MAINTENANCE STATION LOCATION THROUGH OPERATIONS RESEARCH AT THE WYOMING STATE HIGHWAY DEPARTMENT

Robert W. Hayman, Colorado State University; and
Clyde A. Howard, Wyoming State Highway Department

• THE typical roadway maintenance station in the state of Wyoming is charged with the general and total maintenance of assigned roadways from the time of their completion to their eventual replacement or reconstruction.

Snow removal and surface maintenance account for a large percentage of maintenance expenditures. The remainder of the budget is spent on less costly activities such as signing, lighting, and centerline painting.

Over the years, maintenance equipment and procedures have improved, along with the other elements of the highway industry. Despite this progress, maintenance stations have remained essentially the same. Modern highway management has recognized the need for a reevaluation of the maintenance system, particularly with respect to the locations of the stations themselves. Population characteristics have changed, and it is felt that the current requirements for servicing the central portion of the state need particular attention.

A formal study of this problem was initiated in the spring of 1970; the results of the study are reported in this paper. The principal issue is the specification of required locations of maintenance bases in order to provide the required services in the most economical fashion. The study was allowed to assume that any existing station could be removed and that new facilities could be built when such construction was justified by economics and service requirements.

Initially, the study was to be directed to the west-central portion of the state. However, the methodology of the study and the particular techniques developed for producing a solution were to be applicable, whenever possible, to any other region within the state.

The problem was eventually reduced to two mathematical models that were optimized according to the standard techniques of mathematical programming. Computer programs were developed that convert familiar physical parameters, as they apply to any specific case, to the problem form required by the solution methodology.

SELECTION OF THE OBJECTIVE FUNCTION

The technical development of the entire study revolves around determination of the program objective, which was decided on by management and operations research personnel. The objective was as follows: Define the locations of the required maintenance stations, within the boundary of the study, such that the sum of operational and depreciation costs is an absolute minimum. Many other alternatives were considered; among the more notable was the maximization of various service benefits.

IDENTIFICATION OF CRITICAL MAINTENANCE ACTIVITIES

Typical annual budgets were analyzed to determine the current patterns of expenditures, which were classified according to the various maintenance activities. Subsequently, an effort was made to associate each of the activities with some fraction of the cost of the maintenance station itself. For example, some cost fraction of the physical

---

Sponsored by Committee on Maintenance and Operations Costs.

plant exists only for the purpose of housing and maintaining snowplow units; in fact, this particular fraction was 20 to 30 percent. This task was completed in a very subjective manner and remains open to debate.

The next step in attempting to define critical activities was to define, for each of the major activities, the manner in which operating costs varied as a function of location of the operating base for the activity. The most obvious variable was the amount of travel required to reach the work site from a particular base station.

The strategy in all of this is to reduce operation costs for each activity by using the most favorable base location for the activity. Cost savings, if any, could be used for the construction and maintenance of new facilities. Because the optimization objective is to minimize the sum of operating and depreciation costs, we are looking for a physical system configuration that produces a savings that is at least as great as the costs of building and maintaining the required group of physical facilities.

Most of the standard maintenance functions enjoyed little or no operational savings as a function of location of the operating base. In fact, only one set of activities promised to generate sufficient savings to pay for its share of the physical facility; this was the snow removal program. Accordingly, it was determined that the mathematical models for the optimization need only consider this set of activities, together with their proportionate share of the cost of the physical plant.

DEFINITION OF PROGRAM CONSTRAINTS

The primary source of information used for definition of program constraints was Policy and Procedure Directive 70-2, Maintenance Division, Wyoming State Highway Department.

For purposes of the optimization study, maintenance services were divided into two types: sanding and plowing. Separate models were developed to optimize these services independently. In addition to optimizing station location and vehicle assignments, the sanding model provides the optimum locations for stockpiles of sanding material.

Constraints on the Sanding Operation

The language of the Policy and Procedure Directive was abstracted to provide the first four constraints; the last four constraints were identified through interviews with maintenance department personnel.

1. Sanding must begin before the snow has accumulated to ¼-in. depth on the roadway;
2. For a design storm, all roadways entitled to sanding services must be entirely sanded before the snow accumulates to some stated depth depending on the class of service assigned to each roadway;
3. Sanding shall be performed continuously until the entire facility has been sanded or until the snow has accumulated to the maximum depth associated with constraint 2;
4. Sanding material shall be applied to the entire driving surface at the application rate of 2,000 lb per 2-lane mile;
5. The traditional concepts of maintenance district boundaries were to impose no restriction on station location or equipment work assignment;
6. Any sanding unit could be assigned to any work location within the geographic domain of the model;
7. There is no restriction on the number of sanding units assigned to any base station; and
8. Within each of the service classifications, A through E (defined in Policy and Procedure Directive 70-2), provision should be made for service priorities on the basis of relative traffic density.

Several of these constraints are either ambiguous or require further interpretation before they can be paraphrased in mathematical terms. The necessary discussion is given in the subsequent paragraphs.

Of primary importance is the definition of a design storm. It is not expected that constraint 2 could be met for all storm stituations. Consequently, the maximum storm intensity to be accommodated (the design storm) was defined to be a continuous snowfall accumulating at the rate of $\frac{1}{2}$ in. per hour. For any storm of higher intensity, all roadways could not be completely sanded before the snow depth has accumulated to the limiting depth, at which time sanding would be terminated and plowing begun.

It is a physical impossibility to begin sanding all parts of a roadway at the same time. Accordingly, constraint 1 must be interpreted at less than literal value. This condition was redefined to mean that the sanding units would be deployed to their work assignments before the snowfall has accumulated to $\frac{1}{4}$-in. depth.

Constraints 1 and 2, taken together with the definition of the design storm, mean that all sanding must be completed within a specific time period, following the beginning of a storm. For example, for a class A roadway, the maximum snow depth allowed during sanding is 2 in. Accordingly, for the $\frac{1}{2}$-in.-per-hour design storm, sanding must be completed within a 4-hour period measured from the storm beginning.

Service priorities are provided within each class of service (constraint 8) by reducing allowable service times for high-priority roadways. No particular effort was made, in this study, to establish a procedure for setting service priorities on the basis of traffic density, and no specific policy was found to exist within the department. However, the optimization study does provide for this feature by using variations of the allowable service time around the 4-hour nominal time limit.

Other elements given in the list of program constraints are taken at their face value, and no other arguments are imposed on the solution to the problem. The solution is guaranteed to meet these requirements, assuming that equipment performance and cost data are correct.

Constraints on the Plowing Operation

The Policy and Procedure Directive provided the major guidelines in assembling the first four constraints; constraints 5 through 8, as applied to the sanding program, were imposed on the plowing program.

1. Plowing operations begin when snowfall has accumulated to some minimum depth established for each class of service to be provided;
2. For class A facilities, sufficient equipment shall be deployed and remain in continuous service, in order that the roadway be kept bare;
3. The roadway is defined to mean normal driving lanes and passing lanes;
4. For class B service, plowing shall be continuous throughout the storm, and sufficient equipment shall be made available so that the entire roadway may be cleared "soon after the storm subsides";
5. For class C service, sufficient equipment shall be available to clear the entire roadway "soon after the end of the storm";
6. Maintenance station boundaries impose no restriction on the problem solution;
7. Any plow can be assigned to any roadway within the geographic area considered;
8. There are no limitations on the number of plow units that can be based at any given maintenance station; and
9. Service priorities may be applied to any of the facilities falling within service classes A through C.

As in the case of the sanding program, several of the program constraints required translation to more specific form.

First of all, the design storm used for the plow model is a continuous snowfall of $\frac{1}{2}$ in. per hour. It should be pointed out that the duration of the snowstorm is not a factor that the model is required to consider. Service constraints require either continuous service, with an associated continuous result (keep the road bare), or desirable service levels to be achieved following the end of the storm. A collection of equipment designed to keep the roadway bare for 1 hour will also keep it bare throughout a design storm of indefinite duration. The essential difference between servicing a 1-hour storm and a 100-hour storm would be the personnel required to operate the equipment. The 100-hour storm is obviously more costly to service, but the cost differences are independent of the location of the maintenance stations and are therefore of no consequence to the current study.

Constraint 2 was taken directly from the Policy and Procedure Directive and needs considerable restructuring in order to be at all realistic. If the definition of "bare roadway" is that absolutely no snow is allowed to accumulate at any point on the driving surface, then a continuous circulation of plows, moving end-to-end, would not fulfill the requirement for the design storm. A more reasonable requirement would be to limit the average snowfall accumulation to some minimum depth, chosen such that traffic could negotiate the roadway at all times. A satisfactory statement on the average depth requirement is a matter for continued debate; for purposes of this study, the maximum average accumulation of snowfall was taken to be 2 in., and a 4-in. accumulation was the absolute maximum allowed to accumulate at any point on the roadway. The computer programs that solve the problem are designed to accept these numbers as conditions on the solution produced.

Constraints 4 and 5 state that the entire roadway shall be cleared "soon after the storm subsides." Obviously, a strict definition of the word soon must be supplied. The general scheme employed is as follows:

	Class of Service		
Priority	A	B	C
1	2	4	10
2	3	6	14
3	4	8	18

It should be emphasized that the numbers shown here are not the result of current departmental policy. For purposes of the optimization study, the emphasis was placed on developing the mechanics of providing for these management features.

DEVELOPMENT OF THE OPTIMIZATION MODELS

The Policy and Procedure Directive represents a relatively new philosophy for snow control programs in the state of Wyoming. Previous policy was directed almost exclusively to snow control by plowing. It had been suspected that the quality of service could be upgraded at little or no increase in maintenance cost through a more extensive application of abrasive-liquefacient material (referred to as sanding) to the roadway. In Wyoming, it has been found that the roadway frequently may be maintained in satisfactory driving condition through the application of sanding material, with no plowing required. Some storm situations require both sanding and plowing, and some require plowing exclusively. Accordingly, it was decided to build two models—one that optimizes station locations according to the sanding requirements and one that optimizes according to plowing requirements. The management hoped that the station location solutions would be the same in both cases. The models are developed in the following two sections.

Sanding Model

The normal work pattern followed in the case of a general storm is a simple progression down the roadway with each truck returning to the nearest stockpile for reloading when empty.

Figure 1 shows the time required to complete the sanding of a given centerline mileage of roadway when various numbers of trucks are assigned to the task. The completion times shown correspond to that time when the last vehicle has returned to the starting point.

The most critical working relationships in the optimization model require that all of the information shown in Figure 1 be reduced to a single closed form equation.

Figure 1. Sanding completion time.

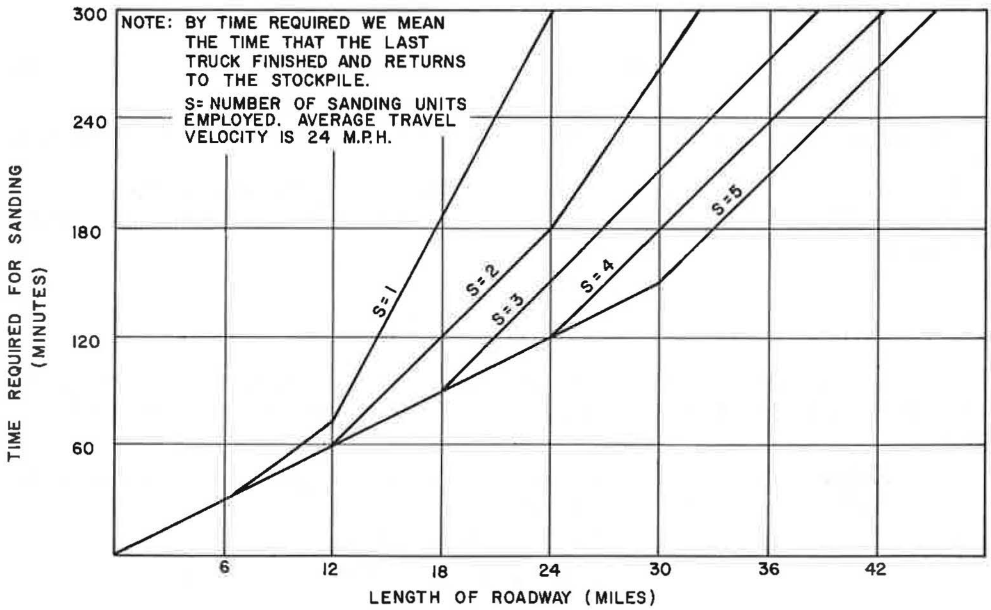

Table 1. Composition of study area, roadway designation, and fixed data applied to the final solutions.

Roadway Number	Centerline Mileage	Service Class	Plow Passes to Clear Roadway	Plow Service Time Limit (hours)	Allowable Snow Depth Before Plow Required (in.)	Service Time Limit for Sanding (min)
1	12.2	B	4	8	2.5	240
2	12.2	B	4	8	2.5	240
3	11.8	B	4	8	2.5	240
4	11.8	B	4	8	2.5	240
5	10.1	B	4	4	2.0	210
6	10.1	B	4	4	2.0	210
7	15.8	B	4	6	2.0	300
8	15.8	B	4	6	2.0	300
9	13.7	B	4	6	2.0	300
10	13.7	B	4	6	2.0	300
11	13.7	B	4	6	2.0	300
12	8.2	B	4	4	2.0	210
13	9.0	C	4	18	10.0	330
14	12.0	B	4	4	2.0	210
15	12.0	B	4	4	2.0	210
16	19.0	C	4	12	10.0	330
17	15.8	C	4	10	10.0	330
18	15.7	C	4	10	10.0	330
19	16.3	B	4	6	2.0	210
20	16.3	B	4	6	2.0	210
21	16.2	B	4	6	2.0	240
22	16.2	B	4	6	2.0	240
23	8.2	B	4	6	2.0	210
24	14.0	B	4	6	2.0	210
25	15.9	C	4	10	10.0	270
26	9.0	B	4	2	2.0	300
27	9.1	B	4	6	2.0	300
28	17.5	C	4	10	10.0	330
29	11.2	B	4	6	2.0	270
30	11.2	B	4	6	2.0	270
31	14.7	B	4	8	2.5	270
32	14.7	B	4	8	2.5	270
33	14.7	B	4	8	2.5	270
34	15.1	B	4	8	2.5	270
35	15.2	B	4	8	2.0	270
36	12.3	B	4	8	2.0	270
37	12.2	B	4	8	2.0	270
38	9.3	B	4	6	2.0	270
39	16.0	B	4	6	2.0	270
40	15.9	C	4	10	10.0	300
41	16.3	B	4	6	2.0	270

In the case of a single sander, the relationship between the total miles driven and the centerline miles sanded may be suitably represented by a quadratic:

$$D = 0.17m^2 + m \tag{1}$$

where D is the total distance driven, in miles, and m is the centerline distance sanded, in miles.

Furthermore, if we assume that the total travel may be equally divided among S sanding units, each traveling at V mph, the total time required to complete the sanding mission would be

$$T = D/VS = (0.17m^2 + m)/VS \tag{2}$$

Equation 2 is not quite complete for working purposes because the sanding units may have to travel some distances between the stockpile and the roadway to be sanded. Therefore,

d = "dead-haul" travel distance in miles between the stockpile and the beginning of the roadway section to be sanded and
n = number of trips required to sand "m" miles of roadway.

To account for n trips over the dead-haul distance, we revise Eq. 1 to give

$$T = 2n\ (d + 0.5m) + 6n/VS \tag{3}$$

Time Constraints

Several constraints previously identified relate to the elapsed time allowed to complete the sanding operation. With proper interpretation and specification of parameters, these conditions may be satisfied with the following development. The necessary terminology is identified as follows:

s = number of proposed locations for maintenance stations;
r = number of roadway sections to be serviced within the domain of the model;
p = number of proposed stockpiles;
S_{ij} = the number of sanding units based at maintenance station i and assigned to work from stockpile j;
S_{jk} = the number of sanding units assembled at stockpile j to effect the servicing of roadway k;
n_{jk} = the number of loads of sanding material to be hauled from stockpile j and distributed on roadway k;
t_{ij} = the time spent by the S_{ij} in traveling from station i to stockpile j;
T_k = time available to complete the sanding of roadway k measured from the beginning of the storm;
M_k = the centerline mileage of roadway to be sanded; and
d_{jk} = the dead-haul travel distance in traveling from stockpile j to roadway k.

The basic strategy is to deploy S_{ij} sanding units from station i to stockpile j. Thereafter, n_{jk} loads of material are to be hauled from stockpile j to roadway k by using S_{jk} sanding units.

In the execution of this strategy, the time of arrival of the sanding vehicles at a particular stockpile will vary, depending on the origin station for the trucks. In other words, t_{ij} will vary with i for any j. Temporarily, assume that the t_{ij} is the same for all i and some particular j. Call this average time $\overline{t}_{ij}$. The time available for productive work on a given roadway, measured from the beginning of a storm, is

$$(\text{time available})_k = T_k - \overline{t}_{ij} \tag{4}$$

S_{jk} sanding units will be deployed from stockpile j to roadway k and will distribute n_{jk} loads of sanding material. The time required for this may be computed from Eq. 3.

In order to accomplish the service within the specified time frame, it is required that

$$2n_{jk}\,(d_{jk} - 0.5M_k) + 6n_{jk} \le (T_k - t_{ij})\,VS_{jk};\; j = 1,\ldots, p \text{ and } k = 1,\ldots, r \qquad (5)$$

The variables in this constraint are the n_{jk} and S_{jk}, of which there are a total of 2 x p x r. The set of these constraints consists of p x r separate inequalities.

The substitution of the travel time averages $\bar{t}_{ij}$ for the actual t_{ij} must now be reconciled. The answer for the substitution is iterative programming. The reason for the substitution is that the model size would have had to be increased to keep a proper count. The number of variables and constraints would have been multiplied by i in the process, and no substantial gain in the accuracy of the model or in the amount of useful physical information would have been derived. As it happens, there are very few cases where more than one station services from a given stockpile, and it was not difficult to iterate to the correct combination.

Model Efficiency Constraints

Although they were not required to satisfy the theoretical behavior of the model, two time constraints were developed for the purpose of accomplishing substantial reduction in the size of the model as it applies in any particular case. It is possible to state, without interacting with other constraints in the model, that

$$T_k - t_{ij} \ge q_1;\; i = 1,\ldots, s \text{ and } j = 1,\ldots, p \text{ and for any } k \qquad (6)$$

and

$$d_{jk}/V \ge q_2 \quad j = 1,\ldots, p \text{ and } k = 1,\ldots, r \qquad (7)$$

The relations in Eq. 6 state that t_{ij}, time spent in traveling from station i to stockpile j, must be something less than the time allotted to sanding the roadways to be serviced from stockpile j. If this is not the case, then no sanding unit should be deployed from station i to stockpile j. In other words, if $T_k - t_{ij} < q_1$, then $S_{ij} = 0$. Similarly, if too much time is consumed by dead-haul in servicing roadway k from stockpile j, there will be none left for the useful work. The time for one-way passage over the dead-haul distance is d_{jk}/V and must actually be consumed $2n_{jk}$ times. Consequently, if $d_{ij}/V < q_2$, then the quantities S_{jk} and n_{jk} could be set to zero. Actually, instead of setting the S_{ij}, S_{jk}, and n_{jk}, and n_{jk} to zero, they are discarded before being built into the final form of the optimization model. The q_1 and q_2 numbers were conservatively chosen so that potentially valid variables were not discarded.

The remaining constraints required for the optimization model follow quite simply.

Work Quantity Constraint

The worst storm situation must be used as a basis of argument; this would be a storm that covers the entire domain of the model at any given time. The set of constraints (defined in Eq. 5) do not, by themselves, require that the total roadway system be covered; they simply provide that any work undertaken must be completed within a given time frame. The requirement for total sanding coverage will be given in terms of the number of loads of material required to service each and every roadway in the system. In order to cover any roadway k with the type of equipment being used, the number of loads of material required are

$$n_k = 0.17\;M_k \qquad (8)$$

These materials may be transported from any stockpile within the system. Accordingly, it is required that

$$\sum_{j=1}^{p} n_{jk} \geq 0.17M_k; \; k = 1,\ldots, r \tag{9}$$

The constraints defined in Eqs. 5 and 9 ensure that all roadways are sanded within the required time allotment.

Equipment Continuity Constraint

So far, S_{jk} units have been used to deliver n_{jk} loads of material from stockpile j to roadway k. It remains to assemble the correct number of sanding units at any stockpile. This is accomplished by dispatching the required number of units from the various maintenance stations. Therefore,

$$\sum_{i=1}^{s} S_{ij} \geq \sum_{k=1}^{r} S_{jk}; \; j = 1,\ldots, p \tag{10}$$

is required.

Constraint Summary and Observations

The sets of constraints defined in Eqs. 5, 10, and 11 complete the constraint requirements for the model.

The total number of variables in the problem are $(s \times p) + (2 \times p \times r)$. There are $(p \times r)$ constraints in the Eq. 5 set, r constraints in the Eq. 9 set, and p constraints in the Eq. 10 set. The rejection of candidate variables based on the relationships shown in Eqs. 6 and 7 is the only way of obtaining a practical solution for geographic areas of any size.

In the section of this paper giving original definition to the constraints on the problem, there were 8 requirements given for the sanding operation. All of these conditions are satisfied through the modeling constraints shown in Eqs. 5, 9, and 10.

In the solution of the model, the variables S_{jk} and S_{ij} were not restricted to integer values. The total number of trucks required at any stockpile would be

$$S_j = \sum_{k=1}^{r} S_{jk}; \; j = 1,\ldots, p \tag{11}$$

and the total trucks required from any maintenance station would be

$$S_i = \sum_{j=1}^{p} S_{ij}; \; i = 1,\ldots, s \tag{12}$$

After the final summations of Eqs. 11 and 12, one must round upward to the nearest integer value.

A final point concerns the original objective of defining the most favorable (economical) locations of the maintenance stations themselves. The station locations are hidden in the variables S_{ij}. If the final solution to the model gives $S_i = 0$, for any i, then a station is not required at location i. Similarly, if $S_j = 0$, for any j, then a stockpile would not be required at location j.

Objective Function

During the investigation, a range of amortization costs was studied for their effect on the final solution, and a higher value was assumed for a new station.

The terminology applied to the constraint development is carried over to this section, with the following additions:

C_i = the station amortization costs to be applied to each of the S_{ij} (dollars per unit);
C_{ij} = unit time cost in traveling from station i to stockpile j and return (dollars per hour);
C_{jk} = unit time cost for trucks plus loaders involved in the sanding mission from stockpile j to roadway k (dollars per hour);
P_j = the unit cost of the sanding material, delivered to stockpile j (dollars per ton);
d_{ij} = the distance traveled from station i to stockpile j; and
V_{ij} = the travel speed from station it to stockpile j.

The total cost Z of a given sanding mission can be computed as follows:

Z = cost in deployment of the S_{ij} (including station amortization)
+ cost of delivering the material from stockpile j to roadway k
+ cost of the sanding material

where

$$\text{Cost of deployment} = \sum_{i=1}^{s} \sum_{j=1}^{p} C_i S_{ij} + \sum_{i=1}^{s} \sum_{j=1}^{p} 2C_{ij} \, (d_{ij}/V_{ij}) \, S_{ij} \qquad (13)$$

$$\text{Cost of delivery} = \sum_{j=1}^{p} \sum_{k=1}^{r} C_{jk} \left[2n_{jk} \, (d_{jk} + 0.5m_k) + 6n_{jk}\right] \Big/ V_{jk} \qquad (14)$$

$$\text{Cost of material} = \sum_{j=1}^{p} \sum_{k=1}^{r} p_j n_{jk} \qquad (15)$$

Only two terms in these expressions should require any discussion. The quantity d_{ij}/V_{ij} in Eq. 13 is the time required to travel between station and stockpile. The travel velocity may be appreciably higher than the effective working velocity, but this is debatable because of weather conditions. The quantity $2n_{jk} \, (d_{jk} + 0.5m) + 6n_{jk}/V_{jk}$ in Eq. 14 represents the total truck-hours spent in servicing the various roadways, regardless of the actual number of vehicles involved.

Finally, we wish to minimize

$$Z = \sum_{i=1}^{s} \sum_{j=1}^{p} \left[C_i + 2C_{ij} \, (d_{ij}/V_{ij})\right] S_{ij} + \qquad (16)$$

$$\sum_{j=1}^{p} \sum_{k=1}^{r} C_{jk} \left[2n_{jk} \, (d_{jk} + 0.5 \, m_k) + 6n_{jk}\right] \Big/ V_{jk} + P_j n_{jk}$$

subject to the constraints given by the relations shown in Eqs. 5, 9, and 10.

PLOWING MODEL

The optimization model for the plowing operation is not as complicated as the one for the sanding operation and will be discussed briefly.

Considerations in the Type of Facility

The Policy and Procedure Directive defines two separate strategies according to classification by type of roadway. Class A facilities are to be kept bare throughout the storm period. Classes B, C, and D must be completely cleared within some reasonable period following storm termination. These two types of treatment require different constraints in the optimization model.

Nomenclature

The following symbology is used for constraint development:

S_{ij} = the number of snowplow units dispatched from station i to clear roadway j;
M_j = centerline mileage for roadway j;
V_t = the average plow velocity in reaching the work site;
V = the average plow speed under working conditions;
T_j = the allowable time for clearing roadway j;
d_{ij} = the distance in miles from station i to the centroid of length of roadway j;
P_j = the number of plow lanes required to clear the roadway, from shoulder to shoulder;
D = critical snow depth;
R = rate of snowfall used for program design purposes;
t = time, in general;
s = the number of potential maintenance stations; and
r = the number of roadway stations to be serviced.

Constraints for Class A Facilities

For a class A roadway, the Policy and Procedure Directive states that the roadway must be kept bare at all times. In reality, this would be a physical impossibility; the specification was modified to require that the snowfall should not be allowed to accumulate beyond a certain critical depth, D. For a design storm, the snowfall intensity is defined as R. Then, the snow accumulation, during any time, t, would be

$$\text{Accumulation} = R\,t \tag{17}$$

Specifically, we wish to know when the snow will accumulate to D, the critical depth. From Eq. 17:

$$t = D/R \tag{18}$$

The distance a plow will travel, at some working speed V, during time, t, is

$$V\,t = V\,D/R \tag{19}$$

If a group of plows, all traveling at V, were to follow one another down the roadway and were spaced according to Eq. 19, the maximum snow accumulation between them would be D. This is the effect desired. The total length of roadway to be plowed, for roadway j, would be

$$P_j\,M_j \tag{20}$$

Therefore, the required number of plows is

$$S_j = P_j M_j R/VD \tag{21}$$

For the optimization model,

$$\sum_{i=1}^{s} S_{ij} \quad P_j M_j R/VD; \qquad j = 1, \ldots, r \tag{22}$$

is required. Furthermore, the S_{ij} defined by Eq. 22 must be available for the duration of the storm.

Constraints for Class B, C, and D Facilities

Class B, C, and D roadways must be cleared within some "reasonable" time following the storm termination; call this time T_j. Any snowplow must be able to reach the work site and complete the assignment in this time. Thus, the time available for work is

$$T_j - d_{ij}/V \tag{23}$$

Once on site, the plow must clear M_j centerline miles on roadway j, and each mile of centerline requires P_j lanes to be cleared. The total mileage to be cleared is therefore $P_j M_j$. Assume that the mileage may be equally distributed among S_{ij} plows; the mileage assignment for each plow is

$$P_j M_j / S_{ij} \tag{24}$$

The time required to accomplish this is

$$t = (P_j M_j / S_{ij})/V \tag{25}$$

and must be accomplished within the time prescribed by Eq. 23. It is therefore required that

$$(P_j M_j / S_j)/V \leq (T_j - d_{ij}/V) \tag{26}$$

and

$$S_j = \sum_{i=1}^{s} S_{ij}; \qquad j = 1, \ldots, r$$

Model Efficiency Constraints

In order for any S_{ij} to have productive work time available, after reaching the work site, it is required that

$$0 < (T_j - d_{ij}/V) \leq q_1 \tag{27}$$

where q_1 is some conservatively chosen time value. This procedure significantly reduces the numbers of the problem variables and, in no way, compromises the final solution.

In the constraint model, the optimization variables are the S_{ij}, and there are s × r potential variables. There will be a total of r constraints.

Objective Function

Once again, the objective is to minimize cost. The total cost Z of a single mission is as follows:

Z = cost of deployment of the S_{ij} to the work site (including station amortization)
+ cost of operating the S_{ij} during the plowing operation (including the operator).

where

$$\text{Cost of development} = \sum_{i=1}^{s} \sum_{j=1}^{r} C_i S_{ij} + \sum_{i=1}^{s} \sum_{j=1}^{r} C_{ij} (d_{ij}/V) S_{ij} \tag{28}$$

where

C_i = station amortization cost (dollars per sanding unit) and
C_{ij} = the hourly time costs for snowplow and operator (dollars per hour).

In the case of the snowplow operation, the operating costs are all the same, once a work site has been reached. Even though this cost is real, it has no relationship to the different choices for the S_{ij}; consequently, the on-site operating costs were not computed. The final form of the objective function, therefore, is

$$\min Z = \sum_{i=1}^{s} \sum_{j=1}^{r} \left[C_i + C_{ij} (d_{ij}/V) \right] S_{ij} \tag{29}$$

APPLICATION AND RESULTS

Existing stations were included as variables to see if the model would include or reject these sites.

For both models, there were 15 proposed station sites and 41 roadway sections. For the sanding model, there were 15 stockpiles located at the station sites and 6 additional stockpiles.

There are originally 2,037 variables and 917 constraints in the sanding model and 615 variables and 41 constraints in the plowing model. To assemble this much data by hand each time the model is run is a difficult task; therefore, two FORTRAN computer programs or model builders were written. These programs compute all coefficients, reject unfeasible combinations of data, and assemble the final matrix of coefficients into a form usable as input to the Simplex Algorithm being used to solve the problem.

Several solutions were made for each model in order to examine the effect of variation in critical data. The fixed data that applied to the several solutions are given in Table 1. Solutions for two variations of the sanding model are given in Table 2. The results for five variations of the plowing model are given in Table 3.

CONCLUSIONS

The results of the study allow the following conclusions to be made:

1. The location of maintenance stations by means of mathematical optimization appears to be feasible. The use of linear programming techniques to produce a solution rather than the use of integer programming techniques is valid when multiple units such as snowplows are used. Rounding up to the next higher integer value provides a measure of reserve that was not considered in either model.

Table 2. Control data and associated optimal solution for the sanding program.

Station Number	Amortized Station Costs (dollars per unit)		Station Required		Number of Sanders Required		Stockpile Serviced	Stockpile Required	Roadway Serviced	Stockpile Number
	S1[a]	S2	S1	S2	S1	S2	S1 and S2	S1 and S2	S1 and S2	
1	5	5	Yes	Yes	1	1	1	Yes	1	1
2	20	30	Yes	Yes	1	1	2	Yes	2, 3	2
3	5	5	Yes	Yes	—	1	3	Yes	4, 5	3
4	20	30	No	No	—	—	—	Yes	6, 7	4
5	20	30	Yes	Yes	—	1	5	Yes	8, 9	5
6	5	5	Yes	Yes	2	2	6, 18	Yes	11	6
7	5	5	Yes	Yes	3	3	4, 7, 16	Yes	12-14, 38	7
8	20	20	Yes	Yes	2	2	8, 21	Yes	15-17, 24, 26	8
9	5	5	Yes	Yes	2	2	9, 20	Yes	20, 21, 23, 25	9
10	20	30	Yes	Yes	1	1	10	Yes	41	10
11	5	5	Yes	Yes	1	1	11	Yes	22	11
12	20	30	Yes	Yes	1	1	12	Yes	29-31, 39, 40	12
13	5	5	Yes	Yes	2	2	13, 19	Yes	33, 34	13
14	20	20	Yes	Yes	1	1	14	Yes	35, 36	14
15	20	20	Yes	Yes	1	1	15	Yes	37	15
								Yes	27, 28	16
								No	—	17
								Yes	10	18
								Yes	32	19
								Yes	19	20
								Yes	18	21

Note: Unit cost of sander was $10 per hour.
Unit cost of operator was $5 per hour.
Effective working speed of sander was 24 mph.
Unit cost of stockpile was $4.80 per hour.
Unit cost of material delivered to stockpile was $2.50 per ton.

[a]S = solution.

Table 3. Control data and associated optimal solution for the plowing program.

Station Number	Amortized Station Costs (dollars per mission)					Station Required					Number of Plows Required					Roadway Serviced				
	S1[a]	S2	S3	S4	S5	S1	S2	S3	S4	S5	S1	S2	S3	S4	S5	S1	S2	S3	S4	S5
1	5	5	5	5	5	Yes	Yes	Yes	Yes	Yes	1	1	1	1	1	1, 2	1, 2	1, 2	1, 2	1-3
2	20	30	30	30	40	No	No	No	No	No	—	—	—	—	—	—	—	—	—	—
3	5	5	5	5	40	Yes	Yes	Yes	Yes	No	1	1	1	1	—	3-5	3-5	3-5	3-5	—
4	20	30	30	30	30	No	No	No	No	No	—	—	—	—	—	—	—	—	—	—
5	25	40	40	40	40	Yes	No	No	Yes	No	1	—	—	1	—	9	—	—	9	—
6	25	40	40	40	25	Yes	Yes	Yes	Yes	Yes	1	1	1	1	1	10, 11	10, 11	10, 11	10, 11	10, 11
7	5	5	5	5	5	Yes	Yes	Yes	Yes	Yes	5	6	6	6	8	6-8, 12-15, 27-30, 38, 40,	6-9, 12-15, 27-30, 38-40	6-9, 12-15 27-30, 38-40	6-9, 12-15 27-30, 38-40	4-9, 12-15, 27-32, 38-40
8	20	30	30	30	30	Yes	No	No	No	No	1	—	—	—	—	26	—	—	—	—
9	5	5	5	5	5	Yes	Yes	Yes	Yes	Yes	4	5	5	5	5	16-21, 23-25	16-21, 23-26, 41	16-21, 23-26, 41	16-21, 23-26	16-21, 23-26, 41
10	20	30	30	30	30	Yes	No	No	Yes	No	1	—	—	1	—	41	—	—	41	—
11	5	5	5	5	5	Yes	Yes	Yes	Yes	Yes	1	1	1	1	1	22	22	22	22	22
12	20	30	30	30	30	Yes	No	No	No	No	1	—	—	—	—	31, 39	—	—	—	—
13	5	5	5	5	25	Yes	Yes	Yes	Yes	Yes	2	3	3	2	1	32-36	31-37	31-37	31-36	33-35
14	25	40	40	40	40	No	No	No	No	Yes	—	—	—	—	1	—	—	—	—	36
15	25	40	40	40	40	Yes	No	No	Yes	Yes	1	—	—	1	1	37	—	—	37	37

Notes: Unit cost of plow was $15 per hour for solutions 1 and 2 and $10 per hour for solutions 3 through 5.
Unit cost of operators was $5 per hour for all solutions.
Plow travel speed (dead-haul) was 40 mph for solutions 1, 2, 3, and 5 and 30 mph for solution 4.
Average working speed of plows was 24 mph for all solutions.

[a]S = solution.

2. The construction of two separate models to solve the problem of station location also appears feasible. More station locations were required to satisfy the requirements of the sanding model than were required by the plowing model. This was undoubtedly due to the great amount of dead haul required in the sanding model.

3. In the plowing model, amortization costs seem to have a greater influence on station location than do the other parameters.

4. Values for points on the perimeter of the models are invalid because work requirements outside of the model area are not considered.

5. In only one case in the plowing model was an existing station location rejected. This was probably caused by assigning a high amortization cost to that station.

6. Optimization techniques should also be applied to other maintenance functions such as sealing and mowing. It is hoped that this paper will provide a stimulus to other agencies to develop these techniques.

REFERENCES

1. Wyoming State Highway Department. Financial Report January 1 to December 31, 1969.
2. Wyoming State Highway Department. An Operations Research Study for the Maintenance Division. Cheyenne, 1970.
3. Wyoming State Highway Department. Optimal Solution for Maintenance Station Location. Cheyenne, 1971.
4. Wyoming State Highway Department. Performance Standards for Snow Removal. Cheyenne, 1971.