \log \left[ (n^2 - 1)/n^2 \right] + 1/n \log \left[ (n + 1/n - 1) \right] \right\}$$  \hspace{1cm} (44)$$

$$\log \left[ (n^2/n^2 - 1) \right] - m/[n(n^2 - 1)] \} \right\}$$

and

$$E(u^2|n) = (1/\bar{L})^2 \left\{ [1/m - n \log \left[ (n^2 - 1)/n^2 \right] - \log \left[ (n + 1)/(n - 1) \right] \right\}$$  \hspace{1cm} (45)$$

$$\left\{ 1/m \log \left[ n^2/(n^2 - 1) \right] - 1/[n(n^2 - 1)] \right\}$$

where we have defined

$$v_b = fL/m$$  \hspace{1cm} (46)$$

DISCUSSION OF ESTIMATES

The estimation of individual vehicle speeds or the space mean speed should be performed on individual vehicle data rather than over some arbitrary fixed sampling period because of the possibility in the latter instance of terminating a sample while a vehicle is directly over the sensor. For example, if the passage of the vehicle is sensed by its arrival at the sensor, the vehicle would be treated as an exceptionally fast vehicle if the sampling period terminated while that vehicle was still over the sensor. It is desirable to avoid the introduction of additional error due to the sampling procedure.
The sampling period determines the number of vehicles whose passages contribute to the time mean or space mean speed. Because the new estimators derived here are all of the form of conditional expectations (to within the approximations made in the 2 previous sections), the expected values of the estimates are the true means, regardless of the sample size. However, the sampling period does affect the variance of the estimates. Precisely for time mean speed, and approximately for space mean speed, the variance of the estimates for N vehicles is 1/N times the variance of the estimate of the speed (or reciprocal speed) for an individual vehicle. Hence, a longer sampling period leads to estimates with smaller variance. If the normal flow is 1,200 vehicles/hour, the variance for the mean speed of a 1-min sample would be about 1/20 of the variance of the estimate of an individual vehicle speed.

In addition to estimator variance, the choice of the sampling period depends on the homogeneity of the traffic flow and the required timeliness of the data. If the estimates are to be used for control purposes, they must be timely. Accurate estimates depicting roadway conditions long past are of little value for control. For these purposes, a 1-min sampling period was employed in the Eisenhower Expressway Study (3) and has been selected for the current surveillance project in Los Angeles (4).

It is evident that the sampling rate affects the accuracy of speed estimators. For example, the variance of the unbiased speed estimator (Eq. 22) is given by

$$\sigma_v^2 = \frac{(fL/n)^2}{[3(n^2 - 1)^2 \log \left[n^2/(n^2 - 1)\right]]}$$

$$= \frac{(fL/n)^2}{[(1/6n^2) + 0 (1/n^4)]}$$

Figure 1 shows the standard deviation $\sigma_v$ with L taken to be 28 ft for sampling rates of 15, 30, and 45 samples/sec. The standard deviation of the error due to the sampling procedure is halved by doubling the sampling rate.

The precise form of the unbiased estimator depends on the actual velocity distribution on the roadway. Real roadways nearly always have a speed limit on them. This causes a truncation of velocity distribution in the region of this speed limit because there are fewer vehicles traveling above this limit than below it. It is possible to pick a speed at which no vehicles are traveling. This condition can be represented by the negative ramp distribution case.

At intermediate speeds the vehicle speeds are more uniformly distributed with some cars going slower than the average and others faster. This region can be represented by a uniform distribution. At low speeds there are more vehicles going faster so that this region can be represented by a positive ramp distribution. Figure 1 shows that the standard deviation of the error gets very small at low speeds so that the low-speed effect will be neglected. The bias caused by the uniform and negative ramp distribution is shown in Figure 2. The unbiased estimator depends on the velocity distribution, and the differences become significant at the higher speeds. An unbiased estimator must consider the effects of the different velocity distributions at the intermediate and high speeds. The selection of the wrong distribution at the low velocity end does not introduce major errors into the system. However, the opposite is true at the upper velocity boundary.

The distribution of vehicle lengths is a major source of variability in the estimate of individual vehicle reciprocal speeds. The effects of the distribution of lengths on the error of the estimate of space mean speed can be reduced by increasing the vehicle's effective length or by decreasing the variance of the car lengths. Because actual vehicle lengths are fixed by the automobile manufacturers, this means that longer sensors reduce the bias due to vehicle length distributions. Similarly, if vehicles can be separated into classes by length, the bias due to the length distributions can be reduced by reducing the standard deviation of lengths possible for each class.

SIMULATION AND EVALUATION OF ESTIMATORS

Four Estimators

There are 2 basic sources of error in the estimation of space mean speed by means of the estimator specified by Eq. 5. The first is due to the method of counting vehicles;
the second is due to the distribution of vehicle speeds and lengths. Four estimators will be described and evaluated here by means of a car-following simulation of traffic. The first, labeled estimator 1, is defined by Eq. 5. The count of vehicles $N$ is based on the number of vehicles arriving at the sensor during the sampling period.

Estimator 2 differs from estimator 1 in that consideration is given to the possibility that a vehicle may be over the sensor at the time the sampling period begins or ends. This estimator uses the average of the counts for arrivals and departures to obtain the vehicle count. If the sum of the arrival and departure counts is even, then the estimates derived from estimators 1 and 2 are identical.

The estimates of space mean speed obtained by estimator 3 exclude data obtained from vehicles that have not completely passed over the sensor in the sampling period. This vehicle is included in the next sampling period. Thus, the sampling period is not necessarily the same for consecutive periods.

Estimator 4 includes corrections for the bias due to the speed and length distributions as described by Eqs. 32, 34, 35, 38, and 41. Otherwise, it is of the same form as estimator 3 so that no errors are incurred by inaccurate vehicle counting.

The sequence of estimators described is successively more sophisticated so that it would be expected that the estimates obtained from estimator 2 are better than those obtained from estimator 1 and so on.

Estimator Evaluation

The estimators described above were evaluated by comparing the space mean speed of a set of vehicles passing over the sensor (as defined by Eq. 2) with the 4 estimates of the same quantity determined from the sensor data. The simulation experiment was set up so that all of the estimators operated on the same sensor data. Thus, there is no question of sample differences causing variations in the accuracy of the estimates. All of the differences in the estimates are due strictly to the estimating procedures.

The sensor data were generated by a freeway and sensor simulation of the car-following type (5) in preference to acquiring real data from an actual roadway because it is impossible to control and accurately measure all of the pertinent variables in a real system. Monte Carlo freeway simulation provides a convenient means to examine the accuracy of various estimating schemes because the precise characteristics and performance of vehicles within the simulation can be readily determined and compared against the estimates derived from the sensor outputs. The effects of variables such as loop field variation, detector sensitivity, vehicle lane position, and vehicle height have been removed so that they do not mask the variations in the estimates associated with the vehicle counting and the vehicle and length distributions. In addition, the traffic was restricted to 1 lane to avoid problems due to lane changing over the sensors.

Simulation Results

The performance of the estimators was measured in terms of the mean difference between the estimated and actual space mean speed and the variance of these differences.

Sensor Experiments

The differences between the actual and estimated space mean speeds are given in Tables 1 and 2. The first tabulation gives the mean and variance of the error for each estimator and represents an external view of the experiment. The second tabulation gives the errors by vehicle. Each vehicle is sensed by a count of $n_1$. The errors of all vehicles that caused a count of $n_1 = m$ were tabulated together so that a posterior velocity distribution and the mean and variance of the errors associated with each element of the distribution could be determined. This provides an internal view of the experiment. Table 1 gives a comparison of the alternate estimators; Table 2 gives the data to determine causes of variation between the predicted and observed results.

The average error and the standard deviation of the error are given in Table 1. The question of bias is examined first. The null hypothesis of zero bias was tested at the 5, 1, and 0.1 percent levels if significant. A 2-sided test was used because the hypothesis of zero bias was examined. The hypothesis of zero bias is accepted for estimator
Figure 1. Effect of sampling rate.

Table 1. Space mean speed estimation errors.

<table>
<thead>
<tr>
<th>Estimator</th>
<th>Standard Deviation</th>
<th>Estimator</th>
<th>Standard Deviation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.592</td>
<td>3</td>
<td>0.491</td>
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<tr>
<td>2</td>
<td>0.522</td>
<td>4</td>
<td>0.179</td>
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</tbody>
</table>

*The effect of the $(\sigma^2/n)^{1/2}$ term was not included.

Table 2. Distribution of estimation errors for individual reciprocal speeds.

<table>
<thead>
<tr>
<th>Number of Presence</th>
<th>Number of Samples</th>
<th>Actual Speed</th>
<th>Reciprocal Average</th>
<th>Estimator 1</th>
<th>Estimator 4</th>
</tr>
</thead>
<tbody>
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<td>4</td>
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<td>84.317</td>
<td>105.0</td>
<td>101.0</td>
<td></td>
</tr>
<tr>
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<td>6</td>
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</tbody>
</table>

Figure 2. Estimators of $1/v$.

Figure 3. Schematic diagram of experiment.

S1, S2 and S3 are pressure tape switches. S1 is used to trigger the oscilloscope. S1 and S3 form the speed trap. S2 determines lateral position.

Figure 4. Effective loop shape.
4 at the 1 percent level of significance. It is rejected for all other estimators. This result agrees with the previous analysis that showed that estimators 1, 2, and 3 were not unbiased. Estimate 4 would be completely unbiased if the assumptions about the velocity and length distributions were met.

For the effectiveness of the alternate estimators in reducing the variance of the errors, the null hypothesis was tested. This tested the assumption that there are no differences between the variance obtained for each estimator. The comparisons were made with estimator 1. The null hypothesis was made at the 5 and 1 percent levels of significance by using the F-test. The hypothesis of no significant differences in variance between estimators is rejected for all estimators at both the 5 and 1 percent levels of significance. This means that estimators 2, 3, and 4 are all more efficient estimators than estimator 1.

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**REFERENCES**


**Appendix**

**DESCRIPTION OF EXPERIMENTS**

The first experiments determined the effective shape of the loop magnetic field. The dependence on speed and the effect of changing detectors were investigated.

A schematic diagram of the experiment is shown in Figure 3. A magnetic loop and 2 pressure tape switches (S1 and S2) were located as shown. The pressure switches were connected with a battery and resistor such that a difference in voltage levels could be observed on a 4-trace oscilloscope whenever the switch was closed. The electronic detector output indicating the presence or absence of a vehicle in the field of the loop was also displayed. The oscilloscope was triggered by the front wheels of the vehicle making contact with the first tape switch, and a Polaroid picture was taken of the traces of the switch and detector outputs. From the photograph, the time occurrence of the relevant events was determined. Knowing the wheelbase of the car and the time required for the car to travel a distance equal to its wheelbase length, we determined the speed. Knowing the vehicle width and velocity, we calculated the position of the vehicle relative to the center of the loop.

For vehicles of known dimension, typical results are shown in Figure 4. The solid line denotes the geometric shape of the loop. The actual (bumper-to-bumper) car length is subtracted from the effective length to allow comparison of the curves for different vehicles. Figure 4 has the significance that, if any part of the vehicle enters the region enclosed by the dotted line, the detector is activated.

The experiment was designed so that actual speeds were determined to within ±0.4 mph and lane position to within ±3 in.

The loop was found to give symmetric results; hence, each data point is shown 4 times in Figure 4. No dependence on vehicle speed between 10 and 60 mph was detected.
The use of 3 different detectors for the same loop resulted in a 4 percent range for the total effective length L. The effective length L varies by less than 2 percent as long as a vehicle remains completely within the 12-ft lane. Comparisons of 4 different vehicles resulted in a range for the effective length of the loop alone of 1.8 ft.

Subsequent experiments are being performed to determine the distribution of effective lengths of vehicles actually using a freeway. The experiment is the same except that an additional pressure tape S3 is utilized to determine the wheelbase length for each vehicle. Two additional loops are located downstream with a 6-ft separation between loops in order to ascertain the difference in operating characteristics of 3 different loop-detector pairs. A 7-channel instrumentation tape recorder is used to record all events. The data are read and processed by a hybrid computer.